DELTA NATURAL GAS COMPANY, INC.
CASE NO. 2021-00185

FIRST ATTORNEY GENERAL DATA REQUEST
DATED JULY 14, 2021

9. Provide any information in Mr. Moul’s or Delta’s possession on state Commission-allowed returns on equity from January 2020 through the most recent month in 2021. Identify whether the allowed returns were based on litigated rate cases and/or settlements.

Response:

Please see attached.

Sponsoring Witness: Dylan D’Ascendis

S&P Global
Market Intelligence

Rate Case History (Past Rate Cases)

List None
Company List All
States All
Years 2021, 2020
Service Type Natural Gas

						Increase Requested						
State	Company	Parent Company Ticker	Docket	Rate Case Service Type	Case Type	Date	Rate Increase ($M)	Return on Original Cost Rate (%)	Return on Equity (%)	Common Equity to Total Capital (%)	Rate Base ($M)	Date
Wyoming	MDU Resources Group	MDU	D-30013-351-GR-19	Natural Gas	Distribution	5/23/2019	1.1	7.75	10.30	52.08	15.38	1/15/2020
New York	Consolidated Edison Co. of NY	ED	C-19-G-0066	Natural Gas	Distribution	1/31/2019	206.2	7.19	9.75	50.00	7,192.55	1/16/2020
Virginia	Roanoke Gas Co.	RGCO	C-PUR-2018-00013	Natural Gas	Distribution	10/10/2018	9.2	7.99	10.70	59.92	126.09	1/24/2020
Washington	Cascade Natural Gas Corp.	MDU	D-UG-190210	Natural Gas	Distribution	3/29/2019	12.7	7.73	10.30	50.00	405.16	2/3/2020
Kansas	Atmos Energy Corp.	ATO	D-19-ATMG-525-RTS	Natural Gas	Distribution	6/28/2019	8.5	7.68	9.90	60.12	243.72	2/24/2020
Utah	Questar Gas Co.	D	D-19-057-02	Natural Gas	Distribution	7/1/2019	17.5	7.73	10.50	55.00	1,804.27	2/25/2020
Massachusetts	Fitchburg Gas & Electric Light	UTL	DPU 19-131	Natural Gas	Distribution	12/17/2019	7.3	8.41	10.50	52.45	95.04	2/28/2020
Washington	Avista Corp.	AVA	D-UG-190335	Natural Gas	Distribution	4/30/2019	12.9	7.52	9.90	50.00	398.99	3/25/2020
Maine	Northern Utilities Inc.	UTL	D-2019-00092	Natural Gas	Distribution	6/28/2019	7.1	8.00	10.50	52.91	231.31	3/26/2020
Texas	Atmos Energy Corp.	ATO	D-GUD-10900	Natural Gas	Distribution	9/27/2019	0.3	7.71	9.80	60.12	37.40	4/21/2020
Colorado	Black Hills Colorado Gas Inc.	BKH	D-19AL-0075G	Natural Gas	Distribution	2/1/2019	3.5	7.32	10.30	50.15	265.29	5/19/2020
Texas	CenterPoint Energy Resources	CNP	D-GUD-10920	Natural Gas	Distribution	11/14/2019	6.8	8.22	10.40	58.00	NA	6/16/2020
Washington	Puget Sound Energy Inc.		D-UG-190530	Natural Gas	Distribution	6/20/2019	65.5	7.48	9.50	48.50	2,113.44	7/8/2020
Texas	Texas Gas Service Co.	OGS	D-GUD-10928	Natural Gas	Distribution	12/20/2019	17.0	7.93	10.00	62.12	473.47	8/4/2020
Michigan	DTE Gas Co.	DTE	C-U-20642	Natural Gas	Distribution	11/25/2019	188.5	5.78	10.50	39.76	5,143.36	8/20/2020
Wyoming	Questar Gas Co.	D	D-30010-187-GR-19	Natural Gas	Distribution	11/1/2019	3.5	7.46	10.50	55.00	62.07	8/21/2020
Michigan	Consumers Energy Co.	CMS	C-U-20650	Natural Gas	Distribution	12/16/2019	229.3	6.09	10.50	42.61	7,605.79	9/10/2020
New Jersey	South Jersey Gas Co.	SJI	D-GR20030243	Natural Gas	Distribution	3/13/2020	73.3	7.38	10.40	54.18	2,220.73	9/23/2020
Nevada	Southwest Gas Corp.	SWX	D-20-02023 (Southern)	Natural Gas	Distribution	2/28/2020	35.8	6.89	10.00	49.26	1,352.57	9/25/2020
Nevada	Southwest Gas Corp.	SWX	D-20-02023 (Northern)	Natural Gas	Distribution	2/28/2020	2.7	7.12	10.00	49.26	156.55	9/25/2020
Massachusetts	Eversource Gas Company of MA	ES	DPU 20-59	Natural Gas	Distribution	7/2/2020	42.8	7.50	9.70	53.25	NA	10/7/2020
Colorado	Public Service Co. of CO	XEL	D-20AL-0049G	Natural Gas	Distribution	2/5/2020	144.5	7.33	9.95	55.81	2,236.46	10/12/2020
Oregon	Northwest Natural Gas Co.	NWN	D-UG-388	Natural Gas	Distribution	12/30/2019	63.3	6.97	9.40	50.00	1,466.23	10/16/2020
Massachusetts	NSTAR Gas Co.	ES	DPU 19-120	Natural Gas	Distribution	11/8/2019	35.0	7.60	10.45	54.84	809.58	10/30/2020
Maryland	Columbia Gas of Maryland Inc	NI	C-9644	Natural Gas	Distribution	5/15/2020	6.3	7.87	10.95	52.64	156.04	11/7/2020
New York	NY State Electric & Gas Corp.	IBE	C-19-G-0379	Natural Gas	Distribution	5/20/2019	4.1	6.61	9.50	50.00	658.31	11/19/2020
New York	Rochester Gas & Electric Co	IBE	C-19-G-0381	Natural Gas	Distribution	5/20/2019	(1.8)	7.07	9.50	50.00	491.38	11/19/2020
Florida	Peoples Gas System	EMA	D-20200051	Natural Gas	Distribution	6/8/2020	85.3	6.63	10.75	54.70	1,578.73	11/19/2020
Wisconsin	Madison Gas and Electric Co.	MGEE	D-3270-UR-123 (Gas)	Natural Gas	Distribution	8/28/2020	6.7	7.08	9.80	55.00	282.36	11/24/2020

Arizona	Southwest Gas Corp.	SWX	D-G-01551A-19-0055	Natural Gas	Distribution	5/1/2019	80.8	7.57	10.15	51.10	2,065.82	12/9/2020
Oregon	Avista Corp.	AVA	D-UG 389	Natural Gas	Distribution	3/16/2020	5.7	7.24	9.40	50.00	304.66	12/10/2020
New Mexico	New Mexico Gas Co.	EMA	C-19-00317-UT	Natural Gas	Distribution	12/23/2019	13.2	7.36	10.20	54.00	741.44	12/16/2020
Maryland	Baltimore Gas and Electric Co.	EXC	C-9645 (Gas)	Natural Gas	Distribution	5/15/2020	91.1	7.09	10.10	52.00	2,972.04	12/16/2020
Wisconsin	Wisconsin Power and Light Co	LNT	D-6680-UR-122 (Gas)	Natural Gas	Distribution	5/1/2020	0.0	NA	NA	NA	NA	12/23/2020
Oregon	Cascade Natural Gas Corp.	MDU	D-UG 390	Natural Gas	Distribution	3/31/2020	4.5	7.08	9.40	50.00	132.61	1/6/2021
Delaware	Delmarva Power & Light Co.	EXC	D-20-0150	Natural Gas	Distribution	2/21/2020	11.6	7.15	10.30	50.37	399.72	1/6/2021
Illinois	Ameren Illinois	AEE	D-20-0308	Natural Gas	Distribution	2/21/2020	97.4	7.64	10.50	54.09	2,119.69	1/13/2021
Nebraska	Black Hills/NE Gas Utility Co	BKH	D-NG-109	Natural Gas	Distribution	6/1/2020	15.7	6.96	10.00	50.00	503.79	1/26/2021
Tennessee	Piedmont Natural Gas Co.	DUK	D-20-00086	Natural Gas	Distribution	7/2/2020	25.8	7.10	10.30	50.50	909.88	2/16/2021
Pennsylvania	Columbia Gas of Pennsylvania	NI	D-R-2020-3018835	Natural Gas	Distribution	4/24/2020	100.4	7.98	10.95	54.19	2,401.43	2/19/2021
District of Columbia	Washington Gas Light Co.	ALA	FC-1162	Natural Gas	Distribution	1/13/2020	39.0	7.56	10.40	52.10	542.57	2/24/2021
California	Southwest Gas Corp.	SWX	A-19-08-015 (SoCal)	Natural Gas	Distribution	8/30/2019	6.8	7.44	10.50	53.00	NA	3/25/2021
California	Southwest Gas Corp.	SWX	A-19-08-015 (NoCal)	Natural Gas	Distribution	8/30/2019	1.5	7.76	10.50	53.00	NA	3/25/2021
California	Southwest Gas Corp.	SWX	A-19-08-015 (LkTah)	Natural Gas	Distribution	8/30/2019	4.5	7.76	10.50	53.00	NA	3/25/2021
Maryland	Washington Gas Light Co.	ALA	C-9651	Natural Gas	Distribution	8/28/2020	28.4	7.73	10.45	54.55	1,225.35	4/9/2021
North Dakota	MDU Resources Group	MDU	C-PU-20-379	Natural Gas	Distribution	8/26/2020	7.7	7.10	9.80	50.31	181.68	5/5/2021
Washington	Cascade Natural Gas Corp.	MDU	D-UG-200568	Natural Gas	Distribution	6/19/2020	7.4	7.22	9.80	50.40	451.94	5/18/2021
New York	Corning Natural Gas Corp.	CNIG	C-20-G-0101	Natural Gas	Distribution	2/27/2020	6.0	7.28	10.20	50.77	71.81	5/19/2021
Pennsylvania	PECO Energy Co.	EXC	D-R-2020-3018929	Natural Gas	Distribution	9/30/2020	66.0	7.63	10.95	53.38	2,463.56	6/17/2021
Kentucky	Louisville Gas & Electric Co.	PPL	C-2020-00350 (gas)	Natural Gas	Distribution	11/25/2020	32.9	7.17	10.00	53.13	1,081.74	6/30/2021

Increase Authorized										
Decision Type	**Rate Increase ($M)**	**Phase-In?**	**Interim Authorized?**	**Return on Original Cost Rate (%)**	**Return on Equity (%)**	**Common Equity to Total Capital (%)**	**Rate Case Test Year End Date**	**Rate Base ($M)**	**Rate Base Valuation Method**	**Rate Case Duration (months)**
Settled	0.8	No	No	7.08	9.35	51.25	12/2018	14.87	Year-end	7
Settled	83.9	Yes	No	6.61	8.80	48.00	12/2020	7,170.73	Average	11
Fully Litigated	7.3	No	Yes	7.28	9.44	59.64	12/2017	125.41	Average	15
Settled	6.5	No	No	7.24	9.40	49.10	12/2018	NA	NA	10
Fully Litigated	3.1	No	No	7.03	9.10	56.32	03/2019	242.31	Year-end	8
Fully Litigated	2.7	Yes	No	7.18	9.50	55.00	12/2020	1,793.54	Average	7
Settled	4.6	Yes	No	7.99	9.70	52.45	12/2018	88.13	Year-end	2
Settled	8.0	No	No	7.21	9.40	48.50	12/2018	NA	NA	11
Fully Litigated	3.6	No	No	7.34	9.48	50.00	12/2018	227.28	Year-end	9
Settled	(0.3)	No	No	7.71	9.80	60.12	NA	NA	NA	6
Fully Litigated	(2.3)	No	No	6.76	9.20	50.15	06/2018	231.33	Average	15
Settled	4.0	No	No	7.38	9.65	56.95	06/2019	280.51	Year-end	7
Fully Litigated	42.9	No	No	7.39	9.40	48.50	12/2018	2,089.02	Year-end	12
Settled	10.3	No	No	7.46	9.50	59.00	06/2019	NA	NA	7
Settled	110.0	No	No	NA	9.90	NA	09/2021	NA	Average	8
Settled	1.5	No	No	7.11	9.35	55.00	12/2019	60.55	Year-end	9
Settled	144.0	No	No	NA	9.90	NA	09/2021	NA	Average	8
Settled	39.5	No	No	6.90	9.60	54.00	06/2020	2,133.63	Year-end	6
Fully Litigated	22.7	No	No	6.52	9.25	49.26	11/2019	1,325.24	Year-end	7
Fully Litigated	0.6	No	No	6.75	9.25	49.26	11/2019	154.97	Year-end	7
Settled	42.8	Yes	No	7.50	9.70	53.25	NA	NA	NA	3
Settled	94.2	No	No	6.84	9.20	55.62	09/2019	2,016.90	Year-end	8
Settled	45.8	No	No	6.97	9.40	50.00	10/2021	1,450.68	Average	9
Fully Litigated	22.8	Yes	No	7.29	9.90	54.77	12/2018	780.12	Year-end	11
Settled	3.3	No	No	7.16	9.60	52.63	05/2020	NA	Average	5
Settled	(0.5)	Yes	No	6.10	8.80	48.00	03/2021	662.11	Average	18
Settled	(1.1)	Yes	No	6.62	8.80	48.00	03/2021	509.47	Average	18
Settled	58.0	No	No	5.93	9.90	54.70	12/2021	1,536.82	Average	5
Settled	6.7	No	No	7.07	9.80	55.00	12/2021	282.36	Average	2

Fully Litigated	36.8	No	No	7.02	9.10	51.10	01/2019	1,930.61	Year-end	19
Settled	4.4	No	No	7.24	9.40	50.00	12/2021	305.03	Year-end	8
Settled	4.5	No	No	6.65	9.38	52.00	12/2021	741.44	Average	11
Fully Litigated	73.9	Yes	No	6.83	9.65	52.00	12/2023	2,443.18	Average	7
Fully Litigated	0.0	No	No	7.14	10.00	52.53	12/2021	480.95	Average	7
Settled	3.2	No	No	7.07	9.40	50.00	12/2020	130.10	Average	9
Settled	6.7	No	Yes	6.80	9.60	50.37	03/2020	NA	Average	10
Fully Litigated	76.1	No	No	7.14	9.67	52.00	12/2021	2,096.11	Average	10
Settled	10.7	No	Yes	6.71	9.50	50.00	12/2019	502.65	Year-end	7
Settled	16.3	No	Yes	6.85	9.80	50.50	12/2021	897.27	Average	7
Fully Litigated	63.5	No	No	7.41	9.86	54.19	12/2021	2,329.12	Year-end	10
Settled	19.5	No	No	7.05	9.25	52.10	12/2019	NA	NA	13
Settled	3.0	No	No	7.11	10.00	52.00	12/2021	285.69	Average	19
Settled	0.0	No	No	7.44	10.00	52.00	12/2021	92.98	Average	19
Settled	3.4	No	No	7.44	10.00	52.00	12/2021	56.82	Average	19
Fully Litigated	13.1	No	No	7.09	9.70	52.03	03/2020	1,212.27	Average	7
Settled	6.9	No	Yes	6.85	9.30	50.31	12/2021	NA	NA	8
Fully Litigated	(0.4)	No	No	6.95	9.40	49.10	12/2019	409.28	Year-end	11
Fully Litigated	(0.8)	No	No	6.28	8.80	48.00	01/2022	69.12	Average	14
Fully Litigated	29.1	No	No	7.26	10.24	53.38	06/2022	2,425.86	Year-end	8
Settled	20.4	No	No	NA	9.43	NA	06/2022	NA	NA	7

DELTA NATURAL GAS COMPANY, INC.
CASE NO. 2021-00185

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DATED JULY 14, 2021

10. Refer to Attachment PRM-10.
    a. To Mr. Moul's knowledge, has his financial risk adjustment ever been accepted in other rate proceedings? If so, provide the docket number, the jurisdiction, and a copy of all Orders accepting Mr. Moul's financial risk adjustment.
    b. Provide the basis for the Hamada calculations, including copies of articles or text support that show the formula used by Mr. Moul and its derivation.
    c. Provide the basis for the M&M calculations, including copies of articles or text support that show the formula used by Mr. Moul and its derivation.

Response:

a. In Mr. D'Ascendis' experience, most Commission Orders are silent on results of individual models and certainly on aspects of individual models, including the adoption of financial risk adjustment in their authorized returns on common equity. Since Mr. D'Ascendis has not performed an exhaustive review of all past state regulatory commission decisions, he is unaware of a regulatory body that has directly accepted a financial risk adjustment.

b. Please see AG 1-10b Attachment 1.

c. Please see AG 1-10c Attachments 1 and 2.

Sponsoring Witness: Dylan D'Ascendis

The Effect of the Firm's Capital Structure on the Systematic Risk of Common Stocks
Author(s): Robert S. Hamada
Reviewed work(s):
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THE EFFECT OF THE FIRM'S CAPITAL STRUCTURE ON THE SYSTEMATIC RISK OF COMMON STOCKS

ROBERT S. HAMADA*

I. INTRODUCTION

ONLY RECENTLY has there been an interest in relating the issues historically associated with corporation finance to those historically associated with investment and portfolio analyses. In fact, rigorous theoretical attempts in this direction were made only since the capital asset pricing model of Sharpe [13], Lintner [6], and Mossin [11], itself an extension of the Markowitz [7] portfolio theory. This study is one of the first empirical works consciously attempting to show and test the relationships between the two fields. In addition, differences in the observed systematic or nondiversifiable risk of common stocks, β, have never really been analyzed before by investigating some of the underlying differences in the firms.

In the capital asset pricing model, it was demonstrated that the efficient set of portfolios to any individual investor will always be some combination of lending at the risk-free rate and the "market portfolio," or borrowing at the risk-free rate and the "market portfolio." At the same time, the Modigliani and Miller (MM) propositions [9, 10] on the effect of corporate leverage are well known to the students of corporation finance. In order for their propositions to hold, personal leverage is required to be a perfect substitute for corporate leverage. If this is true, then corporate borrowing could substitute for personal borrowing in the capital asset pricing model as well.

Both in the pricing model and the MM theory, borrowing, from whatever source, while maintaining a fixed amount of equity, increases the risk to the investor. Therefore, in the mean-standard deviation version of the capital asset pricing model, the covariance of the asset's rate of return with the market portfolio's rate of return (which measures the nondiversifiable risk of the asset—the proxy β will be used to measure this) should be greater for the stock of a firm with a higher debt-equity ratio than for the stock of another firm in the same risk-class with a lower debt-equity ratio.[1]

This study, then, has a number of purposes. First, we shall attempt to link empirically corporation finance issues with portfolio and security analyses through the effect of a firm's leverage on the systematic risk of its common

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1. This very quick summary of the theoretical relationship between what is known as corporation finance and the modern investment and portfolio analyses centered around the capital asset pricing model is more thoroughly presented in [5], along with the necessary assumptions required for this relationship.

stock. Then, we shall attempt to test the MM theory, or at least provide another piece of evidence on this long-standing controversial issue. This test will not rely on an explicit valuation model, such as the MM study of the electric utility industry [8] and the Brown study of the railroad industry [2]. A procedure using systematic risk measures (β s) has been worked out in this paper for this purpose.

If the MM theory is validated by this procedure, then the final purpose of this study is to demonstrate a method for estimating the cost of capital of individual firms to be used by them for scale-changing or nondiversifying investment projects. The primary component of any firm's cost of capital is the capitalization rate for the firm if the firm had no debt and preferred stock in its capital structure. Since most firms do have fixed commitment obligations, this capitalization rate (we shall call it $E(R_A)$; MM denote it ρ^τ) is unobservable. But if the MM theory and the capital asset pricing model are correct, then it is possible to estimate $E(R_A)$ from the systematic risk approach for individual firms, even if these firms are members of a one-firm risk-class.[2]

With this statement of the purposes for this study, we shall, in Section II, discuss the alternative general procedures that are possible for estimating the effect of leverage on systematic risk and select the most feasible ones. The results are presented in Section III. And finally, tests of the MM versus the traditional theories of corporation finance are presented in Section IV.

II. Some Possible Procedures and the Selected Estimating Relationships

There are at least four general procedures that can be used to estimate the effect of the firm's capital structure on the systematic risk of common stocks. The first is the MM valuation model approach. By estimating ρ^τ with an explicit valuation model as they have for the electric utility industry, it is possible to relate this ρ^τ with the use of the capital asset pricing model to a nonleveraged systematic risk measure, ${}_A\beta$. Then the difference between the observed common stock's systematic risk (which we shall denote ${}_B\beta$) and ${}_A\beta$ would be due solely to leverage. But the difficulties of this approach for all firms are many.

The MM valuation model approach requires the specification, in advance, of risk-classes. All firms in a risk-class are then assumed to have the same ρ^τ—the capitalization rate for an all-common equity firm. Unfortunately, there must be enough firms in a risk-class so that a cross-section analysis will yield statistically significant coefficients. There may not be many more risk-classes (with enough observations) now that the electric utility and railroad industries have been studied. In addition, the MM approach requires estimating expected asset earnings and estimating the capitalized growth potential implicit in stock prices. If it is possible to consider growth and expected earnings without having

2. It is, in fact, this last purpose of making applicable and practical some of the implications of the capital asset pricing model for corporation finance issues that provided the initial motivation for this paper. In this context, if one is familiar with the fair rate of return literature for regulated utilities, for example, an industry where debt is so prevalent, adjusting correctly for leverage is not frequently done and can be very critical.

to specify their exact magnitude at a specific point in time, considerable difficulty and possible measurement errors will be avoided.

The second approach is to run a regression between the observed systematic risk of a stock and a number of accounting and leverage variables in an attempt to explain this observed systematic risk. Unfortunately, without a theory, we do not know which variables to include and which variables to exclude and whether the relationship is linear, multiplicative, exponential, curvilinear, etc. Therefore, this method will also not be used.

A third approach is to measure the systematic risk before and after a new debt issue. The difference can then be attributed to the debt issue directly. An attractive feature of this procedure is that a good estimate of the market value of the incremental debt issue can be obtained. A number of disadvantages, unfortunately, are associated with this direct approach. The difference in the systematic risk may be due not only to the additional debt, but also to the reason the debt was issued. It may be used to finance a new investment project, in which case the project's characteristics will also be reflected in the new systematic risk measure. In addition, the new debt issue may have been anticipated by the market if the firm had some long-run target leverage ratio which this issue will help maintain; conversely, the market may not fully consider the new debt issue if it believes the increase in leverage is only temporary. For these reasons, this seemingly attractive procedure will not be employed.

