Effective Load Carrying Capability of Generating Units

L. L. GARVER, MEMBER, IEEE

Abstract—The theory of loss-of-load probability mathematics has been generalized so that the effective load carrying capability of a new generating unit may be estimated using only graphical aids. A parameter m is introduced to characterize the loss-of-load probability as a function of reserve megawatts.

Once m is known or estimated, the effective load carrying capability of a new generating unit may be related to its rating and its forced outage rate. Alternate unit additions may be compared on the basis of their effective capabilities. Comparable expansion patterns may be developed on the basis of equal load carrying capabilities.

Numerical examples are used to illustrate the application of the effective capability concept to the evaluation of changes in the rating of a new unit and to the strategical design of expansion plans.

Introduction

THE THEORY of loss-of-load probability mathematics is generalized in this paper resulting in a graphical method for estimating the effective load carrying capability of a new generating unit. The concept of effective load carrying capability is best illustrated graphically as in Fig. 1. It is the distance in load megawatts between the annual risk functions before and after a unit addition. The measurement of effective load carrying capability is made at some designated level of reliability, often the level calculated for the system in a previous year. The effective capability of a new unit is, therefore the load increase that the system may carry with the designated reliability.

The graphical method for estimating effective capability presented in this paper will aid in the preliminary investigation of generation expansion plans. Illustrations of preliminary planning are presented along with examples of parametric investigations to estimate the effects of a change in the size or forced outage rate of one unit. The estimating method provides insights into how much of a unit's capability is needed to maintain system reliability.

As shown in Fig. 1, the system reliability will be measured in terms of the annual loss-of-load probability. Other measures of reliability could be used to determine the effective capability of a new unit [1]–[9]. The estimating procedure should also give comparable results for these methods [8]–[11].

It is best to begin with a review of the method for establishing the effective load carrying capability of a unit from the results of a series of loss-of-load probability calculations. The remainder of the paper will present the new method of estimation and illustrate its use.

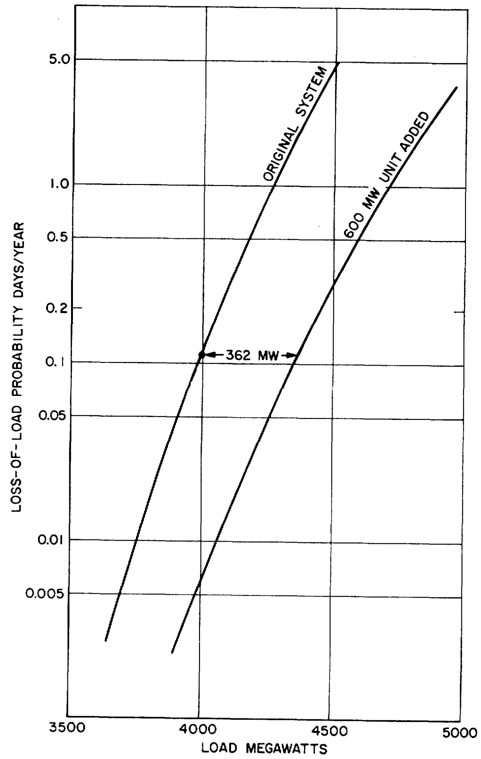

Fig. 1. Annual risk before and after adding a 600-MW unit with five percent forced outage rate.

Effective Capability

Other authors have presented figures similar to Fig. 1 and have also referred to load carrying capability or its counterpart, increased reserve requirements [6], [12], [13]. The steps followed to obtain these results were similar to those given below.

Steps to Determine Effective Capability

1) Determine the annual risk for the year *before* the unit is to be added. This requires a loss-of-load probability calculation based on data describing: a) the capability of each generating unit and its forced outage rate, b) the daily hourly-integrated peak loads, c) maintenance requirements for each unit, and d) other special features such as seasonal deratings, energy interchange contracts.

2) Vary the annual peak load and each daily peak in percent of the annual peak. Calculate the annual risk for a range of loads such as ±20 percent. The graph of the annual risk as a function of the annual peak will produce a curve similar to the original system curve in Fig. 1.

3) Add the new unit into the loss-of-load probability

Paper 31 TP 66-51 recommended and approved by the Power System Engineering Committee of the IEEE Power Group for presentation at the IEEE Winter Power Meeting, New York, N. Y., January 30-February 4, 1966. Manuscript submitted October 25, 1965; made available for printing December 2, 1965.

The author is with the General Electric Company, Schenectady, N. Y.

calculation data, keeping all other data fixed. Again vary the annual peak load with the daily peaks as a percentage and calculate the annual risks for a range of values, perhaps a zero to 40 percent increase over the previous midpoint load. The result when plotted, will form a second curve such as shown in Fig. 1.

4) The megawatt distance between these curves at the risk level determined in Step 1 is the amount of load growth the system can accept and still retain the same reliability the next year as in the starting year.

Illustration

Suppose that a certain system with a 4000-MW annual peak load and a reserve of 600 MW, or 15 percent, has an acceptable level of reliability. The loss-of-load probability calculation for this system resulted in an annual risk of 0.111 days per year. By varying the load up and down in 200-MW steps the *original system* curve in Fig. 1 was determined.

Then without changing any other data, a 600-MW unit with a forced outage rate (f.o.r.) of five percent was added to the generating system. Beginning with the 4000-MW load point the load was increased in 200-MW steps to obtain the *unit added* curve of Fig. 1. At a risk level of 0.111 days per year, i.e., the risk for the original system with a 4000-MW load; the system is able to carry a load of 4362 MW. The effective capability of a 600HMW five percent f.o.r., unit on this system is, therefore, 362 MW—the increase in load carrying capability of the system.

