Kentucky-American Water Company
Case No. 2004-00103
Information Request Response to KAWC
Respondent: OAG Witness Dr. J. Randall Woolridge
Set II

KAWC-II-1. Regarding Exhibit (JRW-3), p. 1 of 1, please provide the C. A. Turner Utility Reports, August 2004, that contain the data described on the exhibit.

Response:

The requested document was provided in response to PSC-I-13 as attachment PSC-I-13A.

KAWC-II-2. Please provide fully functional electronic work papers for the following schedules presented in to Dr. Woolridge's direct testimony. These work papers should be in a format that provides the capability to review all data inputs and calculations that are the basis for the results displayed in the exhibits as filed. In addition, where requested, please provide all source documents underlying the data and results displayed in the exhibits.

(a) Exhibit (JRW-5), pp. 1, 2, and 3.

(b) Exhibit (JRW-5), p. 1 of 5. Also provide all information on how the index of long-term A-rated public utility bonds is constructed, including the specific bonds included in the index and the average maturity of the bonds in the index.

(c) Exhibit (JRW-7), p. 2 of 5. Also provide source documents for data shown on the exhibit.

(d) Exhibit (JRW-7) p. 3 of 5. Please include source data underlying all the results shown in the exhibit.

(e) Page 57. Please provide the data and source documents underlying the graph titled, "Actual Versus Forecasted 5-Year EPS Growth for the S&P 500 2984 - 2003."

Response:

(a) Worksheet included as attachment KAWC-II-2(a).wks.

(b) Worksheet included as attachment KAWC-II-2(b).wks. The yields were obtained off Bloomberg.

(c) Worksheet included as attachment KAWC-II-2(c).wks.

(d) Worksheet included as attachment KAWC-II-2(d).wks.

(e) Worksheet included as attachment KAWC-II-2(e).wks.

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KAWC-II-3. Exhibit (JRW-8), p. 2 of 5. Please provide a copy of the cited paper, "Equity Risk Premium: Expectations Great and Small," Working Paper (version 3.0), Automobile Insurers Bureau of Massachusetts, August 28, 2003.

Response:

The requested document is included at attachment KAWC-II-3A.

Title: Equity Risk Premium: Expectations Great and Small

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Equity Risk Premium: Expectations Great and Small

What I actually think is that our prey, called the equity risk premium, is extremely elusive.

Stephen A. Ross 2001

Abstract:
The Equity Risk Premium (ERP) is an essential building block of the market value of risk. In theory, the collective action of all investors results in an equilibrium expectation for the return on the market portfolio excess of the risk-free return, the equity risk premium. The ability of the valuation actuary to choose a sensible value for the ERP, whether as a required input to CAPM valuation, or any of its descendants, is as important as choosing risk-free rates and risk relatives (betas) to the ERP for the asset at hand. The historical realized ERP for the stock market appears to be at odds with pricing theory parameters for risk aversion. Since 1985, there has been a constant stream of research, each of which reviews theories of estimating market returns, examines historical data periods, or both. Those ERP value estimates vary widely from about minus one percent to about nine percent, based on a geometric or arithmetic averaging, short or long horizons, short or long-run expectations, unconditional or conditional distributions, domestic or international data, data periods, and real or nominal returns. This paper will examine the principal strains of the recent research on the ERP and catalogue the empirical values of the ERP implied by that research. In addition, the paper will supply several time series analyses of the standard Ibbotson Associates 1926-2002 ERP data using short Treasuries for the risk-free rate. Recommendations for ERP values to use in common actuarial valuation problems will be offered.

Acknowledgments
The authors acknowledge the helpful comments from participants in the 2003 Bowles Symposium, Louise Francis, Francis Analytics & Actuarial Data Mining Inc., and four anonymous referees. The authors would also like to thank Jack Wilson for supplying his data series from Wilson & Jones (2002).

Keywords: Equity Risk Premium, Risk Premium Puzzle, Market Return Models, CAPM, Dividend Growth Models, Actuarial Valuations.

Introduction

The Equity Risk Premium (ERP) is an essential building block of the market value of risk. In theory, the collective action of all investors results in an equilibrium expectation for the return on the market portfolio excess of the risk-free return, the equity risk premium. The ability of the valuation actuary to choose a sensible value for the ERP, whether as a required input to CAPM valuation, or any of its descendants[1], is as important as choosing risk-free rates and risk relatives (betas) to the ERP for the asset at hand. Risky discount rates, asset allocation models, and project costs of capital are common actuarial uses of ERP as a benchmark rate.

The equity risk premium should be of particular interest to actuaries. For pensions and annuities backed by bonds and stocks, the actuary needs to have an understanding of the ERP and its variability compared to fixed horizon bonds. Variable products, including Guaranteed Minimum Death Benefits, require accurate projections of returns to ensure adequate future assets. With the latest research producing a relatively low equity risk premium, the rationale for including equities in insurers' asset holdings is being tested. In describing individual investment account guarantees, LaChance and Mitchell (2003) point out an underlying assumption of pension asset investing that, based only on the historical record, future equity returns will continue to outperform bonds; they clarify that those higher expected equity returns come with the additional higher risk of equity returns. Ralfe et al. (2003) support the risky equity view and discuss their pension experience with an all bond portfolio. Recent projections in some literature of a zero or negative equity risk premium challenge the assumptions underlying these views. By reviewing some of the most recent and relevant work on the issue of the equity risk premium, actuaries will have a better understanding of how these values were estimated, critical assumptions that allowed for such a low EPR, and the time period for the projection. Actuaries can then make informed decisions for expected investment results going forward.[2]

In 1985, Mehra and Prescott published their work on the so-called Equity Risk Premium Puzzle: The fact that the historical realized ERP for the stock market 1889-1978 appeared to be at odds with and, relative to Treasury bills, far in excess of asset pricing theory values based on investors with reasonable risk aversion parameters. Since then, there has been a constant stream of research, each of which reviews theories of estimating market returns, examines historical data periods, or both.[3] Those ERP value estimates vary <u>widely</u> from about minus one percent to about nine percent, based on geometric or arithmetic averaging, short or long horizons, short or long-run means, unconditional or conditional expectations, using domestic or international data, differing data periods, and real or nominal returns. Brealey and Myers, in the sixth edition of their standard corporate finance textbook, believe a range of 6% to 8.5% for the US ERP is reasonable for practical project valuation. Is that a fair estimate?

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[1] The multifactor arbitrage pricing theory (APT) of Ross (1976), the three-factor model of Fama and French (1992) and the recent Mamaysky (2002) five-factor model for stocks and bonds are all examples of enhanced CAPM models.