The last approach, which will be used in this study, is to assume the validity of the MM theory from the outset. Then the observed rate of return of a stock can be adjusted to what *it would have been* over the same time period had the firm no debt and preferred stock in its capital structure. The difference between the observed systematic risk, ${}_{B}\beta$, and the systematic risk for this adjusted rate of return time series, ${}_{A}\beta$, can be attributed to leverage, if the MM theory is correct. The final step, then, is to test the MM theory.

To discuss this more specifically, consider the following relationship for the dollar return to the common shareholder from period t − 1 to t:

$$(X - I)_t(1 - \tau)_t - p_t + \Delta G_t = d_t + cg_t \tag{1}$$

where X_t represents earnings before taxes, interest, and preferred dividends and is assumed to be unaffected by fixed commitment obligations; I_t represents interest and other fixed charges paid during the period; τ is the corporation income tax rate; p_t is the preferred dividends paid; ΔG_t represents the change in capitalized growth over the period; and d_t and cg_t are common shareholder dividends and capital gains during the period, respectively.

Equation (1) relates the corporation finance types of variables with the market holding period return important to the investors. The first term on the left-hand-side of (1) is profits after taxes and after interest which is the earnings the common and preferred shareholders receive on their investment for the period. Subtracting out p_t leaves us with the earnings the common shareholder would receive from currently-held assets.

To this must be added any change in capitalized growth since we are trying to explain the common shareholder's market holding period dollar return. ΔG_t

must be added for growth firms to the current period's profits from existing assets since capitalized growth opportunities of the firm—future earnings from new assets over and above the firm's cost of capital which are already reflected in the stock price at $(t-1)$—should change over the period and would accrue to the common shareholder. Assuming shareholders at the start of the period estimated these growth opportunities on average correctly, the expected value of ΔG_t would not be zero, but should be positive. For example, consider growth opportunities five years from now which yield more than the going rate of return and are reflected in today's stock price. These growth opportunities will become one year closer to fruition at time t than at time $t-1$ so that their present value would become larger. ΔG_t then represents this increase in the present value of these future opportunities simply because it is now four years away rather than five.[3]

Since the systematic risk of a common stock is:

$$_B\beta = \frac{\text{cov}\,(R_{B_t}, R_{M_t})}{\sigma^2(R_{M_t})} \tag{2}$$

where R_{B_t} is the common shareholder's rate of return and R_{M_t} is the rate of return on the market portfolio, then substitution of (1) into (2) yields:

$$_B\beta = \frac{\text{cov}\left[\dfrac{(X-I)(1-\tau)_t - p_t + \Delta G_t}{S_{B_{t-1}}}, R_{M_t}\right]}{\sigma^2(R_{M_t})} \tag{2a}$$

where $S_{B_{t-1}}$ denotes the market value of the common stock at the beginning of the period.

The systematic risk for the same firm over the same period *if* there were no debt and preferred stock in its capital structure is:

$$_A\beta = \frac{\text{cov}(R_{A_t}, R_{M_t})}{\sigma^2(R_{M_t})}$$

$$= \frac{\text{cov}\left[\dfrac{X(1-\tau)_t + \Delta G_t}{S_{A_{t-1}}}, R_{M_t}\right]}{\sigma^2(R_{M_t})} \tag{3}$$

where R_{A_t} and $S_{A_{t-1}}$ represent the rate of return and the market value, respectively, to the common shareholder if the firm had no debt and preferred stock. From (3), we can obtain:

$$_A\beta S_{A_{t-1}} = \frac{\text{cov}\,[X(1-\tau)_t + \Delta G_t, R_{M_t}]}{\sigma^2(R_{M_t})} \tag{3a}$$

3. Continual awareness of the difficulties of estimating capitalized growth, or changes in growth, especially in conjunction with leverage considerations, for purposes such as valuation or cost of capital is a characteristic common to students of corporation finance. This is the reason for the emphasis on growth in this paper and for presenting a method to neutralize for differences in growth when comparing rates of return.

Next, by expanding and rearranging (2a), we have:

$$ {}_B\beta S_{B_{t-1}} = \frac{\text{cov}\,[X(1-\tau)_t + \Delta G_t, R_{M_t}]}{\sigma^2(R_{M_t})} - \frac{\text{cov}\,[I(1-\tau)_t, R_{M_t}]}{\sigma^2(R_{M_t})} - \frac{\text{cov}\,(p_t, R_{M_t})}{\sigma^2(R_{M_t})} \quad (2b) $$

If we assume as an empirical approximation that interest and preferred dividends have negligible covariance with the market, at least relative to the (pure equity) common stock's covariance, then substitution of the LHS of (3a) into the RHS of (2b) yields:[4]

$$ {}_B\beta S_{B_{t-1}} = {}_A\beta S_{A_{t-1}} \quad (4) $$

or

$$ {}_A\beta = \left(\frac{S_B}{S_A}\right)_{t-1} {}_B\beta \quad (4a) $$

Because $S_{A_{t-1}}$, the market value of common stock *if* the firm had no debt and preferred stock, is not observable since most firms do have debt and/or preferred stock, a theory is required in order to measure what this quantity *would have been* at $t-1$. The MM theory [10] will be employed for this purpose, that is:

$$ S_{A_{t-1}} = (V - \tau D)_{t-1}. \quad (5) $$

Equation (5) indicates that if the Federal government tax subsidy for debt financing, τD, where D is the market value of debt, is subtracted from the observed market value of the firm, V_{t-1} (where V_{t-1} is the sum of S_B, D and the observed market value of preferred), then the market value of an unleveraged firm is obtained. Underlying (5) is the assumption that the firm is near its target leverage ratio so that no more or no less debt subsidy is capitalized already into the observed stock price. The conditions under which this MM relationship hold are discussed carefully in [4].

It is at this point that problems in obtaining satisfactory estimates of ${}_A\beta$ develop, since (4) theoretically holds only for the next period. As a practical matter, the accepted, and seemingly acceptable, method of obtaining estimates of a stock's systematic risk, ${}_B\beta$, is to run a least squares regression between a stock's and market portfolio's *historical* rates of return. Using past data for ${}_B\beta$, it is not clear which *period's* ratio of market values to apply in (4a) to estimate the firm's systematic risk, ${}_A\beta$. There would be no problem if the market value ratios of debt to equity and preferred stock to equity remained relatively stable over the past for each firm, but a cursory look at these data reveals that this is not true for the large majority of firms in our sample. Should we use the market value ratio required in (4a) that was observed at the start of our regression period, at the end of our regression period, or some kind of average over the period? In addition, since these different observed ratios will give us different estimates for ${}_A\beta$, it is not clear, without some criterion, how we should select from among the various estimates.

4. This general method of arriving at (4) was suggested by the comments of William Sharpe, one of the discussants of this paper at the annual meeting. A much more cumbersome and less general derivation of (4) was in the earlier version.

It is for this purpose—to obtain a standard—that a more cumbersome and more data demanding approach to obtain estimates of ${}_A\beta$ is suggested. Given the large fluctuations in market leverage ratios, intuitively it would appear that the firm's risk is more stable than the common stock's risk. In that event, a leverage-free rate of return time series for each firm should be derived and the market model applied to this time series directly. In this manner, the beta coefficient would give us a *direct* estimate of ${}_A\beta$ which can then be used as a criterion to determine if any of the market value ratios discussed above can be applied to (4a) successfully.

For this purpose, the "would-have-been" rate of return for the common stock if the firm had no debt and preferred is:

$$R_{A_t} = \frac{X_t(1-\tau)_t + \Delta G_t}{S_{A_{t-1}}}. \tag{6}$$

The numerator of (6) can be rearranged to be:

$$X_t(1-\tau)_t + \Delta G_t \equiv [(X-I)_t(1-\tau)_t - p_t + \Delta G_t] + p_t + I_t(1-\tau)_t.$$

Substituting (1):

$$X_t(1-\tau)_t + \Delta G_t = [d_t + cg_t] + p_t + I_t(1-\tau)_t.$$

Therefore, (6) can be written as:

$$R_{A_t} = \frac{d_t + cg_t + p_t + I_t(1-\tau)_t}{S_{A_{t-1}}}. \tag{7}$$

Since $S_{A_{t-1}}$ is unobservable for the firms with leverage, the MM theory, equation (5), will be employed; then:

$$R_{A_t} = \frac{d_t + cg_t + p_t + I_t(1-\tau)_t}{(V-\tau D)_{t-1}}. \tag{8}$$

The observed rate of return on the common stock is, of course:

$$R_{B_t} = \frac{(X-I)_t(1-\tau)_t - p_t + \Delta G_t}{S_{B_{t-1}}} = \frac{d_t + cg_t}{S_{B_{t-1}}}. \tag{9}$$

Equation (8) is the rate of return to the common shareholder of the same firm and over the same period of time as (9). However, in (8) there are the underlying assumptions that the firm never had any debt and preferred stock and that the MM theory is correct; (9) incorporates the exact amount of debt and preferred stock that the firm actually did have over this time period and no leverage assumption is being made. Both (8) and (9) are now in forms where they can be measured with available data. One can note that it is unnecessary to estimate the change in growth, or earnings from current assets, since these should be captured in the market holding period return, $d_t + cg_t$.

Using CRSP data for (9) and both CRSP and Compustat data for the components of (8), a time series of yearly R_{A_t} and R_{B_t} for $t = 1948\text{-}1967$ were derived for 304 different firms. These 304 firms represent an exhaustive sample of the firms with complete data on both tapes for all the years.

A number of "market model" [1, 12] variants were then applied to these data. For each of the 304 firms, the following regressions were run:

$$R_{Ait} = {}_{A}\alpha_i + {}_{A}\beta_i R_{Mt} + {}_{A}\epsilon_{it} \quad (10a)$$

$$R_{Bit} = {}_{B}\alpha_i + {}_{B}\beta_i R_{Mt} + {}_{B}\epsilon_{it} \quad (10b)$$

$$\ln(1 + R_{Ait}) = {}_{AC}\alpha_i + {}_{AC}\beta_i \ln(1 + R_{M_t}) + {}_{AC}\epsilon_{it} \quad (10c)$$

$$\ln(1 + R_{Bit}) = {}_{BC}\alpha_i + {}_{BC}\beta_i \ln(1 + R_{M_t}) + {}_{BC}\epsilon_{it} \quad (10d)$$

$$i = 1, 2, \ldots, 304$$
$$t = 1948\text{-}1967$$

where R_{Mt} is the observed NYSE arithmetic stock market rate of return with dividends reinvested, α_i and β_i are constants for each firm-regression, and the usual conditions are assumed for the properties of the disturbance terms, ϵ_{it}. Equations (10c) and (10d) are the continuously-compounded rate of return versions of (10a) and (10b), respectively.[5]

III. The Results

An abbreviated table of the regression results for each of the four variants, equations (10a)-(10d), summarized across the 304 firms is shown in Table 1.

The first column designated "mean" is the average of the statistic (indicated by the rows) over all 304 firms. Therefore, the mean ${}_{A}\hat{\alpha}$ of 0.0221 is the intercept term of equation (10a) averaged over 304 different firm-regressions. The second and third columns give the deviation measures indicated, of the 304 point estimates of, say, ${}_{A}\hat{\alpha}$. The mean standard error of estimate in the last column is the average over 304 firms of the individual standard errors of estimate.

The major conclusion drawn from Table 1 is the following mean β comparisons:

$${}_{B}\hat{\beta} > {}_{A}\hat{\beta}, \text{ i.e., } 0.9190 > 0.7030$$

$${}_{BC}\hat{\beta} > {}_{AC}\hat{\beta}, \text{ i.e., } 0.9183 > 0.7263.$$

The directional results of these betas, assuming the validity of the MM theory, are not imperceptible and clearly are not negligible differences from the investor's point of view. This is obtained in spite of all the measurement and data problems associated with estimating a time series of the RHS of (8) for

5. Because the R_{M_t} used in equations (10) is defined as the observed stock market return, and since adjusting for capital structure is the major purpose of this exercise, it was decided that the same four regressions should be replicated on a leverage-adjusted stock market rate of return. The major reason for this additional adjustment is the belief that the rates of return over time and their relationship with the market are more stable when we can abstract from all changes in leverage and get at the underlying risk of all firms.

For the 221 firms (out of the total 304) whose fiscal years coincide with the calendar year, average values for the components of the RHS of (8) were obtained for each year so that R_{M_t} could be adjusted in the same way as for the individual firms—a yearly time series of stock market rates of return, if all the firms on the NYSE had no debt and no preferred in their capital structure, was derived. The results, when using this adjusted market portfolio rate of return time series, were not very different from the results of equations (10), and so will not be reported here separately.

TABLE 1
SUMMARY RESULTS OVER 304 FIRMS OF EQUATIONS (10a)-(10d)

	Mean	Mean Absolute Deviation*	Standard Deviation	Mean Standard Error of Estimate
$_A\hat{\alpha}$	0.0221	0.0431	0.0537	0.0558
$_A\hat{\beta}$	0.7030	0.2660	0.3485	0.2130
$_A\hat{R}^2$	0.3799	0.1577	0.1896	
$_A\hat{\rho}$	0.0314			
$_B\hat{\alpha}$	0.0187	0.0571	0.0714	0.0720
$_B\hat{\beta}$	0.9190	0.3550	0.4478	0.2746
$_B\hat{R}^2$	0.3864	0.1578	0.1905	
$_B\hat{\rho}$	0.0281			
$_{AC}\hat{\alpha}$	0.0058	0.0427	0.0535	0.0461
$_{AC}\hat{\beta}$	0.7263	0.2700	0.3442	0.2081
$_{AC}\hat{R}^2$	0.3933	0.1586	0.1909	
$_{AC}\hat{\rho}$	0.0268			
$_{BC}\hat{\alpha}$	−0.0052	0.0580	0.0729	0.0574
$_{BC}\hat{\beta}$	0.9183	0.3426	0.4216	0.2591
$_{BC}\hat{R}^2$	0.4012	0.1602	0.1922	
$_{BC}\hat{\rho}$	0.0262			

* Defined as: $\frac{\sum_{i=1}^{N} |x_i - \bar{x}|}{N}$, where N = 304. $\hat{\rho}$ = first order serial correlation coefficient.

each firm. One of the reasons for the "traditional" theory position on leverage is precisely this point—that small and reasonable amounts of leverage cannot be discerned by the market. In fact, if the MM theory is correct, leverage has explained as much as, roughly, 21 to 24 per cent of the value of the mean β.

We can also note that if the covariance between the asset and market rates of return, as well as the market variance, was constant over time, then the systematic risk from the market model is related to the expected rate of return by the capital asset pricing model. That is:

$$E(R_{A_t}) = R_{F_t} + {}_A\beta[E(R_{M_t}) - R_{F_t}] \quad (11a)$$

$$E(R_{B_t}) = R_{F_t} + {}_B\beta[E(R_{M_t}) - R_{F_t}] \quad (11b)$$

Equation (11a) indicates the relationship between the expected rate of return for the common stock shareholder of a debt-free and preferred-free firm, to the systematic risk, $_A\beta$, as obtained in regressions (10a) or (10c). The LHS of (11a) is the important $\rho\tau$ for the MM cost of capital. The MM theory [9, 10] also predicts that shareholder expected yield must be higher (for the same real firm) when the firm has debt than when it does not. Financial risk is greater, therefore, shareholders require more expected return. Thus, $E(R_{Bt})$ must be greater than $E(R_{At})$. In order for this MM prediction to be true, from (11a) and (11b) it can be observed that $_B\beta$ must be greater than $_A\beta$, which is what we obtained.

Using the results underlying Table 1, namely the firm and stock betas, as the criterion for selecting among the possible observed market value ratios that can be used, if any, for (4), the following cross-section regressions were run:

$$({}_B\beta)_i = a_1 + b_1 \left(\frac{S_A}{S_B} {}_A\beta \right)_i + u_{1i} \qquad i = 1, 2, \ldots, 102 \qquad (12a)$$

$$({}_{BC}\beta)_i = a_2 + b_2 \left(\frac{S_A}{S_B} {}_{AC}\beta \right)_i + u_{2i} \qquad i = 1, 2, \ldots, 102 \qquad (12b)$$

$$({}_A\beta)_i = a_3 + b_3 \left(\frac{S_B}{S_A} {}_B\beta \right)_i + u_{3i} \qquad i = 1, 2, \ldots, 102 \qquad (13a)$$

$$({}_{AC}\beta)_i = a_4 + b_4 \left(\frac{S_B}{S_A} {}_{BC}\beta \right)_i + u_{4i} \qquad i = 1, 2, \ldots, 102 \qquad (13b)$$

Because the preferred stock market values were not as reliable as debt, only the 102 firms (out of 304) that did not have preferred in any of the years were used. The test for the adequacy of this alternative approach, equation (4), to adjust the systematic risk of common stocks for the underlying firm's capital structure, is whether the intercept term, a, is equal to zero, and the slope coefficient, b, is equal to one in the above regressions (as well as, of course, a high R^2)—these requirements are implied by (4). The results of this test would also indicate whether future "market model" studies that only use common stock rates of return without adjusting, or even noting, for the firm's debt-equity ratio will be adequate. The total firm's systematic risk may be stable (as long as the firm stays in the same risk-class), whereas the common stock's systematic risk may not be stable merely because of unanticipated capital structure changes—the data underlying Table 3 indicate that there were very few firms which did not have major changes in their capital structure over the twenty years studied.

The results of these regressions, when using the average S_A and average S_B over the twenty years for each firm, are shown in the first column panel of Table 2. These regressions were then replicated twice, first using the December 31, 1947 values of S_{A_i} and S_{B_i} instead of the twenty-year average for each firm, and then substituting the December 31, 1966 values of S_{A_i} and S_{B_i} for the 1947 values. These results are in the second and third panels of Table 2.[6]

From the first panel of Table 2, it appears that this alternative approach via (4a) for adjusting the systematic risk for the firm's leverage is quite

6. The point should be made that we are not merely regressing a variable on itself in (12) and (13). (12a) and (12b) can be interpreted as correlating the ${}_B\beta_i$ obtained from (10b) and (10d)—the LHS variable in (12a) and (12b)—against the ${}_B\beta_i$ obtained from rearranging (4)—the RHS variable in (12a) and (12b)—to determine whether the use of (4) is as good a means of obtaining ${}_B\beta_i$ as the direct way via the equations (10). We would be regressing a variable on itself only if the ${}_A\beta_i$ were calculated using (4a), and then the ${}_A\beta_i$ thus obtained, inserted into (12a) and (12b).

Instead, we are obtaining ${}_A\beta_i$ using the MM model in *each* of the twenty years so that a leverage-adjusted 20 year time series of R_{A_i} is derived. Of course, if there were no data nor measurement problems, and if the debt-to-equity ratio were perfectly stable over this twenty year period for each firm, then we should obtain perfect correlation in (12a) and (12b), with $a = 0$ and $b = 1$, as (4) would be an identity.

TABLE 2
RESULTS FOR THE EQUATIONS (12a), (12b), (13a), AND (13b)*

	Using 20-Year Average for $\left(\frac{S_A}{S_B}\right)_i$			Using 1947 Value for $\left(\frac{S_A}{S_B}\right)_i$			Using 1966 Value for $\left(\frac{S_A}{S_B}\right)_i$		
	a	b	R^2	a	b	R^2	a	b	R^2
Eq. (12a)	−0.022 (0.021)	1.062 (0.021)	0.962	0.150 (0.048)	0.842 (0.045)	0.781	0.085 (0.041)	0.905 (0.038)	0.849
	constant suppressed	1.042 (0.009)	0.962	constant suppressed	0.966 (0.021)	0.781	constant suppressed	0.976 (0.017)	0.849
Eq. (12b)	−0.003 (0.013)	1.016 (0.013)	0.984	0.159 (0.047)	0.816 (0.044)	0.773	0.124 (0.037)	0.843 (0.034)	0.859
	constant suppressed	1.014 (0.005)	0.984	constant suppressed	0.952 (0.019)	0.773	constant suppressed	0.947 (0.015)	0.859

	Using 20-Year Average for $\left(\frac{S_B}{S_A}\right)_i$			Using 1947 Value for $\left(\frac{S_B}{S_A}\right)_i$			Using 1966 Value for $\left(\frac{S_B}{S_A}\right)_i$		
	a	b	R^2	a	b	R^2	a	b	R^2
Eq. (13a)	0.030 (0.016)	0.931 (0.017)	0.969	0.112 (0.028)	0.843 (0.030)	0.888	0.080 (0.027)	0.898 (0.030)	0.902
	constant suppressed	0.960 (0.007)	0.969	constant suppressed	0.948 (0.015)	0.888	constant suppressed	0.976 (0.014)	0.902
Eq. (13b)	0.007 (0.010)	0.979 (0.011)	0.988	0.119 (0.026)	0.852 (0.028)	0.902	0.063 (0.026)	0.942 (0.029)	0.911
	constant suppressed	1.004 (0.012)	0.911	constant suppressed	0.967 (0.013)	0.902	constant suppressed	1.005 (0.012)	0.911

* Standard error in parentheses.

satisfactory (at least with respect to our sample of firms and years) only if long-run averages of S_A and S_B are used. The second and third panels indicate that the equations (8) and (10) procedure is markedly superior when only one year's market value ratio is used as the adjustment factor. The annual debt-to-equity ratio is much too unstable for this latter procedure.

Thus, when forecasting systematic risk is the primary objective—for example, for portfolio decisions or for estimating the firm's cost of capital to apply to prospective projects—a long-run forecasted leverage adjustment is required. Assuming the firm's risk is more stable than the common stock's risk,[7] and if there is some reason to believe that a better forecast of the firm's future leverage can be obtained than using simply a past year's (or an average of past years') leverage, it should be possible to improve the usual extrapolation forecast of a stock's systematic risk by forecasting the total firm's systematic risk first, and then using the independent leverage estimate as an adjustment.

IV. Tests of the MM vs. Traditional Theories of Corporation Finance

To determine if the difference, $_B\beta - {}_A\beta$, found in this study is indeed the correct effect of leverage, some confirmation of the MM theory (since it was assumed to be correct up to this point) from the systematic risk approach is needed. Since a direct test by this approach seems impossible, an indirect, inferential test is suggested.

The MM theory [9, 10] predicts that for firms in the same risk-class, the capitalization rate if all the firms were financed with only common equity, $E(R_A)$, would be the same—regardless of the actual amount of debt and preferred each individual firm had. This would imply, from (11a), that if $E(R_A)$ must be the same for all firms in a risk-class, so must $_A\beta$. And if these firms had different ratios of fixed commitment obligations to common equity, this difference in financial risk would cause their observed $_B\beta$s to be different.

The major competing theory of corporation finance is what is now known as the "traditional theory," which has contrary implications. This theory predicts that the capitalization rate for common equity, $E(R_B)$, (sometimes called the required or expected stock yield, or expected earnings-price ratio) is constant, as debt is increased, up to some critical leverage point (this point being a function of gambler's ruin and bankruptcy costs).[8] The clear implication of this constant, horizontal, equity yield (or their initial downward sloping cost of capital curve) is that changes in market or covariability risk are assumed not to be discernible to the shareholders as debt is increased. Then the traditional theory is saying that the $_B\beta$s, a measure of this covariability risk, would be the same for all firms in a given risk-class irregardless of differences in leverage, as long as the critical leverage point is not reached.