Further additions of 600-MW units to this system will result in larger effective capabilities for each successive unit. Figure 2 illustrates the results of adding five 600-MW five percent f.o.r. units. Table I summarizes the system load carrying capability, effective capability of each unit, the megawatts of reserve required, and the percent reserve. Kirchmayer and Mellor [12] illustrated the effect of adding a new unit in a figure very similar to Fig. 1. Steinberg and Smith [6] have noted the increases in system reserve requirements due to larger unit additions similar to those shown in Table I. Other examples of the increases in load carrying capability with repetitive additions have been published by Baldwin [13].

In this paper, the intention is to concentrate on the effect of the next unit addtion. The purpose of illustrating the effects of adding five new units is to stress the point that the initial effect, while important, is not the whole story. A complete evaluation of the investment and production cost economics over a period involving the addition of many units is necessary before the study of the next unit can truly be called complete [14], [15].

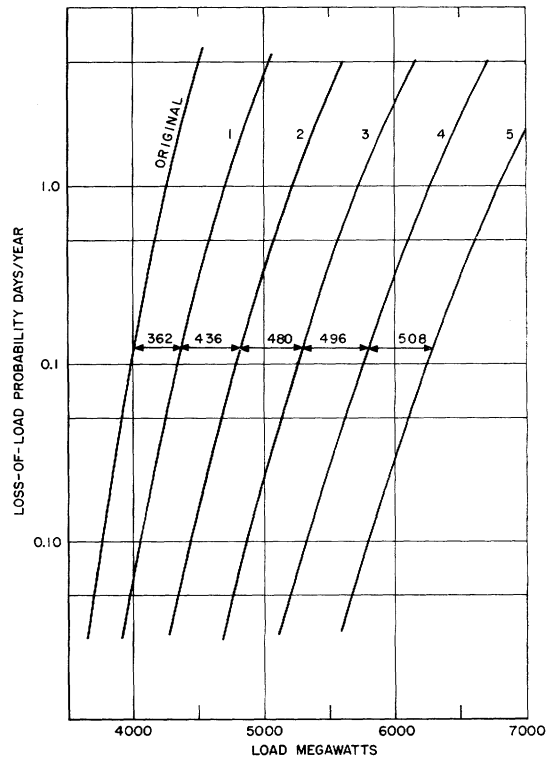

Fig. 2. Annual risk functions adding one to five 600-MW units with five percent forced outage rates.

Effect of Risk Level

The selected risk level has only a minor effect on the load carrying capability. For example, in Fig. 1 if the risk to maintain was 0.2 days per year, nearly twice the previous risk, then the effective capability would be 385 MW, an increase of 23 MW or five percent of the unit rating. If the risk to maintain was 0.05, one-half the original risk, the effective capability would be 340 MW, a decrease of 22 MW. Thus, for estimating purposes, there is no need to be greatly concerned about selecting a precise risk level. If the system is felt to have insufficient reserve, part of the new unit's effective capability may be allocated to improving the deficiency. Similarily, if the system seems to be over built, then part of the load growth may be carried by the present system.

TABLE I
EFFECTS OF ADDING 600-MW UNITS WITH FIVE PERCENT FORCED OUTAGE RATES

No. of Units Added	Installed Capability, MW	Load for Risk 0.111	Effective Capability, MW	E.C. as percent of 600 MW	System Reserve, MW	Reserve in percent of Load
	4600	4000			600	15.0
1	5200	4362	362	60.4	838	19.2
2	5800	4798	436	72.6	1002	21.2
3	6400	5278	480	80.0	1122	21.3
4	7000	5774	496	82.7	1226	21.2
5	7600	6280	508	84.7	1320	21.0

Estimating Effective Capability

The material presented thus far is a review of work already presented by others. Even the term *load carrying capability* is not new. What *is* new is the technique for estimating the effect of a unit addition without making the entire set of probability calculations that are necessary for Fig. 1.

Procedure

The estimating procedure begins after completing the first two steps to determine effective capability, i.e., the determination of the original system risk function.

1) Graph the annual risk as a function of installed annual reserve using semi-log paper. Figure 3 presents the data of the original system function in Fig. 1 plotted vs. reserve.

2) Approximate the annual risk function by a straight line at the designated risk level.

3) Characterize the slope of this straight line by m, the megawatts of load increase necessary to give an annual risk e times larger than the designated risk, where e is the base of the system of natural logarithms, 2.718...

4) Calculate the ratio of the new unit's capability, c, to the characteristic m, giving the parameter c/m.

5) Refer to the generalized graph in Fig. 4 which relates the effective capability c^* to the parameter c/m, to the forced outage rate of the new unit r and to the characteristic m. A multiplication of the result from Fig. 4 by m completes the estimate of effective capability.

The derivation of the functions plotted in Fig. 4 and also in Figs. 5 and 6, is contained in Appendix I.

Examples of Procedures

The effective load carrying capability of a new unit may be estimated once the characteristic m has been determined for the system. The value of m is related to the straight line approximation of the annual risk function plotted on semi-log paper as shown in Fig. 3. The straight line is fit through the risk level to be maintained at a value e times above it, 0.111 and 0.302. The value of m determined graphically is 118 MW. A method for calculating m is given in Appendix II.

To use Fig. 4 in estimating the effective capability c^* of a 600-MW unit, first calculate the c/m ratio.

$$c/m = 600/118 = 5.09.$$

Entering Fig. 4 with this value and the assumed forced outage rate of five percent allows the effective capability to m ratio of 2.89 to be read off. The multiplication by m completes the estimate.

$$c^* = 2.89\ (118) = 341\ \text{MW}.$$

This estimate is within six percent of the value determined in Fig. 1—362 MW.

Estimates of effective capability can be made which are close to the correct value by fitting the approximating straight line, which determines the value of m, through a point farther up on the risk function than e times designated risk. The risk caused by a five percent load increase has proven more successful as the second point in determining m and produced results with five percent of the actual effective capability.