[2] See Appendix D

[3] For example, see Cochrane (1997), Cornell (1999), or Leibowitz (2001).

Current research on the equity risk premium is plentiful (Leibowitz, 2001). This paper covers a selection of mainstream articles and books that describe different approaches to estimating the ex ante equity risk premium. We select examples of the research that cover the most important approaches to the ERP. We begin by describing the methodology of using historical returns to predict future estimates. We identify the many varieties of ERPs in order to alert the reader to the fact that numerical estimates of the ERP that appear different may instead be about the same under a common definition. We examine the well-known Ibbotson Associates 1926-2002 data series for stationarity, i.e. time invariance of the mean ERP. We show by several statistical tests that stationarity cannot be rejected and the best estimate going forward, ceteris paribus, is the realized mean. This paper will examine the principal strains of the recent research on the ERP and catalogue the empirical values of the ERP implied by that research.[4]

We first discuss how the Social Security Administration derives estimates of the equity risk premium. Then, we survey the puzzle research, that is, the literature written in response to the Equity Premium Puzzle suggested by Mehra and Prescott (1985). We cover five major approaches from the literature. Next, we report from two surveys of "experts" on the equity risk premium. Finally, after we describe the main strains of research, we explore some of the implications for practicing actuaries.

We do not discuss the important companion problem of estimating the risk relationship of an individual company, line of insurance, or project with the overall market. Within a CAPM or Fama-French framework, the problem is estimating a market beta.[5] Actuaries should be aware, however, that simple 60-month regression betas are biased low where size or non-synchronous trading is a substantial factor (Kaplan and Peterson (1998), Pratt (1998), p86). Adjustments are made to historical betas in order to remove the bias and derive more accurate estimates. Elton and Gruber (1995) explain that by testing the relationship of beta estimates over time, empirical studies have shown that an adjustment toward the mean should be made to project future betas.[6]

The Equity Risk Premium

Based on the definition in Brealey and Myers, *Principles of Corporate Finance* textbook, the equity risk premium (ERP) is the "expected additional return for making a risky investment rather than a safe one". In other words, the ERP is the difference between the market return and a risk-free return. Market returns include both dividends and capital gains. Because both the historical ERP and the prospective ERP have been referred to simply as the equity risk premium, the terms *ex post* and *ex ante* are used to differentiate between them but are often omitted. Table 1 shows the historical annual

[4] The research catalogued appears as Appendix B.

[5] According to CAPM, investors are compensated only for non-diversifiable, or market, risk. The market beta becomes the measurement of the extent to which returns on an individual security covary with the market. The market beta times the ERP represents the non-diversifiable expected return from an individual security.

[6] Elton and Gruber (1995), p148.

average returns from 1926-2002 for large company equities (S&P 500), Treasury Bills and Bonds, and their arithmetic differences using the Ibbotson data (Ibbotson Associates, 2003).[7]

US Equity Risk Premia 1926-2002 Annual Equity Returns and Premia versus Treasury Bills, Intermediate, and Long Term Bonds			
Horizon	**Equity Returns**	**Risk-Free Return**	**ERP**
Short	12.20%	3.83%	8.37%
Intermediate	12.20%	4.81%	7.40%
Long	12.20%	5.23%	6.97%
Source: Ibbotson Yearbook (2003)			

Table 1

In 1985, Mehra and Prescott introduced the idea of the equity risk premium puzzle. The puzzling result is that the historical realized ERP for the stock market using 1889-1978 data appeared to be at odds with and, relative to Treasury bills, far in excess of asset pricing theory values based on normal parametrizations of risk aversion. When using standard frictionless return models and historical growth rates in consumption, the real risk-free rate, and the equity risk premium, the resulting relative risk aversion parameter appears too high. By choosing a maximum relative risk aversion parameter to be 10 and using the growth in consumption, Mehra and Prescott's model produces an ERP much lower than the historical.[8] Their result inspired a stream of finance literature that attempts to solve the puzzle. Two different research threads have emerged. One thread, including behavioral finance, attempts to explain the historical returns with new models and different assumptions about investors.[9] A second thread is from a group that provides estimates of the ERP that are derived from historical data and/or standard economic models. Some in this latter group argue that historical returns may have been higher than those that should be required in the future. In a curiously asymmetric way, there are no serious studies yet concluding that the historical results are too low to serve as ex ante estimates. Although both groups have made substantial and provocative contributions, the behavioral models do not give any ex ante ERP estimates other than explaining and supporting the historical returns. We presume, until results show otherwise, the behavioralists support the historical average as the ex ante unconditional long-run expectation. Therefore, we focus on the latter to catalogue equity risk premium estimates other than the historical approach, but we will discuss both as important strains for puzzle research.

Equity Risk Premium Types

Many different types of equity risk premium estimates can be given even though they are labeled by the same general term. These estimates vary widely; currently the estimates range from about nine percent to a small negative. When ERP estimates are

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[7] Ibbotson's 1926-2002 series from the 2003 Yearbook, Valuation Edition. The entire series is shown in Appendix A.

[8] Campbell, Lo, and MacKinlay (1997) perform a similar analysis as Mehra and Prescott and find a risk-aversion coefficient of 19, larger than the reasonable level suggested in Mehra and Prescott's paper, pp307-308.

[9] See, for example, Benartzi and Thaler (1995) and Mehra (2002).

given, one should determine the type before comparing to other estimates. We point out seven important types to look for when given an ERP estimate. They include:

- Geometric vs. arithmetic averaging
- Short vs. long investment horizon
- Short vs. long-run expectation
- Unconditional vs. conditional on some related variable
- Domestic US vs. international market data
- Data sources and periods
- Real vs. nominal returns

The average market return and ERP can be stated as a geometric or arithmetic mean return. An arithmetic mean return is a simple average of a series of returns. The geometric mean return is the compound rate of return; it is a measure of the actual average performance of a portfolio over a given time period. Arithmetic returns are the same or higher than geometric returns, so it is not appropriate to make a direct comparison between an arithmetic estimate and a geometric estimate. However, those two returns can be transformed one to the other. For example, arithmetic returns can be approximated from geometric returns by the formula.[10]

$$AR = GR + \frac{\sigma^2}{2}, \sigma^2 \text{ the variance of the (arithmetic) return process}$$

Arithmetic averages of periodic returns are to be preferred when estimating next period returns since they, not geometric averages, reproduce the proper probabilities and means of expected returns.[11] ERPs can be generated by arithmetic differences (Equity – Risk Free) or by geometric differences ([(1 + Equity)/(1 + Risk Free)]-1). Usually, the arithmetic and geometric differences produce similar estimates.[12]