Since there will always be unavoidable errors in estimating the β's of indi-

7. A faint, but possible, empirical indication of this point may be obtained from Table 1. The ratio of the mean point estimate to the mean standard error of estimate is less for the firm β than for the stock β in both the discrete and continuously compounded cases.

8. This interpretation of the traditional theory can be found in [9, especially their figure 2, page 275, and their equation (13) and footnote 24 where reference is made to Durand and Graham and Dodd].

TABLE 3
INDUSTRY MARKET VALUE RATIOS OF PREFERRED STOCK (P) AND DEBT (D) TO COMMON STOCK (S)

Industry Number	Industry	Number of Firms		P/S			D/S			(P + D)/S		
20	Food and Kindred Products	30	Mean*		0.22			0.81			1.04	
			ROM**	0.00		1.18	0.00		3.55	0.00		4.13
			ROCR***	0.00		2.52	0.00		8.10	0.00		10.01
28	Chemicals and Allied Products	30	Mean		0.07			0.25			0.33	
			ROM	0.00		0.51	0.00		0.90	0.00		1.20
			ROCR	0.00		1.54	0.00		2.07	0.00		2.92
29	Petroleum and Coal Products	18	Mean		0.06			0.22			0.27	
			ROM	0.00		0.26	0.00		0.55	0.03		0.57
			ROCR	0.00		0.83	0.00		1.54	0.00		2.30
33	Primary Metals	21	Mean		0.14			0.54			0.68	
			ROM	0.00		1.31	0.00		1.95	0.00		3.04
			ROCR	0.00		4.69	0.00		6.20	0.00		7.49
35	Machinery, except Electrical	28	Mean		0.07			0.33			0.40	
			ROM	0.00		0.49	0.00		1.92	0.00		2.32
			ROCR	0.00		1.28	0.00		6.92	0.00		7.62

TABLE 3 (*Continued*)

Industry Number	Industry	Number of Firms		P/S		D/S		$\frac{P+D}{S}$	
36	Electrical Machinery & Equipment	13	Mean	0.06		0.35		0.41	
			ROM	0.00	0.29	0.00	1.31	0.01	1.33
			ROCR	0.00	1.13	0.00	2.53	0.00	2.53
37	Transportation Equipment	24	Mean	0.08		0.38		0.47	
			ROM	0.00	0.54	0.00	0.93	0.00	1.32
			ROCR	0.00	2.33	0.00	3.76	0.00	6.09
49	Utilities	27	Mean	0.25		1.03		1.28	
			ROM	0.00	0.53	0.49	2.64	0.52	3.12
			ROCR	0.00	3.12	0.12	16.40	0.12	19.52
53	Dep't Stores, Order Houses & Vending Mach. Operators	17	Mean	0.13		0.49		0.62	
			ROM	0.00	0.38	0.01	1.52	0.01	1.87
			ROCR	0.00	1.09	0.00	3.19	0.00	3.66

* "Mean" refers to the average ratio over 20 years and over all firms in the industry.
** "Range of Means" (ROM) refers to the lowest firm's mean (over 20 years) ratio and the highest firm's mean (over 20 years) ratio in the industry.
*** "Range of Company Ranges" (ROCR) refers to the lowest and highest ratio in the industry, regardless of the year.

vidual firms and in specifying a risk-class, we would not expect to find a set of firms with identical systematic risk. But by specifying reasonable a priori risk-classes, if the individual firms had closer or less scattered $_A\beta$s than $_B\beta$s, then this would support the MM theory and contradict the traditional theory. If, instead, the $_B\beta$s were not discernibly more diverse than the $_A\beta$s, and the leverage ratio differed considerably among firms, then this would indicate support for the traditional theory.[9]

In order to test this implication, risk-classes must be first specified. The SEC two-digit industry classification was used for this purpose. Requiring enough firms for statistical reasons in any given industry, nine risk-classes were specified that had at least 13 firms; these nine classes are listed in Table 3 with their various leverage ratios.[10] It is clear from this table that our first requirement is met—that there is a considerable range of leverage ratios among firms in a risk-class and also over the twenty-year period.

Three tests will be performed to distinguish between the MM and traditional theories. The first is simply to calculate the standard deviation of the unbiased β estimates in a risk-class. The second is a chi-square test of the distribution of β's in an industry compared to the distribution of the β's in the total sample. Finally, an analysis of variance test on the estimated variance of the β's between industries, as opposed to within industries, is performed. In all tests, only the point estimate of β (which should be unbiased) for each stock and firm is used.[11]

The first test is reported in Table 4. If we compare the standard deviation of $_{AC}\beta$ with the standard deviation of $_{BC}\beta$ by industries (or risk-classes), we can note that $\sigma(_{AC}\beta)$ is less than $\sigma(_{BC}\beta)$ for eight out of the nine classes. The probability of obtaining this is only 0.0195, given a 50% probability that $\sigma(_{AC}\beta)$ can be larger or smaller than $\sigma(_{BC}\beta)$. These results indicate that the systematic risk of the firms in a given risk-class, if they were all financed only with common equity, is much less diverse than their observed stock's systematic risk. This supports the MM theory, at least in contrast to the traditional theory.[12]

9. The traditional theory also implies that $E(R_A)$ is equal to $E(R_B)$ for all firms. Unfortunately, we do not have a functional relationship between these traditional theory capitalization rates and the measured βs of this study. Clearly, since the $_A\beta$s were obtained assuming the validity of the MM theory, they would not be applicable for the traditional theory. In fact, no relationship between the $_A\beta$ and $_B\beta$ for a given firm, or for firms in a given risk-class, can be specified as was done for the capitalization rates.

10. The tenth largest industry had only eight firms. For our purpose of testing the uniformity of firm βs relative to stock βs within a risk-class, the use of the two-digit industry classification as a proxy does not seem as critical as, for instance, its use for the purpose of performing an MM valuation model study [8] wherein the ρ^T must be pre-specified to be exactly the same for all firms in the industry.

11. Since these βs are estimated in the market model regressions with error, precise testing should incorporate the errors in the β estimation. Unfortunately, to do this is extremely difficult and more importantly, requires the normality assumption for the market model disturbance term. Since there is considerable evidence that is contrary to this required assumption [see 3], our tests will ignore the β measurement error entirely. But ignoring this is partially corrected in our first and third tests since means and variances of these point estimate βs must be calculated, and this procedure will "average out" the individual measurement errors by the factor 1/N.

12. Of course, there could always be another theory, as yet not formulated, which could be even

TABLE 4
MEAN AND STANDARD DEVIATION OF INDUSTRY β'S

Industry Number	Industry	Number of Firms		$_A\beta$	$_B\beta$	$_{AC}\beta$	$_{BC}\beta$
20	Food & Kindred Products	30	Mean β	0.515	0.815	0.528	0.806
			σ(β)	0.232	0.448	0.227	0.424
28	Chemicals & Allied Products	30	Mean β	0.747	0.928	0.785	0.946
			σ(β)	0.237	0.391	0.216	0.329
29	Petroleum & Coal Products	18	Mean β	0.633	0.747	0.656	0.756
			σ(β)	0.144	0.188	0.148	0.176
33	Primary Metals	21	Mean β	1.036	1.399	1.106	1.436
			σ(β)	0.223	0.272	0.197	0.268
35	Machinery, except Electrical	28	Mean β	0.878	1.037	0.917	1.068
			σ(β)	0.262	0.240	0.271	0.259
36	Electrical Machinery and Equipment	13	Mean β	0.940	1.234	0.951	1.164
			σ(β)	0.320	0.505	0.283	0.363
37	Transportation Equipment	24	Mean β	0.860	1.062	0.875	1.048
			σ(β)	0.225	0.313	0.225	0.289
49	Utilities	27	Mean β	0.160	0.255	0.166	0.254
			σ(β)	0.086	0.133	0.098	0.147
53	Department Stores, etc.	17	Mean β	0.652	0.901	0.692	0.923
			σ(β)	0.187	0.282	0.198	0.279

Our second test, the chi-square test, requires us to rank our 300 $_A\beta$s into ten equal categories, each with 30 $_A\beta$s (four miscellaneous firms were taken out randomly). By noting the value of the highest and lowest $_A\beta$ for each of the ten categories, a distribution of the number of $_A\beta$s in each category, by risk-class, can be obtained. This was then repeated for the other three betas. To test whether the distribution for each of the four β's and for each of the risk-classes follows the expected uniform distribution, a chi-square test was performed.[13]

Even with just casual inspection of these distributions of the betas by risk-class, it is clear that two industries, primary metals and utilities, are so highly skewed that they greatly exaggerate our results.[14] Eliminating these

more strongly supported than the MM theory. If we compare $\sigma(_A\beta)$ to $\sigma(_B\beta)$ by risk-classes in Table 4, precisely the same results are obtained as those reported above for the continuously-compounded betas.

13. By risk-classes, seven of the nine chi-square values of $_A\beta$ are larger than those of $_B\beta$, as are eight out of nine for the continuously-compounded betas. This would occur by chance with probabilities of 0.0898 and 0.0195, respectively, if there were a 50% chance that either the firm or stock chi-square value could be larger. Nevertheless, if we inspect the individual chi-square values by risk-class, we note that most of them are large so that the probabilities of obtaining these values are highly unlikely. For all four βs, the distributions for most of the risk-classes are nonuniform.

14. Primary metals have extremely large betas; utilities have extremely small betas.

two industries, and also two miscellaneous firms so that an even 250 firms are in the sample, new upper and lower values of the β's were obtained for each of the ten class intervals and for each of the four β's.

In Table 5, the chi-square values are presented; for the total of all risk-classes, the probability of obtaining a chi-square value less than 120.63 is over 99.95% (for $_A\beta$), whereas the probability of obtaining a chi-square value less than 99.75 is between 99.5% and 99.9% (for $_B\beta$). More sharply contrasting results are obtained when $_{AC}\beta$ is compared to $_{BC}\beta$. For $_{AC}\beta$, the probability of obtaining less than 128.47 is over 99.95%, whereas for $_{BC}\beta$, the probability of obtaining less than 78.65 is only 90.0%. By abstracting from financial risk, the underlying systematic risk is much less scattered when grouped into risk-classes than when leverage is assumed not to affect the systematic risk. The null hypothesis that the β's in a risk-class come from the same distribution as all β's is rejected for $_{AC}\beta$, but not for $_{BC}\beta$ (at the 90% level). Although this, in itself, does not tell us *how* a risk-class differs from the total market, an inspection of the distributions of the betas by risk-class underlying Table 5 does indicate more clustering of the $_{AC}\beta$s than the $_{BC}\beta$s so that the MM theory is again favored over the traditional theory.

The analysis of variance test is our last comparison of the implications of the two theories. The ratio of the estimated variance between industries to the estimated variance within the industries (the F-statistic) when the seven

TABLE 5
CHI-SQUARE RESULTS FOR ALL β'S AND ALL INDUSTRIES
(EXCEPT UTILITIES AND PRIMARY METALS)

Industry		$_A\beta$	$_B\beta$	$_{AC}\beta$	$_{BC}\beta$
Food and Kindred	Chi-Square P {χ^2 < }* =	18.67 95-97.5%	11.33 70-75%	26.00 99.5-99.9%	9.33 50-60%
Chemicals	Chi-Square P {χ^2 < } =	9.33 50-60%	10.67 60-70%	12.00 75-80%	7.33 30-40%
Petroleum	Chi-Square P {χ^2 < } =	17.56 95-97.5%	25.33 99.5-99.9%	18.67 95-97.5%	22.00 99-99.5%
Machinery	Chi-Square P {χ^2 < } =	19.14 97.5-98%	12.00 75-80%	24.86 99.5-99.9%	9.14 50-60%
Electrical Machinery	Chi-Square P {χ^2 < } =	13.92 80-90%	7.77 40-50%	12.38 80-90%	9.31 50-60%
Transportation Equipment	Chi-Square P {χ^2 < } =	15.17 90-95%	16.83 90-95%	13.50 80-90%	6.83 30-40%
Dep't Stores	Chi-Square P {χ^2 < } =	14.18 80-90%	3.59 5-10%	14.18 80-90%	3.59 5-10%
Miscellaneous	Chi-Square P {χ^2 < } =	12.67 80-90%	12.22 80-90%	6.89 30-40%	11.11 70-75%
Total	Chi-Square P {χ^2 < } =	120.63 over 99.95%	99.75 99.5-99.90%	128.47 over 99.95%	78.65 90.0%

* Example: P{$\chi^2 < 18.67$} = 95-97.5% for 9 degrees of freedom.

industries are considered (again, the two obviously skewed industries, primary metals and utilities, were eliminated) is less for $_{B}\beta$ (F = 3.90) than for $_{A}\beta$ (F = 9.99), and less for $_{BC}\beta$ (F = 4.18) than for $_{AC}\beta$ (F = 10.83). The probability of obtaining these F-statistics for $_{A}\beta$ and $_{AC}\beta$ is less than 0.001, but for $_{B}\beta$ and $_{BC}\beta$ greater than or equal to 0.001. These results are consistent with the results obtained from our two previous tests. The MM theory is more compatible with the data than the traditional theory.[15]

V. Conclusions

This study attempted to tie together some of the notions associated with the field of corporation finance with those associated with security and portfolio analyses. Specifically, if the MM corporate tax leverage propositions are correct, then approximately 21 to 24% of the observed systematic risk of common stocks (when averaged over 304 firms) can be explained merely by the added financial risk taken on by the underlying firm with its use of debt and preferred stock. Corporate leverage does count considerably.

To determine whether the MM theory is correct, a number of tests on a contrasting implication of the MM and "traditional" theories of corporation finance were performed. The data confirmed MM's position, at least vis-à-vis our interpretation of the traditional theory's position. This should provide another piece of evidence on this controversial topic.

Finally, if the MM theory and the capital asset pricing model are correct, and if the adjustments made in equations (8) or (4a) result in accurate measures of the systematic risk of a leverage-free firm, the possibility is greater, without resorting to a fullblown risk-class study of the type MM did for the electric utility industry [8], of estimating the cost of capital for individual firms.

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15. All of our tests, it should be emphasized, although consistent, are only inferential. Aside from assuming that the two-digit SEC industry classification is a good proxy for risk-classes and that the errors in estimating the individual βs can be safely ignored, the tests rely on the two theories exhausting all the reasonable theories on leverage. But there is always the use of another line of reasoning. If the results of the MM electric utility study [8] are correct, and if these results can be generalized to all firms and to all risk-classes, then it can be claimed that the MM theory is universally valid. Then our result in Section III does indicate the correct effect of the firm's capital structure on the systematic risk of common stocks.

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THE COST OF CAPITAL, CORPORATION FINANCE AND THE THEORY OF INVESTMENT

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What is the "cost of capital" to a firm in a world in which funds are used to acquire assets whose yields are uncertain; and in which capital can be obtained by many different media, ranging from pure debt instruments, representing money-fixed claims, to pure equity issues, giving holders only the right to a pro-rata share in the uncertain venture? This question has vexed at least three classes of economists: (1) the corporation finance specialist concerned with the techniques of financing firms so as to ensure their survival and growth; (2) the managerial economist concerned with capital budgeting; and (3) the economic theorist concerned with explaining investment behavior at both the micro and macro levels.[1]

In much of his formal analysis, the economic theorist at least has tended to side-step the essence of this cost-of-capital problem by proceeding as though physical assets—like bonds—could be regarded as yielding known, sure streams. Given this assumption, the theorist has concluded that the cost of capital to the owners of a firm is simply the rate of interest on bonds; and has derived the familiar proposition that the firm, acting rationally, will tend to push investment to the point

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[1] The literature bearing on the cost-of-capital problem is far too extensive for listing here. Numerous references to it will be found throughout the paper though we make no claim to completeness. One phase of the problem which we do not consider explicitly, but which has a considerable literature of its own is the relation between the cost of capital and public utility rates. For a recent summary of the "cost-of-capital theory" of rate regulation and a brief discussion of some of its implications, the reader may refer to H. M. Somers [20].

where the marginal yield on physical assets is equal to the market rate of interest.[2] This proposition can be shown to follow from either of two criteria of rational decision-making which are equivalent under certainty, namely (1) the maximization of profits and (2) the maximization of market value.

According to the first criterion, a physical asset is worth acquiring if it will increase the net profit of the owners of the firm. But net profit will increase only if the expected rate of return, or yield, of the asset exceeds the rate of interest. According to the second criterion, an asset is worth acquiring if it increases the value of the owners' equity, *i.e.*, if it adds more to the market value of the firm than the costs of acquisition. But what the asset adds is given by capitalizing the stream it generates at the market rate of interest, and this capitalized value will exceed its cost if and only if the yield of the asset exceeds the rate of interest. Note that, under either formulation, the cost of capital is equal to the rate of interest on bonds, regardless of whether the funds are acquired through debt instruments or through new issues of common stock. Indeed, in a world of sure returns, the distinction between debt and equity funds reduces largely to one of terminology.

It must be acknowledged that some attempt is usually made in this type of analysis to allow for the existence of uncertainty. This attempt typically takes the form of superimposing on the results of the certainty analysis the notion of a "risk discount" to be subtracted from the expected yield (or a "risk premium" to be added to the market rate of interest). Investment decisions are then supposed to be based on a comparison of this "risk adjusted" or "certainty equivalent" yield with the market rate of interest.[3] No satisfactory explanation has yet been provided, however, as to what determines the size of the risk discount and how it varies in response to changes in other variables.

Considered as a convenient approximation, the model of the firm constructed via this certainty—or certainty-equivalent—approach has admittedly been useful in dealing with some of the grosser aspects of the processes of capital accumulation and economic fluctuations. Such a model underlies, for example, the familiar Keynesian aggregate investment function in which aggregate investment is written as a function of the rate of interest—the same riskless rate of interest which appears later in the system in the liquidity-preference equation. Yet few would maintain that this approximation is adequate. At the macroeconomic level there are ample grounds for doubting that the rate of interest has

[2] Or, more accurately, to the marginal cost of borrowed funds since it is customary, at least in advanced analysis, to draw the supply curve of borrowed funds to the firm as a rising one. For an advanced treatment of the certainty case, see F. and V. Lutz [13].

[3] The classic examples of the certainty-equivalent approach are found in J. R. Hicks [8] and O. Lange [11].

as large and as direct an influence on the rate of investment as this analysis would lead us to believe. At the microeconomic level the certainty model has little descriptive value and provides no real guidance to the finance specialist or managerial economist whose main problems cannot be treated in a framework which deals so cavalierly with uncertainty and ignores all forms of financing other than debt issues.[4]

Only recently have economists begun to face up seriously to the problem of the cost of capital *cum* risk. In the process they have found their interests and endeavors merging with those of the finance specialist and the managerial economist who have lived with the problem longer and more intimately. In this joint search to establish the principles which govern rational investment and financial policy in a world of uncertainty two main lines of attack can be discerned. These lines represent, in effect, attempts to extrapolate to the world of uncertainty each of the two criteria—profit maximization and market value maximization—which were seen to have equivalent implications in the special case of certainty. With the recognition of uncertainty this equivalence vanishes. In fact, the profit maximization criterion is no longer even well defined. Under uncertainty there corresponds to each decision of the firm not a unique profit outcome, but a plurality of mutually exclusive outcomes which can at best be described by a subjective probability distribution. The profit outcome, in short, has become a random variable and as such its maximization no longer has an operational meaning. Nor can this difficulty generally be disposed of by using the mathematical expectation of profits as the variable to be maximized. For decisions which affect the expected value will also tend to affect the dispersion and other characteristics of the distribution of outcomes. In particular, the use of debt rather than equity funds to finance a given venture may well increase the expected return to the owners, but only at the cost of increased dispersion of the outcomes.

Under these conditions the profit outcomes of alternative investment and financing decisions can be compared and ranked only in terms of a *subjective* "utility function" of the owners which weighs the expected yield against other characteristics of the distribution. Accordingly, the extrapolation of the profit maximization criterion of the certainty model has tended to evolve into utility maximization, sometimes explicitly, more frequently in a qualitative and heuristic form.[5]

The utility approach undoubtedly represents an advance over the certainty or certainty-equivalent approach. It does at least permit us

[4] Those who have taken a "case-method" course in finance in recent years will recall in this connection the famous Liquigas case of Hunt and Williams, [9, pp. 193–96] a case which is often used to introduce the student to the cost-of-capital problem and to poke a bit of fun at the economist's certainty-model.

[5] For an attempt at a rigorous explicit development of this line of attack, see F. Modigliani and M. Zeman [14].

to explore (within limits) some of the implications of different financing arrangements, and it does give some meaning to the "cost" of different types of funds. However, because the cost of capital has become an essentially subjective concept, the utility approach has serious drawbacks for normative as well as analytical purposes. How, for example, is management to ascertain the risk preferences of its stockholders and to compromise among their tastes? And how can the economist build a meaningful investment function in the face of the fact that any given investment opportunity might or might not be worth exploiting depending on precisely who happen to be the owners of the firm at the moment?

Fortunately, these questions do not have to be answered; for the alternative approach, based on market value maximization, can provide the basis for an operational definition of the cost of capital and a workable theory of investment. Under this approach any investment project and its concomitant financing plan must pass only the following test: Will the project, as financed, raise the market value of the firm's shares? If so, it is worth undertaking; if not, its return is less than the marginal cost of capital to the firm. Note that such a test is entirely independent of the tastes of the current owners, since market prices will reflect not only their preferences but those of all potential owners as well. If any current stockholder disagrees with management and the market over the valuation of the project, he is free to sell out and reinvest elsewhere, but will still benefit from the capital appreciation resulting from management's decision.

The potential advantages of the market-value approach have long been appreciated; yet analytical results have been meager. What appears to be keeping this line of development from achieving its promise is largely the lack of an adequate theory of the effect of financial structure on market valuations, and of how these effects can be inferred from objective market data. It is with the development of such a theory and of its implications for the cost-of-capital problem that we shall be concerned in this paper.

Our procedure will be to develop in Section I the basic theory itself and to give some brief account of its empirical relevance. In Section II, we show how the theory can be used to answer the cost-of-capital question and how it permits us to develop a theory of investment of the firm under conditions of uncertainty. Throughout these sections the approach is essentially a partial-equilibrium one focusing on the firm and "industry." Accordingly, the "prices" of certain income streams will be treated as constant and given from outside the model, just as in the standard Marshallian analysis of the firm and industry the prices of all inputs and of all other products are taken as given. We have chosen to focus at this level rather than on the economy as a whole because it is at the level of the firm and the industry that the interests of the various specialists concerned with the cost-of-capital problem come most closely together. Although the emphasis has thus been placed on partial-equilibrium analysis, the results obtained also provide the essential building blocks for a general equilibrium model which shows how those prices which are here taken as given, are themselves determined. For reasons of space, however, and because the material is of interest in its own right, the presentation of the general equilibrium model which rounds out the analysis must be deferred to a subsequent paper.