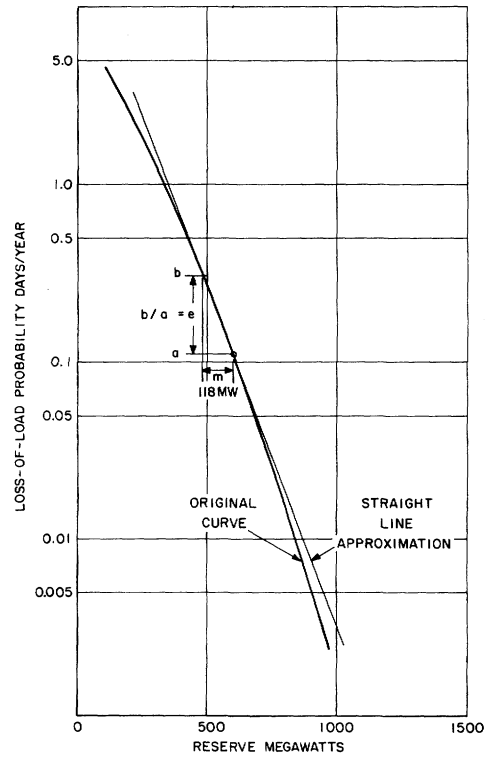

Fig. 3. Approximation of annual risk function by linear exponential function.

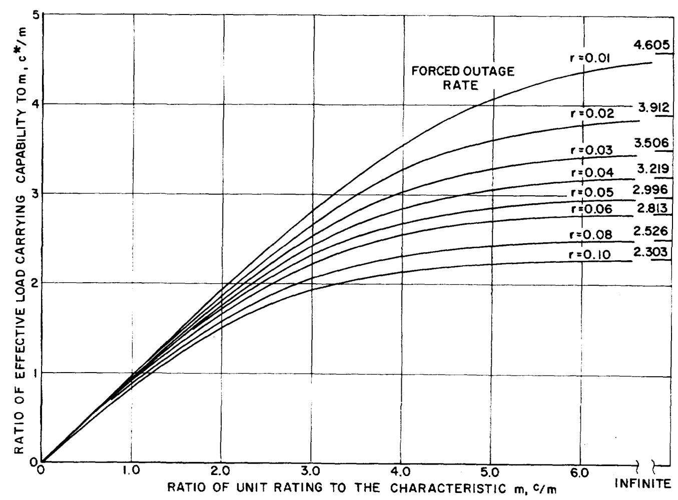

Fig. 4. Ratio of effective capability to m for various forced outage rates.

Alternate Procedure

As an alternate for Step 5 in the estimating procedure when the c/m ratio is 3.0 or less, Fig. 5 may be used to determine the reserve increase as a percent of the capacity of the unit. The effective capability is then the capacity less the reserve increase. Figures 4 and 5 present the same information in two different forms.

For the example of the 600-MW unit with a five percent forced outage rate added to a system with m = 118 MW,

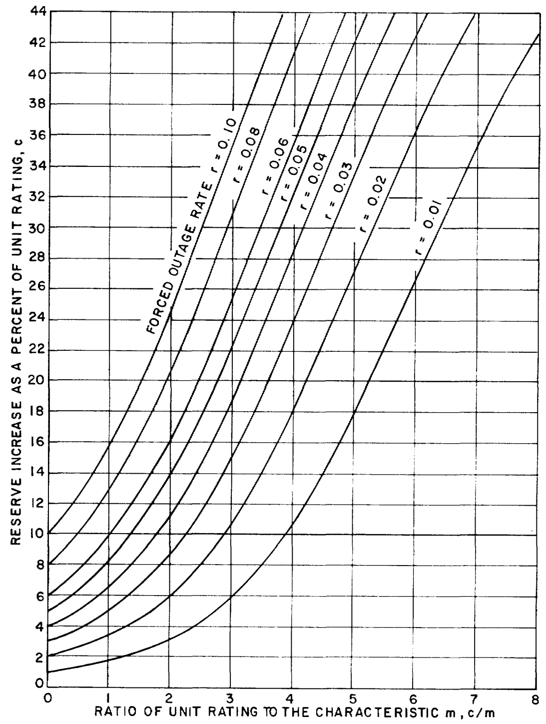

Fig. 5. Increase in reserve as percent of unit rating for various forced outage rates.

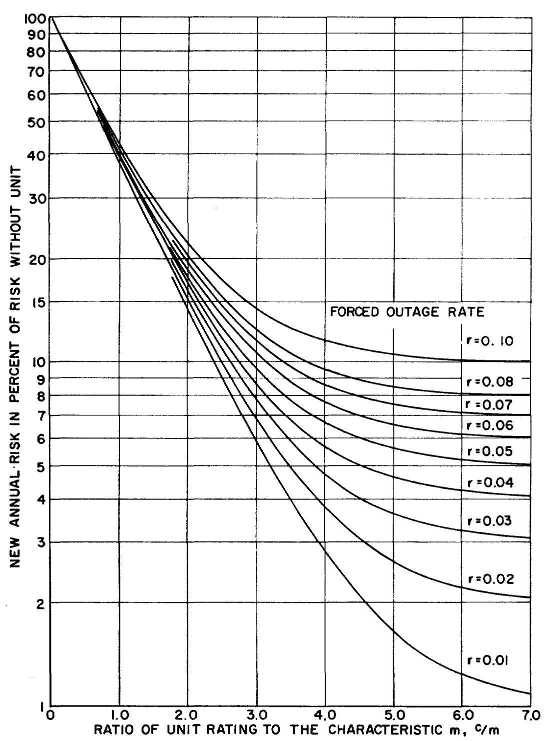

Fig. 6. Annual risk after unit addition as percentage of risk without addition.

Fig. 5 relates a 43.2 percent reserve increase to the c/m ratio of 5.09. The effective capability is thus 56.8 percent of the capacity, again, 341 MW.

Change in Reliability

In these examples, it was assumed that the annual risk was fixed and the load changed when the new unit was added. If we assume that the load remains fixed after a unit is added then the risk will be decreased. The amount of this decrease may be estimated with the aid of Fig. 6. This figure expresses the new loss-of-load probability as a percentage of the previous value when the only change to the system is the addition of a unit of a given c/m ratio and forced outage rate.

