**Title: Equity Risk Premium: Expectations Great and Small**

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## Equity Risk Premium: Expectations Great and Small

What I actually think is that our prey, called the equity risk premium, is extremely elusive.

Stephen A. Ross 2001

***Abstract:***
The Equity Risk Premium (ERP) is an essential building block of the market value of risk. In theory, the collective action of all investors results in an equilibrium expectation for the return on the market portfolio excess of the risk-free return, the equity risk premium. The ability of the valuation actuary to choose a sensible value for the ERP, whether as a required input to CAPM valuation, or any of its descendants, is as important as choosing risk-free rates and risk relatives (betas) to the ERP for the asset at hand. The historical realized ERP for the stock market appears to be at odds with pricing theory parameters for risk aversion. Since 1985, there has been a constant stream of research, each of which reviews theories of estimating market returns, examines historical data periods, or both. Those ERP value estimates vary widely from about minus one percent to about nine percent, based on a geometric or arithmetic averaging, short or long horizons, short or long-run expectations, unconditional or conditional distributions, domestic or international data, data periods, and real or nominal returns. This paper will examine the principal strains of the recent research on the ERP and catalogue the empirical values of the ERP implied by that research. In addition, the paper will supply several time series analyses of the standard Ibbotson Associates 1926-2002 ERP data using short Treasuries for the risk-free rate. Recommendations for ERP values to use in common actuarial valuation problems will be offered.

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**Keywords:** Equity Risk Premium, Risk Premium Puzzle, Market Return Models, CAPM, Dividend Growth Models, Actuarial Valuations.

## Introduction

The Equity Risk Premium (ERP) is an essential building block of the market value of risk. In theory, the collective action of all investors results in an equilibrium expectation for the return on the market portfolio excess of the risk-free return, the equity risk premium. The ability of the valuation actuary to choose a sensible value for the ERP, whether as a required input to CAPM valuation, or any of its descendants[1], is as important as choosing risk-free rates and risk relatives (betas) to the ERP for the asset at hand. Risky discount rates, asset allocation models, and project costs of capital are common actuarial uses of ERP as a benchmark rate.

The equity risk premium should be of particular interest to actuaries. For pensions and annuities backed by bonds and stocks, the actuary needs to have an understanding of the ERP and its variability compared to fixed horizon bonds. Variable products, including Guaranteed Minimum Death Benefits, require accurate projections of returns to ensure adequate future assets. With the latest research producing a relatively low equity risk premium, the rationale for including equities in insurers' asset holdings is being tested. In describing individual investment account guarantees, LaChance and Mitchell (2003) point out an underlying assumption of pension asset investing that, based only on the historical record, future equity returns will continue to outperform bonds; they clarify that those higher expected equity returns come with the additional higher risk of equity returns. Ralfe et al. (2003) support the risky equity view and discuss their pension experience with an all bond portfolio. Recent projections in some literature of a zero or negative equity risk premium challenge the assumptions underlying these views. By reviewing some of the most recent and relevant work on the issue of the equity risk premium, actuaries will have a better understanding of how these values were estimated, critical assumptions that allowed for such a low EPR, and the time period for the projection. Actuaries can then make informed decisions for expected investment results going forward.[2]

In 1985, Mehra and Prescott published their work on the so-called Equity Risk Premium Puzzle: The fact that the historical realized ERP for the stock market 1889-1978 appeared to be at odds with and, relative to Treasury bills, far in excess of asset pricing theory values based on investors with reasonable risk aversion parameters. Since then, there has been a constant stream of research, each of which reviews theories of estimating market returns, examines historical data periods, or both.[3] Those ERP value estimates vary widely from about minus one percent to about nine percent, based on geometric or arithmetic averaging, short or long horizons, short or long-run means, unconditional or conditional expectations, using domestic or international data, differing data periods, and real or nominal returns. Brealey and Myers, in the sixth edition of their standard corporate finance textbook, believe a range of 6% to 8.5% for the US ERP is reasonable for practical project valuation. Is that a fair estimate?

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[1] The multifactor arbitrage pricing theory (APT) of Ross (1976), the three-factor model of Fama and French (1992) and the recent Mamaysky (2002) five-factor model for stocks and bonds are all examples of enhanced CAPM models.

[2] See Appendix D

[3] For example, see Cochrane (1997), Cornell (1999), or Leibowitz (2001).

Current research on the equity risk premium is plentiful (Leibowitz, 2001). This paper covers a selection of mainstream articles and books that describe different approaches to estimating the ex ante equity risk premium. We select examples of the research that cover the most important approaches to the ERP. We begin by describing the methodology of using historical returns to predict future estimates. We identify the many varieties of ERPs in order to alert the reader to the fact that numerical estimates of the ERP that appear different may instead be about the same under a common definition. We examine the well-known Ibbotson Associates 1926-2002 data series for stationarity, i.e. time invariance of the mean ERP. We show by several statistical tests that stationarity cannot be rejected and the best estimate going forward, ceteris paribus, is the realized mean. This paper will examine the principal strains of the recent research on the ERP and catalogue the empirical values of the ERP implied by that research.[4]

We first discuss how the Social Security Administration derives estimates of the equity risk premium. Then, we survey the puzzle research, that is, the literature written in response to the Equity Premium Puzzle suggested by Mehra and Prescott (1985). We cover five major approaches from the literature. Next, we report from two surveys of "experts" on the equity risk premium. Finally, after we describe the main strains of research, we explore some of the implications for practicing actuaries.

We do not discuss the important companion problem of estimating the risk relationship of an individual company, line of insurance, or project with the overall market. Within a CAPM or Fama-French framework, the problem is estimating a market beta.[5] Actuaries should be aware, however, that simple 60-month regression betas are biased low where size or non-synchronous trading is a substantial factor (Kaplan and Peterson (1998), Pratt (1998), p86). Adjustments are made to historical betas in order to remove the bias and derive more accurate estimates. Elton and Gruber (1995) explain that by testing the relationship of beta estimates over time, empirical studies have shown that an adjustment toward the mean should be made to project future betas.[6]

## The Equity Risk Premium

Based on the definition in Brealey and Myers, *Principles of Corporate Finance* textbook, the equity risk premium (ERP) is the "expected additional return for making a risky investment rather than a safe one". In other words, the ERP is the difference between the market return and a risk-free return. Market returns include both dividends and capital gains. Because both the historical ERP and the prospective ERP have been referred to simply as the equity risk premium, the terms *ex post* and *ex ante* are used to differentiate between them but are often omitted. Table 1 shows the historical annual

[4] The research catalogued appears as Appendix B.
[5] According to CAPM, investors are compensated only for non-diversifiable, or market, risk. The market beta becomes the measurement of the extent to which returns on an individual security covary with the market. The market beta times the ERP represents the non-diversifiable expected return from an individual security.
[6] Elton and Gruber (1995), p148.

average returns from 1926-2002 for large company equities (S&P 500), Treasury Bills and Bonds, and their arithmetic differences using the Ibbotson data (Ibbotson Associates, 2003).[7]

| US Equity Risk Premia 1926-2002<br>Annual Equity Returns and Premia versus Treasury Bills, Intermediate, and Long Term Bonds | | | |
|---|---|---|---|
| **Horizon** | **Equity Returns** | **Risk-Free Return** | **ERP** |
| Short | 12.20% | 3.83% | 8.37% |
| Intermediate | 12.20% | 4.81% | 7.40% |
| Long | 12.20% | 5.23% | 6.97% |
| *Source: Ibbotson Yearbook (2003)* | | | |

**Table 1**

In 1985, Mehra and Prescott introduced the idea of the equity risk premium puzzle. The puzzling result is that the historical realized ERP for the stock market using 1889-1978 data appeared to be at odds with and, relative to Treasury bills, far in excess of asset pricing theory values based on normal parametrizations of risk aversion. When using standard frictionless return models and historical growth rates in consumption, the real risk-free rate, and the equity risk premium, the resulting relative risk aversion parameter appears too high. By choosing a maximum relative risk aversion parameter to be 10 and using the growth in consumption, Mehra and Prescott's model produces an ERP much lower than the historical.[8] Their result inspired a stream of finance literature that attempts to solve the puzzle. Two different research threads have emerged. One thread, including behavioral finance, attempts to explain the historical returns with new models and different assumptions about investors.[9] A second thread is from a group that provides estimates of the ERP that are derived from historical data and/or standard economic models. Some in this latter group argue that historical returns may have been higher than those that should be required in the future. In a curiously asymmetric way, there are no serious studies yet concluding that the historical results are too low to serve as ex ante estimates. Although both groups have made substantial and provocative contributions, the behavioral models do not give any ex ante ERP estimates other than explaining and supporting the historical returns. We presume, until results show otherwise, the behavioralists support the historical average as the ex ante unconditional long-run expectation. Therefore, we focus on the latter to catalogue equity risk premium estimates other than the historical approach, but we will discuss both as important strains for puzzle research.

## Equity Risk Premium Types

Many different types of equity risk premium estimates can be given even though they are labeled by the same general term. These estimates vary widely; currently the estimates range from about nine percent to a small negative. When ERP estimates are

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[7] Ibbotson's 1926-2002 series from the 2003 Yearbook, Valuation Edition. The entire series is shown in Appendix A.

[8] Campbell, Lo, and MacKinlay (1997) perform a similar analysis as Mehra and Prescott and find a risk-aversion coefficient of 19, larger than the reasonable level suggested in Mehra and Prescott's paper, pp307-308.

[9] See, for example, Benartzi and Thaler (1995) and Mehra (2002).

given, one should determine the type before comparing to other estimates. We point out seven important types to look for when given an ERP estimate. They include:

- Geometric vs. arithmetic averaging
- Short vs. long investment horizon
- Short vs. long-run expectation
- Unconditional vs. conditional on some related variable
- Domestic US vs. international market data
- Data sources and periods
- Real vs. nominal returns

The average market return and ERP can be stated as a geometric or arithmetic mean return. An arithmetic mean return is a simple average of a series of returns. The geometric mean return is the compound rate of return; it is a measure of the actual average performance of a portfolio over a given time period. Arithmetic returns are the same or higher than geometric returns, so it is not appropriate to make a direct comparison between an arithmetic estimate and a geometric estimate. However, those two returns can be transformed one to the other. For example, arithmetic returns can be approximated from geometric returns by the formula.[10]

$$AR = GR + \frac{s^2}{2}, s^2 \text{ the variance of the (arithmetic) return process}$$

Arithmetic averages of periodic returns are to be preferred when estimating next period returns since they, not geometric averages, reproduce the proper probabilities and means of expected returns.[11] ERPs can be generated by arithmetic differences (Equity – Risk Free) or by geometric differences ([(1 + Equity)/(1 + Risk Free)]-1). Usually, the arithmetic and geometric differences produce similar estimates.[12]

A second important difference in ERP estimate types is the horizon. The horizon indicates the total investment or planning period under consideration. For estimation purposes, the horizon relates to the term or maturity of the risk-free instrument that is used to determine the ERP.[13] The Ibbotson Yearbook (2003) provides definitions for three different horizons.[14] The short-horizon expected ERP[15] is defined as "the large company stock total returns minus U.S. Treasury bill *total* returns". Note, the income return and total return are the same for U.S. Treasury bills. The intermediate-horizon expected ERP is "the large company stock total returns minus intermediate-term government bond *income* returns". Finally, the long-horizon expected ERP is "the large company stock total returns minus long-term government bond *income* returns". For the Ibbotson data, Treasury bills have a maturity of approximately one month; intermediate-term government bonds have a maturity around five years; long-term government bonds

[10] See Welch (2000), Dimson et al. (2002), Ibbotson and Chen (2003).
[11] For example, see Ibbotson Yearbook, Valuation Edition (2003), pp71-73 for a complete discussion of the arithmetic/geometric choice. See also Dimson et al. (2000), p35 and Brennan and Schwartz (1985).
[12] The arithmetic difference is the geometric difference multiplied by 1 + Risk Free.
[13] See Table 1.
[14] See Ibbotson 2003 Yearbook, p177.
[15] Table 1 displays the short horizon ERP calculation for the 1926-2002 Ibbotson Data.

have a maturity of about 20 years. Although the Ibbotson definitions may not apply to other research, we will classify equity risk premium estimates based on these guidelines to establish some consistency among the current research. The reader should note that Ibbotson Associates recommends the income return (or the yield) when using a bond as the risk free rate rather than the total return.[16]

A third type is the length of time of the equity risk premium forecast. We distinguish between short-run and long-run expectations. Short-run expectations refer to the current equity risk premium, or for this paper, a prediction of up to ten years. In contrast, the long-run expectation is a forecast over ten years to as much as seventy-five years for social security purposes. Ten years appears an appropriate breaking point based on the current literature surveyed.