I. *The Valuation of Securities, Leverage, and the Cost of Capital*

A. *The Capitalization Rate for Uncertain Streams*

As a starting point, consider an economy in which all physical assets are owned by corporations. For the moment, assume that these corporations can finance their assets by issuing common stock only; the introduction of bond issues, or their equivalent, as a source of corporate funds is postponed until the next part of this section.

The physical assets held by each firm will yield to the owners of the firm—its stockholders—a stream of "profits" over time; but the elements of this series need not be constant and in any event are uncertain. This stream of income, and hence the stream accruing to any share of common stock, will be regarded as extending indefinitely into the future. We assume, however, that the mean value of the stream over time, or average profit per unit of time, is finite and represents a random variable subject to a (subjective) probability distribution. We shall refer to the average value over time of the stream accruing to a given share as the return of that share; and to the mathematical expectation of this average as the expected return of the share.[6] Although individual investors may have different views as to the shape of the probability distri-

[6] These propositions can be restated analytically as follows: The assets of the ith firm generate a stream:

$$X_i(1),\ X_i(2)\ \cdots\ X_i(T)$$

whose elements are random variables subject to the joint probability distribution:

$$\chi_i[X_i(1),\ X_i(2)\ \cdots\ X_i(t)].$$

The return to the ith firm is defined as:

$$X_i = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} X_i(t).$$

X_i is itself a random variable with a probability distribution $\Phi_i(X_i)$ whose form is determined uniquely by χ_i. The expected return $\bar{X}_i$ is defined as $\bar{X}_i = E(X_i) = \int_{X_i} X_i \Phi_i(X_i) dX_i$. If N_i is the number of shares outstanding, the return of the ith share is $x_i = (1/N)X_i$ with probability distribution $\phi_i(x_i)dx_i = \Phi_i(Nx_i)d(Nx_i)$ and expected value $\bar{x}_i = (1/N)\bar{X}_i$.

bution of the return of any share, we shall assume for simplicity that they are at least in agreement as to the expected return.[7]

This way of characterizing uncertain streams merits brief comment. Notice first that the stream is a stream of profits, not dividends. As will become clear later, as long as management is presumed to be acting in the best interests of the stockholders, retained earnings can be regarded as equivalent to a fully subscribed, pre-emptive issue of common stock. Hence, for present purposes, the division of the stream between cash dividends and retained earnings in any period is a mere detail. Notice also that the uncertainty attaches to the mean value over time of the stream of profits and should not be confused with variability over time of the successive elements of the stream. That variability and uncertainty are two totally different concepts should be clear from the fact that the elements of a stream can be variable even though known with certainty. It can be shown, furthermore, that whether the elements of a stream are sure or uncertain, the effect of variability per se on the valuation of the stream is at best a second-order one which can safely be neglected for our purposes (and indeed most others too).[8]

The next assumption plays a strategic role in the rest of the analysis. We shall assume that firms can be divided into "equivalent return" classes such that the return on the shares issued by any firm in any given class is proportional to (and hence perfectly correlated with) the return on the shares issued by any other firm in the same class. This assumption implies that the various shares within the same class differ, at most, by a "scale factor." Accordingly, if we adjust for the difference in scale, by taking the *ratio* of the return to the expected return, the probability distribution of that ratio is identical for all shares in the class. It follows that all relevant properties of a share are uniquely characterized by specifying (1) the class to which it belongs and (2) its expected return.

The significance of this assumption is that it permits us to classify firms into groups within which the shares of different firms are "homogeneous," that is, perfect substitutes for one another. We have, thus, an analogue to the familiar concept of the industry in which it is the commodity produced by the firms that is taken as homogeneous. To complete this analogy with Marshallian price theory, we shall assume in the

[7] To deal adequately with refinements such as differences among investors in estimates of expected returns would require extensive discussion of the theory of portfolio selection. Brief references to these and related topics will be made in the succeeding article on the general equilibrium model.

[8] The reader may convince himself of this by asking how much he would be willing to rebate to his employer for the privilege of receiving his annual salary in equal monthly installments rather than in irregular amounts over the year. See also J. M. Keynes [10, esp. pp. 53–54].

analysis to follow that the shares concerned are traded in perfect markets under conditions of atomistic competition.[9]

From our definition of homogeneous classes of stock it follows that in equilibrium in a perfect capital market the price per dollar's worth of expected return must be the same for all shares of any given class. Or, equivalently, in any given class the price of every share must be proportional to its expected return. Let us denote this factor of proportionality for any class, say the kth class, by $1/\rho_k$. Then if p_j denotes the price and $\bar{x}_j$ is the expected return per share of the jth firm in class k, we must have:

$$p_j = \frac{1}{\rho_k} \bar{x}_j; \tag{1}$$

or, equivalently,

$$\frac{\bar{x}_j}{p_j} = \rho_k \text{ a constant for all firms } j \text{ in class } k. \tag{2}$$

The constants ρ_k (one for each of the k classes) can be given several economic interpretations: (a) From (2) we see that each ρ_k is the expected rate of return of any share in class k. (b) From (1) $1/\rho_k$ is the price which an investor has to pay for a dollar's worth of expected return in the class k. (c) Again from (1), by analogy with the terminology for perpetual bonds, ρ_k can be regarded as the market rate of capitalization for the expected value of the uncertain streams of the kind generated by the kth class of firms.[10]

B. *Debt Financing and Its Effects on Security Prices*

Having developed an apparatus for dealing with uncertain streams we can now approach the heart of the cost-of-capital problem by dropping the assumption that firms cannot issue bonds. The introduction of debt-financing changes the market for shares in a very fundamental way. Because firms may have different proportions of debt in their capi-

[9] Just what our classes of stocks contain and how the different classes can be identified by outside observers are empirical questions to which we shall return later. For the present, it is sufficient to observe: (1) Our concept of a class, while not identical to that of the industry is at least closely related to it. Certainly the basic characteristics of the probability distributions of the returns on assets will depend to a significant extent on the product sold and the technology used. (2) What are the appropriate class boundaries will depend on the particular problem being studied. An economist concerned with general tendencies in the market, for example, might well be prepared to work with far wider classes than would be appropriate for an investor planning his portfolio, or a firm planning its financial strategy.

[10] We cannot, on the basis of the assumptions so far, make any statements about the relationship or spread between the various ρ's or capitalization rates. Before we could do so we would have to make further specific assumptions about the way investors believe the probability distributions vary from class to class, as well as assumptions about investors' preferences as between the characteristics of different distributions.

tal structure, shares of different companies, even in the same class, can give rise to different probability distributions of returns. In the language of finance, the shares will be subject to different degrees of financial risk or "leverage" and hence they will no longer be perfect substitutes for one another.

To exhibit the mechanism determining the relative prices of shares under these conditions, we make the following two assumptions about the nature of bonds and the bond market, though they are actually stronger than is necessary and will be relaxed later: (1) All bonds (including any debts issued by households for the purpose of carrying shares) are assumed to yield a constant income per unit of time, and this income is regarded as certain by all traders regardless of the issuer. (2) Bonds, like stocks, are traded in a perfect market, where the term perfect is to be taken in its usual sense as implying that any two commodities which are perfect substitutes for each other must sell, in equilibrium, at the same price. It follows from assumption (1) that all bonds are in fact perfect substitutes up to a scale factor. It follows from assumption (2) that they must all sell at the same price per dollar's worth of return, or what amounts to the same thing must yield the same rate of return. This rate of return will be denoted by r and referred to as the rate of interest or, equivalently, as the capitalization rate for sure streams. We now can derive the following two basic propositions with respect to the valuation of securities in companies with different capital structures:

Proposition I. Consider any company j and let $\overline{X}_j$ stand as before for the expected return on the assets owned by the company (that is, its expected profit before deduction of interest). Denote by D_j the market value of the debts of the company; by S_j the market value of its common shares; and by $V_j \equiv S_j + D_j$ the market value of all its securities or, as we shall say, the market value of the firm. Then, our Proposition I asserts that we must have in equilibrium:

$$V_j \equiv (S_j + D_j) = \overline{X}_j/\rho_k, \text{ for any firm } j \text{ in class } k. \quad (3)$$

That is, the *market value of any firm is independent of its capital structure and is given by capitalizing its expected return at the rate ρ_k appropriate to its class.*

This proposition can be stated in an equivalent way in terms of the firm's "average cost of capital," $\overline{X}_j/V_j$, which is the ratio of its expected return to the market value of all its securities. Our proposition then is:

$$\frac{\overline{X}_j}{(S_j + D_j)} \equiv \frac{\overline{X}_j}{V_j} = \rho_k, \text{ for any firm } j, \text{ in class } k. \quad (4)$$

That is, *the average cost of capital to any firm is completely independent of* *its capital structure and is equal to the capitalization rate of a pure equity stream of its class.*

To establish Proposition I we will show that as long as the relations (3) or (4) do not hold between any pair of firms in a class, arbitrage will take place and restore the stated equalities. We use the term arbitrage advisedly. For if Proposition I did not hold, an investor could buy and sell stocks and bonds in such a way as to exchange one income stream for another stream, identical in all relevant respects but selling at a lower price. The exchange would therefore be advantageous to the investor quite independently of his attitudes toward risk.[11] As investors exploit these arbitrage opportunities, the value of the overpriced shares will fall and that of the underpriced shares will rise, thereby tending to eliminate the discrepancy between the market values of the firms.

By way of proof, consider two firms in the same class and assume for simplicity only, that the expected return, X, is the same for both firms. Let company 1 be financed entirely with common stock while company 2 has some debt in its capital structure. Suppose first the value of the levered firm, V_2, to be larger than that of the unlevered one, V_1. Consider an investor holding s_2 dollars' worth of the shares of company 2, representing a fraction α of the total outstanding stock, S_2. The return from this portfolio, denoted by Y_2, will be a fraction α of the income available for the stockholders of company 2, which is equal to the total return X_2 less the interest charge, rD_2. Since under our assumption of homogeneity, the anticipated total return of company 2, X_2, is, under all circumstances, the same as the anticipated total return to company 1, X_1, we can hereafter replace X_2 and X_1 by a common symbol X. Hence, the return from the initial portfolio can be written as:

$$Y_2 = \alpha(X - rD_2). \tag{5}$$

Now suppose the investor sold his αS_2 worth of company 2 shares and acquired instead an amount $s_1 = \alpha(S_2 + D_2)$ of the shares of company 1. He could do so by utilizing the amount αS_2 realized from the sale of his initial holding and borrowing an additional amount αD_2 on his own credit, pledging his new holdings in company 1 as a collateral. He would thus secure for himself a fraction $s_1/S_1 = \alpha(S_2 + D_2)/S_1$ of the shares and earnings of company 1. Making proper allowance for the interest payments on his personal debt αD_2, the return from the new portfolio, Y_1, is given by:

[11] In the language of the theory of choice, the exchanges are movements from inefficient points in the interior to efficient points on the boundary of the investor's opportunity set; and not movements between efficient points along the boundary. Hence for this part of the analysis nothing is involved in the way of specific assumptions about investor attitudes or behavior other than that investors behave consistently and prefer more income to less income, *ceteris paribus*.

(6) $$Y_1 = \frac{\alpha(S_2 + D_2)}{S_1} X - r\alpha D_2 = \alpha \frac{V_2}{V_1} X - r\alpha D_2.$$

Comparing (5) with (6) we see that as long as $V_2 > V_1$ we must have $Y_1 > Y_2$, so that it pays owners of company 2's shares to sell their holdings, thereby depressing S_2 and hence V_2; and to acquire shares of company 1, thereby raising S_1 and thus V_1. We conclude therefore that levered companies cannot command a premium over unlevered companies because investors have the opportunity of putting the equivalent leverage into their portfolio directly by borrowing on personal account.

Consider now the other possibility, namely that the market value of the levered company V_2 is less than V_1. Suppose an investor holds initially an amount s_1 of shares of company 1, representing a fraction α of the total outstanding stock, S_1. His return from this holding is:

$$Y_1 = \frac{s_1}{S_1} X = \alpha X.$$

Suppose he were to exchange this initial holding for another portfolio, also worth s_1, but consisting of s_2 dollars of stock of company 2 and of d dollars of bonds, where s_2 and d are given by:

(7) $$s_2 = \frac{S_2}{V_2} s_1, \qquad d = \frac{D_2}{V_2} s_1.$$

In other words the new portfolio is to consist of stock of company 2 and of bonds in the proportions S_2/V_2 and D_2/V_2, respectively. The return from the stock in the new portfolio will be a fraction s_2/S_2 of the total return to stockholders of company 2, which is $(X - rD_2)$, and the return from the bonds will be rd. Making use of (7), the total return from the portfolio, Y_2, can be expressed as follows:

$$Y_2 = \frac{s_2}{S_2}(X - rD_2) + rd = \frac{s_1}{V_2}(X - rD_2) + r\frac{D_2}{V_2}s_1 = \frac{s_1}{V_2}X = \alpha\frac{S_1}{V_2}X$$

(since $s_1 = \alpha S_1$). Comparing Y_2 with Y_1 we see that, if $V_2 < S_1 \equiv V_1$, then Y_2 will exceed Y_1. Hence it pays the holders of company 1's shares to sell these holdings and replace them with a mixed portfolio containing an appropriate fraction of the shares of company 2.

The acquisition of a mixed portfolio of stock of a levered company j and of bonds in the proportion S_j/V_j and D_j/V_j respectively, may be regarded as an operation which "undoes" the leverage, giving access to an appropriate fraction of the unlevered return X_j. It is this possibility of undoing leverage which prevents the value of levered firms from being consistently less than those of unlevered firms, or more generally prevents the average cost of capital $\bar{X}_j/V_j$ from being systematically higher for levered than for nonlevered companies in the same class.

Since we have already shown that arbitrage will also prevent V_2 from being larger than V_1, we can conclude that in equilibrium we must have $V_2 = V_1$, as stated in Proposition I.

Proposition II. From Proposition I we can derive the following proposition concerning the rate of return on common stock in companies whose capital structure includes some debt: the expected rate of return or yield, i, on the stock of any company j belonging to the kth class is a linear function of leverage as follows:

$$i_j = \rho_k + (\rho_k - r)D_j/S_j. \tag{8}$$

That is, *the expected yield of a share of stock is equal to the appropriate capitalization rate* ρ_k *for a pure equity stream in the class, plus a premium related to financial risk equal to the debt-to-equity ratio times the spread between* ρ_k *and* r. Or equivalently, the market price of any share of stock is given by capitalizing its expected return at the continuously variable rate i_j of (8).[12]

A number of writers have stated close equivalents of our Proposition I although by appealing to intuition rather than by attempting a proof and only to insist immediately that the results were not applicable to the actual capital markets.[13] Proposition II, however, so far as we have been able to discover is new.[14] To establish it we first note that, by definition, the expected rate of return, i, is given by:

$$i_j \equiv \frac{\bar{X}_j - rD_j}{S_j}. \tag{9}$$

From Proposition I, equation (3), we know that:

$$\bar{X}_j = \rho_k(S_j + D_j).$$

Substituting in (9) and simplifying, we obtain equation (8).

[12] To illustrate, suppose $\bar{X} = 1000$, $D = 4000$, $r = 5$ per cent and $\rho_k = 10$ per cent. These values imply that $V = 10{,}000$ and $S = 6000$ by virtue of Proposition I. The expected yield or rate of return per share is then:

$$i = \frac{1000 - 200}{6000} = .1 + (.1 - .05)\frac{4000}{6000} = 13\tfrac{1}{3} \text{ per cent.}$$

[13] See, for example, J. B. Williams [21, esp. pp. 72–73]; David Durand [3]; and W. A. Morton [15]. None of these writers describe in any detail the mechanism which is supposed to keep the average cost of capital constant under changes in capital structure. They seem, however, to be visualizing the equilibrating mechanism in terms of switches by investors between stocks and bonds as the yields of each get out of line with their "riskiness." This is an argument quite different from the pure arbitrage mechanism underlying our proof, and the difference is crucial. Regarding Proposition I as resting on investors' attitudes toward risk leads inevitably to a misunderstanding of many factors influencing relative yields such as, for example, limitations on the portfolio composition of financial institutions. See below, esp. Section I.D.

[14] Morton does make reference to a linear yield function but only " . . . for the sake of simplicity and because the particular function used makes no essential difference in my conclusions" [15, p. 443, note 2].

C. *Some Qualifications and Extensions of the Basic Propositions*

The methods and results developed so far can be extended in a number of useful directions, of which we shall consider here only three: (1) allowing for a corporate profits tax under which interest payments are deductible; (2) recognizing the existence of a multiplicity of bonds and interest rates; and (3) acknowledging the presence of market imperfections which might interfere with the process of arbitrage. The first two will be examined briefly in this section with some further attention given to the tax problem in Section II. Market imperfections will be discussed in Part D of this section in the course of a comparison of our results with those of received doctrines in the field of finance.

Effects of the Present Method of Taxing Corporations. The deduction of interest in computing taxable corporate profits will prevent the arbitrage process from making the value of all firms in a given class proportional to the expected returns generated by their physical assets. Instead, it can be shown (by the same type of proof used for the original version of Proposition I) that the market values of firms in each class must be proportional in equilibrium to their expected return net of taxes (that is, to the sum of the interest paid and expected net stockholder income). This means we must replace each $\bar{X}_j$ in the original versions of Propositions I and II with a new variable $\bar{X}_j^\tau$ representing the total income net of taxes generated by the firm:

$$\bar{X}_j^\tau \equiv (\bar{X}_j - rD_j)(1 - \tau) + rD_j \equiv \bar{\pi}_j^\tau + rD_j, \tag{10}$$

where $\bar{\pi}_j^\tau$ represents the expected net income accruing to the common stockholders and τ stands for the average rate of corporate income tax.[15]

After making these substitutions, the propositions, when adjusted for taxes, continue to have the same form as their originals. That is, Proposition I becomes:

$$\frac{\bar{X}_j^\tau}{V_j} = \rho_k^\tau, \text{ for any firm in class } k, \tag{11}$$

and Proposition II becomes

$$i_j \equiv \frac{\bar{\pi}_j^\tau}{S_j} = \rho_j^\tau + (\rho_k^\tau - r)D_j/S_j \tag{12}$$

where ρ_k^τ is the capitalization rate for income net of taxes in class k.

Although the form of the propositions is unaffected, certain interpretations must be changed. In particular, the after-tax capitalization rate

[15] For simplicity, we shall ignore throughout the tiny element of progression in our present corporate tax and treat τ as a constant independent of $(X_j - rD_j)$.

ρ_k^τ can no longer be identified with the "average cost of capital" which is $\rho_k = \overline{X}_j/V_j$. The difference between ρ_k^τ and the "true" average cost of capital, as we shall see, is a matter of some relevance in connection with investment planning within the firm (Section II). For the description of market behavior, however, which is our immediate concern here, the distinction is not essential. To simplify presentation, therefore, and to preserve continuity with the terminology in the standard literature we shall continue in this section to refer to ρ_k^τ as the average cost of capital, though strictly speaking this identification is correct only in the absence of taxes.

Effects of a Plurality of Bonds and Interest Rates. In existing capital markets we find not one, but a whole family of interest rates varying with maturity, with the technical provisions of the loan and, what is most relevant for present purposes, with the financial condition of the borrower.[16] Economic theory and market experience both suggest that the yields demanded by lenders tend to increase with the debt-equity ratio of the borrowing firm (or individual). If so, and if we can assume as a first approximation that this yield curve, $r = r\ (D/S)$, whatever its precise form, is the same for all borrowers, then we can readily extend our propositions to the case of a rising supply curve for borrowed funds.[17]

Proposition I is actually unaffected in form and interpretation by the fact that the rate of interest may rise with leverage; while the average cost of *borrowed* funds will tend to increase as debt rises, the average cost of funds from *all* sources will still be independent of leverage (apart from the tax effect). This conclusion follows directly from the ability of those who engage in arbitrage to undo the leverage in any financial structure by acquiring an appropriately mixed portfolio of bonds and stocks. Because of this ability, the ratio of earnings (*before* interest charges) to market value—*i.e.*, the average cost of capital from all

[16] We shall not consider here the extension of the analysis to encompass the time structure of interest rates. Although some of the problems posed by the time structure can be handled within our comparative statics framework, an adequate discussion would require a separate paper.

[17] We can also develop a theory of bond valuation along lines essentially parallel to those followed for the case of shares. We conjecture that the curve of bond yields as a function of leverage will turn out to be a nonlinear one in contrast to the linear function of leverage developed for common shares. However, we would also expect that the rate of increase in the yield on new issues would not be substantial in practice. This relatively slow rise would reflect the fact that interest rate increases by themselves can never be completely satisfactory to creditors as compensation for their increased risk. Such increases may simply serve to raise r so high relative to ρ that they become self-defeating by giving rise to a situation in which even normal fluctuations in earnings may force the company into bankruptcy. The difficulty of borrowing more, therefore, tends to show up in the usual case not so much in higher rates as in the form of increasingly stringent restrictions imposed on the company's management and finances by the creditors; and ultimately in a complete inability to obtain new borrowed funds, at least from the institutional investors who normally set the standards in the market for bonds.

sources—must be the same for all firms in a given class.[18] In other words, the increased cost of borrowed funds as leverage increases will tend to be offset by a corresponding reduction in the yield of common stock. This seemingly paradoxical result will be examined more closely below in connection with Proposition II.

A significant modification of Proposition I would be required only if the yield curve $r=r(D/S)$ were different for different borrowers, as might happen if creditors had marked preferences for the securities of a particular class of debtors. If, for example, corporations as a class were able to borrow at lower rates than individuals having equivalent personal leverage, then the average cost of capital to corporations might fall slightly, as leverage increased over some range, in reflection of this differential. In evaluating this possibility, however, remember that the relevant interest rate for our arbitrage operators is the rate on brokers' loans and, historically, that rate has not been noticeably higher than representative corporate rates.[19] The operations of holding companies and investment trusts which can borrow on terms comparable to operating companies represent still another force which could be expected to wipe out any marked or prolonged advantages from holding levered stocks.[20]

Although Proposition I remains unaffected as long as the yield curve is the same for all borrowers, the relation between common stock yields and leverage will no longer be the strictly linear one given by the original Proposition II. If r increases with leverage, the yield i will still tend to

[18] One normally minor qualification might be noted. Once we relax the assumption that all bonds have certain yields, our arbitrage operator faces the danger of something comparable to "gambler's ruin." That is, there is always the possibility that an otherwise sound concern—one whose long-run expected income is greater than its interest liability—might be forced into liquidation as a result of a run of temporary losses. Since reorganization generally involves costs, and because the operation of the firm may be hampered during the period of reorganization with lasting unfavorable effects on earnings prospects, we might perhaps expect heavily levered companies to sell at a slight discount relative to less heavily indebted companies of the same class.