For example, if the 600-MW unit were added to the original system in Fig. 1 and the load did not increase, i.e., remained at 4000 MW, then from Fig. 6 a risk which is 5.6 percent of the previous value ($c/m = 5.09$ and $r = .05$) would be estimated. This compares favorably with the 5.1 percent determined from the digital computer calculation of the annual risk.

Applications

The estimating procedure provides a tool to supplement the use of the digital computer programs which are normally used in calculating system reliability measures for generation expansion studies. Two types of preliminary investigations related to expansion planning will be illustrated by means of numerical examples.

1) Estimate the effects of a change in one unit's capacity or forced outage rate.

2) Prepare preliminary generation expansion plans which are estimated to maintain the present degree of reliability.

Effects of a Change in Assumed Unit Size or Outage Rate

The change in the assumed characteristics of a unit may be viewed as having either of two possible effects: 1) a change in system reliability for a given load level, or 2) a change in load carrying capability for a given risk level. Both of these alternate effects may be estimated using the information in Figs. 4 through 6.

Assume that for a certain system the calculated annual risk in 1972 is 0.053 days per year; the first step is to investigate how sensitive this risk is to the forced outage rate of the 1971 unit and second what the risk would be if the unit were reduced in size.

Suppose in the calculation that the 1971 unit was a 600-MW five percent f.o.r. unit. The 1972 system characteristic m may be found by a straight line approximation to the 1972 curve, as in Fig. 3. Assuming $m = 155$, the c/m is $600/155 = 3.87$. The following information is then available from Figs. 4 through 6:

effective capability $= 2.67\ m = 2.67\ (155) = 413$ MW (constant risk)
percent capacity for reserve = 31.2 percent of 600 MW = 187 MW
new LOLP in percent of old = 7.0 percent (constant load).

If the forced outage rate were one percent higher, six percent instead of five percent, then these same figures would yield this information:

effective capability $= 2.52\ m = 2.52\ (155) = 392$ MW
percent capacity for reserve = 34.6 percent of 600 MW = 208 MW

new LOLP in percent of old = 8.0 percent.

Thus the one percent increase in the forced outage rate would reduce the effective capability by $413 - 392 = 21$ MW or raise the annual risk to

$$0.053\ (0.08/0.07) = 0.053\ (1.14) = 0.0605 \text{ days per year.}$$

If the capacity were reduced by 100 MW, 500 MW instead of 600 MW, and $r = 5$ percent, then the pertinent information would be, ($c/m = 500/155 = 3.23$):

effective capability = $2.43\,m = 2.43\ (155) = 376$ MW
percent capacity for reserve = 24.7 percent of 500 MW = 124 MW
new LOLP in percent of old = 8.7 percent.

Thus the 100-MW decrease in capacity would reduce the effective capability $413 - 376 = 37$ MW or raise the annual risk to

$$0.053\ (0.087/0.07) = 0.053\ (1.24) = 0.066 \text{ days per year.}$$

The general data contained in Figs. 4 and 5 allow a system planner to develop curves comparing the effects of unit size and forced outage rate on the load carrying capability of the next addition to a particular system. The curves in Figs. 7 and 8 illustrate two possibilities for comparison. In Fig. 7 the effective load carrying capability is shown as a function of unit size. In Fig. 8 the effects of the forced outage rates are shown for both a 400-MW and an 800-MW unit. Both curves are for a system with a characteristic m of 125 MW.

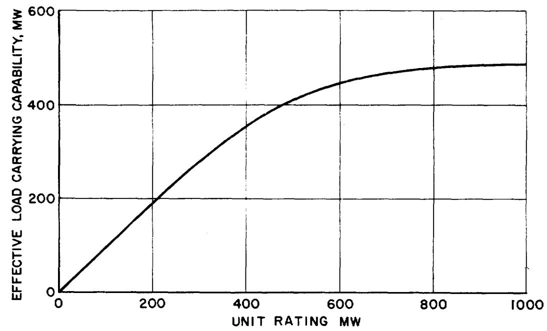

Fig. 7. Effective capability of new unit with two percent forced outage rate added to system, $m = 125$ MW.

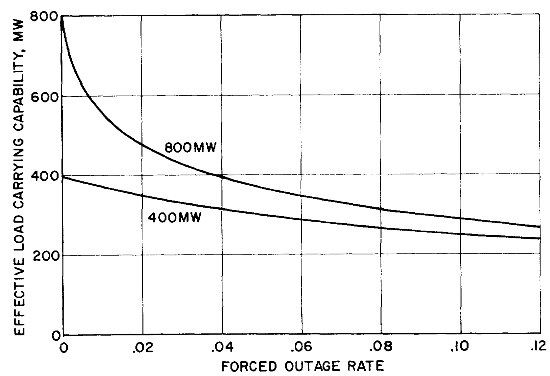

Fig. 8. Effects of outage rate on effective capability, for system with $m = 125$ MW.

Generation Expansion Planning

Generation expansion planning involves the development of alternate patterns of unit sizes and installation dates. Each alternate expansion is designed to meet a criteria of reliability [8], [9]. The estimated effective load carrying capabilities of future units offer a new guide for the preliminary design of alternate expansions.

The system planner may match the effective capability of the unit additions with the forecasted load growth to design alternatives based on constant reliability. The implementation of this concept will be illustrated by designing preliminary expansion plans to meet these three design strategies: 1) add one unit a year matching the effective capability to the load growth, 2) add one unit every two years, 3) add three units of the same size whose combined effective capabilities match four years of load growth.

The implementation of a strategy to add a unit to match the load growth for each year is illustrated by the data in Table II. A system growing at seven percent a year and an initial load of 4000 MW has been assumed. The initial system characteristic m should be determined from a loss-of-load probability calculation as in Fig. 3 or Appendix II. However, assume that such a calculation has not been made and m must be estimated.