A second important difference in ERP estimate types is the horizon. The horizon indicates the total investment or planning period under consideration. For estimation purposes, the horizon relates to the term or maturity of the risk-free instrument that is used to determine the ERP.[13] The Ibbotson Yearbook (2003) provides definitions for three different horizons.[14] The short-horizon expected ERP[15] is defined as "the large company stock total returns minus U.S. Treasury bill *total* returns". Note, the income return and total return are the same for U.S. Treasury bills. The intermediate-horizon expected ERP is "the large company stock total returns minus intermediate-term government bond *income* returns". Finally, the long-horizon expected ERP is "the large company stock total returns minus long-term government bond *income* returns". For the Ibbotson data, Treasury bills have a maturity of approximately one month; intermediate-term government bonds have a maturity around five years; long-term government bonds

[10] See Welch (2000), Dimson et al. (2002), Ibbotson and Chen (2003).
[11] For example, see Ibbotson Yearbook, Valuation Edition (2003), pp71-73 for a complete discussion of the arithmetic/geometric choice. See also Dimson et al. (2000), p35 and Brennan and Schwartz (1985).
[12] The arithmetic difference is the geometric difference multiplied by 1 + Risk Free.
[13] See Table 1.
[14] See Ibbotson 2003 Yearbook, p177.
[15] Table 1 displays the short horizon ERP calculation for the 1926-2002 Ibbotson Data.

have a maturity of about 20 years. Although the Ibbotson definitions may not apply to other research, we will classify equity risk premium estimates based on these guidelines to establish some consistency among the current research. The reader should note that Ibbotson Associates recommends the income return (or the yield) when using a bond as the risk free rate rather than the total return.[16]

A third type is the length of time of the equity risk premium forecast. We distinguish between short-run and long-run expectations. Short-run expectations refer to the current equity risk premium, or for this paper, a prediction of up to ten years. In contrast, the long-run expectation is a forecast over ten years to as much as seventy-five years for social security purposes. Ten years appears an appropriate breaking point based on the current literature surveyed.

The next difference is whether the equity risk premium estimate is unconditional or conditioned on one or more related variables. In defining this type, we refer to an admonition by Constantinides (2002, p1568) of the differences in these estimates:

> "First, I draw a sharp distinction between *conditional, short-term forecasts* of the mean equity return and premium and *estimates of the unconditional mean*. I argue that the currently low conditional short-term forecasts of the return and premium do not lessen the burden on economic theory to explain the large unconditional mean equity return and premium, as measured by their sample average over the past one hundred and thirty years."

Many of the estimates we catalogue below will be conditional ones, conditional on dividend yield, expected earnings, capital gains, or other assumptions about the future.

ERP estimates can also exhibit a US versus international market type depending upon the data used for estimation purposes and the ERP being estimated. Dimson, et al. (2002) notes that at the start of 2000, the US equity market, while dominant, was slightly less than one-half (46.1 %) of the total international market for equities, capitalized at 52.7 trillion dollars. Data from the non-US equity markets are clearly different from US markets and, hence, will produce different estimates for returns and ERP.[17] Results for the entire world equity market will, of course, be a weighted average of the US and non-US estimates.

[16] The reason for this is two-fold. First, when issued, the yield is the expected market return for the entire horizon of the bond. No net capital gains are expected for the market return for the entire horizon of the bond. No capital gains are expected at the default-free maturity. Second, historical annual capital gains on long-term Government Bonds average near zero (0.4%) over the 1926-2002 period (Ibbotson Yearbook, 2003, Table 6-7).

[17] One qualitative difference can arise from the collapse of equity markets during war time.

Worldwide Equity Risk Premia, 1900-2000 Annual Equity Risk Premium Relative to Treasury Bills		
Country	**Geometric Mean**	**Arithmetic Mean**
United States	5.8%	7.7%
World	4.9%	6.2%
Source: Dimson, et al. (2002), pages 166-167		

Table 2

The next type is the data source and period used for the market and ERP estimates. Whether given an historical average of the equity risk premium or an estimate from a model using various historical data, the ERP estimate will be influenced by the length, timing, and source of the underlying data used. The time series compilations are primarily annual or monthly returns. Occasionally, daily returns are analyzed, but not for the purpose of estimating an ERP. Some researchers use as much as 200 years of history; the Ibbotson data currently uses S&P 500 returns from 1926 to the present.[18] As an example, Siegel (2002) examines a series of real US returns beginning in 1802.[19] Siegel uses three sources to obtain the data. For the first period, 1802 to 1870, characterized by stocks of financial organizations involved in banking and insurance, he cites Schwert (1990). The second period, 1871-1925, incorporates Cowles stock indexes compiled in Shiller (1989). The last period, beginning in 1926, uses CRSP data; these are the same data underlying Ibbotson Associates calculations.

Goetzmann et al. (2001) construct a NYSE data series for 1815 to 1925 to add to the 1926-1999 Ibbotson series. They conclude that the pre-1926 and post-1926 data periods show differences in both risk and reward characteristics. They highlight the fact that inclusion of pre-1926 data will generally produce lower estimates of ERPs than relying exclusively on the Ibbotson post-1926 data, similar to that shown in Appendix A. Several studies that rely on pre-1926 data, catalogued in Appendix B, show the magnitudes of these lower estimates.[20] Table 3 displays Siegel's ERPs for three subperiods. He notes that subperiod III, 1926-2001, shows a larger ERP (4.7%), or a smaller real risk-free mean (2.2%), than the prior subperiods[21].

[18] For the Ibbotson analysis of the small stock premium, the NYSE/AMEX/NASDAQ combined data are used with the S&P 500 data falling within deciles 1 and 3 (Ibbotson 2002 Yearbook, pp122-136.)
[19] A more recent alternative is Wilson and Jones (2002) as cited by Dimson et al. (2002), p39.
[20] Using Wilson and Jones' 1871-2002 data series, time series analyses show no significant ERP difference between the 1871-1925 period and the 1926-2002 period; one cannot distinguish the old from the new. The overall average is lower with the additional 1871-1925 data, but on a statistical basis, they are not significantly different. Assuming the equivalency of the two data series for 1871 to 1925 (series of Goetzmann et al. and Wilson & Jones), the risk difference found by Goetzmann et al. must be determined by a significantly different ERP in the pre-1871 data. The 1871-1913 return is prior to personal income tax and appears to be about 35% lower than the 1926-2002 period average of 11.8%, might reflect a zero valuation for income taxes in the pre-1914 returns. Adjusting the pre-1914 data for taxes would most likely make the ERP for the entire period (1871-2002) approximately equal to 7.5%, the 1926-2002 average.
[21] The low risk-free return is indicative of the "risk-free rate puzzle", the twin of the ERP puzzle. For details see Weil (1989).