[19] Under normal conditions, moreover, a substantial part of the arbitrage process could be expected to take the form, not of having the arbitrage operators go into debt on personal account to put the required leverage into their portfolios, but simply of having them reduce the amount of corporate bonds they already hold when they acquire underpriced unlevered stock. Margin requirements are also somewhat less of an obstacle to maintaining any desired degree of leverage in a portfolio than might be thought at first glance. Leverage could be largely restored in the face of higher margin requirements by switching to stocks having more leverage at the corporate level.

[20] An extreme form of inequality between borrowing and lending rates occurs, of course, in the case of preferred stocks, which can not be directly issued by individuals on personal account. Here again, however, we would expect that the operations of investment corporations plus the ability of arbitrage operators to sell off their holdings of preferred stocks would act to prevent the emergence of any substantial premiums (for this reason) on capital structures containing preferred stocks. Nor are preferred stocks so far removed from bonds as to make it impossible for arbitrage operators to approximate closely the risk and leverage of a corporate preferred stock by incurring a somewhat smaller debt on personal account.

rise as D/S increases, but at a decreasing rather than a constant rate. Beyond some high level of leverage, depending on the exact form of the interest function, the yield may even start to fall.[21] The relation between i and D/S could conceivably take the form indicated by the curve MD

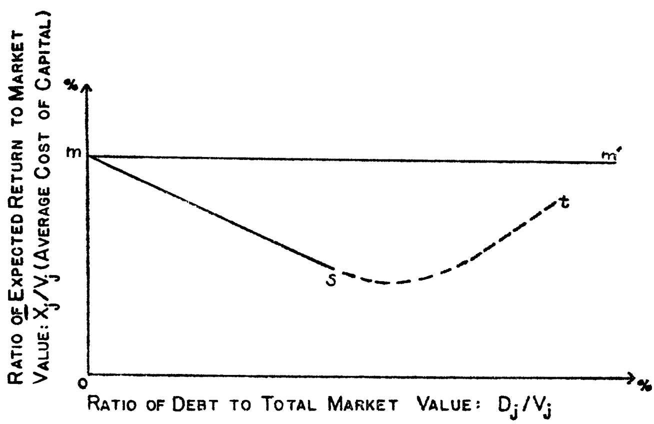

FIGURE 1

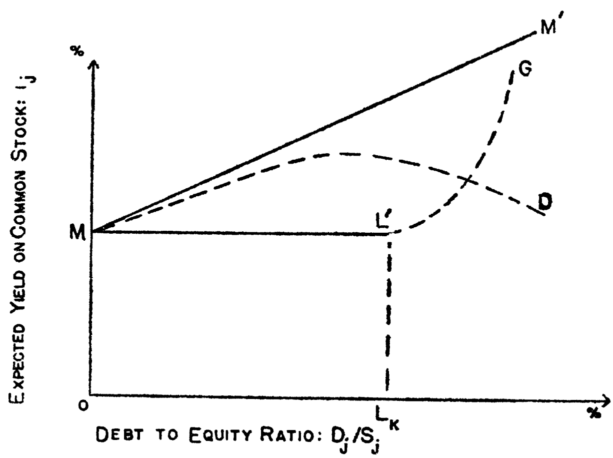

FIGURE 2

in Figure 2, although in practice the curvature would be much less pronounced. By contrast, with a constant rate of interest, the relation would be linear throughout as shown by line MM', Figure 2.

The downward sloping part of the curve MD perhaps requires some

[21] Since new lenders are unlikely to permit this much leverage (*cf.* note 17), this range of the curve is likely to be occupied by companies whose earnings prospects have fallen substantially since the time when their debts were issued.

comment since it may be hard to imagine why investors, other than those who like lotteries, would purchase stocks in this range. Remember, however, that the yield curve of Proposition II is a consequence of the more fundamental Proposition I. Should the demand by the risk-lovers prove insufficient to keep the market to the peculiar yield-curve *MD*, this demand would be reinforced by the action of arbitrage operators. The latter would find it profitable to own a pro-rata share of the firm as a whole by holding its stock *and* bonds, the lower yield of the shares being thus offset by the higher return on bonds.

D. *The Relation of Propositions I and II to Current Doctrines*

The propositions we have developed with respect to the valuation of firms and shares appear to be substantially at variance with current doctrines in the field of finance. The main differences between our view and the current view are summarized graphically in Figures 1 and 2. Our Proposition I [equation (4)] asserts that the average cost of capital, $\overline{X}_j^\tau/V_j$, is a constant for all firms j in class k, independently of their financial structure. This implies that, if we were to take a sample of firms in a given class, and if for each firm we were to plot the ratio of expected return to market value against some measure of leverage or financial structure, the points would tend to fall on a horizontal straight line with intercept ρ_k^τ, like the solid line mm' in Figure 1.[22] From Proposition I we derived Proposition II [equation (8)] which, taking the simplest version with r constant, asserts that, for all firms in a class, the relation between the yield on common stock and financial structure, measured by D_j/S_j, will approximate a straight line with slope $(\rho_k^\tau - r)$ and intercept ρ_k^τ. This relationship is shown as the solid line MM' in Figure 2, to which reference has been made earlier.[23]

By contrast, the conventional view among finance specialists appears to start from the proposition that, other things equal, the earnings-price ratio (or its reciprocal, the times-earnings multiplier) of a firm's common stock will normally be only slightly affected by "moderate" amounts of debt in the firm's capital structure.[24] Translated into our no-

[22] In Figure 1 the measure of leverage used is D_j/V_j (the ratio of debt to market value) rather than D_j/S_j (the ratio of debt to equity), the concept used in the analytical development. The D_j/V_j measure is introduced at this point because it simplifies comparison and contrast of our view with the traditional position.

[23] The line MM' in Figure 2 has been drawn with a positive slope on the assumption that $\rho_k^\tau > r$, a condition which will normally obtain. Our Proposition II as given in equation (8) would continue to be valid, of course, even in the unlikely event that $\rho_k^\tau < r$, but the slope of MM' would be negative.

[24] See, *e.g.*, Graham and Dodd [6, pp. 464–66]. Without doing violence to this position, we can bring out its implications more sharply by ignoring the qualification and treating the yield as a virtual constant over the relevant range. See in this connection the discussion in Durand [3, esp. pp. 225–37] of what he calls the "net income method" of valuation.

tation, it asserts that for any firm j in the class k,

$$\frac{\bar{X}_j^\tau - rD_j}{S_j} \equiv \frac{\bar{\pi}_j^\tau}{S_j} = i_k^*, \text{ a constant for } \frac{D_j}{S_j} \leq L_k \tag{13}$$

or, equivalently,

$$S_j = \bar{\pi}_j^\tau / i_k^*. \tag{14}$$

Here i_k^* represents the capitalization rate or earnings-price ratio on the common stock and L_k denotes some amount of leverage regarded as the maximum "reasonable" amount for firms of the class k. This assumed relationship between yield and leverage is the horizontal solid line ML' of Figure 2. Beyond L', the yield will presumably rise sharply as the market discounts "excessive" trading on the equity. This possibility of a rising range for high leverages is indicated by the broken-line segment $L'G$ in the figure.[25]

If the value of shares were really given by (14) then the over-all market value of the firm must be:

$$V_j \equiv S_j + D_j = \frac{\bar{X}_j^\tau - rD_j}{i_k^*} + D_j = \frac{\bar{X}_j^\tau}{i_k^*} + \frac{(i_k^* - r)D_j}{i_k^*}. \tag{16}$$

That is, for any given level of expected total returns after taxes ($\bar{X}_j^\tau$) and assuming, as seems natural, that $i_k^* > r$, the value of the firm must tend to *rise* with debt;[26] whereas our Proposition I asserts that the value of the firm is completely independent of the capital structure. Another way of contrasting our position with the traditional one is in terms of the cost of capital. Solving (16) for $\bar{X}_j^\tau / V_j$ yields:

$$\bar{X}_j^\tau / V_j = i_k^* - (i_k^* - r) D_j / V_j. \tag{17}$$

According to this equation, the average cost of capital is not independent of capital structure as we have argued, but should tend to *fall* with increasing leverage, at least within the relevant range of moderate debt ratios, as shown by the line *ms* in Figure 1. Or to put it in more familiar terms, debt-financing should be "cheaper" than equity-financing if not carried too far.

When we also allow for the possibility of a rising range of stock yields for large values of leverage, we obtain a U-shaped curve like *mst* in

[25] To make it easier to see some of the implications of this hypothesis as well as to prepare the ground for later statistical testing, it will be helpful to assume that the notion of a critical limit on leverage beyond which yields rise rapidly, can be epitomized by a quadratic relation of the form:

$$\bar{\pi}_j^\tau / S_j = i_k^* + \beta(D_j/S_j) + \alpha(D_j/S_j)^2, \qquad \alpha > 0. \tag{15}$$

[26] For a typical discussion of how a promoter can, supposedly, increase the market value of a firm by recourse to debt issues, see W. J. Eiteman [4, esp. pp. 11–13].

Figure 1.[27] That a yield-curve for stocks of the form $ML'G$ in Figure 2 implies a U-shaped cost-of-capital curve has, of course, been recognized by many writers. A natural further step has been to suggest that the capital structure corresponding to the trough of the U is an "optimal capital structure" towards which management ought to strive in the best interests of the stockholders.[28] According to our model, by contrast, no such optimal structure exists—all structures being equivalent from the point of view of the cost of capital.

Although the falling, or at least U-shaped, cost-of-capital function is in one form or another the dominant view in the literature, the ultimate rationale of that view is by no means clear. The crucial element in the position—that the expected earnings-price ratio of the stock is largely unaffected by leverage up to some conventional limit—is rarely even regarded as something which requires explanation. It is usually simply taken for granted or it is merely asserted that this is the way the market behaves.[29] To the extent that the constant earnings-price ratio has a rationale at all we suspect that it reflects in most cases the feeling that moderate amounts of debt in "sound" corporations do not really add very much to the "riskiness" of the stock. Since the extra risk is slight, it seems natural to suppose that firms will not have to pay noticeably higher yields in order to induce investors to hold the stock.[30]

A more sophisticated line of argument has been advanced by David Durand [3, pp. 231-33]. He suggests that because insurance companies and certain other important institutional investors are restricted to debt securities, nonfinancial corporations are able to borrow from them at interest rates which are lower than would be required to compensate

[27] The U-shaped nature of the cost-of-capital curve can be exhibited explicitly if the yield curve for shares as a function of leverage can be approximated by equation (15) of footnote 25. From that equation, multiplying both sides by S_j we obtain: $\bar{\pi}_j^\tau = \bar{X}_j^\tau - rD_j = i_k^* S_j + \beta D_j + \alpha D_j^2 / S_j$ or, adding and subtracting $i_k^* D_k$ from the right-hand side and collecting terms,

$$\bar{X}_j^\tau = i_k^*(S_j + D_j) + (\beta + r - i_k^*)D_j + \alpha D^2{}_j/S_j. \tag{18}$$

Dividing (18) by V_j gives an expression for the cost of capital:

$$\bar{X}_j^\tau/V_j = i_k^* - (i_k^* - r - \beta)D_j/V_j + \alpha D_j^2/S_jV_j = i_k^* - (i_k^* - r - \beta)D_j/V_j + \alpha(D_j/V_j)^2/(1 - D_j/V_j) \tag{19}$$

which is clearly U-shaped since α is supposed to be positive.

[28] For a typical statement see S. M. Robbins [16, p. 307]. See also Graham and Dodd [6, pp. 468–74].

[29] See *e.g.*, Graham and Dodd [6, p. 466].

[30] A typical statement is the following by Guthmann and Dougall [7, p. 245]: "Theoretically it might be argued that the increased hazard from using bonds and preferred stocks would counterbalance this additional income and so prevent the common stock from being more attractive than when it had a lower return but fewer prior obligations. In practice, the extra earnings from 'trading on the equity' are often regarded by investors as more than sufficient to serve as a 'premium for risk' when the proportions of the several securities are judiciously mixed."

creditors in a free market. Thus, while he would presumably agree with our conclusions that stockholders could not gain from leverage in an unconstrained market, he concludes that they can gain under present institutional arrangements. This gain would arise by virtue of the "safety superpremium" which lenders are willing to pay corporations for the privilege of lending.[31]

The defective link in both the traditional and the Durand version of the argument lies in the confusion between investors' subjective risk preferences and their objective market opportunities. Our Propositions I and II, as noted earlier, do not depend for their validity on any assumption about individual risk preferences. Nor do they involve any assertion as to what is an adequate compensation to investors for assuming a given degree of risk. They rely merely on the fact that a given commodity cannot consistently sell at more than one price in the market; or more precisely that the price of a commodity representing a "bundle" of two other commodities cannot be consistently different from the weighted average of the prices of the two components (the weights being equal to the proportion of the two commodities in the bundle).

An analogy may he helpful at this point. The relations between $1/\rho_k$, the price per dollar of an unlevered stream in class k; $1/r$, the price per dollar of a sure stream, and $1/i_j$, the price per dollar of a levered stream j, in the kth class, are essentially the same as those between, respectively, the price of whole milk, the price of butter fat, and the price of milk which has been thinned out by skimming off some of the butter fat. Our Proposition I states that a firm cannot reduce the cost of capital—*i.e.*, increase the market value of the stream it generates—by securing part of its capital through the sale of bonds, even though debt money appears to be cheaper. This assertion is equivalent to the proposition that, under perfect markets, a dairy farmer cannot in general earn more for the milk he produces by skimming some of the butter fat and selling it separately, even though butter fat per unit weight, sells for more than whole milk. The advantage from skimming the milk rather than selling whole milk would be purely illusory; for what would be gained from selling the high-priced butter fat would be lost in selling the low-priced residue of thinned milk. Similarly our Proposition II—that the price per dollar of a levered stream falls as leverage increases—is an ex-

[31] Like Durand, Morton [15] contends "that the actual market deviates from [Proposition I] by giving a changing over-all cost of money at different points of the [leverage] scale" (p. 443, note 2, inserts ours), but the basis for this contention is nowhere clearly stated. Judging by the great emphasis given to the lack of mobility of investment funds between stocks and bonds and to the psychological and institutional pressures toward debt portfolios (see pp. 444–51 and especially his discussion of the optimal capital structure on p. 453) he would seem to be taking a position very similar to that of Durand above.

act analogue of the statement that the price per gallon of thinned milk falls continuously as more butter fat is skimmed off.[32]

It is clear that this last assertion is true as long as butter fat is worth more per unit weight than whole milk, and it holds even if, for many consumers, taking a little cream out of the milk (adding a little leverage to the stock) does not detract noticeably from the taste (does not add noticeably to the risk). Furthermore the argument remains valid even in the face of instituional limitations of the type envisaged by Durand. For suppose that a large fraction of the population habitually dines in restaurants which are required by law to serve only cream in lieu of milk (entrust their savings to institutional investors who can only buy bonds). To be sure the price of butter fat will then tend to be higher in relation to that of skimmed milk than in the absence such restrictions (the rate of interest will tend to be lower), and this will benefit people who eat at home and who like skim milk (who manage their own portfolio and are able and willing to take risk). But it will still be the case that a farmer cannot gain by skimming some of the butter fat and selling it separately (firm cannot reduce the cost of capital by recourse to borrowed funds).[33]

Our propositions can be regarded as the extension of the classical theory of markets to the particular case of the capital markets. Those who hold the current view—whether they realize it or not—must as-

[32] Let M denote the quantity of whole milk, B/M the proportion of butter fat in the whole milk, and let p_M, p_B and p_α denote, respectively, the price per unit weight of whole milk, butter fat and thinned milk from which a fraction α of the butter fat has been skimmed off. We then have the fundamental perfect market relation:

$$\text{(a)} \qquad p_\alpha(M - \alpha B) + p_B \alpha B = p_M M, \qquad 0 \leq \alpha \leq 1,$$

stating that total receipts will be the same amount $p_M M$, independently of the amount αB of butter fat that may have been sold separately. Since p_M corresponds to $1/\rho$, p_B to $1/r$, p_α to $1/i$, M to $\bar{X}$ and αB to rD, (a) is equivalent to Proposition I, $S+D=\bar{X}/\rho$. From (a) we derive:

$$\text{(b)} \qquad p_\alpha = p_M \frac{M}{M - \alpha B} - p_B \frac{\alpha B}{M - \alpha B}$$

which gives the price of thinned milk as an explicit function of the proportion of butter fat skimmed off; the function decreasing as long as $p_B > p_M$. From (a) also follows:

$$\text{(c)} \qquad 1/p_\alpha = 1/p_M + (1/p_M - 1/p_B) \frac{p_B \alpha B}{p_\alpha(M - \alpha B)}$$

which is the exact analogue of Proposition II, as given by (8).

[33] The reader who likes parables will find that the analogy with interrelated commodity markets can be pushed a good deal farther than we have done in the text. For instance, the effect of changes in the market rate of interest on the over-all cost of capital is the same as the effect of a change in the price of butter on the price of whole milk. Similarly, just as the relation between the prices of skim milk and butter fat influences the kind of cows that will be reared, so the relation between i and r influences the kind of ventures that will be undertaken. If people like butter we shall have Guernseys; if they are willing to pay a high price for safety, this will encourage ventures which promise smaller but less uncertain streams per dollar of physical assets.

sume not merely that there are lags and frictions in the equilibrating process—a feeling we certainly share,[34] claiming for our propositions only that they describe the central tendency around which observations will scatter—but also that there are large and *systematic* imperfections in the market which permanently bias the outcome. This is an assumption that economists, at any rate, will instinctively eye with some skepticism.

In any event, whether such prolonged, systematic departures from equilibrium really exist or whether our propositions are better descriptions of long-run market behavior can be settled only by empirical research. Before going on to the theory of investment it may be helpful, therefore, to look at the evidence.

E. *Some Preliminary Evidence on the Basic Propositions*

Unfortunately the evidence which has been assembled so far is amazingly skimpy. Indeed, we have been able to locate only two recent studies—and these of rather limited scope—which were designed to throw light on the issue. Pending the results of more comprehensive tests which we hope will soon be available, we shall review briefly such evidence as is provided by the two studies in question: (1) an analysis of the relation between security yields and financial structure for some 43 large electric utilities by F. B. Allen [1], and (2) a parallel (unpublished) study by Robert Smith [19], for 42 oil companies designed to test whether Allen's rather striking results would be found in an industry with very different characteristics.[35] The Allen study is based on average figures for the years 1947 and 1948, while the Smith study relates to the single year 1953.

The Effect of Leverage on the Cost of Capital. According to the received view, as shown in equation (17) the average cost of capital, $\overline{X}^{\tau}/V$, should decline linearly with leverage as measured by the ratio D/V, at least through most of the relevant range.[36] According to Proposition I, the average cost of capital within a given class k should tend to have the same value ρ_k^{τ} independently of the degree of leverage. A simple test

[34] Several specific examples of the failure of the arbitrage mechanism can be found in Graham and Dodd [6, *e.g.*, pp. 646–48]. The price discrepancy described on pp. 646–47 is particularly curious since it persists even today despite the fact that a whole generation of security analysts has been brought up on this book!

[35] We wish to express our thanks to both writers for making available to us some of their original worksheets. In addition to these recent studies there is a frequently cited (but apparently seldom read) study by the Federal Communications Commission in 1938 [22] which purports to show the existence of an optimal capital structure or range of structures (in the sense defined above) for public utilities in the 1930's. By current standards for statistical investigations, however, this study cannot be regarded as having any real evidential value for the problem at hand.

[36] We shall simplify our notation in this section by dropping the subscript j used to denote a particular firm wherever this will not lead to confusion.

of the merits of the two alternative hypotheses can thus be carried out by correlating $\overline{X}^{\tau}/V$ with D/V. If the traditional view is correct, the correlation should be significantly negative; if our view represents a better approximation to reality, then the correlation should not be significantly different from zero.

Both studies provide information about the average value of D—the market value of bonds and preferred stock—and of V—the market value of all securities.[37] From these data we can readily compute the ratio D/V and this ratio (expressed as a percentage) is represented by the symbol d in the regression equations below. The measurement of the variable $\overline{X}^{\tau}/V$, however, presents serious difficulties. Strictly speaking, the numerator should measure the expected returns net of taxes, but this is a variable on which no direct information is available. As an approximation, we have followed both authors and used (1) the average value of actual net returns in 1947 and 1948 for Allen's utilities; and (2) actual net returns in 1953 for Smith's oil companies. Net return is defined in both cases as the sum of interest, preferred dividends and stockholders' income net of corporate income taxes. Although this approximation to expected returns is undoubtedly very crude, there is no reason to believe that it will systematically bias the test in so far as the sign of the regression coefficient is concerned. The roughness of the approximation, however, will tend to make for a wide scatter. Also contributing to the scatter is the crudeness of the industrial classification, since especially within the sample of oil companies, the assumption that all the firms belong to the same class in our sense, is at best only approximately valid.

Denoting by x our approximation to $\overline{X}^{\tau}/V$ (expressed, like d, as a percentage), the results of the tests are as follows:

$$\text{Electric Utilities} \quad x = 5.3 + \underset{(\pm .008)}{.006d} \qquad r = .12$$

$$\text{Oil Companies} \quad x = 8.5 + \underset{(\pm .024)}{.006d} \qquad r = .04.$$

The data underlying these equations are also shown in scatter diagram form in Figures 3 and 4.

The results of these tests are clearly favorable to our hypothesis.

[37] Note that for purposes of this test preferred stocks, since they represent an *expected* fixed obligation, are properly classified with bonds even though the tax status of preferred dividends is different from that of interest payments and even though preferred dividends are really fixed only as to their maximum in any year. Some difficulty of classification does arise in the case of convertible preferred stocks (and convertible bonds) selling at a substantial premium, but fortunately very few such issues were involved for the companies included in the two studies. Smith included bank loans and certain other short-term obligations (at book values) in his data on oil company debts and this treatment is perhaps open to some question. However, the amounts involved were relatively small and check computations showed that their elimination would lead to only minor differences in the test results.

