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Robert S. Hamada*

I. Introduction

Only recently has there been an interest in relating the issues historically associated with corporation finance to those historically associated with investment and portfolio analyses. In fact, rigorous theoretical attempts in this direction were made only since the capital asset pricing model of Sharpe [13], Lintner [6], and Mossin [11], itself an extension of the Markowitz [7] portfolio theory. This study is one of the first empirical works consciously attempting to show and test the relationships between the two fields. In addition, differences in the observed systematic or nondiversifiable risk of common stocks, β, have never really been analyzed before by investigating some of the underlying differences in the firms.

In the capital asset pricing model, it was demonstrated that the efficient set of portfolios to any individual investor will always be some combination of lending at the risk-free rate and the "market portfolio," or borrowing at the risk-free rate and the "market portfolio." At the same time, the Modigliani and Miller (MM) propositions [9, 10] on the effect of corporate leverage are well known to the students of corporation finance. In order for their propositions to hold, personal leverage is required to be a perfect substitute for corporate leverage. If this is true, then corporate borrowing could substitute for personal borrowing in the capital asset pricing model as well.

Both in the pricing model and the MM theory, borrowing, from whatever source, while maintaining a fixed amount of equity, increases the risk to the investor. Therefore, in the mean-standard deviation version of the capital asset pricing model, the covariance of the asset's rate of return with the market portfolio's rate of return (which measures the nondiversifiable risk of the asset—the proxy β will be used to measure this) should be greater for the stock of a firm with a higher debt-equity ratio than for the stock of another firm in the same risk-class with a lower debt-equity ratio.[1]

This study, then, has a number of purposes. First, we shall attempt to link empirically corporation finance issues with portfolio and security analyses through the effect of a firm's leverage on the systematic risk of its common

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1. This very quick summary of the theoretical relationship between what is known as corporation finance and the modern investment and portfolio analyses centered around the capital asset pricing model is more thoroughly presented in [5], along with the necessary assumptions required for this relationship.

stock. Then, we shall attempt to test the MM theory, or at least provide another piece of evidence on this long-standing controversial issue. This test will not rely on an explicit valuation model, such as the MM study of the electric utility industry [8] and the Brown study of the railroad industry [2]. A procedure using systematic risk measures (β s) has been worked out in this paper for this purpose.

If the MM theory is validated by this procedure, then the final purpose of this study is to demonstrate a method for estimating the cost of capital of individual firms to be used by them for scale-changing or nondiversifying investment projects. The primary component of any firm's cost of capital is the capitalization rate for the firm if the firm had no debt and preferred stock in its capital structure. Since most firms do have fixed commitment obligations, this capitalization rate (we shall call it $E(R_A)$; MM denote it ρ^τ) is unobservable. But if the MM theory and the capital asset pricing model are correct, then it is possible to estimate $E(R_A)$ from the systematic risk approach for individual firms, even if these firms are members of a one-firm risk-class.[2]

With this statement of the purposes for this study, we shall, in Section II, discuss the alternative general procedures that are possible for estimating the effect of leverage on systematic risk and select the most feasible ones. The results are presented in Section III. And finally, tests of the MM versus the traditional theories of corporation finance are presented in Section IV.

II. Some Possible Procedures and the Selected Estimating Relationships

There are at least four general procedures that can be used to estimate the effect of the firm's capital structure on the systematic risk of common stocks. The first is the MM valuation model approach. By estimating ρ^τ with an explicit valuation model as they have for the electric utility industry, it is possible to relate this ρ^τ with the use of the capital asset pricing model to a nonleveraged systematic risk measure, ${}_A\beta$. Then the difference between the observed common stock's systematic risk (which we shall denote ${}_B\beta$) and ${}_A\beta$ would be due solely to leverage. But the difficulties of this approach for all firms are many.

The MM valuation model approach requires the specification, in advance, of risk-classes. All firms in a risk-class are then assumed to have the same ρ^τ—the capitalization rate for an all-common equity firm. Unfortunately, there must be enough firms in a risk-class so that a cross-section analysis will yield statistically significant coefficients. There may not be many more risk-classes (with enough observations) now that the electric utility and railroad industries have been studied. In addition, the MM approach requires estimating expected asset earnings and estimating the capitalized growth potential implicit in stock prices. If it is possible to consider growth and expected earnings without having

2. It is, in fact, this last purpose of making applicable and practical some of the implications of the capital asset pricing model for corporation finance issues that provided the initial motivation for this paper. In this context, if one is familiar with the fair rate of return literature for regulated utilities, for example, an industry where debt is so prevalent, adjusting correctly for leverage is not frequently done and can be very critical.

to specify their exact magnitude at a specific point in time, considerable difficulty and possible measurement errors will be avoided.

The second approach is to run a regression between the observed systematic risk of a stock and a number of accounting and leverage variables in an attempt to explain this observed systematic risk. Unfortunately, without a theory, we do not know which variables to include and which variables to exclude and whether the relationship is linear, multiplicative, exponential, curvilinear, etc. Therefore, this method will also not be used.

A third approach is to measure the systematic risk before and after a new debt issue. The difference can then be attributed to the debt issue directly. An attractive feature of this procedure is that a good estimate of the market value of the incremental debt issue can be obtained. A number of disadvantages, unfortunately, are associated with this direct approach. The difference in the systematic risk may be due not only to the additional debt, but also to the reason the debt was issued. It may be used to finance a new investment project, in which case the project's characteristics will also be reflected in the new systematic risk measure. In addition, the new debt issue may have been anticipated by the market if the firm had some long-run target leverage ratio which this issue will help maintain; conversely, the market may not fully consider the new debt issue if it believes the increase in leverage is only temporary. For these reasons, this seemingly attractive procedure will not be employed.