Short-Horizon Equity Risk Premium by Subperiods			
	Subperiod I	Subperiod II	Subperiod III
	1802-1870	1871-1925	1926-2001
Real Geometric Stock Returns	7.0%	6.6%	6.9%
Real Geometric Long Term Governments	4.8%	3.7%	2.2%
Equity Risk Premium	2.2%	2.9%	4.7%
Source: Siegel (2002), pages 13 and 15.			

Table 3

Smaller subperiods will show much larger variations in equity, bill and ERP returns. Table 4 displays the Ibbotson returns and short horizon risk premia for subperiods as small as 5 years. The scatter of results is indicative of the underlying large variation (20% sd) in annual data.

Average Short-Horizon Risk Premium over Various Time Period				
		Common Stocks	**U. S. Treasury Bills**	**Short-Horizon**
Year		**Total Annual Returns**	**Total Annual Returns**	**Risk Premium**
All Data	1926-2002	12.20%	3.83%	8.37%
50 Year	1953-2002	12.50%	5.33%	7.17%
40 Year	1963-2002	11.80%	6.11%	5.68%
30 Year	1943-1972	14.55%	2.54%	12.02%
	1973-2002	12.21%	6.61%	5.60%
15 Year	1928-1942	5.84%	0.95%	4.89%
	1943-1957	17.14%	1.20%	15.94%
	1958-1972	11.96%	3.87%	8.09%
	1973-1987	11.42%	8.20%	3.22%
	1988-2002	13.00%	5.03%	7.97%
10 Year	1933-1942	12.88%	0.15%	12.73%
	1943-1952	17.81%	0.81%	17.00%
	1953-1962	15.29%	2.19%	13.11%
	1963-1972	10.55%	4.61%	5.94%
	1973-1982	8.67%	8.50%	0.17%
	1983-1992	16.80%	6.96%	9.84%
	1993-2002	11.17%	4.38%	6.79%
5 Year	1928-1932	- 8.25%	2.55%	-10.80%
	1933-1937	19.82%	0.22%	19.60%
	1938-1942	5.94%	0.07%	5.87%
	1943-1947	15.95%	0.37%	15.57%
	1948-1952	19.68%	1.25%	18.43%
	1953-1957	15.79%	1.97%	13.82%
	1958-1962	14.79%	2.40%	12.39%
	1963-1967	13.13%	3.91%	9.22%
	1968-1972	7.97%	5.31%	2.66%
	1973-1977	2.55%	6.19%	- 3.64%
	1978-1982	14.78%	10.81%	3.97%
	1983-1987	16.93%	7.60%	9.33%
	1988-1992	16.67%	6.33%	10.34%
	1993-1997	21.03%	4.57%	16.46%
	1998-2002	1.31%	4.18%	- 2.88%

Table 4

In calculating an expected market risk premium by averaging historical data, projecting historical data using growth models, or even conducting a survey, one must determine a proxy for the "market". Common proxies for the US market include the S&P 500, the NYSE index, and the NYSE, AMEX, and NASDAQ index.[22] For the purpose of this paper, we use the S&P 500 and its antecedents as the market. However, in the various research surveyed, many different market proxies are assumed. We have already discussed using international versus domestic data when describing different MRP types. With international data, different proxies for other country, region, or world markets are used.[23] For domestic data, different proxies have been used over time as stock market exchanges have expanded.[24] Fortunately, as shown in the Ibbotson Valuation yearbook, the issue of a US market proxy does not have a large effect on the MRP estimate because the various indices are highly correlated. For example, the S&P 500 and the NYSE have a correlation of 0.95, the S&P 500 and NYSE/AMEX/NASDAQ 0.97, and the NYSE and NYSE/AMEX/NASDAQ 0.90.[25] Therefore, the market proxy selected is one reason for slight differences in the estimates of the market risk premium.

As a final note, stock returns and risk-free rates can be stated in nominal or real terms. Nominal includes inflation; real removes inflation. The equity risk premium should not be affected by inflation because either the stock return and risk-free rate both include the effects of inflation (both stated in nominal terms) or neither have inflation (both stated in real terms). If both returns are nominal, the difference in the returns is generally assumed to remove inflation. Otherwise, both terms are real, so inflation is removed prior to finding the equity risk premium. While numerical differences in the real and nominal approaches may exist, their magnitudes are expected to be small.

Equity Risk Premia 1926-2002

As an example of the importance of knowing the types of equity risk premium estimates under consideration, Table 5 displays ERP returns that each use the same historical data, but are based on arithmetic or geometric returns and the type of horizon. The ERP estimates are quite different.[26]

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22 2003 Ibbotson Valuation Yearbook, p92.

23 For example, Dimson (2002) and Claus and Thomas (2001) use international market data.

24 For a data series that is a mixture of the NYSE exchange, NYSE, AMEX, and NASDAQ stock exchange, and the Wilshire 5000, see Dimson (2002), p306.

25 2003 Ibbotson Valuation Yearbook, p93; using data from October 1997 to September 2002.

26 The nominal and real ERPs are identical in Table 5 because the ERPs are calculated as arithmetic differences, and the same value of inflation will reduce the market return and the risk-free return equally. Geometric differences would produce minimally different estimates for the same types.

ERP using same historical data (1926-2002)		
RFR Description	**ERP Description**	**ERP Historical Return**
Short nominal	Arithmetic Short-horizon	8.4%
Short nominal	Geometric Short-horizon	6.4%
Short real	Arithmetic Short-horizon	8.4%
Short real	Geometric Short-horizon	6.4%
Intermediate nominal	Arithmetic Inter-horizon	7.4%
Intermediate nominal	Geometric Inter-horizon	5.4%
Intermediate real	Arithmetic Inter-horizon	7.4%
Intermediate real	Geometric Inter-horizon	5.4%
Long nominal	Arithmetic Long-horizon	7.0%
Long nominal	Geometric Long-horizon	5.0%
Long real	Arithmetic Long-horizon	7.0%
Long real	Geometric Long-horizon	5.0%