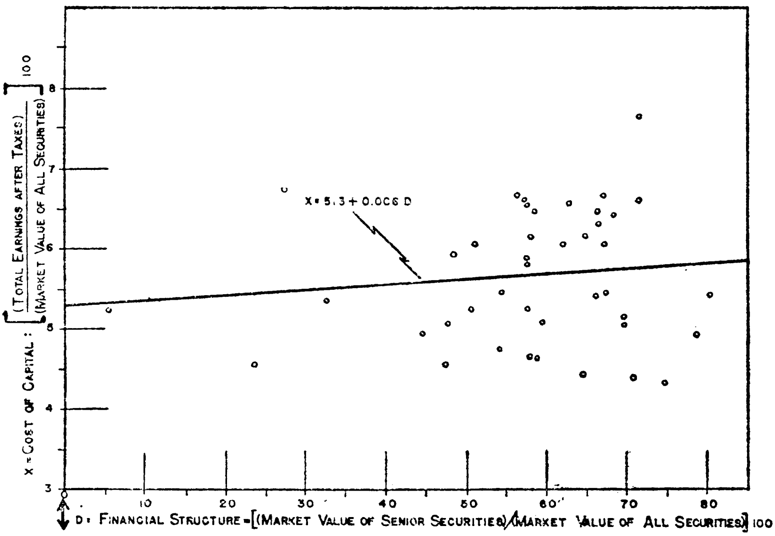

FIGURE 3. COST OF CAPITAL IN RELATION TO FINANCIAL STRUCTURE FOR 43 ELECTRIC UTILITIES, 1947–48

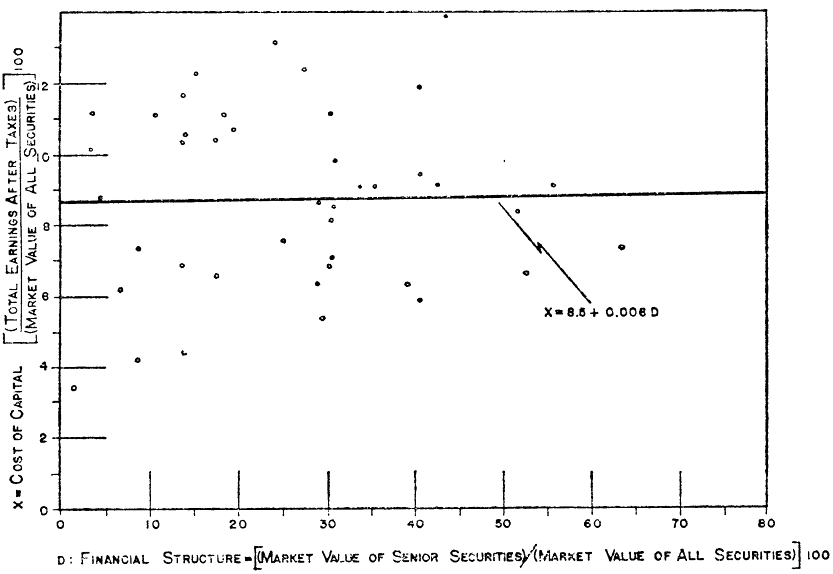

FIGURE 4. COST OF CAPITAL IN RELATION TO FINANCIAL STRUCTURE FOR 42 OIL COMPANIES, 1953

Both correlation coefficients are very close to zero and not statistically significant. Furthermore, the implications of the traditional view fail to be supported even with respect to the sign of the correlation. The data in short provide no evidence of any tendency for the cost of capital to fall as the debt ratio increases.[38]

It should also be apparent from the scatter diagrams that there is no hint of a curvilinear, U-shaped, relation of the kind which is widely believed to hold between the cost of capital and leverage. This graphical impression was confirmed by statistical tests which showed that for both industries the curvature was not significantly different from zero, its sign actually being opposite to that hypothesized.[39]

Note also that according to our model, the constant terms of the regression equations are measures of ρ_k^τ, the capitalization rates for unlevered streams and hence the average cost of capital in the classes in question. The estimates of 8.5 per cent for the oil companies as against 5.3 per cent for electric utilities appear to accord well with a priori expectations, both in absolute value and relative spread.

The Effect of Leverage on Common Stock Yields. According to our Proposition II—see equation 12 and Figure 2—the expected yield on common stock, $\bar{\pi}^\tau/S$, in any given class, should tend to increase with leverage as measured by the ratio D/S. The relation should tend to be linear and with positive slope through most of the relevant range (as in the curve MM' of Figure 2), though it might tend to flatten out if we move

[38] It may be argued that a test of the kind used is biased against the traditional view. The fact that both sides of the regression equation are divided by the variable V which may be subject to random variation might tend to impart a positive bias to the correlation. As a check on the results presented in the text, we have, therefore, carried out a supplementary test based on equation (16). This equation shows that, if the traditional view is correct, the market value of a company should, foı given $\bar{X}^\tau$, increase with debt through most of the relevant range; according to our model the market value should be uncorrelated with D, given $\bar{X}^\tau$. Because of wide variations in the size of the firms included in our samples, all variables must be divided by a suitable scale factor in order to avoid spurious results in carrying out a test of equation (16). The factor we have used is the book value of the firm denoted by A. The hypothesis tested thus takes the specific form:

$$V/A = a + b(\bar{X}^\tau/A) + c(D/A)$$

and the numerator of the ratio X^τ/A is again approximated by actual net returns. The partial correlation between V/A and D/A should now be positive according to the traditional view and zero according to our model. Although division by A should, if anything, bias the results in favor of the traditional hypothesis, the partial correlation turns out to be only .03 for the oil companies and −.28 for the electric utilities. Neither of these coefficients is significantly different from zero and the larger one even has the wrong sign.

[39] The tests consisted of fitting to the data the equation (19) of footnote 27. As shown there, it follows from the U-shaped hypothesis that the coefficient α of the variable $(D/V)^2/(1-D/V)$, denoted hereafter by d^*, should be significant and positive. The following regression equations and partials were obtained:

Electric Utilities $x = 5.0 + .017d - .003d^*;\ r_{xd^*.d} = -.15$

Oil Companies $x = 8.0 + .05d - .03d^*;\ r_{xd^*.d} = -.14.$

far enough to the right (as in the curve MD'), to the extent that high leverage tends to drive up the cost of senior capital. According to the conventional view, the yield curve as a function of leverage should be a horizontal straight line (like ML') through most of the relevant range; far enough to the right, the yield may tend to rise at an increasing rate. Here again, a straight-forward correlation—in this case between $\bar{\pi}^\tau/S$ and D/S—can provide a test of the two positions. If our view is correct, the correlation should be significantly positive; if the traditional view is correct, the correlation should be negligible.

Subject to the same qualifications noted above in connection with $\bar{X}^\tau$, we can approximate $\bar{\pi}^\tau$ by actual stockholder net income.[40] Letting z denote in each case the approximation to $\bar{\pi}^\tau/S$ (expressed as a percentage) and letting h denote the ratio D/S (also in percentage terms) the following results are obtained:

$$\text{Electric Utilities} \quad z = 6.6 + \underset{(+.004)}{.017h} \qquad r = .53$$

$$\text{Oil Companies} \quad z = 8.9 + \underset{(\pm .012)}{.051h} \qquad r = .53.$$

These results are shown in scatter diagram form in Figures 5 and 6.

Here again the implications of our analysis seem to be borne out by the data. Both correlation coefficients are positive and highly significant when account is taken of the substantial sample size. Furthermore, the estimates of the coefficients of the equations seem to accord reasonably well with our hypothesis. According to equation (12) the constant term should be the value of $\rho_k{}^\tau$ for the given class while the slope should be $(\rho_k{}^\tau - r)$. From the test of Proposition I we have seen that for the oil companies the mean value of $\rho_k{}^\tau$ could be estimated at around 8.7. Since the average yield of senior capital during the period covered was in the order of $3\frac{1}{2}$ per cent, we should expect a constant term of about 8.7 per cent and a slope of just over 5 per cent. These values closely approximate the regression estimates of 8.9 per cent and 5.1 per cent respectively. For the electric utilities, the yield of senior capital was also on the order of $3\frac{1}{2}$ per cent during the test years, but since the estimate of the mean value of $\rho_k{}^\tau$ from the test of Proposition I was 5.6 per cent,

[40] As indicated earlier, Smith's data were for the single year 1953. Since the use of a single year's profits as a measure of expected profits might be open to objection we collected profit data for 1952 for the same companies and based the computation of $\bar{\pi}^\tau/S$ on the average of the two years. The value of $\bar{\pi}^\tau/S$ was obtained from the formula:

$$\left(\text{net earnings in } 1952 \cdot \frac{\text{assets in '53}}{\text{assets in '52}} + \text{net earnings in '1953}\right)\frac{1}{2}$$
$$\div \text{ (average market value of common stock in '53).}$$

The asset adjustment was introduced as rough allowance for the effects of possible growth in the size of the firm. It might be added that the correlation computed with $\bar{\pi}^\tau/S$ based on net profits in 1953 alone was found to be only slightly smaller, namely .50.

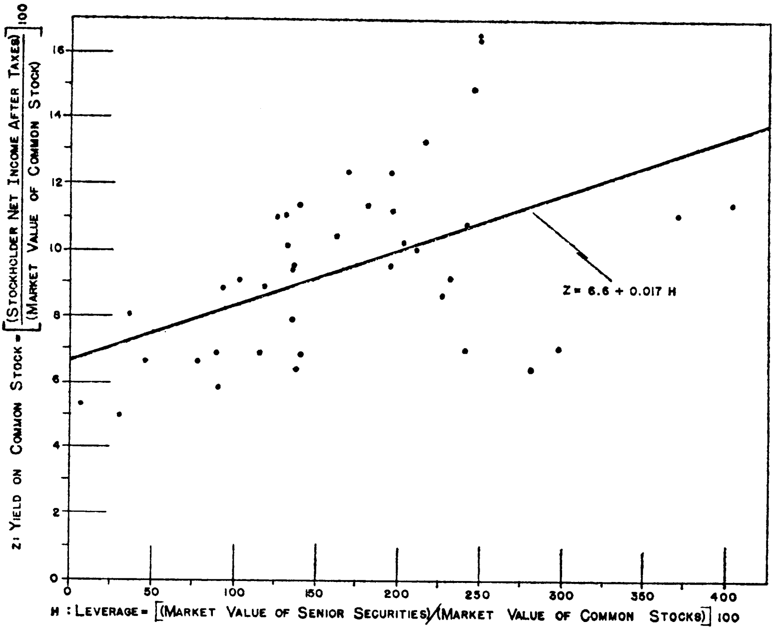

FIGURE 5. YIELD ON COMMON STOCK IN RELATION TO LEVERAGE FOR 43 ELECTRIC UTILITIES, 1947–48

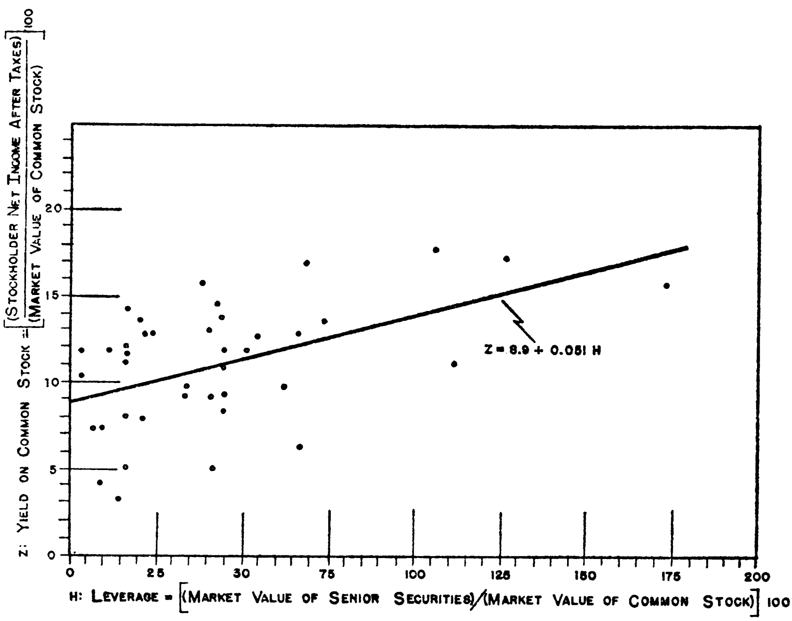

FIGURE 6. YIELD ON COMMON STOCK IN RELATION TO LEVERAGE FOR 42 OIL COMPANIES, 1952–53

the slope should be just above 2 per cent. The actual regression estimate for the slope of 1.7 per cent is thus somewhat low, but still within one standard error of its theoretical value. Because of this underestimate of the slope and because of the large mean value of leverage ($\bar{h}=160$ per cent) the regression estimate of the constant term, 6.6 per cent, is somewhat high, although not significantly different from the value of 5.6 per cent obtained in the test of Proposition I.

When we add a square term to the above equations to test for the presence and direction of curvature we obtain the following estimates:

$$\text{Electric Utilities} \quad z = 4.6 + .004h - .007h^2$$

$$\text{Oil Companies} \quad z = 8.5 + .072h - .016h^2.$$

For both cases the curvature is negative. In fact, for the electric utilities, where the observations cover a wider range of leverage ratios, the negative coefficient of the square term is actually significant at the 5 per cent level. Negative curvature, as we have seen, runs directly counter to the traditional hypothesis, whereas it can be readily accounted for by our model in terms of rising cost of borrowed funds.[41]

In summary, the empirical evidence we have reviewed seems to be broadly consistent with our model and largely inconsistent with traditional views. Needless to say much more extensive testing will be required before we can firmly conclude that our theory describes market behavior. Caution is indicated especially with regard to our test of Proposition II, partly because of possible statistical pitfalls[42] and partly because not all the factors that might have a systematic effect on stock yields have been considered. In particular, no attempt was made to test the possible influence of the dividend pay-out ratio whose role has tended to receive a great deal of attention in current research and thinking. There are two reasons for this omission. First, our main objective has been to assess the prima facie tenability of *our* model, and in this model, based as it is on rational behavior by investors, dividends per se play no role. Second, in a world in which the policy of dividend stabilization is widespread, there is no simple way of disentangling the true effect of dividend payments on stock prices from their apparent effect,

[41] That the yield of senior capital tended to rise for utilities as leverage increased is clearly shown in several of the scatter diagrams presented in the published version of Allen's study. This significant negative curvature between stock yields and leverage for utilities may be partly responsible for the fact, previously noted, that the constant in the linear regression is somewhat higher and the slope somewhat lower than implied by equation (12). Note also in connection with the estimate of ρ_k^7 that the introduction of the quadratic term reduces the constant considerably, pushing it in fact below the a priori expectation of 5.6, though the difference is again not statistically significant.

[42] In our test, *e.g.*, the two variables z and h are both ratios with S appearing in the denominator, which may tend to impart a positive bias to the correlation (*cf.* note 38). Attempts were made to develop alternative tests, but although various possibilities were explored, we have so far been unable to find satisfactory alternatives.

the latter reflecting only the role of dividends as a proxy measure of long-term earning anticipations.[43] The difficulties just mentioned are further compounded by possible interrelations between dividend policy and leverage.[44]

II. *Implications of the Analysis for the Theory of Investment*

A. *Capital Structure and Investment Policy*

On the basis of our propositions with respect to cost of capital and financial structure (and for the moment neglecting taxes), we can derive the following simple rule for optimal investment policy by the firm:

Proposition III. If a firm in class k is acting in the best interest of the stockholders at the time of the decision, it will exploit an investment opportunity if and only if the rate of return on the investment, say ρ^*, is as large as or larger than ρ_k. That is, *the cut-off point for investment in the firm will in all cases be* ρ_k *and will be completely unaffected by the type of security used to finance the investment.* Equivalently, we may say that regardless of the financing used, the marginal cost of capital to a firm is equal to the average cost of capital, which is in turn equal to the capitalization rate for an unlevered stream in the class to which the firm belongs.[45]

To establish this result we will consider the three major financing alternatives open to the firm—bonds, retained earnings, and common stock issues—and show that in each case an investment is worth undertaking if, and only if, $\rho^* \geqq \rho_k$.[46]

Consider first the case of an investment financed by the sale of bonds. We know from Proposition I that the market value of the firm before the investment was undertaken was:[47]

$$V_0 = \overline{X}_0/\rho_k \tag{20}$$

[43] We suggest that failure to appreciate this difficulty is responsible for many fallacious, or at least unwarranted, conclusions about the role of dividends.

[44] In the sample of electric utilities, there is a substantial negative correlation between yields and pay-out ratios, but also between pay-out ratios and leverage, suggesting that either the association of yields and leverage or of yields and pay-out ratios may be (at least partly) spurious. These difficulties however do not arise in the case of the oil industry sample. A preliminary analysis indicates that there is here no significant relation between leverage and pay-out ratios and also no significant correlation (either gross or partial) between yields and pay-out ratios.

[45] The analysis developed in this paper is essentially a comparative-statics, not a dynamic analysis. This note of caution applies with special force to Proposition III. Such problems as those posed by expected changes in r and in ρ_k over time will not be treated here. Although they are in principle amenable to analysis within the general framework we have laid out, such an undertaking is sufficiently complex to deserve separate treatment. *Cf.* note 17.

[46] The extension of the proof to other types of financing, such as the sale of preferred stock or the issuance of stock rights is straightforward.

[47] Since no confusion is likely to arise, we have again, for simplicity, eliminated the subscripts identifying the firm in the equations to follow. Except for ρ_k, the subscripts now refer to time periods.

and that the value of the common stock was:

(21) $$S_0 = V_0 - D_0.$$

If now the firm borrows I dollars to finance an investment yielding ρ^* its market value will become:

(22) $$V_1 = \frac{\overline{X}_0 + \rho^* I}{\rho_k} = V_0 + \frac{\rho^* I}{\rho_k}$$

and the value of its common stock will be:

(23) $$S_1 = V_1 - (D_0 + I) = V_0 + \frac{\rho^* I}{\rho_k} - D_0 - I$$

or using equation 21,

(24) $$S_1 = S_0 + \frac{\rho^* I}{\rho_k} - I.$$

Hence $S_1 \gtreqless S_0$ as $\rho^* \gtreqless \rho_k$.[48]

To illustrate, suppose the capitalization rate for uncertain streams in the kth class is 10 per cent and the rate of interest is 4 per cent. Then if a given company had an expected income of 1,000 and if it were financed entirely by common stock we know from Proposition I that the market value of its stock would be 10,000. Assume now that the managers of the firm discover an investment opportunity which will require an outlay of 100 and which is expected to yield 8 per cent. At first sight this might appear to be a profitable opportunity since the expected return is double the interest cost. If, however, the management borrows the necessary 100 at 4 per cent, the total expected income of the company rises to 1,008 and the market value of the firm to 10,080. But the firm now will have 100 of bonds in its capital structure so that, paradoxically, the market value of the stock must actually be reduced from 10,000 to 9,980 as a consequence of this apparently profitable investment. Or, to put it another way, the gains from being able to tap cheap, borrowed funds are more than offset for the stockholders by the market's discounting of the stock for the added leverage assumed.

Consider next the case of retained earnings. Suppose that in the course of its operations the firm acquired I dollars of cash (without impairing

[48] In the case of bond-financing the rate of interest on bonds does not enter explicitly into the decision (assuming the firm borrows at the market rate of interest). This is true, moreover, given the conditions outlined in Section I.C, even though interest rates may be an increasing function of debt outstanding. To the extent that the firm borrowed at a rate other than the market rate the two I's in equation (24) would no longer be identical and an additional gain or loss, as the case might be, would accrue to the shareholders. It might also be noted in passing that permitting the two I's in (24) to take on different values provides a simple method for introducing underwriting expenses into the analysis.

the earning power of its assets). If the cash is distributed as a dividend to the stockholders their wealth W_0, after the distribution will be:

$$W_0 = S_0 + I = \frac{\overline{X}_0}{\rho_k} - D_0 + I \tag{25}$$

where $\overline{X}_0$ represents the expected return from the assets exclusive of the amount I in question. If however the funds are retained by the company and used to finance new assets whose expected rate of return is ρ^*, then the stockholders' wealth would become:

$$W_1 = S_1 = \frac{\overline{X}_0 + \rho^* I}{\rho_k} - D_0 = S_0 + \frac{\rho^* I}{\rho_k}\,. \tag{26}$$

Clearly $W_1 \gtreqless W_0$ as $\rho^* \gtreqless \rho_k$ so that an investment financed by retained earnings raises the net worth of the owners if and only if $\rho^* > \rho_k$.[49]

Consider finally, the case of common-stock financing. Let P_0 denote the current market price per share of stock and assume, for simplicity, that this price reflects currently expected earnings only, that is, it does not reflect any future increase in earnings as a result of the investment under consideration.[50] Then if N is the original number of shares, the price per share is:

$$P_0 = S_0/N \tag{27}$$

and the number of new shares, M, needed to finance an investment of I dollars is given by:

$$M = \frac{I}{P_0}\,. \tag{28}$$

As a result of the investment the market value of the stock becomes:

$$S_1 = \frac{\overline{X}_0 + \rho^* I}{\rho_k} - D_0 = S_0 + \frac{\rho^* I}{\rho_k} = NP_0 + \frac{\rho^* I}{\rho_k}$$

and the price per share:

$$P_1 = \frac{S_1}{N + M} = \frac{1}{N + M}\left[NP_0 + \frac{\rho^* I}{\rho_k}\right]. \tag{29}$$

[49] The conclusion that ρ_k is the cut-off point for investments financed from internal funds applies not only to undistributed net profits, but to depreciation allowances (and even to the funds represented by the current sale value of any asset or collection of assets). Since the owners can earn ρ_k by investing funds elsewhere in the class, partial or total liquidating distributions should be made whenever the firm cannot achieve a marginal internal rate of return equal to ρ_k.

[50] If we assumed that the market price of the stock did reflect the expected higher future earnings (as would be the case if our original set of assumptions above were strictly followed) the analysis would differ slightly in detail, but not in essentials. The cut-off point for new investment would still be ρ_k, but where $\rho^* > \rho_k$ the gain to the original owners would be larger than if the stock price were based on the pre-investment expectations only.

Since by equation (28), $I = MP_0$, we can add MP_0 and subtract I from the quantity in bracket, obtaining:

$$(30)\qquad \begin{aligned} P_1 &= \frac{1}{N+M}\left[(N+M)P_0 + \frac{\rho^* - \rho_k}{\rho_k} I\right] \\ &= P_0 + \frac{1}{N+M}\,\frac{\rho^* - \rho_k}{\rho_k} I > P_0 \text{ if,} \end{aligned}$$

and only if, $\rho^* > \rho_k$.

Thus an investment financed by common stock is advantageous to the current stockholders if and only if its yield exceeds the capitalization rate ρ_k.

Once again a numerical example may help to illustrate the result and make it clear why the relevant cut-off rate is ρ_k and not the current yield on common stock, i. Suppose that ρ_k is 10 per cent, r is 4 per cent, that the original expected income of our company is 1,000 and that management has the opportunity of investing 100 having an expected yield of 12 per cent. If the original capital structure is 50 per cent debt and 50 per cent equity, and 1,000 shares of stock are initially outstanding, then, by Proposition I, the market value of the common stock must be 5,000 or 5 per share. Furthermore, since the interest bill is $.04 \times 5{,}000 = 200$, the yield on common stock is $800/5{,}000 = 16$ per cent. It may then appear that financing the additional investment of 100 by issuing 20 shares to outsiders at 5 per share would dilute the equity of the original owners since the 100 promises to yield 12 per cent whereas the common stock is currently yielding 16 per cent. Actually, however, the income of the company would rise to 1,012; the value of the firm to 10,120; and the value of the common stock to 5,120. Since there are now 1,020 shares, each would be worth 5.02 and the wealth of the original stockholders would thus have been increased. What has happened is that the dilution in expected earnings per share (from .80 to .796) has been more than offset, in its effect upon the market price of the shares, by the decrease in leverage.