The last approach, which will be used in this study, is to assume the validity of the MM theory from the outset. Then the observed rate of return of a stock can be adjusted to what *it would have been* over the same time period had the firm no debt and preferred stock in its capital structure. The difference between the observed systematic risk, ${}_{B}\beta$, and the systematic risk for this adjusted rate of return time series, ${}_{A}\beta$, can be attributed to leverage, if the MM theory is correct. The final step, then, is to test the MM theory.

To discuss this more specifically, consider the following relationship for the dollar return to the common shareholder from period $t-1$ to t:

$$(X-I)_t(1-\tau)_t - p_t + \Delta G_t = d_t + cg_t \tag{1}$$

where X_t represents earnings before taxes, interest, and preferred dividends and is assumed to be unaffected by fixed commitment obligations; I_t represents interest and other fixed charges paid during the period; τ is the corporation income tax rate; p_t is the preferred dividends paid; ΔG_t represents the change in capitalized growth over the period; and d_t and cg_t are common shareholder dividends and capital gains during the period, respectively.

Equation (1) relates the corporation finance types of variables with the market holding period return important to the investors. The first term on the left-hand-side of (1) is profits after taxes and after interest which is the earnings the common and preferred shareholders receive on their investment for the period. Subtracting out p_t leaves us with the earnings the common shareholder would receive from currently-held assets.

To this must be added any change in capitalized growth since we are trying to explain the common shareholder's market holding period dollar return. ΔG_t

must be added for growth firms to the current period's profits from existing assets since capitalized growth opportunities of the firm—future earnings from new assets over and above the firm's cost of capital which are already reflected in the stock price at $(t-1)$—should change over the period and would accrue to the common shareholder. Assuming shareholders at the start of the period estimated these growth opportunities on average correctly, the expected value of ΔG_t would not be zero, but should be positive. For example, consider growth opportunities five years from now which yield more than the going rate of return and are reflected in today's stock price. These growth opportunities will become one year closer to fruition at time t than at time $t-1$ so that their present value would become larger. ΔG_t then represents this increase in the present value of these future opportunities simply because it is now four years away rather than five.[3]

Since the systematic risk of a common stock is:

$$_B\beta = \frac{\text{cov}\,(R_{B_t}, R_{M_t})}{\sigma^2(R_{M_t})} \tag{2}$$

where R_{B_t} is the common shareholder's rate of return and R_{M_t} is the rate of return on the market portfolio, then substitution of (1) into (2) yields:

$$_B\beta = \frac{\text{cov}\left[\dfrac{(X-I)(1-\tau)_t - p_t + \Delta G_t}{S_{B_{t-1}}}, R_{M_t}\right]}{\sigma^2(R_{M_t})} \tag{2a}$$

where $S_{B_{t-1}}$ denotes the market value of the common stock at the beginning of the period.

The systematic risk for the same firm over the same period *if* there were no debt and preferred stock in its capital structure is:

$$_A\beta = \frac{\text{cov}(R_{A_t}, R_{M_t})}{\sigma^2(R_{M_t})}$$

$$= \frac{\text{cov}\left[\dfrac{X(1-\tau)_t + \Delta G_t}{S_{A_{t-1}}}, R_{M_t}\right]}{\sigma^2(R_{M_t})} \tag{3}$$

where R_{A_t} and $S_{A_{t-1}}$ represent the rate of return and the market value, respectively, to the common shareholder if the firm had no debt and preferred stock. From (3), we can obtain:

$$_A\beta S_{A_{t-1}} = \frac{\text{cov}\,[X(1-\tau)_t + \Delta G_t, R_{M_t}]}{\sigma^2(R_{M_t})} \tag{3a}$$

3. Continual awareness of the difficulties of estimating capitalized growth, or changes in growth, especially in conjunction with leverage considerations, for purposes such as valuation or cost of capital is a characteristic common to students of corporation finance. This is the reason for the emphasis on growth in this paper and for presenting a method to neutralize for differences in growth when comparing rates of return.

Next, by expanding and rearranging (2a), we have:

$$_B\beta S_{B_{t-1}} = \frac{\text{cov}\,[X(1-\tau)_t + \Delta G_t, R_{M_t}]}{\sigma^2(R_{M_t})} - \frac{\text{cov}\,[I(1-\tau)_t, R_{M_t}]}{\sigma^2(R_{M_t})} - \frac{\text{cov}\,(p_t, R_{M_t})}{\sigma^2(R_{M_t})} \tag{2b}$$

If we assume as an empirical approximation that interest and preferred dividends have negligible covariance with the market, at least relative to the (pure equity) common stock's covariance, then substitution of the LHS of (3a) into the RHS of (2b) yields:[4]

$$_B\beta S_{B_{t-1}} = {_A\beta} S_{A_{t-1}} \tag{4}$$

or

$$_A\beta = \left(\frac{S_B}{S_A}\right)_{t-1} {_B\beta} \tag{4a}$$

Because $S_{A_{t-1}}$, the market value of common stock *if* the firm had no debt and preferred stock, is not observable since most firms do have debt and/or preferred stock, a theory is required in order to measure what this quantity *would have been* at $t-1$. The MM theory [10] will be employed for this purpose, that is:

$$S_{A_{t-1}} = (V - \tau D)_{t-1}. \tag{5}$$

Equation (5) indicates that if the Federal government tax subsidy for debt financing, τD, where D is the market value of debt, is subtracted from the observed market value of the firm, V_{t-1} (where V_{t-1} is the sum of S_B, D and the observed market value of preferred), then the market value of an unleveraged firm is obtained. Underlying (5) is the assumption that the firm is near its target leverage ratio so that no more or no less debt subsidy is capitalized already into the observed stock price. The conditions under which this MM relationship hold are discussed carefully in [4].