The next difference is whether the equity risk premium estimate is unconditional or conditioned on one or more related variables. In defining this type, we refer to an admonition by Constantinides (2002, p1568) of the differences in these estimates:

> "First, I draw a sharp distinction between *conditional, short-term forecasts* of the mean equity return and premium and *estimates of the unconditional mean*. I argue that the currently low conditional short-term forecasts of the return and premium do not lessen the burden on economic theory to explain the large unconditional mean equity return and premium, as measured by their sample average over the past one hundred and thirty years."

Many of the estimates we catalogue below will be conditional ones, conditional on dividend yield, expected earnings, capital gains, or other assumptions about the future.

ERP estimates can also exhibit a US versus international market type depending upon the data used for estimation purposes and the ERP being estimated. Dimson, et al. (2002) notes that at the start of 2000, the US equity market, while dominant, was slightly less than one-half (46.1%) of the total international market for equities, capitalized at 52.7 trillion dollars. Data from the non-US equity markets are clearly different from US markets and, hence, will produce different estimates for returns and ERP.[17] Results for the entire world equity market will, of course, be a weighted average of the US and non-US estimates.

[16] The reason for this is two-fold. First, when issued, the yield is the expected market return for the entire horizon of the bond. No net capital gains are expected for the market return for the entire horizon of the bond. No capital gains are expected at the default-free maturity. Second, historical annual capital gains on long-term Government Bonds average near zero (0.4%) over the 1926-2002 period (Ibbotson Yearbook, 2003, Table 6-7).

[17] One qualitative difference can arise from the collapse of equity markets during war time.

| Worldwide Equity Risk Premia, 1900-2000<br>Annual Equity Risk Premium Relative to Treasury Bills | | |
|---|---|---|
| **Country** | **Geometric Mean** | **Arithmetic Mean** |
| United States | 5.8% | 7.7% |
| World | 4.9% | 6.2% |
| *Source: Dimson, et al. (2002), pages 166-167* | | |

**Table 2**

The next type is the data source and period used for the market and ERP estimates. Whether given an historical average of the equity risk premium or an estimate from a model using various historical data, the ERP estimate will be influenced by the length, timing, and source of the underlying data used. The time series compilations are primarily annual or monthly returns. Occasionally, daily returns are analyzed, but not for the purpose of estimating an ERP. Some researchers use as much as 200 years of history; the Ibbotson data currently uses S&P 500 returns from 1926 to the present.[18] As an example, Siegel (2002) examines a series of real US returns beginning in 1802.[19] Siegel uses three sources to obtain the data. For the first period, 1802 to 1870, characterized by stocks of financial organizations involved in banking and insurance, he cites Schwert (1990). The second period, 1871-1925, incorporates Cowles stock indexes compiled in Shiller (1989). The last period, beginning in 1926, uses CRSP data; these are the same data underlying Ibbotson Associates calculations.

Goetzmann et al. (2001) construct a NYSE data series for 1815 to 1925 to add to the 1926-1999 Ibbotson series. They conclude that the pre-1926 and post-1926 data periods show differences in both risk and reward characteristics. They highlight the fact that inclusion of pre-1926 data will generally produce lower estimates of ERPs than relying exclusively on the Ibbotson post-1926 data, similar to that shown in Appendix A. Several studies that rely on pre-1926 data, catalogued in Appendix B, show the magnitudes of these lower estimates.[20] Table 3 displays Siegel's ERPs for three subperiods. He notes that subperiod III, 1926-2001, shows a larger ERP (4.7%), or a smaller real risk-free mean (2.2%), than the prior subperiods[21].

[18] For the Ibbotson analysis of the small stock premium, the NYSE/AMEX/NASDAQ combined data are used with the S&P 500 data falling within deciles 1 and 3 (Ibbotson 2002 Yearbook, pp122-136.)
[19] A more recent alternative is Wilson and Jones (2002) as cited by Dimson et al. (2002), p39.
[20] Using Wilson and Jones' 1871-2002 data series, time series analyses show no significant ERP difference between the 1871-1925 period and the 1926-2002 period; one cannot distinguish the old from the new. The overall average is lower with the additional 1871-1925 data, but on a statistical basis, they are not significantly different. Assuming the equivalency of the two data series for 1871 to 1925 (series of Goetzmann et al. and Wilson & Jones), the risk difference found by Goetzmann et al. must be determined by a significantly different ERP in the pre-1871 data. The 1871-1913 return is prior to personal income tax and appears to be about 35% lower than the 1926-2002 period average of 11.8%, might reflect a zero valuation for income taxes in the pre-1914 returns. Adjusting the pre-1914 data for taxes would most likely make the ERP for the entire period (1871-2002) approximately equal to 7.5%, the 1926-2002 average.
[21] The low risk-free return is indicative of the "risk-free rate puzzle", the twin of the ERP puzzle. For details see Weil (1989).

| Short-Horizon Equity Risk Premium by Subperiods | | | |
|---|---|---|---|
| | **Subperiod I** | **Subperiod II** | **Subperiod III** |
| | **1802-1870** | **1871-1925** | **1926-2001** |
| Real Geometric Stock Returns | 7.0% | 6.6% | 6.9% |
| Real Geometric Long Term Governments | 4.8% | 3.7% | 2.2% |
| Equity Risk Premium | 2.2% | 2.9% | 4.7% |
| *Source: Siegel (2002), pages 13 and 15.* | | | |

**Table 3**

Smaller subperiods will show much larger variations in equity, bill and ERP returns. Table 4 displays the Ibbotson returns and short horizon risk premia for subperiods as small as 5 years. The scatter of results is indicative of the underlying large variation (20% sd) in annual data.

| Average Short-Horizon Risk Premium over Various Time Period | | | | |
|---|---|---|---|---|
| | | Common Stocks | U. S. Treasury Bills | Short-Horizon |
| **Year** | | **Total Annual Returns** | **Total Annual Returns** | **Risk Premium** |
| | | | | |
| All Data | 1926-2002 | 12.20% | 3.83% | 8.37% |
| | | | | |
| 50 Year | 1953-2002 | 12.50% | 5.33% | 7.17% |
| | | | | |
| 40 Year | 1963-2002 | 11.80% | 6.11% | 5.68% |
| | | | | |
| 30 Year | 1943-1972 | 14.55% | 2.54% | 12.02% |
| | 1973-2002 | 12.21% | 6.61% | 5.60% |
| | | | | |
| 15 Year | 1928-1942 | 5.84% | 0.95% | 4.89% |
| | 1943-1957 | 17.14% | 1.20% | 15.94% |
| | 1958-1972 | 11.96% | 3.87% | 8.09% |
| | 1973-1987 | 11.42% | 8.20% | 3.22% |
| | 1988-2002 | 13.00% | 5.03% | 7.97% |
| | | | | |
| 10 Year | 1933-1942 | 12.88% | 0.15% | 12.73% |
| | 1943-1952 | 17.81% | 0.81% | 17.00% |
| | 1953-1962 | 15.29% | 2.19% | 13.11% |
| | 1963-1972 | 10.55% | 4.61% | 5.94% |
| | 1973-1982 | 8.67% | 8.50% | 0.17% |
| | 1983-1992 | 16.80% | 6.96% | 9.84% |
| | 1993-2002 | 11.17% | 4.38% | 6.79% |
| | | | | |
| 5 Year | 1928-1932 | - 8.25% | 2.55% | -10.80% |
| | 1933-1937 | 19.82% | 0.22% | 19.60% |
| | 1938-1942 | 5.94% | 0.07% | 5.87% |
| | 1943-1947 | 15.95% | 0.37% | 15.57% |
| | 1948-1952 | 19.68% | 1.25% | 18.43% |
| | 1953-1957 | 15.79% | 1.97% | 13.82% |
| | 1958-1962 | 14.79% | 2.40% | 12.39% |
| | 1963-1967 | 13.13% | 3.91% | 9.22% |
| | 1968-1972 | 7.97% | 5.31% | 2.66% |
| | 1973-1977 | 2.55% | 6.19% | - 3.64% |
| | 1978-1982 | 14.78% | 10.81% | 3.97% |
| | 1983-1987 | 16.93% | 7.60% | 9.33% |
| | 1988-1992 | 16.67% | 6.33% | 10.34% |
| | 1993-1997 | 21.03% | 4.57% | 16.46% |
| | 1998-2002 | 1.31% | 4.18% | - 2.88% |

**Table 4**

In calculating an expected market risk premium by averaging historical data, projecting historical data using growth models, or even conducting a survey, one must determine a proxy for the "market". Common proxies for the US market include the S&P 500, the NYSE index, and the NYSE, AMEX, and NASDAQ index.[22] For the purpose of this paper, we use the S&P 500 and its antecedents as the market. However, in the various research surveyed, many different market proxies are assumed. We have already discussed using international versus domestic data when describing different MRP types. With international data, different proxies for other country, region, or world markets are used.[23] For domestic data, different proxies have been used over time as stock market exchanges have expanded.[24] Fortunately, as shown in the Ibbotson Valuation yearbook, the issue of a US market proxy does not have a large effect on the MRP estimate because the various indices are highly correlated. For example, the S&P 500 and the NYSE have a correlation of 0.95, the S&P 500 and NYSE/AMEX/NASDAQ 0.97, and the NYSE and NYSE/AMEX/NASDAQ 0.90.[25] Therefore, the market proxy selected is one reason for slight differences in the estimates of the market risk premium.

As a final note, stock returns and risk-free rates can be stated in <u>nominal or real</u> terms. Nominal includes inflation; real removes inflation. The equity risk premium should not be affected by inflation because either the stock return and risk-free rate both include the effects of inflation (both stated in nominal terms) or neither have inflation (both stated in real terms). If both returns are nominal, the difference in the returns is generally assumed to remove inflation. Otherwise, both terms are real, so inflation is removed prior to finding the equity risk premium. While numerical differences in the real and nominal approaches may exist, their magnitudes are expected to be small.