Our conclusion is, once again, at variance with conventional views,[51] so much so as to be easily misinterpreted. Read hastily, Proposition III seems to imply that the capital structure of a firm is a matter of indifference; and that, consequently, one of the core problems of corporate finance—the problem of the optimal capital structure for a firm—is no problem at all. It may be helpful, therefore, to clear up such possible misundertandings.

[51] In the matter of investment policy under uncertainty there is no single position which represents "accepted" doctrine. For a sample of current formulations, all very different from ours, see Joel Dean [2, esp. Ch. 3], M. Gordon and E. Shapiro [5], and Harry Roberts [17].

B. *Proposition III and Financial Planning by Firms*

Misinterpretation of the scope of Proposition III can be avoided by remembering that this Proposition tells us only that the type of instrument used to finance an investment is irrelevant to the question of whether or not the investment is worth while. This does not mean that the owners (or the managers) have no grounds whatever for preferring one financing plan to another; or that there are no other policy or technical issues in finance at the level of the firm.

That grounds for preferring one type of financial structure to another will still exist within the framework of our model can readily be seen for the case of common-stock financing. In general, except for something like a widely publicized oil-strike, we would expect the market to place very heavy weight on current and recent past earnings in forming expectations as to future returns. Hence, if the owners of a firm discovered a major investment opportunity which they felt would yield much more than ρ_k, they might well prefer not to finance it via common stock at the then ruling price, because this price may fail to capitalize the new venture. A better course would be a pre-emptive issue of stock (and in this connection it should be remembered that stockholders are free to borrow and buy). Another possibility would be to finance the project initially with debt. Once the project had reflected itself in increased actual earnings, the debt could be retired either with an equity issue at much better prices or through retained earnings. Still another possibility along the same lines might be to combine the two steps by means of a convertible debenture or preferred stock, perhaps with a progressively declining conversion rate. Even such a double-stage financing plan may possibly be regarded as yielding too large a share to outsiders since the new stockholders are, in effect, being given an interest in any similar opportunities the firm may discover in the future. If there is a reasonable prospect that even larger opportunities may arise in the near future and if there is some danger that borrowing now would preclude more borrowing later, the owners might find their interests best protected by splitting off the current opportunity into a separate subsidiary with independent financing. Clearly the problems involved in making the crucial estimates and in planning the optimal financial strategy are by no means trivial, even though they should have no bearing on the basic decision to invest (as long as $\rho^* \geqq \rho_k$).[52]

Another reason why the alternatives in financial plans may not be a matter of indifference arises from the fact that managers are concerned

[52] Nor can we rule out the possibility that the existing owners, if unable to use a financing plan which protects their interest, may actually prefer to pass up an otherwise profitable venture rather than give outsiders an "excessive" share of the business. It is presumably in situations of this kind that we could justifiably speak of a shortage of "equity capital," though this kind of market imperfection is likely to be of significance only for small or new firms.

with more than simply furthering the interest of the owners. Such other objectives of the management—which need not be necessarily in conflict with those of the owners—are much more likely to be served by some types of financing arrangements than others. In many forms of borrowing agreements, for example, creditors are able to stipulate terms which the current management may regard as infringing on its prerogatives or restricting its freedom to maneuver. The creditors might even be able to insist on having a direct voice in the formation of policy.[53] To the extent, therefore, that financial policies have these implications for the management of the firm, something like the utility approach described in the introductory section becomes relevant to financial (as opposed to investment) decision-making. It is, however, the utility functions of the managers per se and not of the owners that are now involved.[54]

In summary, many of the specific considerations which bulk so large in traditional discussions of corporate finance can readily be superimposed on our simple framework without forcing any drastic (and certainly no systematic) alteration of the conclusion which is our principal concern, namely that for investment decisions, the marginal cost of capital is ρ_k.

C. *The Effect of the Corporate Income Tax on Investment Decisions*

In Section I it was shown that when an unintegrated corporate income tax is introduced, the original version of our Proposition I,

$$\overline{X}/V = \rho_k = \text{a constant}$$

must be rewritten as:

$$\frac{(\overline{X} - rD)(1 - \tau) + rD}{V} \equiv \frac{\overline{X}^\tau}{V} = \rho_k^\tau = \text{a constant.} \tag{11}$$

Throughout Section I we found it convenient to refer to $\overline{X}^\tau/V$ as the cost of capital. The appropriate measure of the cost of capital relevant

[53] Similar considerations are involved in the matter of dividend policy. Even though the stockholders may be indifferent as to payout policy as long as investment policy is optimal, the management need not be so. Retained earnings involve far fewer threats to control than any of the alternative sources of funds and, of course, involve no underwriting expense or risk. But against these advantages management must balance the fact that sharp changes in dividend rates, which heavy reliance on retained earnings might imply, may give the impression that a firm's finances are being poorly managed, with consequent threats to the control and professional standing of the management.

[54] In principle, at least, this introduction of management's risk preferences with respect to financing methods would do much to reconcile the apparent conflict between Proposition III and such empirical findings as those of Modigliani and Zeman [14] on the close relation between interest rates and the ratio of new debt to new equity issues; or of John Lintner [12] on the considerable stability in target and actual dividend-payout ratios.

to investment decisions, however, is the ratio of the expected return *before* taxes to the market value, *i.e.*, $\bar{X}/V$. From (11) above we find:

$$\frac{\bar{X}}{V} = \frac{\rho_k^\tau - \tau_r(D/V)}{1-\tau} = \frac{\rho_k^\tau}{1-\tau}\left[1 - \frac{\tau r D}{\rho_k^\tau V}\right], \tag{31}$$

which shows that the cost of capital now depends on the debt ratio, decreasing, as D/V rises, at the constant rate $\tau r/(1-\tau)$.[55] Thus, with a corporate income tax under which interest is a deductible expense, gains can accrue to stockholders from having debt in the capital structure, even when capital markets are perfect. The gains however are small, as can be seen from (31), and as will be shown more explicitly below.

From (31) we can develop the tax-adjusted counterpart of Proposition III by interpreting the term D/V in that equation as the proportion of debt used in any additional financing of V dollars. For example, in the case where the financing is entirely by new common stock, $D=0$ and the required rate of return ρ_k^S on a venture so financed becomes:

$$\rho_k^S = \frac{\rho_k^\tau}{1-\tau}. \tag{32}$$

For the other extreme of pure debt financing $D=V$ and the required rate of return, ρ_k^D, becomes:

$$\rho_k^D = \frac{\rho_k^\tau}{1-\tau}\left[1 - \tau\frac{r}{\rho_k^\tau}\right] = \rho_k^S\left[1 - \tau\frac{r}{\rho_k^\tau}\right] = \rho_k^S - \frac{\tau}{1-\tau}r.^{56} \tag{33}$$

For investments financed out of retained earnings, the problem of defining the required rate of return is more difficult since it involves a comparison of the tax consequences to the individual stockholder of receiving a dividend versus having a capital gain. Depending on the time of realization, a capital gain produced by retained earnings may be taxed either at ordinary income tax rates, 50 per cent of these rates, 25 per

[55] Equation (31) is amenable, in principle, to statistical tests similar to those described in Section I.E. However we have not made any systematic attempt to carry out such tests so far, because neither the Allen nor the Smith study provides the required information. Actually, Smith's data included a very crude estimate of tax liability, and, using this estimate, we did in fact obtain a negative relation between $\bar{X}/V$ and D/V. However, the correlation (−.28) turned out to be significant only at about the 10 per cent level. While this result is not conclusive, it should be remembered that, according to our theory, the slope of the regression equation should be in any event quite small. In fact, with a value of τ in the order of .5, and values of ρ_k^τ and r in the order of 8.5 and 3.5 per cent respectively (*cf.* Section I.E) an increase in D/V from 0 to 60 per cent (which is, approximately, the range of variation of this variable in the sample) should tend to reduce the average cost of capital only from about 17 to about 15 per cent.

[56] This conclusion does not extend to preferred stocks even though they have been classed with debt issues previously. Since preferred dividends except for a portion of those of public utilities are not in general deductible from the corporate tax, the cut-off point for new financing via preferred stock is exactly the same as that for common stock.

cent, or zero, if held till death. The rate on any dividends received in the event of a distribution will also be a variable depending on the amount of other income received by the stockholder, and with the added complications introduced by the current dividend-credit provisions. If we assume that the managers proceed on the basis of reasonable estimates as to the average values of the relevant tax rates for the owners, then the required return for retained earnings ρ_k^R can be shown to be:

$$\rho_k^R = \rho_k^\tau \frac{1}{1-\tau} \frac{1-\tau_d}{1-\tau_g} = \frac{1-\tau_d}{1-\tau_g} \rho_k^s \tag{34}$$

where τ_d is the assumed rate of personal income tax on dividends and τ_g is the assumed rate of tax on capital gains.

A numerical illustration may perhaps be helpful in clarifying the relationship between these required rates of return. If we take the following round numbers as representative order-of-magnitude values under present conditions: an after-tax capitalization rate ρ_k^τ of 10 per cent, a rate of interest on bonds of 4 per cent, a corporate tax rate of 50 per cent, a marginal personal income tax rate on dividends of 40 per cent (corresponding to an income of about $25,000 on a joint return), and a capital gains rate of 20 per cent (one-half the marginal rate on dividends), then the required rates of return would be: (1) 20 per cent for investments financed entirely by issuance of new common shares; (2) 16 per cent for investments financed entirely by new debt; and (3) 15 per cent for investments financed wholly from internal funds.

These results would seem to have considerable significance for current discussions of the effect of the corporate income tax on financial policy and on investment. Although we cannot explore the implications of the results in any detail here, we should at least like to call attention to the remarkably small difference between the "cost" of equity funds and debt funds. With the numerical values assumed, equity money turned out to be only 25 per cent more expensive than debt money, rather than something on the order of 5 times as expensive as is commonly supposed to be the case.[57] The reason for the wide difference is that the traditional

[57] See *e.g.*, D. T. Smith [18]. It should also be pointed out that our tax system acts in other ways to reduce the gains from debt financing. Heavy reliance on debt in the capital structure, for example, commits a company to paying out a substantial proportion of its income in the form of interest payments taxable to the owners under the personal income tax. A debt-free company, by contrast, can reinvest in the business all of its (smaller) net income and to this extent subject the owners only to the low capital gains rate (or possibly no tax at all by virtue of the loophole at death). Thus, we should expect a high degree of leverage to be of value to the owners, even in the case of closely held corporations, primarily in cases where their firm was not expected to have much need for additional funds to expand assets and earnings in the future. To the extent that opportunities for growth were available, as they presumably would be for most successful corporations, the interest of the stockholders would tend to be better served by a structure which permitted maximum use of retained earnings.

view starts from the position that debt funds are several times cheaper than equity funds even in the absence of taxes, with taxes serving simply to magnify the cost ratio in proportion to the corporate rate. By contrast, in our model in which the repercussions of debt financing on the value of shares are taken into account, the *only* difference in cost is that due to the tax effect, and its magnitude is simply the tax on the "grossed up" interest payment. Not only is this magnitude likely to be small but our analysis yields the further paradoxical implication that the stockholders' gain from, and hence incentive to use, debt financing is actually smaller the lower the rate of interest. In the extreme case where the firm could borrow for practically nothing, the advantage of debt financing would also be practically nothing.

III. *Conclusion*

With the development of Proposition III the main objectives we outlined in our introductory discussion have been reached. We have in our Propositions I and II at least the foundations of a theory of the valuation of firms and shares in a world of uncertainty. We have shown, moreover, how this theory can lead to an operational definition of the cost of capital and how that concept can be used in turn as a basis for rational investment decision-making within the firm. Needless to say, however, much remains to be done before the cost of capital can be put away on the shelf among the solved problems. Our approach has been that of static, partial equilibrium analysis. It has assumed among other things a state of atomistic competition in the capital markets and an ease of access to those markets which only a relatively small (though important) group of firms even come close to possessing. These and other drastic simplifications have been necessary in order to come to grips with the problem at all. Having served their purpose they can now be relaxed in the direction of greater realism and relevance, a task in which we hope others interested in this area will wish to share.

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equanimity a writing-down of the value of their reserves, or unless one is prepared to forego the possibility of exchange-rate adjustment, any major extension of the gold exchange standard is dependent upon the introduction of guarantees. It is misleading to suggest that the multiple key-currency system is an alternative to a guarantee, as implied by Roosa [6, pp. 5-7 and 9-12].

IV. *Conclusion*

The most noteworthy conclusion to be drawn from this analysis is that the successful operation of a multiple key-currency system would require both exchange guarantees and continuing cooperation between central bankers of a type that would effectively limit their choice as to the form in which they hold their reserves. Yet these are two of the conditions whose undesirability has frequently been held to be an obstacle to implementation of the alternative proposal to create a world central bank. The multiple key-currency proposal represents an attempt to avoid the impracticality supposedly associated with a world central bank, but if both proposals in fact depend on the fulfillment of similar conditions, it is difficult to convince oneself that the sacrifice of the additional liquidity that an almost closed system would permit is worth while. Unless, of course, the object of the exercise is to reinforce discipline rather than to expand liquidity.

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Corporate Income Taxes and the Cost of Capital: A Correction

The purpose of this communication is to correct an error in our paper "The Cost of Capital, Corporation Finance and the Theory of Investment" (this *Review*, June 1958). In our discussion of the effects of the present method of taxing corporations on the valuation of firms, we said (p. 272):

> The deduction of interest in computing taxable corporate profits will prevent the arbitrage process from making the value of all firms in a given class proportional to the expected returns generated by their

physical assets. Instead, it can be shown (by the same type of proof used for the original version of Proposition I) that *the market values of firms in each class must be proportional in equilibrium to their expected returns net of taxes (that is, to the sum of the interest paid and expected net stockholder income).* (Italics added.)

The statement in italics, unfortunately, is wrong. For even though one firm may have an *expected* return after taxes (our $\bar{X}^{\tau}$) twice that of another firm in the same risk-equivalent class, it will not be the case that the *actual* return after taxes (our X^{τ}) of the first firm will always be twice that of the second, if the two firms have different degrees of leverage.[1] And since the distribution of returns after taxes of the two firms will not be proportional, there can be no "arbitrage" process which forces their values to be proportional to their expected after-tax returns.[2] In fact, it can be shown—and this time it really will be shown—that "arbitrage" will make values within any class a function not only of expected after-tax returns, but of the tax rate and the degree of leverage. This means, among other things, that the tax advantages of debt financing are somewhat greater than we originally suggested and, to this extent, the quantitative difference between the valuations implied by our position and by the traditional view is narrowed. It still remains true, however, that under our analysis the tax advantages of debt are the *only* permanent advantages so that the gulf between the two views in matters of interpretation and policy is as wide as ever.

I. *Taxes, Leverage, and the Probability Distribution of After-Tax Returns*

To see how the distribution of after-tax earnings is affected by leverage, let us again denote by the random variable X the (long-run average) earnings before interest and taxes generated by the currently owned assets of a given firm in some stated risk class, k.[3] From our definition of a risk class it follows that X can be expressed in the form $\bar{X}Z$, where $\bar{X}$ is the expected value of X, and the random variable $Z = X/\bar{X}$, having the same value for all firms in class k, is a drawing from a distribution, say $f_k(Z)$. Hence the

[1] With some exceptions, which will be noted when they occur, we shall preserve here both the notation and the terminology of the original paper. A working knowledge of both on the part of the reader will be presumed.

[2] Barring, of course, the trivial case of universal linear utility functions. Note that in deference to Professor Durand (see his Comment on our paper and our reply, this *Review*, Sept. 1959, *49*, 639–69) we here and throughout use quotation marks when referring to arbitrage.

[3] Thus our X corresponds essentially to the familiar EBIT concept of the finance literature. The use of EBIT and related "income" concepts as the basis of valuation is strictly valid only when the underlying real assets are assumed to have perpetual lives. In such a case, of course, EBIT and "cash flow" are one and the same. This was, in effect, the interpretation of X we used in the original paper and we shall retain it here both to preserve continuity and for the considerable simplification it permits in the exposition. We should point out, however, that the perpetuity interpretation is much less restrictive than might appear at first glance. Before-tax cash flow and EBIT can also safely be equated even where assets have finite lives as soon as these assets attain a steady state age distribution in which annual replacements equal annual depreciation. The subject of finite lives of assets will be further discussed in connection with the problem of the cut-off rate for investment decisions.

random variable X^τ, measuring the after-tax return, can be expressed as:

(1) $X^\tau = (1 - \tau)(X - R) + R = (1 - \tau)X + \tau R = (1 - \tau)\overline{X}Z + \tau R$

where τ is the marginal corporate income tax rate (assumed equal to the average), and R is the interest bill. Since $E(X^\tau) \equiv \overline{X}^\tau = (1-\tau)\overline{X} + \tau R$ we can substitute $\overline{X}^\tau - \tau R$ for $(1-\tau)\overline{X}$ in (1) to obtain:

$$(2) \qquad X^\tau = (\overline{X}^\tau - \tau R)Z + \tau R = \overline{X}^\tau\left(1 - \frac{\tau R}{\overline{X}^\tau}\right)Z + \tau R.$$

Thus, if the tax rate is other than zero, the shape of the distribution of X^τ will depend not only on the "scale" of the stream $\overline{X}^\tau$ and on the distribution of Z, but also on the tax rate and the degree of leverage (one measure of which is $R/\overline{X}^\tau$). For example, if Var $(Z) = \sigma^2$, we have:

$$\text{Var}\,(X^\tau) = \sigma^2(\overline{X}^\tau)^2\left(1 - \tau\frac{R}{\overline{X}^\tau}\right)^2$$

implying that for given $\overline{X}^\tau$ the variance of after-tax returns is smaller, the higher τ and the degree of leverage.[4]

II. *The Valuation of After-Tax Returns*

Note from equation (1) that, from the investor's point of view, the long-run average stream of after-tax returns appears as a sum of two components: (1) an uncertain stream $(1-\tau)\overline{X}Z$; and (2) a sure stream τR.[5] This suggests that the equilibrium market value of the combined stream can be found by capitalizing each component separately. More precisely, let ρ^τ be the rate at which the market capitalizes the expected returns net of tax of an unlevered company of size $\overline{X}$ in class k, i.e.,

$$\rho^\tau = \frac{(1-\tau)\overline{X}}{V_U} \quad \text{or} \quad V_U = \frac{(1-\tau)\overline{X}}{\rho^\tau};$$[6]

[4] It may seem paradoxical at first to say that leverage *reduces* the variability of outcomes, but remember we are here discussing the variability of total returns, interest plus net profits. The variability of stockholder net profits will, of course, be greater in the presence than in the absence of leverage, though relatively less so than in an otherwise comparable world of no taxes. The reasons for this will become clearer after the discussion in the next section.

[5] The statement that τR—the tax saving per period on the interest payments—is a sure stream is subject to two qualifications. First, it must be the case that firms can always obtain the tax benefit of their interest deductions either by offsetting them directly against other taxable income in the year incurred; or, in the event no such income is available in any given year, by carrying them backward or forward against past or future taxable earnings; or, in the extreme case, by merger of the firm with (or its sale to) another firm that can utilize the deduction. Second, it must be assumed that the tax rate will remain the same. To the extent that neither of these conditions holds exactly then some uncertainty attaches even to the tax savings, though, of course, it is of a different kind and order from that attaching to the stream generated by the assets. For simplicity, however, we shall here ignore these possible elements of delay or of uncertainty in the tax saving; but it should be kept in mind that this neglect means that the subsequent valuation formulas overstate, if anything, the value of the tax saving for any given permanent level of debt.

[6] Note that here, as in our original paper, we neglect dividend policy and "growth" in the

and let r be the rate at which the market capitalizes the sure streams generated by debts. For simplicity, assume this rate of interest is a constant independent of the size of the debt so that

$$r = \frac{R}{D} \quad \text{or} \quad D = \frac{R}{r}.$$[7]

Then we would expect the value of a levered firm of size $\bar{X}$, with a permanent level of debt D_L in its capital structure, to be given by:

$$V_L = \frac{(1-\tau)\bar{X}}{\rho^\tau} + \frac{\tau R}{r} = V_U + \tau D_L. \tag{3}$$[8]

In our original paper we asserted instead that, within a risk class, market value would be proportional to expected after-tax return $\bar{X}^\tau$ (cf. our original equation [11]), which would imply:

$$V_L = \frac{\bar{X}^\tau}{\rho^\tau} = \frac{(1-\tau)\bar{X}}{\rho^\tau} + \frac{\tau R}{\rho^\tau} = V_U + \frac{r}{\rho^\tau}\tau D_L. \tag{4}$$

We will now show that if (3) does not hold, investors can secure a more efficient portfolio by switching from relatively overvalued to relatively undervalued firms. Suppose first that unlevered firms are overvalued or that

$$V_L - \tau D_L < V_U.$$

An investor holding m dollars of stock in the unlevered company has a right to the fraction m/V_U of the eventual outcome, i.e., has the uncertain income

$$Y_U = \left(\frac{m}{V_U}\right)(1-\tau)\bar{X}Z.$$

Consider now an alternative portfolio obtained by investing m dollars as follows: (1) the portion,

$$m\left(\frac{S_L}{S_L + (1-\tau)D_L}\right),$$

is invested in the stock of the levered firm, S_L; and (2) the remaining portion,

$$m\left(\frac{(1-\tau)D_L}{S_L + (1-\tau)D_L}\right),$$

sense of opportunities to invest at a rate of return greater than the market rate of return. These subjects are treated extensively in our paper, "Dividend Policy, Growth and the Valuation of Shares," *Jour. Bus.*, Univ. Chicago, Oct. 1961, 411–33.

[7] Here and throughout, the corresponding formulas when the rate of interest rises with leverage can be obtained merely by substituting $r(L)$ for r, where L is some suitable measure of leverage.

[8] The assumption that the debt is permanent is not necessary for the analysis. It is employed here both to maintain continuity with the original model and because it gives an upper bound on the value of the tax saving. See in this connection footnote 5 and footnote 9.

is invested in its bonds. The stock component entitles the holder to a fraction,

$$\frac{m}{S_L + (1-\tau)D_L},$$

of the net profits of the levered company or

$$\left(\frac{m}{S_L + (1-\tau)D_L}\right)[(1-\tau)(\overline{X}Z - R_L)].$$

The holding of bonds yields

$$\left(\frac{m}{S_L + (1-\tau)D_L}\right)[(1-\tau)R_L].$$

Hence the total outcome is

$$Y_L = \left(\frac{m}{(S_L + (1-\tau)D_L)}\right)[(1-\tau)\overline{X}Z]$$

and this will dominate the uncertain income Y_U if (and only if)

$$S_L + (1-\tau)D_L \equiv S_L + D_L - \tau D_L \equiv V_L - \tau D_L < V_U.$$

Thus, in equilibrium, V_U cannot exceed $V_L - \tau D_L$, for if it did investors would have an incentive to sell shares in the unlevered company and purchase the shares (and bonds) of the levered company.