It is at this point that problems in obtaining satisfactory estimates of $_A\beta$ develop, since (4) theoretically holds only for the next period. As a practical matter, the accepted, and seemingly acceptable, method of obtaining estimates of a stock's systematic risk, $_B\beta$, is to run a least squares regression between a stock's and market portfolio's *historical* rates of return. Using past data for $_B\beta$, it is not clear which *period's* ratio of market values to apply in (4a) to estimate the firm's systematic risk, $_A\beta$. There would be no problem if the market value ratios of debt to equity and preferred stock to equity remained relatively stable over the past for each firm, but a cursory look at these data reveals that this is not true for the large majority of firms in our sample. Should we use the market value ratio required in (4a) that was observed at the start of our regression period, at the end of our regression period, or some kind of average over the period? In addition, since these different observed ratios will give us different estimates for $_A\beta$, it is not clear, without some criterion, how we should select from among the various estimates.

4. This general method of arriving at (4) was suggested by the comments of William Sharpe, one of the discussants of this paper at the annual meeting. A much more cumbersome and less general derivation of (4) was in the earlier version.

It is for this purpose—to obtain a standard—that a more cumbersome and more data demanding approach to obtain estimates of ${}_A\beta$ is suggested. Given the large fluctuations in market leverage ratios, intuitively it would appear that the firm's risk is more stable than the common stock's risk. In that event, a leverage-free rate of return time series for each firm should be derived and the market model applied to this time series directly. In this manner, the beta coefficient would give us a *direct* estimate of ${}_A\beta$ which can then be used as a criterion to determine if any of the market value ratios discussed above can be applied to (4a) successfully.

For this purpose, the "would-have-been" rate of return for the common stock if the firm had no debt and preferred is:

$$R_{A_t} = \frac{X_t(1-\tau)_t + \Delta G_t}{S_{A_{t-1}}}. \qquad (6)$$

The numerator of (6) can be rearranged to be:

$$X_t(1-\tau)_t + \Delta G_t \equiv [(X-I)_t(1-\tau)_t - p_t + \Delta G_t] + p_t + I_t(1-\tau)_t.$$

Substituting (1):

$$X_t(1-\tau)_t + \Delta G_t = [d_t + cg_t] + p_t + I_t(1-\tau)_t.$$

Therefore, (6) can be written as:

$$R_{A_t} = \frac{d_t + cg_t + p_t + I_t(1-\tau)_t}{S_{A_{t-1}}}. \qquad (7)$$

Since $S_{A_{t-1}}$ is unobservable for the firms with leverage, the MM theory, equation (5), will be employed; then:

$$R_{A_t} = \frac{d_t + cg_t + p_t + I_t(1-\tau)_t}{(V-\tau D)_{t-1}}. \qquad (8)$$

The observed rate of return on the common stock is, of course:

$$R_{B_t} = \frac{(X-I)_t(1-\tau)_t - p_t + \Delta G_t}{S_{B_{t-1}}} = \frac{d_t + cg_t}{S_{B_{t-1}}}. \qquad (9)$$

Equation (8) is the rate of return to the common shareholder of the same firm and over the same period of time as (9). However, in (8) there are the underlying assumptions that the firm never had any debt and preferred stock and that the MM theory is correct; (9) incorporates the exact amount of debt and preferred stock that the firm actually did have over this time period and no leverage assumption is being made. Both (8) and (9) are now in forms where they can be measured with available data. One can note that it is unnecessary to estimate the change in growth, or earnings from current assets, since these should be captured in the market holding period return, $d_t + cg_t$.

Using CRSP data for (9) and both CRSP and Compustat data for the components of (8), a time series of yearly R_{A_t} and R_{B_t} for $t = 1948\text{-}1967$ were derived for 304 different firms. These 304 firms represent an exhaustive sample of the firms with complete data on both tapes for all the years.

A number of "market model" [1, 12] variants were then applied to these data. For each of the 304 firms, the following regressions were run:

$$R_{Ait} = {}_A\alpha_i + {}_A\beta_i R_{Mt} + {}_A\epsilon_{it} \quad (10a)$$

$$R_{Bit} = {}_B\alpha_i + {}_B\beta_i R_{Mt} + {}_B\epsilon_{it} \quad (10b)$$

$$\ln(1 + R_{Ait}) = {}_{AC}\alpha_i + {}_{AC}\beta_i \ln(1 + R_{M_t}) + {}_{AC}\epsilon_{it} \quad (10c)$$

$$\ln(1 + R_{Bit}) = {}_{BC}\alpha_i + {}_{BC}\beta_i \ln(1 + R_{M_t}) + {}_{BC}\epsilon_{it} \quad (10d)$$

$$i = 1, 2, \ldots, 304$$
$$t = 1948\text{-}1967$$

where R_{Mt} is the observed NYSE arithmetic stock market rate of return with dividends reinvested, α_i and β_i are constants for each firm-regression, and the usual conditions are assumed for the properties of the disturbance terms, ϵ_{it}. Equations (10c) and (10d) are the continuously-compounded rate of return versions of (10a) and (10b), respectively.[5]

III. The Results

An abbreviated table of the regression results for each of the four variants, equations (10a)-(10d), summarized across the 304 firms is shown in Table 1.