## Equity Risk Premia 1926-2002

As an example of the importance of knowing the types of equity risk premium estimates under consideration, Table 5 displays ERP returns that each use the same historical data, but are based on arithmetic or geometric returns and the type of horizon. The ERP estimates are quite different.[26]

[22] 2003 Ibbotson Valuation Yearbook, p92.
[23] For example, Dimson (2002) and Claus and Thomas (2001) use international market data.
[24] For a data series that is a mixture of the NYSE exchange, NYSE, AMEX, and NASDAQ stock exchange, and the Wilshire 5000, see Dimson (2002), p306.
[25] 2003 Ibbotson Valuation Yearbook, p93; using data from October 1997 to September 2002.
[26] The nominal and real ERPs are identical in Table 5 because the ERPs are calculated as arithmetic differences, and the same value of inflation will reduce the market return and the risk-free return equally. Geometric differences would produce minimally different estimates for the same types.

| ERP using same historical data (1926-2002) | | |
|---|---|---|
| **RFR Description** | **ERP Description** | **ERP Historical Return** |
| Short nominal | Arithmetic Short-horizon | 8.4% |
| Short nominal | Geometric Short-horizon | 6.4% |
| Short real | Arithmetic Short-horizon | 8.4% |
| Short real | Geometric Short-horizon | 6.4% |
| Intermediate nominal | Arithmetic Inter-horizon | 7.4% |
| Intermediate nominal | Geometric Inter-horizon | 5.4% |
| Intermediate real | Arithmetic Inter-horizon | 7.4% |
| Intermediate real | Geometric Inter-horizon | 5.4% |
| Long nominal | Arithmetic Long-horizon | 7.0% |
| Long nominal | Geometric Long-horizon | 5.0% |
| Long real | Arithmetic Long-horizon | 7.0% |
| Long real | Geometric Long-horizon | 5.0% |

**Table 5**

## Historical Methods

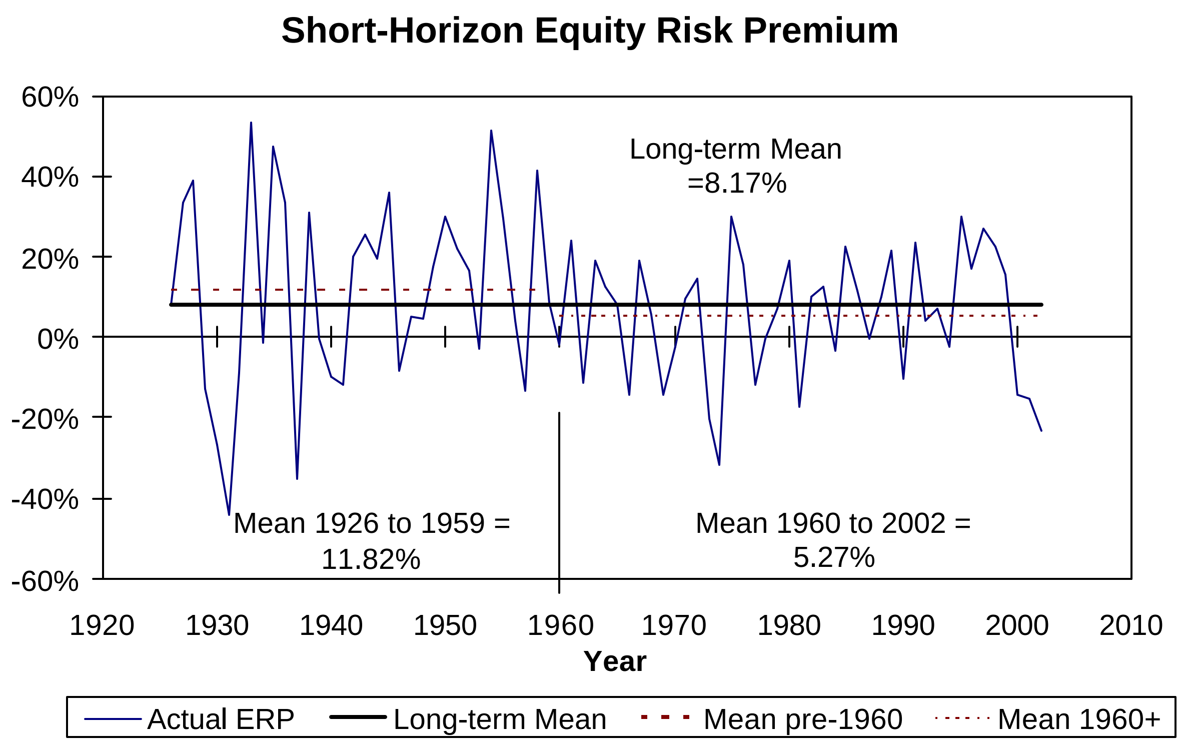
The historical methodology uses averages of past returns to forecast future returns. Different time periods may be selected, but the two most common periods arise from data provided by either Ibbotson or Siegel. The Ibbotson series begins in 1926 and is updated each year. The Siegel series begins in 1802 with the most recent compilation using returns through 2001. Appendix A provides equity risk premium estimates using Ibbotson data for the 1926-2002 period that we use in this paper for most illustrations. We begin with a look at the ERP history through a time series analysis of the Ibbotson data.

## Time Series Analysis

Much of the analysis addressing the equity risk premium puzzle relies on the annual time series of market, risk-free and risk premium returns. Two opposite views can be taken of these data. One view would have the 1926-2002 Ibbotson data, or the 1802-2001 Siegel data, represent one data point; i.e., we have observed one path for the ERP through time from the many possible 77 or 200 year paths. This view rests upon the existence or assumption of a stochastic process with (possibly) inter-temporal correlations. While mathematically sophisticated, this model is particularly unhelpful without some testable hint at the details of the generating stochastic process. The practical view is that the observed returns are random samples from annual distributions that are iid, independent and identically distributed about the mean. The obvious advantage is that we have at hand 77 or 200 observations on the iid process to analyze. We adopt the latter view.

Some analyses adopt the assumption of stationarity of ERP, i.e., the true mean does not change with time. Figure 1 displays the Ibbotson ERP data and highlights two subperiods, 1926-1959 and 1960-2002.[27] While the mean ERP for the two subperiods appear quite different (11.82% vs. 5.27%), the large variance of the process (std dev 20.24%) should make them indistinguishable statistically speaking.

[27] The ERP shown here are the geometric differences (calculated) rather than the simple arithmetic differences in Table 1; i.e. ERP = $[(1+r_m)/(1+r_f)] - 1$. The test results are qualitatively the same for the arithmetic differences.




Source: Ibbotson Yearbook (2003), Geometric Differences

**Figure 1**

## T-Tests

The standard T-test can be used for the null hypothesis $H_o$ : mean 1960-2002 = 8.17%, the 77 year mean.[28] The outcome of the test is shown in Table 6; the null hypothesis cannot be rejected.

| T-Test Under the Null Hypothesis that ERP (1960-2002) = ERP (1926-2002) = 8.17% | |
|---|---|
| Sample mean 1960-2002 | 5.27% |
| Sample s.d. 1960-2002 | 15.83% |
| T value (DF=42) | -1.20 |
| PR > \|T\| | 0.2374 |
| Confidence Interval 95% | (0.0040, 0.1014) |
| Confidence Interval 90% | (0.0121, 0.0933) |

**Table 6**

Another T-Test can be used to test whether the subperiod means are different in the presence of unequal variances.[29] The result is similar to Table 6 and the difference of subperiod means equal to zero cannot be rejected.[30]

[28] Standard statistical procedures in SAS 8.1 have been used for all tests.
[29] Equality of variances is rejected at the one percent level by an F test (F=2.39, DF=33,42)
[30] t-value 1.35, PR> |T| = 0.1850 with the Cochran method.

## Time Trends

The supposition of stationarity of the ERP series can be supported by ANOVA regressions. The results of regressing the ERP series on time is shown in Table 7.

| ERP ANOVA Regressions on Time | | |
|---|---|---|
| **Period** | **Time Coefficient** | **P-Value** |
| 1926-1959 | 0.004 | 0.355 |
| 1960-2002 | 0.001 | 0.749 |
| 1926-2002 | -0.001 | 0.443 |

**Table 7**

There are no significant time trends in the Ibbotson ERP data.[31]

## ARIMA Model

Time series analysis using the well established Box-Jenkins approach can be used to predict future series values through the lag correlation structure.[32] The SAS ARIMA procedure applied to the full 77 time series data shows:

(1) No significant autocorrelation lags.
(2) An identification of the series as white noise.
(3) ARIMA projection of year 78+ ERP is 8.17%, the 77 year average.

All of the above single time series tests point to the reasonability of the stationarity assumption for (at least) the Ibbotson ERP 77 year series.[33]

## Social Security Administration

In the current debate on whether to allow private accounts that may invest in equities, the Office of the Chief Actuary of the Social Security Administration has selected certain assumptions to assess various proposals (Goss, 2001). The relevant selection is to use 7 percent as the real (geometric) annual rate of return for equities.[34] This assumption is based on the historical return of the 20th century. SSA received further support that showed the historical return for the last 200 years is consistent with this estimate, along with the Ibbotson series beginning in 1926. For SSA, the calculation of the equity risk premium uses a long-run real yield on Treasury bonds as the risk-free rate. From the assumptions in the 1995 Trustees Report, the long-run real yield on Treasury bonds that the Advisory Council proposals use is 2.3%. Using a future Treasury securities real yield of 2.3% produces a geometric equity risk premium of 4.7% over long-term Treasury securities. More recently, the Treasury securities assumption has increased to 3%[35], yielding a 4% geometric ERP over long-term Treasury securities.

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[31] The result is confirmed by a separate Chow test on the two subperiods.
[32] See Harvey (1990), p30.
[33] The same tests applied to the Wilson and Jones 1871-2002 data series show similar results: Neither the 1871-1925 period nor the 1926-2002 period is different from the overall 1871-2002 period. The overall period and subperiods also show no trends over time.
[34] Compare Table 3, subperiod III.
[35] 1999 Social Security Trustees Report.

At the request of the Office of the Chief Actuary of the Social Security Administration (OCACT), John Campbell, Peter Diamond, and John Shoven were engaged to give their expert opinions on the assumptions Social Security mode. Each economist begins with the Social Security assumptions and then explains any difference he feels would be more appropriate.

In John Campbell's response, he considers valuation ratios as a comparison to the returns from the historical approach (Campbell 2001). The current valuation ratios are at unusual levels, with a low dividend-price ratio and high price-earnings ratio. He reasons that the prices are what have dramatically changed these ratios. Campbell presents two views as to the effect of valuation ratios in their current state. One view is that valuations will remain at the current level, suggesting much lower expected returns. The second view is a correction to the ratios, resulting in less favorable returns until the ratios readjust. He decides to give some weight to both possibilities, so he lowers the geometric equity return estimate to 5-5.5% from 7%. For the risk-free rate, he uses the yield on the long-term inflation-indexed bonds[36] of 3.5% or the OCACT assumption of 3%. Therefore, his geometric equity premium estimate is around 1.5 to 2.5%.

Peter Diamond uses the Gordon growth formula to calculate an estimate of the equity return (Diamond 2001). The classic Gordon Dividend Growth model is[37]:

$$K = (D_1 / P_0) + g$$

$K$ = Expected Return or Discount Rate $\quad P_0$ = Price this period
$D_1$ = Expected Dividend next period $\quad g$ = Expected growth in dividends in perpetuity

Based on his analysis, he feels that the equity return assumption of 7% for the next 75 years is not consistent with a reasonable level of stock value compared to GDP. Even when increasing the GDP growth assumption, he still does not feel that the equity return is plausible. By reasoning that the next decade of returns will be lower than normal, only then is the equity return beyond that time frame consistent with the historical return. By considering the next 75 years together, he would lower the overall projected equity return to 6-6.5%. He argues that the stock market is overvalued, and a correction is required before the long-run historical return is a reasonable projection for the future. By using the OCACT assumption of 3.0% for the long-term real yield on Treasury bonds, Diamond estimates a geometric equity risk premium of about 3-3.5%.

John Shoven begins by explaining why the traditional Gordon growth model is not appropriate, and he suggests a modernized Gordon model that allows share repurchases to be included instead of only using the dividend yield and growth rate (Shoven 2001). By assuming a long-term price-earnings ratio between its current and historical value, he comes up with an estimate for the long-term real equity return of 6.125%. Using his general estimate of 6-6.5% for the equity return and the OCACT assumptions for the long-term bond yield, he projects a long-term equity risk premium of approximately 3-3.5%. All the SSA experts begin by accepting the long-run historical

[36] See discussion of current yields on TIPS below.
[37] Brealey and Myers (2000), p67.

ERP analyses and then modifying that by changes in the risk-free rate or by decreases in the long-term ERP based on their own personal assessments. We now turn to the major strains in ERP puzzle research.

## ERP Puzzle Research

Campbell and Shiller (2001) begin with the assumption of mean reversion of dividend/price and price/earnings ratios. Next, they explain the result of prior research which finds that the dividend-price ratio predicts future prices, and historically, the price corrects the ratio when it diverts from the mean.[38] Based on this result, they then use regressions of the dividend-price ratio and the price-smoothed-earnings[39] ratio to predict future stock prices out ten years. Both regressions predict large losses in stock prices for the ten year horizon. Although Campbell and Shiller do not rerun the regression on the dividend-price ratio to incorporate share repurchases, they point out that the dividend-price ratio should be upwardly adjusted, but the adjustment only moves the ratio to the lower range of the historical fluctuations (as opposed to the mean). They conclude that the valuation ratios indicate a bear market in the near future[40]. They predict for the next ten year period negative real stock returns. They caution that because valuation ratios have changed so much from their normal level, they may not completely revert to the historical mean, but this does not change their pessimism about the next decade of stock market returns.