An approximate method for estimating the value of m is given in Appendix III. Suppose that the generating system is composed of 4600 MW of capability: 3000 MW in units less than 299 MW in rating, 1200 MW in units between 300 MW and 399 MW, and a 400-MW unit. The forced outage rates assumed for existing and future units are shown in Table III. The estimate of m from (22) of Appendix III is

$$m \approx 3000\ (0.02) + 1200\ (0.03) + 400\ (0.04) = 112 \text{ MW.}$$

In Table II each increment of load growth is first converted into the required load carrying capability to m ratio, c^*/m. For example the load growth for the first year is forecasted to be 280 MW and the required c^*/m is thus 2 80/112 = 2.50. The rating of the unit was found by looking up the c/m ratio in Fig. 4 corresponding to a c^*/m of 2.50 and an f.o.r. of three percent. The result was a c/m of 2.90 which when multiplied by the current value of m, 112 MW, gave a unit size of 325 MW.

Each addition of the system will change the value of m, (note Fig. 2). The amount of change was approximated using (22), which for the addition of one unit is just the rating times its forced outage rate. The change in m for the first year was estimated to be $(325)\ (0.03) = 9.75$.

In the fifth year, a three percent f.o.r. unit would have to be rated at 421.5 MW. Since this is in the four percent f.o.r. range, as shown in Table III, the c/m ratio was determined from the four percent line in Fig. 4. Similarly, in the eighth year the units become so large that the five percent line must be used. Table IV presents the reserve and percent reserve for the expansion of Table II.

TABLE II
SELECTING UNITS TO MATCH THE LOAD GROWTH

Year	Load, MW	Load Growth, MW	Needed, c^*/m	Forced Outage Rate	c/m from Fig. 4	Unit Rating, MW	Change in m	New m, MW
0	4000							112
1	4280	280	2.50	0.03	2.90	325	9.75	121.75
2	4580	300	2.47	0.03	2.85	347	10.41	132.16
3	4900	320	2.42	0.03	2.80	370	11.10	143.26
4	5243	343	2.41	0.03	2.78	390	11.70	154.96
5	5610	367	2.37	0.03	2.72	422	(must change f.o.r.)	
				0.04	2.90	450	17.98	172.94
6	6003	393	2.27	0.04	2.72	470	18.80	191.74
7	6423	420	2.19	0.04	2.60	498	19.92	211.66
8	6873	450	2.13	0.04	2.51	532	(must change f.o.r.)	
				0.05	2.62	555	27.75	239.41
9	7354	481	2.01	0.05	2.43	582	29.10	268.51
10	7869	515	1.92	0.05	2.28	625		

TABLE III
FORCED OUTAGE RATES ASSUMED FOR EXAMPLES

Range of Unit Ratings, MW	Forced Outage Rates
0–299	0.02
300–399	0.03
400–499	0.04
500–700	0.05

TABLE IV
EXPANSION PLAN TO MATCH CAPACITY TO LOAD GROWTH

Year	Load, MW	Capacity Added, MW	Total Capacity, MW	Installed Reserve, MW	Reserve in Percent of Load
0	4000		4600	600	15.00
1	4280	325	4925	645	15.07
2	4580	347	5272	692	15.11
3	4900	370	5642	742	15.14
4	5243	390	6032	789	15.05
5	5610	450	6482	872	15.54
6	6003	470	6952	949	15.81
7	6423	498	7450	1027	15.99
8	6873	555	8005	1132	16.47
9	7354	582	8587	1233	16.77
10	7869	625	9212	1343	17.07

TABLE V
ATTEMPT TO MATCH THREE UNITS TO FOUR YEARS OF LOAD GROWTH

Unit size, MW	650
Forced outage rates, percent	5
Change in m, MW	32.5

Unit Number	Value of m	c/m	c^*/m (Fig. 4)	Effective Capability
1	112	5.8	2.9	329
2	144.5	4.5	2.8	405
3	177	3.7	2.6	460

Total effective capability, MW	1194
Four year load growth, MW	1213
Mismatch, MW	19

Suppose the strategy were changed to a new unit every *two* years. The load growth for two years is 280 + 300 = 580 MW. The effective capability to m ratio, c^*/m, of the new unit must be 580/112 = 5.18. However, it is evident from Fig. 4, that no single unit with a forced outage rate of even one percent could carry that load growth with present system reliability. Thus, two units must be installed in the first two years to maintain system reliability.

A strategy of adding three identical units in four years will require a cut and try procedure. Assume a unit size and calculate the load carrying capabilities for all three. If the total load carrying capability is below the 4-year load growth, increase the unit size and try again. In Table V, the calculations are shown for the addition of three 650-MW units at five percent forced outage rate. The effective capability of the three fell 19-MW short of the 4-year load growth, 1213 MW. Another trial at 675-MW may prove successful.

The examples illustrate the manner in which the concepts of effective capability and the general curves of Figs. 4 through 6 can aid in generation expansion planning.

CONCLUSION

A procedure for estimating the effective load carrying capability of a generating unit has been presented. Its use makes possible preliminary investigations to supplement the detailed calculation of system reliability associated with a generation expansion study. The estimating procedure uses a graphical relationship between effective capability and the characteristics of the unit with a system parameter m. The parameter m has been introduced as a single number to characterize the annual risk function of the system. Though the value of m should be determined from a calculation of the loss-of-load probability function, a method for approximating its value is also presented.

The estimating procedure has been applied to two types of investigations associated with generation expansion planning—changing one unit's characteristics in a plan already studied in detail, and preparing expansion plans to meet certain design strategies while maintaining constant reliability. Numerical examples illustrated these two uses of the estimating procedure.