The first column designated "mean" is the average of the statistic (indicated by the rows) over all 304 firms. Therefore, the mean ${}_A\hat{\alpha}$ of 0.0221 is the intercept term of equation (10a) averaged over 304 different firm-regressions. The second and third columns give the deviation measures indicated, of the 304 point estimates of, say, ${}_A\hat{\alpha}$. The mean standard error of estimate in the last column is the average over 304 firms of the individual standard errors of estimate.

The major conclusion drawn from Table 1 is the following mean β comparisons:

$${}_B\hat{\beta} > {}_A\hat{\beta}, \text{i.e., } 0.9190 > 0.7030$$

$${}_{BC}\hat{\beta} > {}_{AC}\hat{\beta}, \text{i.e., } 0.9183 > 0.7263.$$

The directional results of these betas, assuming the validity of the MM theory, are not imperceptible and clearly are not negligible differences from the investor's point of view. This is obtained in spite of all the measurement and data problems associated with estimating a time series of the RHS of (8) for

5. Because the R_{M_t} used in equations (10) is defined as the observed stock market return, and since adjusting for capital structure is the major purpose of this exercise, it was decided that the same four regressions should be replicated on a leverage-adjusted stock market rate of return. The major reason for this additional adjustment is the belief that the rates of return over time and their relationship with the market are more stable when we can abstract from all changes in leverage and get at the underlying risk of all firms.

For the 221 firms (out of the total 304) whose fiscal years coincide with the calendar year, average values for the components of the RHS of (8) were obtained for each year so that R_{M_t} could be adjusted in the same way as for the individual firms—a yearly time series of stock market rates of return, if all the firms on the NYSE had no debt and no preferred in their capital structure, was derived. The results, when using this adjusted market portfolio rate of return time series, were not very different from the results of equations (10), and so will not be reported here separately.

TABLE 1
SUMMARY RESULTS OVER 304 FIRMS OF EQUATIONS (10a)-(10d)

	Mean	Mean Absolute Deviation*	Standard Deviation	Mean Standard Error of Estimate
$_A\hat{\alpha}$	0.0221	0.0431	0.0537	0.0558
$_A\hat{\beta}$	0.7030	0.2660	0.3485	0.2130
$_A\hat{R}^2$	0.3799	0.1577	0.1896	
$_A\hat{\rho}$	0.0314			
$_B\hat{\alpha}$	0.0187	0.0571	0.0714	0.0720
$_B\hat{\beta}$	0.9190	0.3550	0.4478	0.2746
$_B\hat{R}^2$	0.3864	0.1578	0.1905	
$_B\hat{\rho}$	0.0281			
$_{AC}\hat{\alpha}$	0.0058	0.0427	0.0535	0.0461
$_{AC}\hat{\beta}$	0.7263	0.2700	0.3442	0.2081
$_{AC}\hat{R}^2$	0.3933	0.1586	0.1909	
$_{AC}\hat{\rho}$	0.0268			
$_{BC}\hat{\alpha}$	−0.0052	0.0580	0.0729	0.0574
$_{BC}\hat{\beta}$	0.9183	0.3426	0.4216	0.2591
$_{BC}\hat{R}^2$	0.4012	0.1602	0.1922	
$_{BC}\hat{\rho}$	0.0262			

* Defined as: $\frac{\sum_{i=1}^{N} |x_i - \bar{x}|}{N}$, where $N = 304$. $\hat{\rho}$ = first order serial correlation coefficient.

each firm. One of the reasons for the "traditional" theory position on leverage is precisely this point—that small and reasonable amounts of leverage cannot be discerned by the market. In fact, if the MM theory is correct, leverage has explained as much as, roughly, 21 to 24 per cent of the value of the mean β.

We can also note that if the covariance between the asset and market rates of return, as well as the market variance, was constant over time, then the systematic risk from the market model is related to the expected rate of return by the capital asset pricing model. That is:

$$E(R_{A_t}) = R_{F_t} + {}_A\beta[E(R_{M_t}) - R_{F_t}] \quad (11a)$$

$$E(R_{B_t}) = R_{F_t} + {}_B\beta[E(R_{M_t}) - R_{F_t}] \quad (11b)$$

Equation (11a) indicates the relationship between the expected rate of return for the common stock shareholder of a debt-free and preferred-free firm, to the systematic risk, $_A\beta$, as obtained in regressions (10a) or (10c). The LHS of (11a) is the important $\rho\tau$ for the MM cost of capital. The MM theory [9, 10] also predicts that shareholder expected yield must be higher (for the same real firm) when the firm has debt than when it does not. Financial risk is greater, therefore, shareholders require more expected return. Thus, $E(R_{B_t})$ must be greater than $E(R_{A_t})$. In order for this MM prediction to be true, from (11a) and (11b) it can be observed that $_B\beta$ must be greater than $_A\beta$, which is what we obtained.