Arnott and Ryan (2001) take the perspective of fiduciaries, such as pension fund managers, with an investment portfolio. They begin by breaking down the historical stock returns (past 74 years since December 1925) by analyzing dividend yields and real dividend growth. They point out that the historical dividend yield is much higher than the current dividend yield of about 1.2%. They argue that the changes from stock repurchases, reinvestment, and mergers and acquisitions, which affect the lower dividend yield, can be represented by a higher dividend growth rate. However, they cap real dividend or earnings growth at the level of real economic growth. They add the dividend yield and the growth in real dividends to come up with an estimate for the future equity return; the current dividend yield of 1.2% and the economic growth rate of 2.0% add to the 3.2% estimated real stock return. This method corresponds to the dividend growth model or earnings growth model and does not take into account changing valuation levels. They cite a TIPS yield of 4.1% for the real risk-free rate return.[41] These two estimates yield a negative geometric long-horizon conditional equity risk premium.

Arnott and Bernstein (2002) begin by arguing that in 1926 investors were not expecting the realized, historical compensation that they later received from stocks. They cite bonds' reaction to inflation, increasing valuations, survivorship bias[42], and changes in

[38] Campbell and Shiller (1989).
[39] Earnings are "smoothed" by using ten year averages.
[40] The stock market correction from year-end 1999 to year-end 2002 is a decrease of 37.6% or 14.6% per year. Presumably, the "next ten years" refers to 2000 to 2010.
[41] See the current TIPS yield discussion near end of paper.
[42] See Brown et al. (1992, 1995) for details on potential survivorship bias.

regulation as positive events that helped investors during this period. They only use the dividend growth model to predict a future expected return for investors. They do not agree that the earnings growth model is better than the dividend growth model both because earnings are reported using accounting methods and earnings data before 1870 are inaccurate. Even if the earnings growth model is chosen instead, they find that the earnings growth rate from 1870 only grows 0.3% faster than dividends, so their results would not change much. Because of the Modigliani-Miller theorem[43], a change in dividend policy should not change the value of the firm. They conclude that managers benefited in the "era of 'robber baron' capitalism" instead of the conclusion reached by others that the dividend growth model under-represents the value of the firm.

By holding valuations constant and using the dividend yield and real growth of dividends, Arnott and Bernstein calculate the equity return that an investor might have expected during the historical time period starting in 1802. They use an expected dividend yield of 5.0%, close to the historical average of 1810 to 2001. For the real growth of dividends, they choose the real per capita GDP growth less a reduction for entrepreneurial activity in the economy plus stock repurchases. They conclude that the net adjustment is negative, so the real GDP growth is reduced from 2.5-3% to only 1%. A fair expectation of the stock return for the historical period is close to 6.1% by adding 5.0% for the dividend yield and a net real GDP per capita growth of 1.1%. They use a TIPS yield of 3.7% for the real risk-free rate, which yields a geometric intermediate-horizon equity risk premium of 2.4% as a fair expectation for investors in the past. They consider this a "normal" equity risk premium estimate. They also opine that the current ERP is zero; i.e. they expect stocks and (risk-free) bonds to return the same amounts.

Fama and French (2002) use both the dividend growth model and the earnings growth model to investigate three periods of historical returns: 1872 to 2000, 1872 to 1950, and 1951 to 2000. Their ultimate aim is to find an unconditional equity risk premium. They cite that by assuming the dividend-price ratio and the earnings-price ratio follow a mean reversion process, the result follows that the dividend growth model or earnings growth model produce approximations of the unconditional equity return. Fama and French's analysis of the earlier period of 1872 to 1950 shows that the historical average equity return and the estimate from the dividend growth model are about the same. In contrast, they find that the 1951 to 2000 period has different estimates for returns when comparing the historical average and the growth models' estimates. The difference in the historical average and the model estimates for 1951 to 2000 is interpreted to be "unexpected capital gains" over this period. They find that the unadjusted growth model estimates of the ERP, 2.55% from the dividend model and 4.32% from the earnings model, fall short of the realized average excess return for 1951-2000. Fama and French prefer estimates from growth models instead of the historical method because of the lower standard error using the dividend growth model. Fama and French provide 3.83% as the unconditional expected equity risk premium return (referred to as the annual bias-adjusted ERP estimate) using the dividend growth model with underlying data from 1951 to 2000. They give 4.78% as the unconditional expected equity risk

[43] Brealey and Myers (2000), p447. See also discussion in Ibbotson and Chen (2003).

premium return using the earnings growth model with data from 1951 to 2000. Note that using a one-month Treasury bill instead of commercial paper for the risk-free rate would increase the ERP by about 1% to nearly 6% for the 1951-2000 period.

Ibbotson and Chen (2003) examine the historical real geometric long-run market and long risk-free returns using their "building block" methodology.[44] They use the full 1926-2000 Ibbotson Associates data and consider as building blocks all of the fundamental variables of the prior researchers. Those blocks include (not all simultaneously):

- Inflation
- Real risk-free rates (long)
- Real capital gains
- Growth of real earnings per share
- Growth of real dividends
- Growth in payout ratio (dividend/earnings)
- Growth in book value
- Growth in ROE
- Growth in price/earnings ratio
- Growth in real GDP/population
- Growth in equities excess of GDP/POP
- Reinvestment

Their calculations show that a forecast real geometric long run return of 9.4% is a reasonable extrapolation of the historical data underlying a realized 1926-2000 return of 10.7%, yielding a long horizon arithmetic ERP of 6%, or a short horizon arithmetic ERP of about 7.5%.

The authors construct six building block methods; i.e., they use combinations of historic estimates to produce an expected geometric equity return. They highlight the importance of using both dividends and capital gains by invoking the Modigliani-Miller theorem. The methods, and their component building blocks are:

- Method 1: Inflation, real risk free rate, realized ERP
- Method 2: Inflation, income, capital gains and reinvestment
- Method 3: Inflation, income, growth in price/earnings, growth in real earnings per share and reinvestment.
- Method 4: Inflation, growth rate of price/earnings, growth rate of real dividends, growth rate of payout ratio dividend yield and reinvestment
- Method 5: Inflation, income growth rate of price/earnings, growth of real book value, ROE growth and reinvestment
- Method 6: Inflation, income, growth in real GDP/POP, growth in equities excess GDP/POP and reinvestment.

[44] See Appendix D for a summary of their building block estimates. See also Pratt (1998) for a discussion of the Building Block, or Build-Up Model, cost of capital estimation method.

All six methods reproduce the historical long horizon geometric mean of 10.70% as shown in Appendix D. Since the source of most other researchers' lower ERP is the dividend yield, the authors recast the historical results in terms of ex ante forecasts for the next 75 years. Their estimate of 9.37% using supply side methods 3 and 4 is approximately 130 basis points lower than the historical result. Within their methods, they also show how the substantially lower expectation of 5.44% for the long mean geometric return is calculated by omitting one or more relevant variables. Underlying these ex ante methods are the assumptions of stationarity of the mean ERP return and market efficiency, the absence of the assumption that the market has mispriced equities. All of their methods are aimed at producing an unconditioned estimate of the ex ante ERP.

As opposed to short-run, conditional estimates from Campbell and Shiller and others, Constantinides (2002) seeks to estimate the unconditional equity risk premium, more in line with the goal of Fama and French (2002) and Ibbotson and Chen (2003). He begins with the premise that the unconditional ERP can be estimated from the historical average using the assumption that the ERP follows a stationary path. He suggests most of the other research produces conditional estimates, conditioned upon beliefs about the future paths of fundamentals such as dividend growth, price-earnings ratio and the like. While interesting in themselves, they add little to the estimation of the unconditional mean ERP.

Constantinides uses the historical return and adjusts downward by the growth in the price-earnings ratio to calculate the unconditional equity risk premium. He removes the growth in the price-earnings ratio because he is assuming no change in valuations in the unconditional state. He gives estimates using three periods. For 1872-2000, he uses the historical equity risk premium which is 6.9%, and after amortizing the growth in the price-dividend ratio or price-earnings ratio over a period as long as 129 years, the effect of the potential reduction is no change. Therefore, he finds an unconditional arithmetic, short-horizon equity risk premium of 6.9% using the 1872-2000 underlying data. For 1951-2000, he again starts with the historical equity risk premium which is 8.7% and lowers this estimate by the growth in the price-earnings ratio of 2.7% to find an unconditional arithmetic, short-horizon equity risk premium of 6.0%. For 1926-2000, he uses the historical equity risk premium which is 9.3% and reduces this estimate by the growth in the price-earnings ratio of 1.3% to find an unconditional arithmetic, short-horizon equity risk premium of 8.0%. He appeals to behavioral finance to offer explanations for such high unconditional equity risk premium estimates.

From the perspective of giving practical investor advice, Malkiel (1999) discusses "the age of the millennium" to give some indication of what investors might expect for the future. He specifically estimates a reasonable expectation for the first few decades of the twenty-first century. He estimates the future bond returns by giving estimates if bonds are held to maturity with corporate bonds of 6.5-7%, long-term zero-coupon Treasury bonds of about 5.25%, and TIPS with a 3.75% return. Depending on the desired level of risk, Malkiel indicates bondholders should be more favorably compensated in the future compared to the historical returns from 1926 to 1998. Malkiel uses the earnings growth model to predict future equity returns. He uses the current dividend yield of 1.5% and an earnings growth estimate of 6.5%, yielding an 8% equity return estimate compared with an 11% historical return. Malkiel's estimated range of the equity risk premium is from 1% to 4.25%, depending on the risk-free instrument selected. Although his equity risk premium is lower than the historical return, his selection of a relatively high earnings growth rate is similar to Ibbotson and Chen's forecasted models. In contrast with Ibbotson and Chen, Malkiel allows for a changing equity risk premium and advises investors to not rely solely on the past "age of exuberance" as a guide for the future. Malkiel points out the impact of changes in valuation ratios, but he does not attempt to predict future valuation levels.

Finally, Mehra (2002) summarizes the results of the research since the ERP puzzle was posed. The essence of the puzzle is the inconsistency of the ERPs produced by descriptive and prescriptive economic models of asset pricing on the one hand and the historical ERPs realized in the US market on the other. Mehra and Prescott (1985) speculated that the inconsistency could arise from the inadequacy of standard models to incorporate market imperfections and transaction costs. Failure of the models to reflect reality rather than failure of the market to follow the theory seems to be Mehra's conclusion as of 2002. Mehra points to two promising threads of model-modifying research. Campbell and Cochrane (1999) incorporate economic cycles and changing risk aversion while Constantinides et al. (2002) propose a life cycle investing modification, replacing the representative agent by segmenting investors into young, middle aged, and older cohorts. Mehra sums up by offering:

> "Before we dismiss the premium, we not only need to have an understanding of the observed phenomena but also why the future is likely to be different. In the absence of this, we can make the following claim based on what we know. Over the long horizon the equity premium is likely to be similar to what it has been in the past and the returns to investment in equity will continue to substantially dominate those in bonds for investors with a long planning horizon."

## **Financial Analyst Estimates**

Claus and Thomas (2001) and Harris and Marston (2001) both provide equity premium estimates using financial analysts' forecasts. However, their results are rather different. Claus and Thomas use an abnormal earnings model with data from 1985 to 1998 to calculate an equity risk premium as opposed to using the more common dividend growth model. Financial analysts project five year estimates of future earnings growth rates. When using this five year growth rate for the dividend growth rate in perpetuity in the Gordon growth model, Claus and Thomas explain that there is a potential upward bias in estimates for the equity risk premium. Therefore, they choose to use the abnormal earnings model instead and only let earnings grow at the level of inflation after five years. The abnormal earnings model replaces dividends with "abnormal earnings" and discounts each flow separately instead of using a perpetuity. The average estimate that they find is 3.39% for the equity risk premium. Although it is generally recognized that financial analysts' estimates have an upward bias, Claus and Thomas propose that in the current literature, financial analysts' forecasts have underestimated short-term earnings in order for management to achieve earnings estimates in the slower economy. Claus and Thomas conclude that their findings of the ERP using data from the past fifteen years are not in line with historical values.