Table 5

Historical Methods

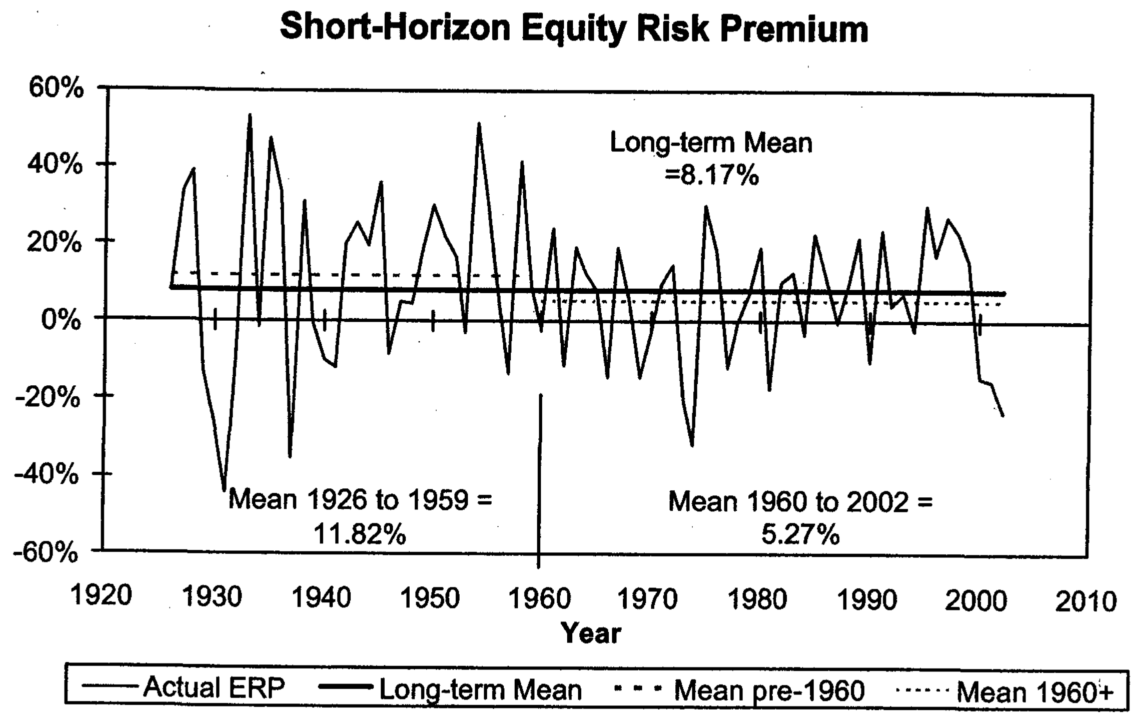
The historical methodology uses averages of past returns to forecast future returns. Different time periods may be selected, but the two most common periods arise from data provided by either Ibbotson or Siegel. The Ibbotson series begins in 1926 and is updated each year. The Siegel series begins in 1802 with the most recent compilation using returns through 2001. Appendix A provides equity risk premium estimates using Ibbotson data for the 1926-2002 period that we use in this paper for most illustrations. We begin with a look at the ERP history through a time series analysis of the Ibbotson data.

Time Series Analysis

Much of the analysis addressing the equity risk premium puzzle relies on the annual time series of market, risk-free and risk premium returns. Two opposite views can be taken of these data. One view would have the 1926-2002 Ibbotson data, or the 1802-2001 Siegel data, represent one data point; i.e., we have observed one path for the ERP through time from the many possible 77 or 200 year paths. This view rests upon the existence or assumption of a stochastic process with (possibly) inter-temporal correlations. While mathematically sophisticated, this model is particularly unhelpful without some testable hint at the details of the generating stochastic process. The practical view is that the observed returns are random samples from annual distributions that are iid, independent and identically distributed about the mean. The obvious advantage is that we have at hand 77 or 200 observations on the iid process to analyze. We adopt the latter view.

Some analyses adopt the assumption of stationarity of ERP, i.e., the true mean does not change with time. Figure 1 displays the Ibbotson ERP data and highlights two subperiods, 1926-1959 and 1960-2002.[27] While the mean ERP for the two subperiods appear quite different (11.82% vs. 5.27%), the large variance of the process (std dev 20.24%) should make them indistinguishable statistically speaking.

[27] The ERP shown here are the geometric differences (calculated) rather than the simple arithmetic differences in Table 1; i.e. ERP = $[(1+r_m)/(1+r_f)] - 1$. The test results are qualitatively the same for the arithmetic differences.



Source: Ibbotson Yearbook (2003), Geometric Differences

Figure 1

T-Tests

The standard T-test can be used for the null hypothesis H_o : mean 1960-2002 = 8.17%, the 77 year mean.[28] The outcome of the test is shown in Table 6; the null hypothesis cannot be rejected.

T-Test Under the Null Hypothesis that ERP (1960-2002) = ERP (1926-2002) = 8.17%	
Sample mean 1960-2002	5.27%
Sample s.d. 1960-2002	15.83%
T value (DF=42)	-1.20
PR > |T|	0.2374
Confidence Interval 95%	(0.0040, 0.1014)
Confidence Interval 90%	(0.0121, 0.0933)

Table 6

Another T-Test can be used to test whether the subperiod means are different in the presence of unequal variances.[29] The result is similar to Table 6 and the difference of subperiod means equal to zero cannot be rejected.[30]

[28] Standard statistical procedures in SAS 8.1 have been used for all tests.
[29] Equality of variances is rejected at the one percent level by an F test (F=2.39, DF=33,42)
[30] t-value 1.35, PR> |T| = 0.1850 with the Cochran method.

Time Trends

The supposition of stationarity of the ERP series can be supported by ANOVA regressions. The results of regressing the ERP series on time is shown in Table 7.

ERP ANOVA Regressions on Time		
Period	**Time Coefficient**	**P-Value**
1926-1959	0.004	0.355
1960-2002	0.001	0.749
1926-2002	-0.001	0.443

Table 7

There are no significant time trends in the Ibbotson ERP data.[31]

ARIMA Model

Time series analysis using the well established Box-Jenkins approach can be used to predict future series values through the lag correlation structure.[32] The SAS ARIMA procedure applied to the full 77 time series data shows:

(1) No significant autocorrelation lags.
(2) An identification of the series as white noise.
(3) ARIMA projection of year 78+ ERP is 8.17%, the 77 year average.

All of the above single time series tests point to the reasonability of the stationarity assumption for (at least) the Ibbotson ERP 77 year series.[33]

Social Security Administration

In the current debate on whether to allow private accounts that may invest in equities, the Office of the Chief Actuary of the Social Security Administration has selected certain assumptions to assess various proposals (Goss, 2001). The relevant selection is to use 7 percent as the real (geometric) annual rate of return for equities.[34] This assumption is based on the historical return of the 20th century. SSA received further support that showed the historical return for the last 200 years is consistent with this estimate, along with the Ibbotson series beginning in 1926. For SSA, the calculation of the equity risk premium uses a long-run real yield on Treasury bonds as the risk-free rate. From the assumptions in the 1995 Trustees Report, the long-run real yield on Treasury bonds that the Advisory Council proposals use is 2.3%. Using a future Treasury securities real yield of 2.3% produces a geometric equity risk premium of 4.7% over long-term Treasury securities. More recently, the Treasury securities assumption has increased to 3%[35], yielding a 4% geometric ERP over long-term Treasury securities.