Using the results underlying Table 1, namely the firm and stock betas, as the criterion for selecting among the possible observed market value ratios that can be used, if any, for (4), the following cross-section regressions were run:

$$({}_B\beta)_i = a_1 + b_1 \left(\frac{S_A}{S_B} {}_A\beta \right)_i + u_{1i} \qquad i = 1, 2, \ldots, 102 \qquad (12a)$$

$$({}_{BC}\beta)_i = a_2 + b_2 \left(\frac{S_A}{S_B} {}_{AC}\beta \right)_i + u_{2i} \qquad i = 1, 2, \ldots, 102 \qquad (12b)$$

$$({}_A\beta)_i = a_3 + b_3 \left(\frac{S_B}{S_A} {}_B\beta \right)_i + u_{3i} \qquad i = 1, 2, \ldots, 102 \qquad (13a)$$

$$({}_{AC}\beta)_i = a_4 + b_4 \left(\frac{S_B}{S_A} {}_{BC}\beta \right)_i + u_{4i} \qquad i = 1, 2, \ldots, 102 \qquad (13b)$$

Because the preferred stock market values were not as reliable as debt, only the 102 firms (out of 304) that did not have preferred in any of the years were used. The test for the adequacy of this alternative approach, equation (4), to adjust the systematic risk of common stocks for the underlying firm's capital structure, is whether the intercept term, a, is equal to zero, and the slope coefficient, b, is equal to one in the above regressions (as well as, of course, a high R^2)—these requirements are implied by (4). The results of this test would also indicate whether future "market model" studies that only use common stock rates of return without adjusting, or even noting, for the firm's debt-equity ratio will be adequate. The total firm's systematic risk may be stable (as long as the firm stays in the same risk-class), whereas the common stock's systematic risk may not be stable merely because of unanticipated capital structure changes—the data underlying Table 3 indicate that there were very few firms which did not have major changes in their capital structure over the twenty years studied.

The results of these regressions, when using the average S_A and average S_B over the twenty years for each firm, are shown in the first column panel of Table 2. These regressions were then replicated twice, first using the December 31, 1947 values of S_{A_i} and S_{B_i} instead of the twenty-year average for each firm, and then substituting the December 31, 1966 values of S_{A_i} and S_{B_i} for the 1947 values. These results are in the second and third panels of Table 2.[6]

From the first panel of Table 2, it appears that this alternative approach via (4a) for adjusting the systematic risk for the firm's leverage is quite

6. The point should be made that we are not merely regressing a variable on itself in (12) and (13). (12a) and (12b) can be interpreted as correlating the ${}_B\beta_i$ obtained from (10b) and (10d)—the LHS variable in (12a) and (12b)—against the ${}_B\beta_i$ obtained from rearranging (4)—the RHS variable in (12a) and (12b)—to determine whether the use of (4) is as good a means of obtaining ${}_B\beta_i$ as the direct way via the equations (10). We would be regressing a variable on itself only if the ${}_A\beta_i$ were calculated using (4a), and then the ${}_A\beta_i$ thus obtained, inserted into (12a) and (12b).

Instead, we are obtaining ${}_A\beta_i$ using the MM model in *each* of the twenty years so that a leverage-adjusted 20 year time series of R_{A_i} is derived. Of course, if there were no data nor measurement problems, and if the debt-to-equity ratio were perfectly stable over this twenty year period for each firm, then we should obtain perfect correlation in (12a) and (12b), with $a = 0$ and $b = 1$, as (4) would be an identity.

TABLE 2

RESULTS FOR THE EQUATIONS (12a), (12b), (13a), AND (13b)*

	Using 20-Year Average for $\left(\frac{S_A}{S_B}\right)_i$			Using 1947 Value for $\left(\frac{S_A}{S_B}\right)_i$			Using 1966 Value for $\left(\frac{S_A}{S_B}\right)_i$		
	a	b	R^2	a	b	R^2	a	b	R^2
Eq. (12a)	−0.022 (0.021)	1.062 (0.021)	0.962	0.150 (0.048)	0.842 (0.045)	0.781	0.085 (0.041)	0.905 (0.038)	0.849
	constant suppressed	1.042 (0.009)	0.962	constant suppressed	0.966 (0.021)	0.781	constant suppressed	0.976 (0.017)	0.849
Eq. (12b)	−0.003 (0.013)	1.016 (0.013)	0.984	0.159 (0.047)	0.816 (0.044)	0.773	0.124 (0.037)	0.843 (0.034)	0.859
	constant suppressed	1.014 (0.005)	0.984	constant suppressed	0.952 (0.019)	0.773	constant suppressed	0.947 (0.015)	0.859

	Using 20-Year Average for $\left(\frac{S_B}{S_A}\right)_i$			Using 1947 Value for $\left(\frac{S_B}{S_A}\right)_i$			Using 1966 Value for $\left(\frac{S_B}{S_A}\right)_i$		
	a	b	R^2	a	b	R^2	a	b	R^2
Eq. (13a)	0.030 (0.016)	0.931 (0.017)	0.969	0.112 (0.028)	0.843 (0.030)	0.888	0.080 (0.027)	0.898 (0.030)	0.902
	constant suppressed	0.960 (0.007)	0.969	constant suppressed	0.948 (0.015)	0.888	constant suppressed	0.976 (0.014)	0.902
Eq. (13b)	0.007 (0.010)	0.979 (0.011)	0.988	0.119 (0.026)	0.852 (0.028)	0.902	0.063 (0.026)	0.942 (0.029)	0.911
	constant suppressed	1.004 (0.012)	0.911	constant suppressed	0.967 (0.013)	0.902	constant suppressed	1.005 (0.012)	0.911

* Standard error in parentheses.

satisfactory (at least with respect to our sample of firms and years) only if long-run averages of S_A and S_B are used. The second and third panels indicate that the equations (8) and (10) procedure is markedly superior when only one year's market value ratio is used as the adjustment factor. The annual debt-to-equity ratio is much too unstable for this latter procedure.

Thus, when forecasting systematic risk is the primary objective—for example, for portfolio decisions or for estimating the firm's cost of capital to apply to prospective projects—a long-run forecasted leverage adjustment is required. Assuming the firm's risk is more stable than the common stock's risk,[7] and if there is some reason to believe that a better forecast of the firm's future leverage can be obtained than using simply a past year's (or an average of past years') leverage, it should be possible to improve the usual extrapolation forecast of a stock's systematic risk by forecasting the total firm's systematic risk first, and then using the independent leverage estimate as an adjustment.