Harris and Marston use the dividend growth model with data from 1982 to 1998. They assume that the dividend growth rate should correspond to investor expectations. By using financial analysts' longest estimates (five years) of earnings growth in the model, they attempt to estimate these expectations. They argue that if investors are in accord with the optimism shown in analysts' estimates, even biased estimates do not pose a drawback because these market sentiments will be reflected in actual returns. Harris and Marston find an equity risk premium estimate of 7.14%. They find fluctuations in the equity risk premium over time. Because their estimates are close to historical returns, they contend that investors continue to require a high equity risk premium.

**Survey Methods**

One method to estimate the ex ante equity risk premium is to find the consensus view of experts. John Graham and Campbell Harvey perform a survey of Chief Financial Officers to determine the average cost of capital used by firms. Ivo Welch surveys financial economists to determine the equity risk premium that academic experts in this area would estimate.

Graham and Harvey administer surveys from the second quarter of 2000 to the third quarter of 2002 (Graham and Harvey, 2002). For their survey format, they show the current ten year bond yield and then ask CFOs to provide their estimate of the S&P 500 return for the next year and over the next ten years. CFOs are actively involved in setting a company's individual hurdle[45] rate and are therefore considered knowledgeable about investors' expectations.[46] When comparing the survey responses of the one and ten year returns, the one year returns have so much volatility that they conclude that the ten-year equity risk premium is the more important and appropriate return of the two when making financial decisions such as hurdle rates and estimating cost of capital. The average ten-year equity risk premium estimate varies from 3% to 4.7%.

The most current Welch survey compiles the consensus view of about five hundred financial economists (Welch 2001). The average arithmetic estimate for the 30-year equity risk premium relative to Treasury bills is 5.5%; the one-year arithmetic equity risk premium consensus is 3.4%. Welch deduces from the average 30-year geometric

[45] A "hurdle" rate is a benchmark cost of capital used to evaluate projects to accept (expected returns greater than hurdle rate) or reject (expected returns less than hurdle rate).
[46] Graham and Harvey claim three-fourths of the CFOs use CAPM to estimate hurdle rates.

equity return estimate of 9.1% that the arithmetic equity return forecast is approximately 10%.[47]

Welch's survey question allows the participants to self select into different categories based upon their knowledge of ERP. The results indicate that the responses of the less ERP knowledgeable participants showed more pessimism than those of the self reported experts. The experts gave 30-year estimates that are 30 to 150 basis points above the estimates of the non-expert group.

| **Differences in Forecasts across Expertise Level** | | | | |
|---|---|---|---|---|
| **Relative Expertise** | **Statistic** | **Stock Market** | **Equity Premium** | |
| | | | | |
| | | 30-Year Geometric | 30-Year Arithmetic | 30-Year Geometric |
| 188 Less Involved | Mean | 8.5% | 4.9% | 4.4% |
| | Median | 8% | 5% | 4% |
| | IQ Range | 6%-10% | 3%-6% | 2%-5.5% |
| 235 Average | Mean | 9.2% | 5.8% | 4.8% |
| | Median | 9% | 5% | 4% |
| | IQ Range | 7.5%-10% | 3.5%-7% | 3%-6% |
| 72 Experts | Mean | 10.1% | 6.2% | 5.4% |
| | Median | 9% | 5.4% | 5% |
| | IQ Range | 8%-11% | 4%-7.5% | 3.4%-6% |
| *Data Source: Welch (2001), Table 5* | | | | |

**Table 8**

Table 8 shows that there may be a "lemming" effect, especially among economists who are not directly involved in the ERP question. Stated differently, all the academic and popular press, together with the prior Welch survey[48] could condition the non-expert, the "less involved", that the expected ERP was lower than historic levels.

## The Behavioral Approach

Benartzi and Thaler (1995) analyze the equity risk premium puzzle from the point of view of prospect theory (Kahneman and Tversky; 1979). Prospect theory[49] has "loss aversion", the fact that individuals are more sensitive to potential loss than gain, as one of its central tenets. Once an asymmetry in risk aversion is introduced into the model of the rational representative investor or agent, the unusual risk aversion problem raised initially by Mehra and Prescott (1985) can be "explained" within this behavioral model of decision-making under uncertainty. Stated differently, given the historical ERP series, there exists a model of investor behavior that can produce those or similar results. Benartzi and Thaler combine loss aversion with "mental accounting", the behavioral process people use to evaluate their status relative to gains and losses compared to expectations, utility and wealth, to get "myopic loss aversion". In particular, mental

[47] For the Ibbotson 1926-2002 data, the arithmetic return is about 190 basis points higher than the geometric return rather than the inferred 90 basis points. This suggests the participant's beliefs may not be internally consistent.
[48] The prior Welch survey in 1998 had a consensus ERP of about 7%.
[49] A current survey of the applications of prospect theory to finance can be found in Benartzi et al. (2001).

accounting for a portfolio needs to take place infrequently because of loss aversion, in order to reduce the chances of observing loss versus gain. The authors concede that there is a puzzle with the standard expected utility-maximizing paradigm but that the myopic loss aversion view may resolve the puzzle. The authors' views are not free of controversy; any progress along those lines is sure to match the advance of behavioral economics in the large.

The adoption of other behavioral aspects of investing may also provide support for the historical patterns of ERPs we see from 1802-2002. For example, as the true nature of risk and rewards has been uncovered by the virtual army of 20$^{th}$ century researchers, and as institutional investors held sway in the latter fifty years of the century, the demand for higher rewards seen in the later historical data may be a natural and rational response to the new and expanded information set. Dimson et al. (2002, Figure 4-6) displays increasing real US equity returns of 6.7, 7.4, 8.2 and 10.2 for periods of 101, 75, 50 and 25 years ending in 2001 consistent with this "risk-learning" view.

**Next Ten Years**

The "next ten years" is an issue that experts reviewing Social Security assumptions and Campbell and Shiller address either explicitly or implicitly. Experts evaluating Social Security's proposals predicted that the "next ten years", indicating a period beginning around 2000, of returns were likely to be below the historical return. However, a historical return was recommended as appropriate for the remaining 65 of the 75 years to be projected. For Campbell and Shiller (2001), the period they discuss is approximately 2000-2010. Based on the current state of valuation ratios, they predict lower stock market returns over "the next ten years". These expert predictions, and other pessimistic low estimates, have already come to fruition as market results 2000 through 2002.[50] The US equities market has decreased 37.6% since 1999, or an annual decrease of 14.6%. Although these forecasts have proved to be accurate in the short term, for future long-run projections, the market is not at the same valuation today as it was when these conditional estimates were originally given. Therefore, actuaries should be wary of using the low long-run estimates made prior to the large market correction of 2000-2002.

**Treasury Inflation Protection Securities (TIPS)**

Several of the ERP researchers refer to TIPS when considering the real risk free rates. Historically, they adjust Treasury yields downward to a real rate by an estimate of inflation, presumably for the term of the Treasury security. As Table 3 shows using the Siegel data, the modern era data show a low real long-term risk-free rate of return (2.2%). This contrasts with the initial[51] TIPS issue yields of 3.375%. Some researchers use those TIPS yields as (market) forecasts of real risk-free returns for intermediate and long-horizon, together with reduced (real) equity returns to produce low estimates of ex ante ERPs. None consider the volatility of TIPS as indicative of the accuracy of their ERP estimate.

[50] The Social Security Advisory Board will revisit the seventy five year rate of return assumption during 2003, Social Security Advisory Board (2002).
[51] TIPS were introduced by the Treasury in 1996 with the first issue in January, 1997.

Table 9 shows a recent market valuation of ten and thirty year TIPS issued in 1998-2002.

| Inflation-Indexed Treasury Securities | | |
|---|---|---|
| **Maturity** | **Coupon Issue Rate** | **Yield to Maturity** |
| 1/11 | 3.500 | 1.763 |
| 1/12 | 3.375 | 1.831 |
| 7/12 | 3.000 | 1.878 |
| 4/28 | 3.625 | 2.498 |
| 4/29 | 3.875 | 2.490 |
| 4/32 | 3.375 | 2.408 |
| Source: WSJ 1 2/24/2003 | | |

**Table 9**

Note the large 90-180 basis point decrease in the current "real" yields from the issue yields as recent as ten months ago. While there can be several explanations for the change (revaluation of the inflation option, flight to Treasury quality, paucity of 30 year Treasuries), the use of these current "real" risk free yields, with fixed expected returns, would raise ERPs by at least one percent.

## Conclusion

This paper has sought to bring the essence of recent research on the equity risk premium to practicing actuaries. The researchers covered here face the same ubiquitous problems that actuaries face daily: Do I rely on past data to forecast the future (costs, premiums, investments) or do I analyze the past and apply informed judgment as to future differences, if any, to arrive at actuarially fair forecasts? Most of the ERP estimates lower than the unconditional historical estimate have an undue reliance on recent lower dividend yields (without a recognition of capital gains[52]) and/or on data prior to 1926.

Despite a spate of research suggesting ex ante ERPs lower than recent realized ERPs, actuaries should be aware of the range of estimates covered here (Appendix B); be aware of the underlying assumptions, data and terminology; and be aware that their independent analysis is required before adopting an estimate other than the historical average. We believe that the Ibbotson-Chen (2003) layout, reproduced here as Appendix D, offers the actuary both an understanding of the fundamental components of the historical ERP and the opportunity to change the estimates based upon good judgment and supportable beliefs. We believe that reliance solely on "expert" survey averages, whether of financial analysts, academic economists, or CFOs, is fraught with risks of statistical bias to fair estimates of the forward ERP.

[52] Under the current US tax code, capital gains are tax-advantaged relative to dividend income for the vast majority of equity holders (households and mutual funds are 55% of the total equity holders, Federal Flow of Funds, 2002 Q3, Table L-213). Curiously, the reverse is true for property-liability insurers because of the 70% stock dividend exclusion afforded insurers.

It is dangerous for actuaries to engage in simplistic analyses of historical ERPs to generate ex ante forecasts that differ from the realized mean.[53] The research we have catalogued in Appendix B, the common level ERPs estimated in Appendix C, and the building block (historical) approach of Ibboston and Chen in Appendix D all discuss important concepts related to both ex post and ex ante ERPs and cannot be ignored in reaching an informed estimate. For example, Richard Wendt, writing in a 2002 issue of Risks and Rewards, a newsletter of the Society of Actuaries, concludes that a linear relationship is a better predictor of future returns than a "constant" ERP based on the average historical return. He arrives at this conclusion by estimating a regression equation[54] relating long bond yields with 15-year geometric mean market returns starting monthly in 1960. First, there is no significant relationship between short, intermediate or long-term income returns over 1926-2002 (or 1960-2002) and ERPs, as evidenced by simple regressions using Ibbotson data.[55] Second, if the linear structural equation indeed held, there would be no need for an ERP since the (15-year) return could be predicted within small error bars. Third, there is always a negative bias introduced when geometric averages are used as dependent variables (Brennan and Schwartz, 1985). Finally, the results are likely to be spurious due to the high autocorrelations of the target and independent variables; an autocorrelation correction would eliminate any significant relationship of long-yields to the ERP.

Actuaries should also be aware of the variability of both the ERP and risk-free rate estimates discussed in this paper (see Tables 4 and 9). All too often, return estimates are made without noting the error bars and that can lead to unexpected "surprises". As one example, recent research by Francis Longstaff (2002), proposes that a 1991-2001 "flight to quality" has created a valuation premium (and lowered yields) in the entire yield curve of Treasuries. He finds a 10 to 16 basis point liquidity premium throughout the zero coupon Treasury yield curve. He translates that into a 10% to 15% pricing difference at the long end. This would imply a simple CAPM market estimate for the long horizon might be biased low.

Finally, actuaries should know that the research catalogued in Appendix B is not definitive. No simple model of ERP estimation has been universally accepted. Undoubtedly, there will be still more empirical and theoretical research into this data rich financial topic. We await the potential advances in understanding the return process that the behavioral view may uncover.

**Post Script: Appendices A-D**

We provide four appendices that catalogue the ERP approaches and estimates discussed in the paper. Actuaries, in particular, should find the numerical values, and descriptions of assumptions underlying those values, helpful for valuation work that

[53] ERPs are derived from historical or expected after corporate tax returns. Pre-tax returns depend uniquely on the tax schedule for the differing sources of income.