[31] The result is confirmed by a separate Chow test on the two subperiods.
[32] See Harvey (1990), p30.
[33] The same tests applied to the Wilson and Jones 1871-2002 data series show similar results: Neither the 1871-1925 period nor the 1926-2002 period is different from the overall 1871-2002 period. The overall period and subperiods also show no trends over time.
[34] Compare Table 3, subperiod III.
[35] 1999 Social Security Trustees Report.

At the request of the Office of the Chief Actuary of the Social Security Administration (OCACT), John Campbell, Peter Diamond, and John Shoven were engaged to give their expert opinions on the assumptions Social Security mode. Each economist begins with the Social Security assumptions and then explains any difference he feels would be more appropriate.

In John Campbell's response, he considers valuation ratios as a comparison to the returns from the historical approach (Campbell 2001). The current valuation ratios are at unusual levels, with a low dividend-price ratio and high price-earnings ratio. He reasons that the prices are what have dramatically changed these ratios. Campbell presents two views as to the effect of valuation ratios in their current state. One view is that valuations will remain at the current level, suggesting much lower expected returns. The second view is a correction to the ratios, resulting in less favorable returns until the ratios readjust. He decides to give some weight to both possibilities, so he lowers the geometric equity return estimate to 5-5.5% from 7%. For the risk-free rate, he uses the yield on the long-term inflation-indexed bonds[36] of 3.5% or the OCACT assumption of 3%. Therefore, his geometric equity premium estimate is around 1.5 to 2.5%.

Peter Diamond uses the Gordon growth formula to calculate an estimate of the equity return (Diamond 2001). The classic Gordon Dividend Growth model is[37]:

$$K = (D_1 / P_0) + g$$

K = Expected Return or Discount Rate P_0 = Price this period
D_1 = Expected Dividend next period g = Expected growth in dividends in perpetuity

Based on his analysis, he feels that the equity return assumption of 7% for the next 75 years is not consistent with a reasonable level of stock value compared to GDP. Even when increasing the GDP growth assumption, he still does not feel that the equity return is plausible. By reasoning that the next decade of returns will be lower than normal, only then is the equity return beyond that time frame consistent with the historical return. By considering the next 75 years together, he would lower the overall projected equity return to 6-6.5%. He argues that the stock market is overvalued, and a correction is required before the long-run historical return is a reasonable projection for the future. By using the OCACT assumption of 3.0% for the long-term real yield on Treasury bonds, Diamond estimates a geometric equity risk premium of about 3-3.5%.

John Shoven begins by explaining why the traditional Gordon growth model is not appropriate, and he suggests a modernized Gordon model that allows share repurchases to be included instead of only using the dividend yield and growth rate (Shoven 2001). By assuming a long-term price-earnings ratio between its current and historical value, he comes up with an estimate for the long-term real equity return of 6.125%. Using his general estimate of 6-6.5% for the equity return and the OCACT assumptions for the long-term bond yield, he projects a long-term equity risk premium of approximately 3-3.5%. All the SSA experts begin by accepting the long-run historical

[36] See discussion of current yields on TIPS below.
[37] Brealey and Myers (2000), p67.

ERP analyses and then modifying that by changes in the risk-free rate or by decreases in the long-term ERP based on their own personal assessments. We now turn to the major strains in ERP puzzle research.

ERP Puzzle Research

Campbell and Shiller (2001) begin with the assumption of mean reversion of dividend/price and price/earnings ratios. Next, they explain the result of prior research which finds that the dividend-price ratio predicts future prices, and historically, the price corrects the ratio when it diverts from the mean.[38] Based on this result, they then use regressions of the dividend-price ratio and the price-smoothed-earnings[39] ratio to predict future stock prices out ten years. Both regressions predict large losses in stock prices for the ten year horizon. Although Campbell and Shiller do not rerun the regression on the dividend-price ratio to incorporate share repurchases, they point out that the dividend-price ratio should be upwardly adjusted, but the adjustment only moves the ratio to the lower range of the historical fluctuations (as opposed to the mean). They conclude that the valuation ratios indicate a bear market in the near future[40]. They predict for the next ten year period negative real stock returns. They caution that because valuation ratios have changed so much from their normal level, they may not completely revert to the historical mean, but this does not change their pessimism about the next decade of stock market returns.

Arnott and Ryan (2001) take the perspective of fiduciaries, such as pension fund managers, with an investment portfolio. They begin by breaking down the historical stock returns (past 74 years since December 1925) by analyzing dividend yields and real dividend growth. They point out that the historical dividend yield is much higher than the current dividend yield of about 1.2%. They argue that the changes from stock repurchases, reinvestment, and mergers and acquisitions, which affect the lower dividend yield, can be represented by a higher dividend growth rate. However, they cap real dividend or earnings growth at the level of real economic growth. They add the dividend yield and the growth in real dividends to come up with an estimate for the future equity return; the current dividend yield of 1.2% and the economic growth rate of 2.0% add to the 3.2% estimated real stock return. This method corresponds to the dividend growth model or earnings growth model and does not take into account changing valuation levels. They cite a TIPS yield of 4.1% for the real risk-free rate return.[41] These two estimates yield a negative geometric long-horizon conditional equity risk premium.

Arnott and Bernstein (2002) begin by arguing that in 1926 investors were not expecting the realized, historical compensation that they later received from stocks. They cite bonds' reaction to inflation, increasing valuations, survivorship bias[42], and changes in

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[38] Campbell and Shiller (1989).
[39] Earnings are "smoothed" by using ten year averages.
[40] The stock market correction from year-end 1999 to year-end 2002 is a decrease of 37.6% or 14.6% per year. Presumably, the "next ten years" refers to 2000 to 2010.
[41] See the current TIPS yield discussion near end of paper.
[42] See Brown et al. (1992, 1995) for details on potential survivorship bias.

regulation as positive events that helped investors during this period. They only use the dividend growth model to predict a future expected return for investors. They do not agree that the earnings growth model is better than the dividend growth model both because earnings are reported using accounting methods and earnings data before 1870 are inaccurate. Even if the earnings growth model is chosen instead, they find that the earnings growth rate from 1870 only grows 0.3% faster than dividends, so their results would not change much. Because of the Modigliani-Miller theorem[43], a change in dividend policy should not change the value of the firm. They conclude that managers benefited in the "era of 'robber baron' capitalism" instead of the conclusion reached by others that the dividend growth model under-represents the value of the firm.