IV. Tests of the MM vs. Traditional Theories of Corporation Finance

To determine if the difference, ${}_B\beta - {}_A\beta$, found in this study is indeed the correct effect of leverage, some confirmation of the MM theory (since it was assumed to be correct up to this point) from the systematic risk approach is needed. Since a direct test by this approach seems impossible, an indirect, inferential test is suggested.

The MM theory [9, 10] predicts that for firms in the same risk-class, the capitalization rate if all the firms were financed with only common equity, $E(R_A)$, would be the same—regardless of the actual amount of debt and preferred each individual firm had. This would imply, from (11a), that if $E(R_A)$ must be the same for all firms in a risk-class, so must ${}_A\beta$. And if these firms had different ratios of fixed commitment obligations to common equity, this difference in financial risk would cause their observed ${}_B\beta$s to be different.

The major competing theory of corporation finance is what is now known as the "traditional theory," which has contrary implications. This theory predicts that the capitalization rate for common equity, $E(R_B)$, (sometimes called the required or expected stock yield, or expected earnings-price ratio) is constant, as debt is increased, up to some critical leverage point (this point being a function of gambler's ruin and bankruptcy costs).[8] The clear implication of this constant, horizontal, equity yield (or their initial downward sloping cost of capital curve) is that changes in market or covariability risk are assumed not to be discernible to the shareholders as debt is increased. Then the traditional theory is saying that the ${}_B\beta$s, a measure of this covariability risk, would be the same for all firms in a given risk-class irregardless of differences in leverage, as long as the critical leverage point is not reached.

Since there will always be unavoidable errors in estimating the β's of indi-

7. A faint, but possible, empirical indication of this point may be obtained from Table 1. The ratio of the mean point estimate to the mean standard error of estimate is less for the firm β than for the stock β in both the discrete and continuously compounded cases.

8. This interpretation of the traditional theory can be found in [9, especially their figure 2, page 275, and their equation (13) and footnote 24 where reference is made to Durand and Graham and Dodd].

TABLE 3

Industry Market Value Ratios of Preferred Stock (P) and Debt (D) to Common Stock (S)

Industry Number	Industry	Number of Firms			P/S			D/S			$\frac{P+D}{S}$	
20	Food and Kindred Products	30	Mean*		0.22			0.81			1.04	
			ROM**	0.00		1.18	0.00		3.55	0.00		4.13
			ROCR***	0.00		2.52	0.00		8.10	0.00		10.01
28	Chemicals and Allied Products	30	Mean		0.07			0.25			0.33	
			ROM	0.00		0.51	0.00		0.90	0.00		1.20
			ROCR	0.00		1.54	0.00		2.07	0.00		2.92
29	Petroleum and Coal Products	18	Mean		0.06			0.22			0.27	
			ROM	0.00		0.26	0.00		0.55	0.03		0.57
			ROCR	0.00		0.83	0.00		1.54	0.00		2.30
33	Primary Metals	21	Mean		0.14			0.54			0.68	
			ROM	0.00		1.31	0.00		1.95	0.00		3.04
			ROCR	0.00		4.69	0.00		6.20	0.00		7.49
35	Machinery, except Electrical	28	Mean		0.07			0.33			0.40	
			ROM	0.00		0.49	0.00		1.92	0.00		2.32
			ROCR	0.00		1.28	0.00		6.92	0.00		7.62

TABLE 3 (*Continued*)

Industry Number	Industry	Number of Firms		P/S			D/S			$\frac{P+D}{S}$		
36	Electrical Machinery & Equipment	13	Mean		0.06			0.35			0.41	
			ROM	0.00		0.29	0.00		1.31	0.01		1.33
			ROCR	0.00		1.13	0.00		2.53	0.00		2.53
37	Transportation Equipment	24	Mean		0.08			0.38			0.47	
			ROM	0.00		0.54	0.00		0.93	0.00		1.32
			ROCR	0.00		2.33	0.00		3.76	0.00		6.09
49	Utilities	27	Mean		0.25			1.03			1.28	
			ROM	0.00		0.53	0.49		2.64	0.52		3.12
			ROCR	0.00		3.12	0.12		16.40	0.12		19.52
53	Dep't Stores, Order Houses & Vending Mach. Operators	17	Mean		0.13			0.49			0.62	
			ROM	0.00		0.38	0.01		1.52	0.01		1.87
			ROCR	0.00		1.09	0.00		3.19	0.00		3.66

* "Mean" refers to the average ratio over 20 years and over all firms in the industry.
** "Range of Means" (ROM) refers to the lowest firm's mean (over 20 years) ratio and the highest firm's mean (over 20 years) ratio in the industry.
*** "Range of Company Ranges" (ROCR) refers to the lowest and highest ratio in the industry, regardless of the year.

vidual firms and in specifying a risk-class, we would not expect to find a set of firms with identical systematic risk. But by specifying reasonable a priori risk-classes, if the individual firms had closer or less scattered $_A\beta$s than $_B\beta$s, then this would support the MM theory and contradict the traditional theory. If, instead, the $_B\beta$s were not discernibly more diverse than the $_A\beta$s, and the leverage ratio differed considerably among firms, then this would indicate support for the traditional theory.[9]

In order to test this implication, risk-classes must be first specified. The SEC two-digit industry classification was used for this purpose. Requiring enough firms for statistical reasons in any given industry, nine risk-classes were specified that had at least 13 firms; these nine classes are listed in Table 3 with their various leverage ratios.[10] It is clear from this table that our first requirement is met—that there is a considerable range of leverage ratios among firms in a risk-class and also over the twenty-year period.