Nomenclature

- A_x annual risk or loss-of-load probability in days per year for a reserve of x MW
- A_x' new annual risk for reserve x with a new unit added to the system
- a_i ratio of day i peak load to annual peak load
- B a constant in the approximation of A_x by an exponential function
- c rating of a new generating unit, megawatts
- c^* effective load carrying capability of a unit with rating c
- e base of the system of natural logarithms, 2.718...
- ln( ) natural logarithmic function
- m the MW of load increase that will give annual risk increase e times greater than before—called the system characteristic m
- n total number of generating units in a system
- k number of daily hourly-integrated peak loads considered in an annual risk calculation
- P_{x_i} probability of having x_i MW or greater capacity on forced outage
- r forced outage rate of a generating unit
- x annual installed reserve, megawatts
- x_i installed reserve for day i, megawatts
- y increase in annual installed reserve in megawatts necessary after a unit is added to maintain the same reliability as before the addition

Appendix I
Estimating the Effects of a Unit Addition

Annual Risk as a Function of Reserve

The first step in the development of the estimating procedure for the effective load carrying capability of a new unit is to change the independent variable of the annual risk from the load to the reserve. Reserve is a function of both the installed capacity and the annual peak load. The annual risk is also a function of both of these variables.

In Fig. 3 the original system curve of Fig. 1 is plotted vs. system reserve instead of system load.

Risk Function After a Unit Addition

The second step in the development is to express the annual risk after a unit addition in terms of the annual risk function before the addition. Recall from the fundamentals of the loss-of-load probability calculation that the annual risk is the sum of the daily risks. Each daily risk is the cumulative probability of having a total amount of capacity on forced outage greater than the reserve for that day

$$\text{annual risk} = \sum_{i=1}^{k} \text{daily risk}_i$$
$$\text{daily risk}_i = P_{x_i}$$

where P_{x_i} is the cumulative probability of having x_i MW or greater on forced outage. The expression for the annual risk becomes

$$A_x = \sum_{i=1}^{k} P_{x_i} \tag{1}$$

where x is the annual installed reserve and x_i is the available reserve on day i.

When a new unit with rating c and forced outage rate r is added to the system the cumulative probability of outage x is

$$P_x' = (1-r)\, P_x + rP_{x-c}. \tag{2}$$

The new cumulative outage probability is the sum of the two components corresponding to the two possible conditions for the new unit, i.e., in service or on forced outage. The first component assumes that the new unit is in service, a probability of $(1-r)$, and the capacity outage of x MW or greater if it is to occur will be in the old system. The second component assumes that the new unit is on forced outage, a probability of r, and thus an outage of only x-c MW or greater in the old system will cause a total outage of x or greater.

Expression (2) may be substituted into (1) for each daily reserve x_i giving the following expression:

$$\begin{aligned} A_x' &= \sum_{i=1}^{k} [(1-r)\, P_{x_i} + rP_{x_i-c}] \\ &= (1-r) \quad (\textstyle\sum_{i=1}^{k} P_{x_i}) + r(\sum_{i=1}^{k} P_{x_i-c}). \end{aligned} \tag{3}$$

The first term in (3) is the old annual risk for reserve x multiplied by the innage rate for the new unit. The second term is the annual risk for reserve x but with each day's reserve decreased by the rating of the new unit c and multiplied by the forced outage rate of the unit.

Approximation 1: The second term in (3) will be replaced by the annual risk for a load increase of c MW multiplied by r. This amounts to replacing

$$x_i - c$$

with the expression

$$x_i - a_i c.$$

Thus each day's reserve is decreased by a percentage of c instead of the full amount, causing a smaller value for the second term in expression (3). The result of the first approximation is the following expression for the new annual risk in terms of the old values:

$$A_x' = (1-r)\, A_x + rA_{x-c}. \tag{4}$$

To gauge the error introduced by this approximation, we note that in the example of Fig. 1 the annual risk for a load increase of 200 MW was 0.604 days per year while the annual risk for a capacity decrease of 200 MW was 0.631 days per year.

Straight Line Approximation

The third step in the development of the estimate of load carrying capability requires that the annual risk as a function of reserve be approximated by a straight line on semi-log paper.

Approximation 2: Assume that the annual risk expressed in terms of the installed reserve has the following form:

$$A_x = Be^{-x/m} \tag{5}$$

where

B = a constant that need not be evaluated
m = the system characteristic and has the dimension of megawatts
e = the base of the natural system of logarithms, 2.718...
x = the installed reserve on the system, megawatts.

This assumption is shown graphically in Fig. 3. To determine the constant m we require only two points on the straight line. Assume that the two points are

$$A_{600} = 0.111 \text{ days per year}$$

$$A_{400} = 0.604 \text{ days per year.}$$

Dividing the larger risk by the smaller yields

$$\frac{A_{400}}{A_{600}} = \frac{Be^{-400/m}}{Be^{-600/m}} = e^{-(400-600)/m} = e^{200/m}.$$

Also,

$$\frac{A_{400}}{A_{600}} = \frac{0.604}{0.111} = 5.44.$$

Equating the two expressions and taking the natural logarithm of both sides gives

$$200/m = \ln(5.44).$$

Solving for m gives

$$m = \frac{200}{\ln(5.44)} = \frac{200}{1.69} = 118 \text{ MW}.$$

The general expression for m in terms of any two given points y MW apart, A_x and A_{x-y}, is

$$m = \frac{y}{\ln(A_{x-y}/A_x)}. \tag{6}$$

To gauge the effect of this approximation, refer to Fig. 3. The approximation gives higher values of annual risk than those actually calculated. The effects of Approximations 1 and 2 tend to offset one another since 1 gives values too low while 2 gives values too high. Experience in selecting the straight line approximation will enable a planner to estimate the effective capabilities quite close to those determined from a computer calculation of annual risks.

Derivation of Generalized Expressions

The fourth and final step in the development is to derive expressions for the new annual risk, the reserve increase in percent of the new capacity and the load carrying capability based on the previous annual risk function.