By holding valuations constant and using the dividend yield and real growth of dividends, Arnott and Bernstein calculate the equity return that an investor might have expected during the historical time period starting in 1802. They use an expected dividend yield of 5.0%, close to the historical average of 1810 to 2001. For the real growth of dividends, they choose the real per capita GDP growth less a reduction for entrepreneurial activity in the economy plus stock repurchases. They conclude that the net adjustment is negative, so the real GDP growth is reduced from 2.5-3% to only 1%. A fair expectation of the stock return for the historical period is close to 6.1% by adding 5.0% for the dividend yield and a net real GDP per capita growth of 1.1%. They use a TIPS yield of 3.7% for the real risk-free rate, which yields a geometric intermediate-horizon equity risk premium of 2.4% as a fair expectation for investors in the past. They consider this a "normal" equity risk premium estimate. They also opine that the current ERP is zero; i.e. they expect stocks and (risk-free) bonds to return the same amounts.

Fama and French (2002) use both the dividend growth model and the earnings growth model to investigate three periods of historical returns: 1872 to 2000, 1872 to 1950, and 1951 to 2000. Their ultimate aim is to find an unconditional equity risk premium. They cite that by assuming the dividend-price ratio and the earnings-price ratio follow a mean reversion process, the result follows that the dividend growth model or earnings growth model produce approximations of the unconditional equity return. Fama and French's analysis of the earlier period of 1872 to 1950 shows that the historical average equity return and the estimate from the dividend growth model are about the same. In contrast, they find that the 1951 to 2000 period has different estimates for returns when comparing the historical average and the growth models' estimates. The difference in the historical average and the model estimates for 1951 to 2000 is interpreted to be "unexpected capital gains" over this period. They find that the unadjusted growth model estimates of the ERP, 2.55% from the dividend model and 4.32% from the earnings model, fall short of the realized average excess return for 1951-2000. Fama and French prefer estimates from growth models instead of the historical method because of the lower standard error using the dividend growth model. Fama and French provide 3.83% as the unconditional expected equity risk premium return (referred to as the annual bias-adjusted ERP estimate) using the dividend growth model with underlying data from 1951 to 2000. They give 4.78% as the unconditional expected equity risk

[43] Brealey and Myers (2000), p447. See also discussion in Ibbotson and Chen (2003).

premium return using the earnings growth model with data from 1951 to 2000. Note that using a one-month Treasury bill instead of commercial paper for the risk-free rate would increase the ERP by about 1% to nearly 6% for the 1951-2000 period.

Ibbotson and Chen (2003) examine the historical real geometric long-run market and long risk-free returns using their "building block" methodology.[44] They use the full 1926-2000 Ibbotson Associates data and consider as building blocks all of the fundamental variables of the prior researchers. Those blocks include (not all simultaneously):

- Inflation
- Real risk-free rates (long)
- Real capital gains
- Growth of real earnings per share
- Growth of real dividends
- Growth in payout ratio (dividend/earnings)
- Growth in book value
- Growth in ROE
- Growth in price/earnings ratio
- Growth in real GDP/population
- Growth in equities excess of GDP/POP
- Reinvestment

Their calculations show that a forecast real geometric long run return of 9.4% is a reasonable extrapolation of the historical data underlying a realized 1926-2000 return of 10.7%, yielding a long horizon arithmetic ERP of 6%, or a short horizon arithmetic ERP of about 7.5%.

The authors construct six building block methods; i.e., they use combinations of historic estimates to produce an expected geometric equity return. They highlight the importance of using both dividends and capital gains by invoking the Modigliani-Miller theorem. The methods, and their component building blocks are:

- Method 1: Inflation, real risk free rate, realized ERP
- Method 2: Inflation, income, capital gains and reinvestment
- Method 3: Inflation, income, growth in price/earnings, growth in real earnings per share and reinvestment.
- Method 4: Inflation, growth rate of price/earnings, growth rate of real dividends, growth rate of payout ratio dividend yield and reinvestment
- Method 5: Inflation, income growth rate of price/earnings, growth of real book value, ROE growth and reinvestment
- Method 6: Inflation, income, growth in real GDP/POP, growth in equities excess GDP/POP and reinvestment.

[44] See Appendix D for a summary of their building block estimates. See also Pratt (1998) for a discussion of the Building Block, or Build-Up Model, cost of capital estimation method.

All six methods reproduce the historical long horizon geometric mean of 10.70% as shown in Appendix D. Since the source of most other researchers' lower ERP is the dividend yield, the authors recast the historical results in terms of ex ante forecasts for the next 75 years. Their estimate of 9.37% using supply side methods 3 and 4 is approximately 130 basis points lower than the historical result. Within their methods, they also show how the substantially lower expectation of 5.44% for the long mean geometric return is calculated by omitting one or more relevant variables. Underlying these ex ante methods are the assumptions of stationarity of the mean ERP return and market efficiency, the absence of the assumption that the market has mispriced equities. All of their methods are aimed at producing an unconditioned estimate of the ex ante ERP.

As opposed to short-run, conditional estimates from Campbell and Shiller and others, Constantinides (2002) seeks to estimate the unconditional equity risk premium, more in line with the goal of Fama and French (2002) and Ibbotson and Chen (2003). He begins with the premise that the unconditional ERP can be estimated from the historical average using the assumption that the ERP follows a stationary path. He suggests most of the other research produces conditional estimates, conditioned upon beliefs about the future paths of fundamentals such as dividend growth, price-earnings ratio and the like. While interesting in themselves, they add little to the estimation of the unconditional mean ERP.

Constantinides uses the historical return and adjusts downward by the growth in the price-earnings ratio to calculate the unconditional equity risk premium. He removes the growth in the price-earnings ratio because he is assuming no change in valuations in the unconditional state. He gives estimates using three periods. For 1872-2000, he uses the historical equity risk premium which is 6.9%, and after amortizing the growth in the price-dividend ratio or price-earnings ratio over a period as long as 129 years, the effect of the potential reduction is no change. Therefore, he finds an unconditional arithmetic, short-horizon equity risk premium of 6.9% using the 1872-2000 underlying data. For 1951-2000, he again starts with the historical equity risk premium which is 8.7% and lowers this estimate by the growth in the price-earnings ratio of 2.7% to find an unconditional arithmetic, short-horizon equity risk premium of 6.0%. For 1926-2000, he uses the historical equity risk premium which is 9.3% and reduces this estimate by the growth in the price-earnings ratio of 1.3% to find an unconditional arithmetic, short-horizon equity risk premium of 8.0%. He appeals to behavioral finance to offer explanations for such high unconditional equity risk premium estimates.