[54] 15-year mean returns = 2.032 (Long Government Bond Yield) – 0.0242, $R^2$ = 0.882.

[55] The p-values on the yield-variables in an ERP/Yield regression using 1926-2002 annual data are 0.1324, 0.2246, and 0.3604 for short, intermediate and long term yields respectively with adjusted r square virtually zero.

adjusts for risk. Appendix A provides the annual Ibbotson data from 1926 through 2002 from Ibbotson Associates referred to throughout this paper. The equity risk-premium shown is a simple difference of the arithmetic stock returns and the arithmetic U.S. Treasury Bills total returns. Appendix B is a compilation of articles and books related to the equity risk premium. The puzzle research section contains the articles and books that were most related to addressing the equity risk premium puzzle. Page 1 of Appendix B gives each source, along with risk-free rate and equity risk premium estimates. Then, each source's estimate is classified by type (indicated with an X for the appropriate type). Page 2 of Appendix B shows further details collected from each source. This page adds the data period used, if applicable, and the projection period. We also list the general methodology used in the reference. The final three pages of Appendix B provide the footnotes which give additional details on the sources' intent.

Appendix C adjusts all the equity risk premium estimates to a short-horizon, arithmetic, unconditional ERP estimate. We begin with the authors' estimates for a stock return (the risk-free rate plus the ERP estimate). Next, we make adjustments if the ERP "type" given by the author(s) is not given in this format. For example, to adjust from a geometric to an arithmetic ERP estimate, we adjust upwards by the 1926-2002 historical difference in the arithmetic large company stocks' total return and the geometric large company stocks' total return of 2%. Next, if the estimate is given in real instead of nominal terms, we adjust the stock return estimate upwards by 3.1%, the 1926-2002 historical return for inflation.

We make an approximate adjustment to move the estimate from a conditional to unconditional estimate based on Fama and French (2002). Using the results for the 1951-2000 period shown in Table 4 of their paper and the standard deviations provided in Table 1, we have four adjustments based on their data. For the 1951-2000 period, Fama and French use an adjustment of 1.28% for the dividend growth model and 0.46% for the earnings growth model. Following a similar calculation, the 1872-2000 period would require a 0.82% adjustment using a dividend growth model; the 1872-1950 period would require a 0.54% adjustment using a dividend growth model. Earnings growth models were used by Fama and French only for the 1951-2000 data period. Therefore, we selected the lowest adjustment (0.46%) as a minimum adjustment from a conditional estimate to an unconditional estimate. Finally, we subtract the 1926-2002 historical U.S. Treasury Bills' total return to arrive at an adjusted equity risk premium.

These adjustments are only approximations because the various sources rely on different underlying data, but the changes in the ERP estimate should reflect the underlying concept that different "types" of ERPs cannot be directly compared and require some attempt to normalize the various estimates.

Page 1 of Appendix D is a table from Ibbotson and Chen which breaks down historical returns using various methods that correspond to their 2003 paper (reprinted with permission of Ibbotson Associates). The bottom portion provides forward-looking estimates. Page 2 of Appendix D is provided to show the formulas that Ibbotson and Chen develop within their paper.

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| Appendix A<br>Ibbotson Market Data 1926-2002* | | | |
|---|---|---|---|
| | **Common Stocks** | **U. S. Treasury Bills** | **Arithmetic Short-Horizon** |
| **Year** | **Total Annual Returns** | **Total Annual Returns** | **Equity Risk Premia** |
| 1926 | 11.62% | 3.27% | 8.35% |
| 1927 | 37.49% | 3.12% | 34.37% |
| 1928 | 43.61% | 3.56% | 40.05% |
| 1929 | - 8.42% | 4.75% | -13.17% |
| 1930 | -24.90% | 2.41% | -27.31% |
| 1931 | -43.34% | 1.07% | -44.41% |
| 1932 | - 8.19% | 0.96% | - 9.15% |
| 1933 | 53.99% | 0.30% | 53.69% |
| 1934 | - 1.44% | 0.16% | - 1.60% |
| 1935 | 47.67% | 0.17% | 47.50% |
| 1936 | 33.92% | 0.18% | 33.74% |
| 1937 | -35.03% | 0.31% | -35.34% |
| 1938 | 31.12% | - 0.02% | 31.14% |
| 1939 | - 0.41% | 0.02% | - 0.43% |
| 1940 | - 9.78% | 0.00% | - 9.78% |
| 1941 | -11.59% | 0.06% | -11.65% |
| 1942 | 20.34% | 0.27% | 20.07% |
| 1943 | 25.90% | 0.35% | 25.55% |
| 1944 | 19.75% | 0.33% | 19.42% |
| 1945 | 36.44% | 0.33% | 36.11% |
| 1946 | - 8.07% | 0.35% | - 8.42% |
| 1947 | 5.71% | 0.50% | 5.21% |
| 1948 | 5.50% | 0.81% | 4.69% |
| 1949 | 18.79% | 1.10% | 17.69% |
| 1950 | 31.71% | 1.20% | 30.51% |
| 1951 | 24.02% | 1.49% | 22.53% |
| 1952 | 18.37% | 1.66% | 16.71% |
| 1953 | - 0.99% | 1.82% | - 2.81% |
| 1954 | 52.62% | 0.86% | 51.76% |
| 1955 | 31.56% | 1.57% | 29.99% |
| 1956 | 6.56% | 2.46% | 4.10% |

| Appendix A<br>Ibbotson Market Data 1926-2002* | | | |
|---|---|---|---|
| | **Common Stocks** | **U. S. Treasury Bills** | **Arithmetic Short-Horizon** |
| **Year** | **Total Annual Returns** | **Total Annual Returns** | **Equity Risk Premia** |
| 1957 | -10.78% | 3.14% | -13.92% |
| 1958 | 43.36% | 1.54% | 41.82% |
| 1959 | 11.96% | 2.95% | 9.01% |
| 1960 | 0.47% | 2.66% | - 2.19% |
| 1961 | 26.89% | 2.13% | 24.76% |
| 1962 | - 8.73% | 2.73% | -11.46% |
| 1963 | 22.80% | 3.12% | 19.68% |
| 1964 | 16.48% | 3.54% | 12.94% |
| 1965 | 12.45% | 3.93% | 8.52% |
| 1966 | -10.06% | 4.76% | -14.82% |
| 1967 | 23.98% | 4.21% | 19.77% |
| 1968 | 11.06% | 5.21% | 5.85% |
| 1969 | - 8.50% | 6.58% | -15.08% |
| 1970 | 4.01% | 6.52% | - 2.51% |
| 1971 | 14.31% | 4.39% | 9.92% |
| 1972 | 18.98% | 3.84% | 15.14% |
| 1973 | -14.66% | 6.93% | -21.59% |
| 1974 | -26.47% | 8.00% | -34.47% |
| 1975 | 37.20% | 5.80% | 31.40% |
| 1976 | 23.84% | 5.08% | 18.76% |
| 1977 | - 7.18% | 5.12% | -12.30% |
| 1978 | 6.56% | 7.18% | - 0.62% |
| 1979 | 18.44% | 10.38% | 8.06% |
| 1980 | 32.42% | 11.24% | 21.18% |
| 1981 | - 4.91% | 14.71% | -19.62% |
| 1982 | 21.41% | 10.54% | 10.87% |
| 1983 | 22.51% | 8.80% | 13.71% |
| 1984 | 6.27% | 9.85% | - 3.58% |
| 1985 | 32.16% | 7.72% | 24.44% |
| 1986 | 18.47% | 6.16% | 12.31% |
| 1987 | 5.23% | 5.47% | - 0.24% |
| 1988 | 16.81% | 6.35% | 10.46% |
| 1989 | 31.49% | 8.37% | 23.12% |

**Appendix A**
**Ibbotson Market Data 1926-2002***

| Year | Common Stocks Total Annual Returns | U. S. Treasury Bills Total Annual Returns | Arithmetic Short-Horizon Equity Risk Premia |
|---|---|---|---|
| 1990 | - 3.17% | 7.81% | -10.98% |
| 1991 | 30.55% | 5.60% | 24.95% |
| 1992 | 7.67% | 3.51% | 4.16% |
| 1993 | 9.99% | 2.90% | 7.09% |
| 1994 | 1.31% | 3.90% | - 2.59% |
| 1995 | 37.43% | 5.60% | 31.83% |
| 1996 | 23.07% | 5.21% | 17.86% |
| 1997 | 33.36% | 5.26% | 28.10% |
| 1998 | 28.58% | 4.86% | 23.72% |
| 1999 | 21.04% | 4.68% | 16.36% |
| 2000 | - 9.11% | 5.89% | -15.00% |
| 2001 | -11.88% | 3.83% | -15.71% |
| 2002 | -22.10% | 1.65% | -23.75% |
| mean= | **12.20%** | **3.83%** | **8.37%** |
| Standard Dev= | **20.49%** | **3.15%** | **20.78%** |

* *2003 SBBI Yearbook pages 38 and 39*

## Appendix B

| Source | Risk-free-Rate | ERP Estimate | Real risk-free rate | Nominal risk-free rate | Geometric | Arithmetic | Long-horizon | Short-horizon | Short-run expectation | Long-run expectation | Conditional | Unconditional |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Historical** | | | | | | | | | | | | |
| Ibbotson Associates | 3.8% [7] | 8.4% [31] | | X | | X | | X | | X | | X |
| **Social Security** | | | | | | | | | | | | |
| Office of the Chief Actuary [1] | 2.3%,3.0% [8] | 4.7%,4.0% [32] | X | | X | | X | | | X | | X |
| John Campbell [2] | 3% to 3.5% [9] | 1.5-2.5%, 3-4% [33] | X | | X | X | X | X | | X | X | |
| Peter Diamond | 2.2% [10] | <4.8% [34] | X | | X | | X | | | X | X | |
| Peter Diamond [3] | 3.0% [11] | 3.0% to 3.5% [35] | X | | X | | X | | | X | X | |
| John Shoven [4] | 3.0%,3.5% [12] | 3.0% to 3.5% [36] | X | | X | | X | | | X | X | |
| **Puzzle Research** | | | | | | | | | | | | |
| Robert Arnott and Peter Bernstein | 3.7% [13] | 2.4% [37] | X | | X | | X | | | X | X | |
| Robert Arnott and Ronald Ryan | 4.1% [14] | -0.9% [38] | X | | X | | X | | | X | X | |
| John Campbell and Robert Shiller | N/A | Negative [39] | X | | ? | | ? | | X | | X | |
| James Claus and Jacob Thomas | 7.64% [15] | 3.39% or less [40] | | X | | X | X | | | X | X | |
| George Constantinides | 2.0% [16] | 6.9% [41] | X | | | X | | X | | X | | X |
| Bradford Cornell | 5.6%, 3.8% [17] | 3.5-5.5%, 5-7% [42] | | X | | X | X | X | | X | X | |
| Dimson, Marsh, & Staunton | 1.0% [18] | 5.4% [43] | X | | | X | | X | | X | X | |
| Eugene Fama and Kenneth French | 3.24% [19] | 3.83% & 4.78% [44] | X | | | X | | X | | X | | X |
| Robert Harris and Felicia Marston | 8.53% [20] | 7.14% [45] | | X | | X | X | | X | | X | |
| Roger Ibbotson and Peng Chen | 2.05% [21] | 4% and 6% [46] | X | | X | X | X | | | X | | X |
| Jeremy Siegel | 4.0% [22] | -0.9% to -0.3% [47] | X | | X | | X | | | X | X | |
| Jeremy Siegel | 3.5% [23] | 2-3% [48] | X | | X | | X | | | ? | X | |
| **Surveys** | | | | | | | | | | | | |
| John Graham and Campbell Harvey | ? by survey [24] | 3-4.7% [49] | | X | | ? | X | | X | | X | |
| Ivo Welch | N/A [25] | 7% [50] | | X | | X | | X | | X | X | |
| Ivo Welch [5] | 5% [26] | 5.0% to 5.5% [51] | | X | | X | | X | | X | X | |
| **Misc.** | | | | | | | | | | | | |
| Barclays Global Investors | 5% [27] | 2.5%, 3.25% [52] | | X | X | | X | | X | | X | |
| Richard Brealey and Stewart Myers | N/A [28] | 6 to 8.5% [53] | | X | | X | | X | | X | | X |
| Burton Malkiel | 5.25% [29] | 2.75% [54] | | X | X | | X | | | X | X | |
| Richard Wendt [6] | 5.5% [30] | 3.3% [55] | | X | | X | X | | | X | X | |