Three tests will be performed to distinguish between the MM and traditional theories. The first is simply to calculate the standard deviation of the unbiased β estimates in a risk-class. The second is a chi-square test of the distribution of β's in an industry compared to the distribution of the β's in the total sample. Finally, an analysis of variance test on the estimated variance of the β's between industries, as opposed to within industries, is performed. In all tests, only the point estimate of β (which should be unbiased) for each stock and firm is used.[11]

The first test is reported in Table 4. If we compare the standard deviation of $_{AC}\beta$ with the standard deviation of $_{BC}\beta$ by industries (or risk-classes), we can note that $\sigma(_{AC}\beta)$ is less than $\sigma(_{BC}\beta)$ for eight out of the nine classes. The probability of obtaining this is only 0.0195, given a 50% probability that $\sigma(_{AC}\beta)$ can be larger or smaller than $\sigma(_{BC}\beta)$. These results indicate that the systematic risk of the firms in a given risk-class, if they were all financed only with common equity, is much less diverse than their observed stock's systematic risk. This supports the MM theory, at least in contrast to the traditional theory.[12]

9. The traditional theory also implies that $E(R_A)$ is equal to $E(R_B)$ for all firms. Unfortunately, we do not have a functional relationship between these traditional theory capitalization rates and the measured βs of this study. Clearly, since the $_A\beta$s were obtained assuming the validity of the MM theory, they would not be applicable for the traditional theory. In fact, no relationship between the $_A\beta$ and $_B\beta$ for a given firm, or for firms in a given risk-class, can be specified as was done for the capitalization rates.

10. The tenth largest industry had only eight firms. For our purpose of testing the uniformity of firm βs relative to stock βs within a risk-class, the use of the two-digit industry classification as a proxy does not seem as critical as, for instance, its use for the purpose of performing an MM valuation model study [8] wherein the ρ^T must be pre-specified to be exactly the same for all firms in the industry.

11. Since these βs are estimated in the market model regressions with error, precise testing should incorporate the errors in the β estimation. Unfortunately, to do this is extremely difficult and more importantly, requires the normality assumption for the market model disturbance term. Since there is considerable evidence that is contrary to this required assumption [see 3], our tests will ignore the β measurement error entirely. But ignoring this is partially corrected in our first and third tests since means and variances of these point estimate βs must be calculated, and this procedure will "average out" the individual measurement errors by the factor 1/N.

12. Of course, there could always be another theory, as yet not formulated, which could be even

TABLE 4
MEAN AND STANDARD DEVIATION OF INDUSTRY β'S

Industry Number	Industry	Number of Firms		$_A\beta$	$_B\beta$	$_{AC}\beta$	$_{BC}\beta$
20	Food & Kindred Products	30	Mean β	0.515	0.815	0.528	0.806
			σ(β)	0.232	0.448	0.227	0.424
28	Chemicals & Allied Products	30	Mean β	0.747	0.928	0.785	0.946
			σ(β)	0.237	0.391	0.216	0.329
29	Petroleum & Coal Products	18	Mean β	0.633	0.747	0.656	0.756
			σ(β)	0.144	0.188	0.148	0.176
33	Primary Metals	21	Mean β	1.036	1.399	1.106	1.436
			σ(β)	0.223	0.272	0.197	0.268
35	Machinery, except Electrical	28	Mean β	0.878	1.037	0.917	1.068
			σ(β)	0.262	0.240	0.271	0.259
36	Electrical Machinery and Equipment	13	Mean β	0.940	1.234	0.951	1.164
			σ(β)	0.320	0.505	0.283	0.363
37	Transportation Equipment	24	Mean β	0.860	1.062	0.875	1.048
			σ(β)	0.225	0.313	0.225	0.289
49	Utilities	27	Mean β	0.160	0.255	0.166	0.254
			σ(β)	0.086	0.133	0.098	0.147
53	Department Stores, etc.	17	Mean β	0.652	0.901	0.692	0.923
			σ(β)	0.187	0.282	0.198	0.279

Our second test, the chi-square test, requires us to rank our 300 $_A\beta$s into ten equal categories, each with 30 $_A\beta$s (four miscellaneous firms were taken out randomly). By noting the value of the highest and lowest $_A\beta$ for each of the ten categories, a distribution of the number of $_A\beta$s in each category, by risk-class, can be obtained. This was then repeated for the other three betas. To test whether the distribution for each of the four β's and for each of the risk-classes follows the expected uniform distribution, a chi-square test was performed.[13]

Even with just casual inspection of these distributions of the betas by risk-class, it is clear that two industries, primary metals and utilities, are so highly skewed that they greatly exaggerate our results.[14] Eliminating these

more strongly supported than the MM theory. If we compare $\sigma(_A\beta)$ to $\sigma(_B\beta)$ by risk-classes in Table 4, precisely the same results are obtained as those reported above for the continuously-compounded betas.

13. By risk-classes, seven of the nine chi-square values of $_A\beta$ are larger than those of $_B\beta$, as are eight out of nine for the continuously-compounded betas. This would occur by chance with probabilities of 0.0898 and 0.0195, respectively, if there were a 50% chance that either the firm or stock chi-square value could be larger. Nevertheless, if we inspect the individual chi-square values by risk-class, we note that most of them are large so that the probabilities of obtaining these values are highly unlikely. For all four βs, the distributions for most of the risk-classes are nonuniform.