The annual risk after the addition of a unit and no change in load will be the value at a reserve of $x+c$ MW. Substituting $x+c$ for x in (4) yields

$$A'_{x+c} = (1-r)\, A_{x+c} + rA_x. \tag{7}$$

Expressing A'_{x+c} in terms of A_x may be accomplished by using (5).

$$A_{x+c} = Be^{-(x+c)/m} = (Be^{-x/m})\, e^{-c/m} = A_x(e^{-c/m}). \tag{8}$$

Substituting (8) into (7) gives the desired expression for the new annual risk:

$$A_{x+c}' = [(1-r)\, e^{-c/m} + r]\, A_x. \tag{9}$$

The general curves in Fig. 6 were calculated by solving (10) for the new annual risk in percent of the old value and evaluating the expression for a range of c/m and r.

$$\frac{100\, A_{x+c}'}{A_x} = 100\, [(1-r)\, e^{-c/m} + r]. \tag{10}$$

To derive the expression for the reserve increase as a percentage of the new capacity, let y be the reserve increase necessary to maintain the same annual risk.

$$A_{x+y}' = A_x. \tag{11}$$

The expression for the new annual risk in terms of the old function is found by substituting $x+y$ into (4):

$$A_{x+y}' = (1-r)\, A_{x+y} + rA_{x+y-c}. \tag{12}$$

Express each term in this equation as a product involving A_x by using the assumption in (5).

$$A_{x+y} = Be^{-(x+y)/m} = e^{-(y/m)}\, A_x \tag{13}$$

$$A_{x+y-c} = Be^{-(x+y-c)/m} = e^{-(y-c)/m} A_x. \tag{14}$$

Substituting (13) and (14) into (12) and collecting terms yields

$$A_{x+y}' = [(1-r)e^{-y/m} + re^{-(y-c)/m}]A_x. \tag{15}$$

Substituting into (11) and dividing through by A_x gives

$$[(1-r) + re^{c/m}]e^{-y/m} = 1. \tag{16}$$

Take the natural log of both sides and recall that the log of a product is the sum of the logs.

$$\ln[(1-r) + re^{c/m}] + (-y/m) = \ln(1) = 0.$$

Solving for y:

$$y = m \ln[(1-r) + re^{c/m}] \tag{17}$$

and expressing the reserve increase y as a percentage of the new capacity c

$$100\, y/c = 100\, (m/c) \ln\, [(1-r) + re^{c/m}]. \tag{18}$$

Expression (18) has been evaluated for a range of values for c/m and r with the results plotted in Fig. 5.

The load carrying capability of the new unit is the difference between its capacity c and the required reserve of (17).

$$c^* = c - y = c - m \ln[(1-r) + re^{c/m}]. \tag{19}$$

To generalize this expression it was divided by the system characteristic m.

$$c^*/m = (c/m) - \ln[(1-r) + re^{c/m}]. \tag{20}$$

Expression (20) has been evaluated for a range of values of c/m and r and the results are shown in Fig. 4.

Appendix II
Calculating the Value of m

The value of m may be determined analytically instead of graphically as illustrated in Fig. 3. The value of m for the line through any two points on the annual risk curve may be calculated by assuming that the two known points are 0.397 days per year for a reserve of 450 MW and 0.111 days per year for a reserve of 600 MW. Thus, the value of m is

$$m = (600\text{-}450)/\ln(0.397/0.111) = 150/\ln(3.57) = 150/1.272 = 118 \text{ MW}.$$

In general,

$$m = (b - a)/\ln(A_a/A_b) \tag{21}$$

where

A_a = the annual risk for a reserve of a MW
A_b = the annual risk for a reserve of b MW
ln = the natural logarithmic function.

Appendix III
Estimating the Value of m

In order to design a generation expansion alternative before the initial calculation of the loss-of-load probability, some estimate of the characteristic m is necessary. Also the value of m changes after each unit addition and these changes should also be approximated.

The value of m is determined by the change in the annual risk due to a change in the forecasted loads. The annual peak load change which gives an annual risk e times greater than before is the value of m for the system.

The annual risk is a function of every individual unit on the system: its rating and forced outage rate and the annual load shape. Any simple estimate of how this function will change with a change in the load forecast can only be a very rough estimate and should be checked with a computer calculation.

It appears from experience that a rough gauge of the value of m is the sum of each unit's rating times its forced outage rate.

$$\text{estimate of } m = \textstyle\sum_{i=1}^{n} c_i r_i \tag{22}$$

where

c_i = the megawatt capacity of the i^{th} unit
r_i = the forced outage rate of the i^{th} unit
n = the number of units presently in the system.

This expression, though only a rough approximation, does illustrate the behavior of the characteristic m. A unit with no forced outage rate does not affect the slope of the annual risk characteristic. The larger the unit or the larger its forced outage rate, the greater its effect on the slope m.

Thus, the first large unit on a system while not having a large percentage of load carrying capability, will have a great effect on the characteristic m and prepare the system to make better use of the second and third units.

Acknowledgment

The author wishes to acknowledge the assistance of Misses G. A. Cary and R. B. Roginska for their work in computing the data for this paper.