Long-run expectation considered to be a forecast of more than 10 years.
Short-run expectation considered to be a forecast of 10 years or less.

| Source | Risk-free Rate | ERP Estimate | Data Period | Methodology |
|---|---|---|---|---|
| | | | | |
| **Historical** | | | | |
| Ibbotson Associates | 3.8% [7] | 8.4% [31] | 1926-2002 | Historical |
| **Social Security** | | | | |
| Office of the Chief Actuary [1] | 2.3%, 3.0% [8] | 4.7%, 4.0% [32] | 1900-1995, Projecting out 75 years | Historical |
| John Campbell [2] | 3% to 3.5% [9] | 1.5-2.5%, 3-4% [33] | Projecting out 75 years | Historical & Ratios (Div/Price & Earn Gr) |
| Peter Diamond | 2.2% [10] | <4.8% [34] | Last 200 yrs for eq/ 75 for bonds, Proj 75 yrs | Fundamentals: Div Yld, GDP Gr |
| Peter Diamond [3] | 3.0% [11] | 3.0% to 3.5% [35] | Projecting out 75 years | Fundamentals: Div/Price |
| John Shoven [4] | 3.0%, 3.5% [12] | 3.0% to 3.5% [36] | Projecting out 75 years | Fundamentals: P/E, GDP Gr |
| **Puzzle Research** | | | | |
| Robert Arnott and Peter Bernstein | 3.7% [13] | 2.4% [37] | 1802 to 2001, normal | Fundamentals: Div Yld & Gr |
| Robert Arnott and Ronald Ryan | 4.1% [14] | -0.9% [38] | Past 74 years, 74 year projection [56] | Fundamentals: Div Yld & Gr |
| John Campbell and Robert Shiller | N/A | Negative [39] | 1871 to 2000, ten-year projection | Ratios: P/E and Div/Price |
| James Claus and Jacob Thomas | 7.64% [15] | 3.39% or less [40] | 1985-1998, long-term | Abnormal Earnings model |
| George Constantinides | 2.0% [16] | 6.9% [41] | 1872 to 2000, long-term | Hist. and Fund.: Price/Div & P/E |
| Bradford Cornell | 5.6%, 3.8% [17] | 3.5-5.5%, 5-7% [42] | 1926-1997, long run forward-looking | Weighing theoretical and empirical evid |
| Dimson, Marsh, & Staunton | 1.0% [18] | 5.4% [43] | 1900-2000, prospective | Adj hist ret, Var of Gordon gr model |
| Eugene Fama and Kenneth French | 3.24% [19] | 3.83% & 4.78% [44] | Estimate for 1951-2000, long-term | Fundamentals: Dividends and Earnings |
| Robert Harris and Felicia Marston | 8.53% [20] | 7.14% [45] | 1982-1998, expectational | Fin analysts' est, div gr model |
| Roger Ibbotson and Peng Chen | 2.05% [21] | 4% and 6% [46] | 1926-2000, long-term | Historical and supply side approaches |
| Jeremy Siegel | 4.0% [22] | -0.9% to -0.3% [47] | 1871 to 1998, forward-looking | Fundamentals: P/E, Div Yld, Div Gr |
| Jeremy Siegel | 3.5% [23] | 2-3% [48] | 1802-2001, forward-looking | Earnings yield |
| **Surveys** | | | | |
| John Graham and Campbell Harvey | ? by survey [24] | 3-4.7% [49] | 2Q 2000 thru 3Q 2002, 1 & 10 year proj | Survey of CFO's |
| Ivo Welch | N/A [25] | 7% [50] | 30-Year forecast, surveys in 97/98 & 99 | Survey of financial economists |
| Ivo Welch [5] | 5% [26] | 5.0% to 5.5% [51] | 30-Year forecast, survey around August 2001 | Survey of financial economists |
| **Misc.** | | | | |
| Barclays Global Investors | 5% [27] | 2.5%, 3.25% [52] | Long-run (10-year) expected return | Fundamentals: Inc, Earn Gr, & Repricing |
| Richard Brealey and Stewart Myers | N/A [28] | 6 to 8.5% [53] | 1926-1997 | Predominantly Historical |
| Burton Malkiel | 5.25% [29] | 2.75% [54] | 1926 to 1997, estimate millennium [57] | Fundamentals: Div Yld, Earn Gr |
| Richard Wendt [6] | 5.5% [30] | 3.3% [55] | 1960-2000, estimate for 2001-2015 period | Linear regression model |

**Footnotes:**

[1] Social Security Administration.

[2] Presented to the Social Security Advisory Board.

[3] Presented to the Social Security Advisory Board. Update of 1999 article.

[4] Presented to the Social Security Advisory Board.

[5] Update to Welch 2000.

[6] Newsletter of the Investment Section of the Society of Actuaries.

[7] Arithmetic mean of U.S. Treasury bills annual total returns from 1926-2002.

[8] 2.3% Long-run real yield on Treasury bonds; used for Advisory Council proposals. 3.0% Long-term real yield on Treasury bonds; used in 1999 Social Security Trustees Report.

[9] Estimate for safe real interest rates in the future based on yield of long-term inflation-indexed Treasury securities of 3.5% and short-term real interest rates recently averaging about 3%.

[10] Real long-term bond yield using 75 year historical average.

[11] Real yield on long-term Treasuries (assumption by OCACT).

[12] 3.0% is the OCACT assumption. 3.5% is the real return on long-run (30-year) inflation-indexed Treasury securities.

[13] Long-term expected real geometric bond return (10 year-horizon).

[14] The yield on US government inflation-indexed bonds (starting bond real yield in Jan 2000).

[15] Average 10-year Government T-bond yield between 1985 and 1998 (yield of 11.43% in 1985 to 5.64% in 1998. The mean 30-year risk-free rate for each year of the U.S. sample period is 31 basis points higher than the mean 10-year risk-free rate.

[16] Rolled-over real arithmetic return of three-month Treasury bills and certificates.

[17] Historical 20-year Treasury bond return of 5.6%. Yield on 20-year Treasury bonds in 1998 was approximately 6%. Historical 1-month Treasury bill return of 3.8%. Yield on 1-month Treasury bills in 1998 was approximately 4%.

[18] United States historical arithmetic real Treasury bill return over 1900-2000 period. 0.9% geometric Treasury bill return.

[19] Average real return on six-month commercial paper (proxy for risk-free interest rate). Substituting the one-month Treasury bill rate for the six-month commercial paper rate causes estimates of the annual equity premium for 1951-2000 to rise by about 1.00%.

[20] Average yield to maturity on long-term U.S. government bonds, 1982-1998.

[21] Real, geometric risk-free rate. Geometric risk-free rate with inflation (nominal) 5.13%. Nominal yield equivalent to historical geometric long-term government bond income return for 1926-2000.

[22] The ten- and thirty-year TIPS bond yielded 4.0% in August 1999.

[23] Return on inflation-indexed securities.

[24] Current 10-year Treasury bond yield. Survey administered from June 6, 2000 to June 4, 2002. The rate on the 10-year Treasury bond changes in each survey. For example, in the Dec. 1, 2000 survey, the current annual yield on the 10-year Treasury bond was 5.5%. For the June 6, 2001 survey, the current annual yield on the 10-year Treasury bond was 5.3%.

[25] Arithmetic per-annum average return on rolled-over 30-day T-bills.

[26] Average forecast of arithmetic risk-free rate of about 5% by deducting ERP from market return.

[27] Current nominal 10-year bond yield.

[28] Return on Treasury bills. Treasury bills yield of about 5 percent in mid-1998. Average historical return on Treasury bills 3.8 percent.

[29] Good quality corporate bonds will earn approximately 6.5% to 7%. Long-term zero-coupon Treasury bonds will earn about 5.25%. Long-term TIPS will earn a real return of 3.75%.

[30] 1/1/01 Long T-Bond yield; uses initial bond yields in predictive model.

[31] Arithmetic short-horizon expected equity risk premium. Arithmetic intermediate-horizon expected equity risk premium 7.4%. Arithmetic long-horizon expected equity risk premium 7.0%. Geometric short-horizon expected equity risk premium 6.4%.

[32] Geometric equity premium over long-term Treasury securities. OCACT assumes a constant geometric real 7.0% stock return.

[33] Long-run average equity premium of 1.5% to 2.5% in geometric terms and 3% to 4% in arithmetic terms.

[34] Lower return over the next decade, followed by a geometric, real 7.0% stock return for remaining 65 years or lower rate of return for entire 75-year period (obscures pattern of returns).

[35] Most likely poor return over the next decade followed by a return to historic yields. Working from OCACT stock return assumption, he gives a single rate of return on equities for projection purposes of 6.0 to 6.5% (geometric, real).

[36] Geometric real stock return over the geometric real return on long-term government bonds.

[37] Expected geometric return over long-term government bonds. Their current risk premium is approximately zero, and their recommended expectation for the future real return for both stocks and bonds is 2-4 percent. The "normal" level of the risk premium is modest (2.4 percent or quite possibly less).

[38] Geometric real returns on stocks are likely to be in the 3%-4% range for the foreseeable future (10-20 years).

[39] Substantial declines in real stock prices, and real stock returns below zero, over the next ten years (2001-2010).

[40] The equity premium for each year between 1985 and 1998 in the United States. Similar results for five other markets.

[41] Unconditional, arithmetic mean aggregate equity premium over the 1872-2000 period. Over the period 1951 to 2000, the adjusted estimate of the unconditional mean premium is 6.0%. The corresponding estimate over the 1926 to 2000 period is 8.0%. Sharp distinction between conditional, short-term forecasts of the mean equity return and premium and estimates of the unconditional mean.

[42] Long run arithmetic future ERP of 3.5% to 5.5% over Treasury bonds and 5% to 7% over Treasury bills. Compares estimates to historical returns of 7.4% for bond premium and 9.2% for bill premium.

[43] 5.4% United States arithmetic expected future ERP relative to bills. 4.0% World (16 countries) arithmetic expected future ERP relative to bills. 4.1% United States geometric expected future ERP relative to bills. 3.0% World (16 countries) geometric expected future ERP relative to bills.

[44] 3.83% unconditional expected annual simple equity premium return (referred to as the annual-bias adjusted estimate of the annual equity premium) using dividend growth model. 4.78% unconditional expected annual simple equity premium return (referred to as the annual-bias adjusted estimate of the annual equity premium) using earnings growth model. Compares these results against historical real equity risk premium of 7.43% for 1951-2000.

[45] Average expectational risk premium. Because of the possible bias of analysts' optimism, the estimates are interpreted as "upper bounds" for the market premium. The average expectational risk premium is approximately equal to the arithmetic (7.5%) long-term differential between returns on stocks and long-term government bonds.

[46] 4% geometric (real) and 6% arithmetic (real). Forward looking long-horizon sustainable equity risk premium.

[47] Using the dividend discount model, the forward-looking real long-term geometric return on equity is 3.3%. Based on the earnings yield, the forward-looking real long-term geometric return on equity is between 3.1% and 3.7%.

[48] Future geometric equity premium. Future real return on equities of about 6%.

[49] The 10-year premium. The one-year risk premium averages between 0.4 and 5.2% depending on the quarter surveyed.

[50] Arithmetic 30-year forecast relative to short-term bills; 10-year same estimate. Second survey 6.8% for 30 and 10-year estimate. 1-year horizon between 0.5% and 1.5% lower. Geometric 30-year forecast around 5.2% (50% responded to this question).

[51] Arithmetic 30-year equity premium (relative to short-term T-bills). Geometric about 50 basis points below arithmetic. Arithmetic 1-year equity premium 3 to 3.5%.