14. Primary metals have extremely large betas; utilities have extremely small betas.

two industries, and also two miscellaneous firms so that an even 250 firms are in the sample, new upper and lower values of the β's were obtained for each of the ten class intervals and for each of the four β's.

In Table 5, the chi-square values are presented; for the total of all risk-classes, the probability of obtaining a chi-square value less than 120.63 is over 99.95% (for $_A\beta$), whereas the probability of obtaining a chi-square value less than 99.75 is between 99.5% and 99.9% (for $_B\beta$). More sharply contrasting results are obtained when $_{AC}\beta$ is compared to $_{BC}\beta$. For $_{AC}\beta$, the probability of obtaining less than 128.47 is over 99.95%, whereas for $_{BC}\beta$, the probability of obtaining less than 78.65 is only 90.0%. By abstracting from financial risk, the underlying systematic risk is much less scattered when grouped into risk-classes than when leverage is assumed not to affect the systematic risk. The null hypothesis that the β's in a risk-class come from the same distribution as all β's is rejected for $_{AC}\beta$, but not for $_{BC}\beta$ (at the 90% level). Although this, in itself, does not tell us *how* a risk-class differs from the total market, an inspection of the distributions of the betas by risk-class underlying Table 5 does indicate more clustering of the $_{AC}\beta$s than the $_{BC}\beta$s so that the MM theory is again favored over the traditional theory.

The analysis of variance test is our last comparison of the implications of the two theories. The ratio of the estimated variance between industries to the estimated variance within the industries (the F-statistic) when the seven

TABLE 5
CHI-SQUARE RESULTS FOR ALL β'S AND ALL INDUSTRIES (EXCEPT UTILITIES AND PRIMARY METALS)

Industry		$_A\beta$	$_B\beta$	$_{AC}\beta$	$_{BC}\beta$
Food and Kindred	Chi-Square	18.67	11.33	26.00	9.33
	$P\{\chi^2 < \}^* =$	95-97.5%	70-75%	99.5-99.9%	50-60%
Chemicals	Chi-Square	9.33	10.67	12.00	7.33
	$P\{\chi^2 < \} =$	50-60%	60-70%	75-80%	30-40%
Petroleum	Chi-Square	17.56	25.33	18.67	22.00
	$P\{\chi^2 < \} =$	95-97.5%	99.5-99.9%	95-97.5%	99-99.5%
Machinery	Chi-Square	19.14	12.00	24.86	9.14
	$P\{\chi^2 < \} =$	97.5-98%	75-80%	99.5-99.9%	50-60%
Electrical Machinery	Chi-Square	13.92	7.77	12.38	9.31
	$P\{\chi^2 < \} =$	80-90%	40-50%	80-90%	50-60%
Transportation Equipment	Chi-Square	15.17	16.83	13.50	6.83
	$P\{\chi^2 < \} =$	90-95%	90-95%	80-90%	30-40%
Dep't Stores	Chi-Square	14.18	3.59	14.18	3.59
	$P\{\chi^2 < \} =$	80-90%	5-10%	80-90%	5-10%
Miscellaneous	Chi-Square	12.67	12.22	6.89	11.11
	$P\{\chi^2 < \} =$	80-90%	80-90%	30-40%	70-75%
Total	Chi-Square	120.63	99.75	128.47	78.65
	$P\{\chi^2 < \} =$	over 99.95%	99.5-99.90%	over 99.95%	90.0%

* Example: $P\{\chi^2 < 18.67\} = 95\text{-}97.5\%$ for 9 degrees of freedom.

industries are considered (again, the two obviously skewed industries, primary metals and utilities, were eliminated) is less for ${}_{B}\beta$ (F = 3.90) than for ${}_{A}\beta$ (F = 9.99), and less for ${}_{BC}\beta$ (F = 4.18) than for ${}_{AC}\beta$ (F = 10.83). The probability of obtaining these F-statistics for ${}_{A}\beta$ and ${}_{AC}\beta$ is less than 0.001, but for ${}_{B}\beta$ and ${}_{BC}\beta$ greater than or equal to 0.001. These results are consistent with the results obtained from our two previous tests. The MM theory is more compatible with the data than the traditional theory.[15]

V. Conclusions

This study attempted to tie together some of the notions associated with the field of corporation finance with those associated with security and portfolio analyses. Specifically, if the MM corporate tax leverage propositions are correct, then approximately 21 to 24% of the observed systematic risk of common stocks (when averaged over 304 firms) can be explained merely by the added financial risk taken on by the underlying firm with its use of debt and preferred stock. Corporate leverage does count considerably.

To determine whether the MM theory is correct, a number of tests on a contrasting implication of the MM and "traditional" theories of corporation finance were performed. The data confirmed MM's position, at least vis-à-vis our interpretation of the traditional theory's position. This should provide another piece of evidence on this controversial topic.

Finally, if the MM theory and the capital asset pricing model are correct, and if the adjustments made in equations (8) or (4a) result in accurate measures of the systematic risk of a leverage-free firm, the possibility is greater, without resorting to a fullblown risk-class study of the type MM did for the electric utility industry [8], of estimating the cost of capital for individual firms.

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15. All of our tests, it should be emphasized, although consistent, are only inferential. Aside from assuming that the two-digit SEC industry classification is a good proxy for risk-classes and that the errors in estimating the individual βs can be safely ignored, the tests rely on the two theories exhausting all the reasonable theories on leverage. But there is always the use of another line of reasoning. If the results of the MM electric utility study [8] are correct, and if these results can be generalized to all firms and to all risk-classes, then it can be claimed that the MM theory is universally valid. Then our result in Section III does indicate the correct effect of the firm's capital structure on the systematic risk of common stocks.

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