Suppose now that $V_L - \tau D_L > V_U$. An investment of m dollars in the stock of the levered firm entitles the holder to the outcome

$$\begin{aligned} Y_L &= (m/S_L)[(1-\tau)(\overline{X}Z - R_L)] \\ &= (m/S_L)(1-\tau)\overline{X}Z - (m/S_L)(1-\tau)R_L. \end{aligned}$$

Consider the following alternative portfolio: (1) borrow an amount $(m/S_L)(1-\tau)D_L$ for which the interest cost will be $(m/S_L)(1-\tau)R_L$ (assuming, of course, that individuals and corporations can borrow at the same rate, r); and (2) invest m plus the amount borrowed, i.e.,

$$m + \frac{m(1-\tau)D_L}{S_L} = m\frac{S_L + (1-\tau)D_L}{S_L} = (m/S_L)[V_L - \tau D_L]$$

in the stock of the unlevered firm. The outcome so secured will be

$$(m/S_L)\left(\frac{V_L - \tau D_L}{V_U}\right)(1-\tau)\overline{X}Z.$$

Subtracting the interest charges on the borrowed funds leaves an income of

$$Y_U = (m/S_L)\left(\frac{V_L - \tau D_L}{V_U}\right)(1-\tau)\overline{X}Z - (m/S_L)(1-\tau)R_L$$

which will dominate Y_L if (and only if) $V_L - \tau D_L > V_U$. Thus, in equilibrium, both $V_L - \tau D_L > V_U$ and $V_L - \tau D_L < V_U$ are ruled out and (3) must hold.

III. *Some Implications of Formula* (3)

To see what is involved in replacing (4) with (3) as the rule of valuation, note first that both expressions make the value of the firm a function of leverage and the tax rate. The difference between them is a matter of the size and source of the tax advantages of debt financing. Under our original formulation, values within a class were strictly proportional to expected earnings after taxes. Hence the tax advantage of debt was due solely to the fact that the deductibility of interest payments implied a higher level of after-tax income for any given level of before-tax earnings (i.e., higher by the amount τR since $\overline{X}^{\tau}=(1-\tau)\overline{X}+\tau R$). Under the corrected rule (3), however, there is an additional gain due to the fact that the extra after-tax earnings, τR, represent a sure income in contrast to the uncertain outcome $(1-\tau)\overline{X}$. Hence τR is capitalized at the more favorable certainty rate, $1/r$, rather than at the rate for uncertain streams, $1/\rho^{\tau}$.[9]

Since the difference between (3) and (4) is solely a matter of the rate at which the tax savings on interest payments are capitalized, the required changes in all formulas and expressions derived from (4) are reasonably straightforward. Consider, first, the before-tax earnings yield, i.e., the ratio of expected earnings before interest and taxes to the value of the firm.[10] Dividing both sides of (3) by V and by $(1-\tau)$ and simplifying we obtain:

$$\text{(31.c)} \qquad \frac{\overline{X}}{V}=\frac{\rho^{\tau}}{1-\tau}\left[1-\tau\frac{D}{V}\right]$$

which replaces our original equation (31) (p. 294). The new relation differs from the old in that the coefficient of D/V in the original (31) was smaller by a factor of r/ρ^{τ}.

Consider next the after-tax earnings yield, i.e., the ratio of interest payments plus profits after taxes to total market value.[11] This concept was discussed extensively in our paper because it helps to bring out more clearly the differences between our position and the traditional view, and because it facilitates the construction of empirical tests of the two hypotheses about the valuation process. To see what the new equation (3) implies for this yield we need merely substitute $\overline{X}^{\tau}-\tau R$ for $(1-\tau)\overline{X}$ in (3) obtaining:

[9] Remember, however, that in one sense formula (3) gives only an upper bound on the value of the firm since $\tau R/r=\tau D$ is an exact measure of the value of the tax saving only where both the tax rate and the level of debt are assumed to be fixed forever (and where the firm is certain to be able to use its interest deduction to reduce taxable income either directly or via transfer of the loss to another firm). Alternative versions of (3) can readily be developed for cases in which the debt is not assumed to be permanent, but rather to be outstanding only for some specified finite length of time. For reasons of space, we shall not pursue this line of inquiry here beyond observing that the shorter the debt period considered, the closer does the valuation formula approach our original (4). Hence, the latter is perhaps still of some interest if only as a lower bound.

[10] Following usage common in the field of finance we referred to this yield as the "average cost of capital." We feel now, however, that the term "before-tax earnings yield" would be preferable both because it is more immediately descriptive and because it releases the term "cost of capital" for use in discussions of optimal investment policy (in accord with standard usage in the capital budgeting literature).

[11] We referred to this yield as the "after-tax cost of capital." Cf. the previous footnote.

(5) $$V = \frac{\bar{X}^\tau - \tau R}{\rho^\tau} + \tau D = \frac{\bar{X}^\tau}{\rho^\tau} + \tau \frac{\rho^\tau - r}{\rho^\tau} D,$$

from which it follows that the after-tax earnings yield must be:

(11.c) $$\frac{\bar{X}^\tau}{V} = \rho^\tau - \tau(\rho^\tau - r) D/V.$$

This replaces our original equation (11) (p. 272) in which we had simply $\bar{X}^\tau/V = \rho^\tau$. Thus, in contrast to our earlier result, the corrected version (11.c) implies that even the after-tax yield is affected by leverage. The predicted rate of decrease of $\bar{X}^\tau/V$ with D/V, however, is still considerably smaller than under the naive traditional view, which, as we showed, implied essentially $\bar{X}^\tau/V = \rho^\tau\text{-}(\rho^\tau - r)D/V$. See our equation (17) and the discussion immediately preceding it (p. 277).[12] And, of course, (11.c) implies that the effect of leverage on $\bar{X}^\tau/V$ is *solely* a matter of the deductibility of interest payments whereas, under the traditional view, going into debt would lower the cost of capital regardless of the method of taxing corporate earnings.

Finally, we have the matter of the after-tax yield on *equity* capital, i.e., the ratio of net profits after taxes to the value of the shares.[13] By subtracting D from both sides of (5) and breaking $\bar{X}^\tau$ into its two components—expected net profits after taxes, $\bar{\pi}^\tau$, and interest payments, $R = rD$—we obtain after simplifying:

(6) $$S = V - D = \frac{\bar{\pi}^\tau}{\rho^\tau} - (1 - \tau)\left(\frac{\rho^\tau - r}{\rho^\tau}\right)D.$$

From (6) it follows that the after-tax yield on equity capital must be:

(12.c) $$\frac{\bar{\pi}^\tau}{S} = \rho^\tau + (1 - \tau)[\rho^\tau - r] D/S$$

which replaces our original equation (12), $\bar{\pi}^\tau/S = \rho^\tau + (\rho^\tau - r)D/S$ (p. 272). The new (12.c) implies an increase in the after-tax yield on equity capital as leverage increases which is smaller than that of our original (12) by a factor of $(1 - \tau)$. But again, the linear increasing relation of the corrected (12.c) is still fundamentally different from the naive traditional view which asserts the cost of equity capital to be completely independent of leverage (at least as long as leverage remains within "conventional" industry limits).

IV. *Taxes and the Cost of Capital*

From these corrected valuation formulas we can readily derive corrected measures of the cost of capital in the capital budgeting sense of the minimum prospective yield an investment project must offer to be just worth

[12] The i_k^* of (17) is the same as ρ^τ in the present context, each measuring the ratio of net profits to the value of the shares (and hence of the whole firm) in an unlevered company of the class.

[13] We referred to this yield as the "after-tax cost of equity capital." Cf. footnote 9.

undertaking from the standpoint of the present stockholders. If we interpret earnings streams as perpetuities, as we did in the original paper, then we actually have two equally good ways of defining this minimum yield: either by the required increase in before-tax earnings, $d\overline{X}$, or by the required increase in earnings net of taxes, $d\overline{X}(1-\tau)$.[14] To conserve space, however, as well as to maintain continuity with the original paper, we shall concentrate here on the before-tax case with only brief footnote references to the net-of-tax concept.

Analytically, the derivation of the cost of capital in the above sense amounts to finding the minimum value of $d\overline{X}/dI$ for which $dV=dI$, where I denotes the level of new investment.[15] By differentiating (3) we see that:

$$\text{(7)} \qquad \frac{dV}{dI} = \frac{1-\tau}{\rho^{\tau}} \frac{d\overline{X}}{dI} + \tau \frac{dD}{dI} \geq 1 \qquad \text{if } \frac{d\overline{X}}{dI} \geq \frac{1 - \tau \dfrac{dD}{dI}}{1-\tau} \rho^{\tau}.$$

Hence the before tax required rate of return cannot be defined without reference to financial policy. In particular, for an investment considered as being financed entirely by new equity capital $dD/dI=0$ and the required rate of return or marginal cost of equity financing (neglecting flotation costs) would be:

$$\rho^{S} = \frac{\rho^{\tau}}{1-\tau} \cdot$$

This result is the same as that in the original paper (see equation [32], p. 294) and is applicable to any other sources of financing where the remuneration to the suppliers of capital is not deductible for tax purposes. It applies, therefore, to preferred stock (except for certain partially deductible issues of public utilities) and would apply also to retained earnings were it not for the favorable tax treatment of capital gains under the personal income tax.

For investments considered as being financed entirely by new debt capital $dI=dD$ and we find from (7) that:

$$\text{(33.c)} \qquad \rho^{D} = \rho^{\tau}$$

which replaces our original equation (33) in which we had:

$$\text{(33)} \qquad \rho^{D} = \rho^{S} - \frac{\tau}{1-\tau} r.$$

[14] Note that we use the term "earnings net of taxes" rather than "earnings after taxes." We feel that to avoid confusion the latter term should be reserved to describe what will actually appear in the firm's accounting statements, namely the net cash flow including the tax savings on the interest (our $\overline{X}^{\tau}$). Since financing sources cannot in general be allocated to particular investments (see below), the after-tax or accounting concept is not useful for capital budgeting purposes, although it can be extremely useful for valuation equations as we saw in the previous section.

[15] Remember that when we speak of the minimum required yield on an investment we are referring in principle only to investments which increase the *scale* of the firm. That is, the new

Thus for borrowed funds (or any other tax-deductible source of capital) the marginal cost or before-tax required rate of return is simply the market rate of capitalization for net of tax unlevered streams and is thus independent of both the tax rate and the interest rate. This required rate is lower than that implied by our original (33), but still considerably higher than that implied by the traditional view (see esp. pp. 276–77 of our paper) under which the before-tax cost of borrowed funds is simply the interest rate, r.

Having derived the above expressions for the marginal costs of debt and equity financing it may be well to warn readers at this point that these expressions represent at best only the hypothetical extremes insofar as costs are concerned and that neither is directly usable as a cut-off criterion for investment planning. In particular, care must be taken to avoid falling into the famous "Liquigas" fallacy of concluding that if a firm intends to float a bond issue in some given year then its cut-off rate should be set that year at ρ^D; while, if the next issue is to be an equity one, the cut-off is ρ^S. The point is, of course, that no investment can meaningfully be regarded as 100 per cent equity financed if the firm makes any use of debt capital—and most firms do, not only for the tax savings, but for many other reasons having nothing to do with "cost" in the present static sense (cf. our original paper pp. 292–93). And no investment can meaningfully be regarded as 100 per cent debt financed when lenders impose strict limitations on the maximum amount a firm can borrow relative to its equity (and when most firms actually plan on normally borrowing less than this external maximum so as to leave themselves with an emergency reserve of unused borrowing power). Since the firm's long-run capital structure will thus contain both debt and equity capital, investment planning must recognize that, over the long pull, *all* of the firm's assets are really financed by a mixture of debt and equity capital even though only one kind of capital may be raised in any particular year. More precisely, if L^* denotes the firm's long-run "target" debt ratio (around which its actual debt ratio will fluctuate as it "alternately" floats debt issues and retires them with internal or external equity) then the firm can assume, to a first approximation at least, that for any particular investment $dD/dI = L^*$. Hence, the relevant marginal cost of capital for investment planning, which we shall here denote by ρ^*, is:

$$\rho^* = \frac{1 - \tau L^*}{1 - \tau} \rho^\tau = \rho^S - \frac{\tau}{1 - \tau} \rho^D L^* = \rho^S(1 - L^*) + \rho^D L^*.$$

That is, the appropriate cost of capital for (repetitive) investment decisions over time is, to a first approximation, a weighted average of the costs of debt and equity financing, the weights being the proportions of each in the "target" capital structure.[16]

assets must be in the same "class" as the old. See in this connection, J. Hirshleifer, "Risk, the Discount Rate and Investment Decisions," *Am. Econ. Rev.*, May 1961, *51*, 112–20 (especially pp. 119–20). See also footnote 16.

[16] From the formulas in the text one can readily derive corresponding expressions for the required net-of-tax yield, or net-of-tax cost of capital for any given financing policy. Specifi-

V. *Some Concluding Observations*

Such, then, are the major corrections that must be made to the various formulas and valuation expressions in our earlier paper. In general, we can say that the force of these corrections has been to increase somewhat the estimate of the tax advantages of debt financing under our model and consequently to reduce somewhat the quantitative difference between the estimates of the effects of leverage under our model and under the naive traditional view. It may be useful to remind readers once again that the existence of a tax advantage for debt financing—even the larger advantage of the corrected version—does not necessarily mean that corporations should at all times seek to use the maximum possible amount of debt in their capital structures. For one thing, other forms of financing, notably retained earnings, may in some circumstances be cheaper still when the tax status of investors under the personal income tax is taken into account. More important, there are, as we pointed out, limitations imposed by lenders (see pp. 292–93), as well as many other dimensions (and kinds of costs) in real-world problems of financial strategy which are not fully comprehended within the framework of static equilibrium models, either our own or those of the traditional variety. These additional considerations, which are typically grouped under the rubric of "the need for preserving flexibility," will normally imply the maintenance by the corporation of a substantial reserve of untapped borrowing power. The tax advantage of debt may well tend to lower the optimal size of that reserve, but it is hard to believe that advantages of the size contemplated under our model could justify any substantial reduction, let alone their complete elimination. Nor do the data

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cally, let $\tilde{\rho}(L)$ denote the required net-of-tax yield for investment financed with a proportion of debt $L=dD/dI$. (More generally L denotes the proportion financed with tax deductible sources of capital.) Then from (7) we find:

$$\tilde{\rho}(L) = (1-\tau)\frac{d\bar{X}}{dI} = (1-L\tau)\rho^{\tau} \tag{8}$$

and the various costs can be found by substituting the appropriate value for L. In particular, if we substitute in this formula the "target" leverage ratio, L^*, we obtain:

$$\tilde{\rho}^* \equiv \tilde{\rho}(L^*) = (1-\tau L^*)\rho^{\tau}$$

and $\tilde{\rho}^*$ measures the average net-of-tax cost of capital in the sense described above.

Although the before-tax and the net-of-tax approaches to the cost of capital provide equally good criteria for investment decisions when assets are assumed to generate perpetual (i.e., non-depreciating) streams, such is not the case when assets are assumed to have finite lives (even when it is also assumed that the firm's assets are in a steady state age distribution so that our X or EBIT is approximately the same as the net cash flow before taxes). See footnote 3 above. In the latter event, the correct method for determining the desirability of an investment would be, in principle, to discount the net-of-tax stream at the net-of-tax cost of capital. Only under this net-of-tax approach would it be possible to take into account the deductibility of depreciation (and also to choose the most advantageous depreciation policy for tax purposes). Note that we say that the net-of-tax approach is correct "in principle" because, strictly speaking, nothing in our analysis (or anyone else's, for that matter) has yet established that it is indeed legitimate to "discount" an uncertain stream. One can hope that subsequent research will show the analogy to discounting under the certainty case is a valid one; but, at the moment, this is still only a hope.

indicate that there has in fact been a substantial increase in the use of debt (except relative to preferred stock) by the corporate sector during the recent high tax years.[17]

As to the differences between our modified model and the traditional one, we feel that they are still large in quantitative terms and still very much worth trying to detect. It is not only a matter of the two views having different implications for corporate financial policy (or even for national tax policy). But since the two positions rest on fundamentally different views about investor behavior and the functioning of the capital markets, the results of tests between them may have an important bearing on issues ranging far beyond the immediate one of the effects of leverage on the cost of capital.

FRANCO MODIGLIANI AND MERTON H. MILLER*

[17] See, e.g., Merton H. Miller, "The Corporate Income Tax and Corporate Financial Policies," in *Staff Reports to the Commission on Money and Credit* (forthcoming).

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Consumption, Savings and Windfall Gains: Comment

In her recent article in this *Review* [3], Margaret Reid attempted to answer previous articles by Bodkin [1] and Jones [2] challenging the validity of the permanent income hypothesis. Bodkin and Jones used income and expenditure data for those consumer units who had received the soldiers' bonus (National Service Life Insurance dividends) during 1950, the year of the urban consumption survey [4]. These bonuses were regarded as windfall gains for the purposes of their analyses.

Professor Reid used data from the same survey, but her windfall gains were represented by "other money receipts." These are defined as "inheritances and occasional large gifts of money from persons outside the family . . . and net receipts from the settlement of fire and accident policies" [4, Vol. 1, p. xxix]. She assumed that the soldiers' bonus was included, and that it accounted for about one-half of other money receipts. Here she made an unfortunate mistake in interpreting the data for the main critical purpose of her article.

The soldiers' bonus is not part of "other money receipts" (O) but rather a part of "disposable money income" (Y). It is the main part of an item in the disposable money income category called "military pay, allotments, and pensions" [4, Vol. 11, p. xxix].

This would appear to alter completely the relationship of Professor Reid's main findings to the Bodkin results and to change the windfall interpretation of the O variable. Surely, fire and accident policy settlements are not windfall income, but rather a (partial) recovery of real assets previously lost. Likewise, inheritances are probably best considered as a long-anticipated increase in assets—not an increase in transitory income.

The discovery of this error probably does not affect whatever importance Professor Reid's secondary finding may have: ". . . the need, in any study of

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DATED JULY 14, 2021

11. Refer to page 26, lines 12 through 17 of Mr. Moul's Direct Testimony.
    a. Provide support for Mr. Moul's statement that a "leverage adjustment properly accounts for the risk differential between market-value and book-value capital structures." Provide copies of any articles, studies, textbook excerpts, or other documentary support.
    b. Provide support that a leverage adjustment "must" be made to the DCF in a utility regulatory context. Provide copies of any articles, studies, textbook excerpts, Commission orders, or other documentary support.

Response:

a. Dr. Roger Morin notes:

> The third and perhaps most important reason for caution and skepticism is that application of the DCF model produces estimates of common equity cost that are consistent with investors' expected return only when stock price and book value are reasonably similar, that is, when the M/B is close to unity. As shown below, application of the standard DCF model to utility stocks understates the investor's expected return when the market-to-book (M/B) ratio of a given stock exceeds unity. This was particularly relevant in the capital market environment of the 1990s and 2000s where utility stocks were trading at M/B ratios well above unity and have been for nearly two decades. The converse is also true, that is, the DCF model overstates that investor's return when the stock's M/B ratio is less than unity. The reason for the distortion is that the DCF market return is applied to a book value rate base by the regulator, that is, a utility's earnings are limited to earnings on a book value rate base.[1]

b. Please see response to (a) above. Further, as noted on page 27, lines 2 through 12, because the rate-setting process uses book value capitalization, a leverage adjustment synchronizes the DCF, which is based on market values with the book values used in the rate-setting process.

Sponsoring Witness: Dylan D'Ascendis

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[1] Roger A. Morin, Ph.D., New Regulatory Finance, Public Utility Reports, Inc., 2006, at 434 (AG 1-11a Attachment 1)

NEW
REGULATORY
FINANCE

Roger A. Morin, PhD

2006
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TABLE 15-1
EFFECT OF MARKET-TO-BOOK RATIO ON MARKET RETURN

	Situation 1	Situation 2	Situation 3
1 Initial purchase price	$25.00	$50.00	$100.00
2 Initial book value	$50.00	$50.00	$50.00
3 **Initial M/B**	**0.50**	**1.00**	**2.00**
4 **DCF Return 10% = 5% + 5%**	**10.00%**	**10.00%**	**10.00%**
5 Dollar Return	$5.00	$5.00	$5.00
6 Dollar Dividends 5% Yield	$1.25	$2.50	$5.00
7 Dollar Growth 5% Growth	$3.75	$2.50	$0.00
8 **Market Return**	**20.00%**	**10.00%**	**5.00%**

But what if investors expect an increase in the price/earnings ratio from 12.5 to 13.5? Then, the growth in value is from $100 to $114.48, or 13.5 times next year's earnings of $8.48, for a total return of 18.5% (dividend yield of 4%, plus growth in value of 14.5%). The orthodox DCF model would indicate returns of 10%, whereas the investors' true expected return is 18.5%. Investor-expected returns are substantially understated whenever investors anticipate increases in relative market valuation, and conversely.

The third and perhaps most important reason for caution and skepticism is that application of the DCF model produces estimates of common equity cost that are consistent with investors' expected return only when stock price and book value are reasonably similar, that is, when the M/B is close to unity. As shown below, application of the standard DCF model to utility stocks understates the investor's expected return when the market-to-book (M/B) ratio of a given stock exceeds unity. This was particularly relevant in the capital market environment of the 1990s and 2000s where utility stocks were trading at M/B ratios well above unity and have been for nearly two decades. The converse is also true, that is, the DCF model overstates the investor's return when the stock's M/B ratio is less than unity. The reason for the distortion is that the DCF market return is applied to a book value rate base by the regulator, that is, a utility's earnings are limited to earnings on a book value rate base.

The simple numerical illustration shown in Table 15-1 demonstrates the impact of M/B ratios on the DCF market return. The example shows the result of applying a market value cost rate to book value rate base under three different M/B scenarios. The three columns correspond to three M/B situations: the stock trades below, equal to, and above book value, respectively. The latter situation is noteworthy and representative of the capital market environment of the last two decades. As shown in the third column, the DCF cost rate of 10%, made up of a 5% dividend yield and a 5% growth rate, is applied to

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12. Refer to page 27, lines 12 through 14 of Mr. Moul's Direct Testimony. Provide support for Mr. Moul's statement that the leverage adjustment uses "well recognized analytical procedures that are widely accepted in the financial literature." Provide copies of any articles, studies, textbook excerpts, or other documentary support.

Response:

Please refer to the responses to AG 1-10 and 1-11.

Sponsoring Witness: Dylan D'Ascendis