[52] 2.5% current (conditional) geometric equity risk premium. 3.25% long-run, geometric normal or equilibrium equity risk premium.

[53] Extra arithmetic return versus Treasury bills. "Brealey and Myers have no official position on the exact market risk premium, but we believe a range of 6 to 8.5 percent is reasonable for the United States. We are most comfortable with figures towards the upper end of the range."

[54] The projected geometric (nominal) total return for the S&P 500 is 8 percent per year.

[55] Arithmetic mean 15 year horizon.

[56] 74 years since Dec 1925 and 74 years starting Jan 2000.

[57] Estimate the early decades of the twenty-first century.

## Appendix C
## Estimating a Short-Horizon Arithmetic Unconditional Equity Risk Premium

| Source | Risk-free Rate | ERP Estimate | Stock Return Estimate | Geometric to arithmetic | Real to nominal | Conditional to unconditional [60] | Fixed short-horizon RFR | Short-horizon arithmetic unconditional ERP estimate |
|---|---|---|---|---|---|---|---|---|
| | I | II | III | IV | V | VI | VII | VIII |
| **Historical** | | | | | | | | |
| Ibbotson Associates | 3.8% [7] | 8.4% [31] | 12.2% | 0.0% | 0.0% | 0.00% | 3.8% | 8.4% |
| **Social Security** | | | | | | | | |
| Office of the Chief Actuary [1] | 2.3%,3.0% [8] | 4.7%,4.0% [32] | 7.0% | 2.0% | 3.1% | 0.00% | 3.8% | 8.3% |
| John Campbell [2] | 3% to 3.5% [9] | 1.5-2.5%, 3-4% [33] | 6.0%-7.5% | 0.0% | 3.1% | 0.46% | 3.8% | 5.8%-7.3% |
| Peter Diamond | 2.2% [10] | <4.8% [34] | <7.0% | 2.0% | 3.1% | 0.46% | 3.8% | <8.8% |
| Peter Diamond [3] | 3.0% [11] | 3.0% to 3.5% [35] | 6.0%-6.5% | 2.0% | 3.1% | 0.46% | 3.8% | 7.8%-8.3% |
| John Shoven [4] | 3.0%,3.5% [12] | 3.0% to 3.5% [36] | 6.0%-7.0% | 2.0% | 3.1% | 0.46% | 3.8% | 7.8%-8.8% |
| **Puzzle Research** | | | | | | | | |
| Robert Arnott and Peter Bernstein | 3.7% [13] | 2.4% [37] | 6.1% | 2.0% | 3.1% | 0.46% | 3.8% | 7.9% |
| Robert Arnott and Ronald Ryan | 4.1% [14] | -0.9% [38] | 3.2% | 2.0% | 3.1% | 0.46% | 3.8% | 5.0% |
| John Campbell and Robert Shiller | N/A | Negative [39] | Negative | N/A | N/A | N/A | N/A | N/A |
| James Claus and Jacob Thomas | 7.64% [15] | 3.39% or less [40] | 11.03% | 0.0% | 0.0% | 0.46% | 3.8% | 7.69% |
| George Constantinides | 2.0% [16] | 6.9% [41] | 8.9% | 0.0% | 3.1% | 0.00% | 3.8% | 8.2% |
| Bradford Cornell | 5.6%, 3.8% [17] | 3.5-5.5%, 5-7% [42] | 8.8%-10.8% | 0.0% | 0.0% | 0.46% | 3.8% | 5.5%-7.5% |
| Dimson, Marsh, & Staunton | 1.0% [18] | 5.4% [43] | 6.4% [58] | 0.0% | 3.1% | 0.46% | 3.8% | 6.2% [61] |
| Eugene Fama and Kenneth French | 3.24% [19] | 3.83% & 4.78% [44] | 7.07%-8.02% | 0.0% | 3.1% | 0.00% | 3.8% | 6.37%-7.32% |
| Robert Harris and Felicia Marston | 8.53% [20] | 7.14% [45] | 12.34% [59] | 0.0% | 0.0% | 0.46% | 3.8% | 9.00% |
| Roger Ibbotson and Peng Chen | 2.05% [21] | 4% and 6% [46] | 8.05% | 0.0% | 3.1% | 0.00% | 3.8% | 7.35% |
| Jeremy Siegel | 4.0% [22] | -0.9% to -0.3% [47] | 3.1%-3.7% | 2.0% | 3.1% | 0.46% | 3.8% | 4.9%-5.5% |
| Jeremy Siegel | 3.5% [23] | 2-3% [48] | 5.5%-6.5% | 2.0% | 3.1% | 0.46% | 3.8% | 7.3%-8.3% |
| **Surveys** | | | | | | | | |
| John Graham and Campbell Harvey | ? by survey [24] | 3-4.7% [49] | 8.3%-10.2% | N/A | 0.0% | 0.46% | 3.8% | 5.0%-6.9% |
| Ivo Welch | N/A [25] | 7% [50] | N/A | 0.0% | 0.0% | 0.46% | 0.0% | 7.5% |
| Ivo Welch [5] | 5% [26] | 5.0% to 5.5% [51] | 10.0%-10.5% | 0.0% | 0.0% | 0.46% | 3.8% | 6.7%-7.2% |
| **Misc.** | | | | | | | | |
| Barclays Global Investors | 5% [27] | 2.5%, 3.25% [52] | 7.5%,8.25% | 2.0% | 0.0% | 0.46% | 3.8% | 6.16%-6.91% |
| Richard Brealey and Stewart Myers | N/A [28] | 6 to 8.5% [53] | N/A | 0.0% | 0.0% | 0.00% | 0.0% | 6.0%-8.5% |
| Burton Malkiel | 5.25% [29] | 2.75% [54] | 8.0% | 2.0% | 0.0% | 0.46% | 3.8% | 6.7% |
| Richard Wendt [6] | 5.5% [30] | 3.3% [55] | 8.8% | 0.0% | 0.0% | 0.46% | 3.8% | 5.5% |

Column formulas:
III = I + II
VIII = III + IV + V + VI –VII

Source for adjustments:
2003 Ibbotson Yearbook Table 2-1 page 33
Fama French 2002 (see footnote 60)

Footnotes (1-57 from Appendix B):
[58] World estimate of 5.0%.
[59] Long risk-free of 5.2% plus 7.14%.
[60] For the 1951-2000 period, Fama and French (2002) adjust the conditional dividend growth model estimate upwards by 1.28% for an unconditional estimate, and they make a 0.46% upwards adjustment to the earnings growth model. We select the smaller of the two as an approximate minimum adjustment. For the longer period of 1872-2000, a comparable adjustment would be 0.82% for the dividend growth model and 0.54% for the 1872-1950 period using a dividend growth model. Earnings growth rates are shown by Fama and French only for the 1951-2000 period.
[61] World estimate of 4.8%.

## Appendix D

**Historical and Forecasted Equity Returns- All Ibbotson and Chen Models (Percent).**

| Method/ Model | Sum | Inflation | Real Risk-Free Rate | Equity Risk Premium | Real Capital Gain | g(Real EPS) | g(Real Div) | - g (Pay out Ratio) | g (BV) | g (ROE) | g P/E) | g(Real GDP/ POP) | g(FS-GDP/ POP) | Income Return | Re-Investment + Interaction | Additional Growth | Forecast Earnings Growth |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Column # | I | II | III | IV | V | VI | VII | VIII | IX | X | XI | XII | XIII | XIV | XV | XVI | XVII |
| **Historical** | | | | | | | | | | | | | | | | | |
| Method 1 | 10.70 | 3.08 | 2.05 | 5.24 | | | | | | | | | | | 0.33 | | |
| Method 2 | 10.70 | 3.08 | | | 3.02 | | | | | | | | | 4.28 | 0.32 | | |
| Method 3 | 10.70 | 3.08 | | | | 1.75 | | | | | 1.25 | | | 4.28 | 0.34 | | |
| Method 4 | 10.70 | 3.08 | | | | | 1.23 | 0.51 | | | 1.25 | | | 4.28 | 0.35 | | |
| Method 5 | 10.70 | 3.08 | | | | | | | 1.46 | 0.31 | 1.25 | | | 4.28 | 0.31 | | |
| Method 6 | 10.70 | 3.08 | | | | | | | | | | 2.04 | 0.96 | 4.28 | 0.32 | | |
| **Forecast with Historical Dividend Yield** | | | | | | | | | | | | | | | | | |
| Model 3F | 9.37 | 3.08 | | | | 1.75 | | | | | | | | 4.28 | 0.26 | | |
| Model 3F (ERP) | 9.37 | 3.08 | 2.05 | 3.97 | | | | | | | | | | | 0.27 | | |
| **Forecast with Current Dividend Yield** | | | | | | | | | | | | | | | | | |
| Model 4F | 5.44 | 3.08 | | | | | 1.23 | | | | | | | 1.10 [a] | 0.03 | | |
| Model 4F (ERP) | 5.44 | 3.08 | 2.05 | 0.24 | | | | | | | | | | | 0.07 | | |
| Model $4F_2$ | 9.37 | 3.08 | | | | | 1.23 | 0.51 | | | | | | 2.05 [b] | 0.21 | 2.28 | |
| Model $4F_2$ (FG) | 9.37 | 3.08 | | | | | | | | | | | | 1.10 [a] | 0.21 | | 4.98 |

*Source:* The data and format was made available by Ibbotson/Chen and is reprinted with permission by Ibbotson Associates. Corresponds to Ibbotson/Chen Table 2 Exhibit; column numbers have been added.

[a] 2000 dividend yield

[b] Assuming the historical average dividend-payout ratio, the 2000 dividend yield is adjusted up 0.95 pps.

| | Formula* | Description of Method |
|---|---|---|
| **Historical** | | |
| Method 1 | I=(1+II)*(1+III)*(1+IV)-1 | Building Blocks Method: inflation, real risk-free rate, and equity risk premium. |
| Method 2 | I=[(1+II)*(1+V)-1]+XIV+XV | Capital Gain and Income Method: inflation, real capital gain, and income return. |
| Method 3 | I=[(1+II)*(1+VI)*(1+XI)-1]+XIV+XV | Earnings Model: inflation, growth in earnings per share, growth in price to earnings ratio, and income return. |
| Method 4 | I=[(1+II)*(1+XI)*(1+VII)/(1-VIII)-1]+XIV+XV | Dividends Model: inflation, growth rate of price earnings ratio, growth rate of the dollar amount of dividends after inflation, growth rate of payout ratio, and dividend yield (income return). |
| Method 5 | I=[(1+II)*(1+XI)*(1+IX)*(1+X)-1]+XIV+XV | Return on Book Equity Model: inflation, growth rate of price earnings ratio, growth rate of book value, growth rate of ROE, and income return. |
| Method 6 | I=[(1+II)*(1+XII)*(1+XIII)-1]+XIV+XV | GDP Per Capita Model: inflation, real growth rate of the overall economic productivity (GDP per capita), increase of the equity market relative to the overall economic productivity, and income return. |
| **Forecast with Historical Dividend Yield** | | |
| Model 3F | I=[(1+II)*(1+VI)-1]+XIV+XV | Forward-looking Earnings Model: inflation, growth in real earnings per share, and income return. |
| Model 3F (ERP) | IV=(1+I)/[(1+II)*(1+III)]-1 | Using Model 3F result to calculate ERP. |
| **Forecast with Current Dividend Yield** | | |
| Model 4F | I=[(1+II)*(1+VII)-1]+XIV+XV | Forward-looking Dividends Model: inflation, growth in real dividend, and dividend yield (income return); also referred to as Gordon model. |
| Model 4F (ERP) | IV=(1+I)/[(1+II)*(1+III)]-1 | Using Model 4F result to calculate ERP. |
| Model $4F_2$ | I=[(1+II)*(1+VII)*(1+VIII)-1]+XIV+XV+XVI | Attempt to reconcile Model 4F and Model 3F. |
| Model $4F_2$ (FG) | XVII=[(1+I)/(1+II)-1]-XIV-XV | Using Method $4F_2$ result to calculate forecasted earnings. |

Explanation of Ibbotson/Chen Table 2 Exhibit; using column numbers to represent formula.