References

[1] G. Calabrese, "Determination of reserve capacity by the probability method," *Trans. AIEE*, pt. II, vol. 69, pp. 1681–1989, 1950.
[2] C. D. Galloway and L. K. Kirchmayer, "Digital computer aids economic-probabilistic study of generation systems," *AIEE Trans. (Power Apparatus and Systems)*, vol. 77, pp. 564–577, August 1958.
[3] C. Kist and G. J. Thomas, "Probability calculations for system generation reserves," *AIEE Trans. (Power Apparatus and Systems)*, vol. 77, pp. 515–520, August 1958.
[4] H. D. Limmer, "Determination of reserve and interconnection requirements," *AIEE Trans. (Power Apparatus and Systems)*, vol. 77, pp. 544–550, August 1958.
[5] A. L. Miller, "Details of outage probability calculations," *AIEE Trans. (Power Apparatus and Systems)*, vol. 77, pp. 551–557, August 1958.
[6] M. J. Steinberg and V. M. Cook, "Evaluations of unit capacity additions," *AIEE Trans. (Power Apparatus and Systems)*, vol. 75, pp. 169–179, April 1956.
[7] C. W. Watchorn, "Elements of system capacity requirements," *Trans. AIEE*, vol. 70, pp. 1163–1185, 1951.
[8] AIEE Working Group Report, "Application of probability methods to generating capacity problems," *AIEE Trans. (Power Apparatus and Systems)*, vol. 79, pp. 1165–1182, 1960.
[9] FPC Special Technical Subcommittee, "Determination of reserve requirements for interconnected systems," Federal Power Commission National Power Survey Advisory Committee, Rept. 5, February 1963.
[10] H. Halperin and H. A. Adler, "Determination of reserve-generating capability," *AIEE Trans. (Power Apparatus and Systems)*, vol. 77, pp. 530–544, August 1958.
[11] E. S. Vassell and N. Tibberts, "An approach to the analysis of generation-capacity reserve requirements," *IEEE Trans. on Power Apparatus and Systems*, PAS-84, pp. 64–79, January 1965.
[12] L. K. Kirchmayer and A. G. Mellor, "Economic choice of generator unit size," *ASME Trans.*, vol. 80, pp. 1015–1026, July 1958.
[13] C. J. Baldwin "Planning for power pools and EHV ties," *Proc. of the American Power Conference*, vol. 25, pp. 823–833, 1963.
[14] C. D. Galloway and L. L. Garver, "Computer design of single-area generation expansions," *IEEE Trans. on Power Apparatus and Systems*, vol. 83, pp. 305–311, April 1964.
[15] E. S. Bailey, Jr., C. D. Galloway, E. S. Hawkins, A. J. Wood, "Generation planning programs for interconnected systems," *IEEE Trans. on Power Apparatus and Systems*, special supplement, pp. 761–788, 1963.

Discussion

J. H. Ashby (Technical Services Inc., Dallas, Texas): This paper is a very timely and valuable contribution to the methods of analyzing power system performance by the use of probability mathematics. The author has given the system planner a most useful tool by which he may extrapolate the results of a given probability study, or perhaps, what is more important, acquire a better perspective of his system's projected performance under the influence of many variables.

Manuscript received February 3, 1966.

The companies with which the author and I are associated have had several recent occasions to make good use of the methods presented in this paper. In 1964 and 1965 eight loss-of-load probability studies were performed, using the General Electric Program, to examine the Dallas Power and Light Company, Texas Electric Service Company, and Texas Power and Light Company. This examination concerned the companies system service reliability in the past and under projected conditions of variable forced outage rates, as well as constant and increasing generating unit size. After these studies were made, certain changes in forecast demands and the size of two future generating units called for a restudy of the affected years. In each case, the methods of this paper permitted an easy determination of change in the system's risk index from the available computer output of previous studies. When the system plan later became firm, another computer study was made and confirmed the validity of the approximating methods.

In all of these studies, two load levels were prescribed for each year—one for the normal forecast, and the other for 106 percent of this to account for the effect of an extremely hot summer. The computer output then gave the two points of data to permit plotting curves for each year similar to those of Fig. 3 of this paper. The reserve required each year to maintain the selected risk level of 0.1 day per year could then be read off and used as a measure of the relative reliability of the planned expansion pattern. In this process, it was noted that as unit size increased, loss-of-load probability became less sensitive to change in reserve, i.e., the slope of the system characteristic became less steep. This bears out the increasing value of m shown in Table II of the paper. It has been found in subsequent calculations using the more precise statement of (6) that the value of m can vary from 150 MW to nearly 300 MW as unit size increases, depending on the timing of such units and their forced outage rates.

C. W. Watchorn (Pennsylvania Power and Light Company, Allentown, Pa.): The paper presents a very interesting and valuable basis for extending the results of a computer study of installed generating capacity requirements to future conditions other than the specific ones for which the study was made without necessitating further computer studies. When a report is being studied such latitudes are always valuable because of the many questions that often arise about situations having conditions different from those considered in the initial study, for which it is highly desirable to obtain quick and, at least, reasonably accurate answers. The method described in the paper provides a means for doing just this with respect to capacity to be added in the future.

A simplified method for doing the same thing, plus a great deal more, i.e., providing a simple basis for making the basic study itself, for studying the effect of the removal, and for changing the forced outage rates of already installed as well as future units, with a simpler computer program than the one upon which this paper is based, was presented about ten years ago.[1] This method has been found to be very valuable not only for making investigations along the lines described in the subject paper but also for making the basic primary studies of installed capacity requirements.

The two principal draw backs to this method are: 1) it requires so few computer computations that the results can hardly be said to have been determined by a computer, and 2) no theoretical basis has been presented for the validity of the method although the results have been found to almost exactly check those determined by computer calculations for the same problem to well within the degree of practical accuracy requirements. The derivation of the method was entirely empirical, but it is no less believed to be the result of some rapidly converging mathematical process.

[1] C. W. Watchorn, "A simplified basis for applying probability methods to the determination of installed generating capacity requirements," *AIEE Trans. (Power Apparatus and Systems)*, vol. 76, pp. 829–832, October 1957.

Manuscript received February 17, 1966.

L. L. Garver: I appreciate learning from Mr. Ashby that the results of this paper have been successfully applied to their generation expansion planning problem. Mr. Watchorn puts his finger on the real need for an estimating procedure—data changes. Input data assumptions are forever being adjusted, and it is desirable to quickly extrapolate to new situations from the results of previous investigations.

The pioneering work of Mr. Watchorn is in evidence here with the reference to his 10-year-old publication on a method of estimating system reliability. However, there does not appear to be any simularity between our two methods.

Finally, I want to thank J. H. Ashby and C. W. Watchorn for their interest in this paper as evidenced by their discussions.

Manuscript received March 4, 1966.