From the perspective of giving practical investor advice, Malkiel (1999) discusses "the age of the millennium" to give some indication of what investors might expect for the future. He specifically estimates a reasonable expectation for the first few decades of the twenty-first century. He estimates the future bond returns by giving estimates if bonds are held to maturity with corporate bonds of 6.5-7%, long-term zero-coupon Treasury bonds of about 5.25%, and TIPS with a 3.75% return. Depending on the desired level of risk, Malkiel indicates bondholders should be more favorably compensated in the future compared to the historical returns from 1926 to 1998. Malkiel uses the earnings growth model to predict future equity returns. He uses the current dividend yield of 1.5% and an earnings growth estimate of 6.5%, yielding an 8% equity return estimate compared with an 11% historical return. Malkiel's estimated range of the equity risk premium is from 1% to 4.25%, depending on the risk-free instrument selected. Although his equity risk premium is lower than the historical return, his selection of a relatively high earnings growth rate is similar to Ibbotson and Chen's forecasted models. In contrast with Ibbotson and Chen, Malkiel allows for a changing equity risk premium and advises investors to not rely solely on the past "age of exuberance" as a guide for the future. Malkiel points out the impact of changes in valuation ratios, but he does not attempt to predict future valuation levels.

Finally, Mehra (2002) summarizes the results of the research since the ERP puzzle was posed. The essence of the puzzle is the inconsistency of the ERPs produced by descriptive and prescriptive economic models of asset pricing on the one hand and the historical ERPs realized in the US market on the other. Mehra and Prescott (1985) speculated that the inconsistency could arise from the inadequacy of standard models to incorporate market imperfections and transaction costs. Failure of the models to reflect reality rather than failure of the market to follow the theory seems to be Mehra's conclusion as of 2002. Mehra points to two promising threads of model-modifying research. Campbell and Cochrane (1999) incorporate economic cycles and changing risk aversion while Constantinides et al. (2002) propose a life cycle investing modification, replacing the representative agent by segmenting investors into young, middle aged, and older cohorts. Mehra sums up by offering:

> "Before we dismiss the premium, we not only need to have an understanding of the observed phenomena but also why the future is likely to be different. In the absence of this, we can make the following claim based on what we know. Over the long horizon the equity premium is likely to be similar to what it has been in the past and the returns to investment in equity will continue to substantially dominate those in bonds for investors with a long planning horizon."

Financial Analyst Estimates

Claus and Thomas (2001) and Harris and Marston (2001) both provide equity premium estimates using financial analysts' forecasts. However, their results are rather different. Claus and Thomas use an abnormal earnings model with data from 1985 to 1998 to calculate an equity risk premium as opposed to using the more common dividend growth model. Financial analysts project five year estimates of future earnings growth rates. When using this five year growth rate for the dividend growth rate in perpetuity in the Gordon growth model, Claus and Thomas explain that there is a potential upward bias in estimates for the equity risk premium. Therefore, they choose to use the abnormal earnings model instead and only let earnings grow at the level of inflation after five years. The abnormal earnings model replaces dividends with "abnormal earnings" and discounts each flow separately instead of using a perpetuity. The average estimate that they find is 3.39% for the equity risk premium. Although it is generally recognized that financial analysts' estimates have an upward bias, Claus and Thomas propose that in the current literature, financial analysts' forecasts have underestimated short-term earnings in order for management to achieve earnings estimates in the slower economy. Claus and Thomas conclude that their findings of the ERP using data from the past fifteen years are not in line with historical values.

Harris and Marston use the dividend growth model with data from 1982 to 1998. They assume that the dividend growth rate should correspond to investor expectations. By using financial analysts' longest estimates (five years) of earnings growth in the model, they attempt to estimate these expectations. They argue that if investors are in accord with the optimism shown in analysts' estimates, even biased estimates do not pose a drawback because these market sentiments will be reflected in actual returns. Harris and Marston find an equity risk premium estimate of 7.14%. They find fluctuations in the equity risk premium over time. Because their estimates are close to historical returns, they contend that investors continue to require a high equity risk premium.

Survey Methods

One method to estimate the ex ante equity risk premium is to find the consensus view of experts. John Graham and Campbell Harvey perform a survey of Chief Financial Officers to determine the average cost of capital used by firms. Ivo Welch surveys financial economists to determine the equity risk premium that academic experts in this area would estimate.

Graham and Harvey administer surveys from the second quarter of 2000 to the third quarter of 2002 (Graham and Harvey, 2002). For their survey format, they show the current ten year bond yield and then ask CFOs to provide their estimate of the S&P 500 return for the next year and over the next ten years. CFOs are actively involved in setting a company's individual hurdle[45] rate and are therefore considered knowledgeable about investors' expectations.[46] When comparing the survey responses of the one and ten year returns, the one year returns have so much volatility that they conclude that the ten-year equity risk premium is the more important and appropriate return of the two when making financial decisions such as hurdle rates and estimating cost of capital. The average ten-year equity risk premium estimate varies from 3% to 4.7%.

The most current Welch survey compiles the consensus view of about five hundred financial economists (Welch 2001). The average arithmetic estimate for the 30-year equity risk premium relative to Treasury bills is 5.5%; the one-year arithmetic equity risk premium consensus is 3.4%. Welch deduces from the average 30-year geometric

[45] A "hurdle" rate is a benchmark cost of capital used to evaluate projects to accept (expected returns greater than hurdle rate) or reject (expected returns less than hurdle rate).
[46] Graham and Harvey claim three-fourths of the CFOs use CAPM to estimate hurdle rates.

equity return estimate of 9.1% that the arithmetic equity return forecast is approximately 10%.[47]

Welch's survey question allows the participants to self select into different categories based upon their knowledge of ERP. The results indicate that the responses of the less ERP knowledgeable participants showed more pessimism than those of the self reported experts. The experts gave 30-year estimates that are 30 to 150 basis points above the estimates of the non-expert group.

Differences In Forecasts across Expertise Level				
Relative Expertise	**Statistic**	**Stock Market**	**Equity Premium**	
		30-Year Geometric	30-Year Arithmetic	30-Year Geometric
188 Less Involved	Mean	8.5%	4.9%	4.4%
	Median	8%	5%	4%
	IQ Range	6%-10%	3%-6%	2%-5.5%
235 Average	Mean	9.2%	5.8%	4.8%
	Median	9%	5%	4%
	IQ Range	7.5%-10%	3.5%-7%	3%-6%
72 Experts	Mean	10.1%	6.2%	5.4%
	Median	9%	5.4%	5%
	IQ Range	8%-11%	4%-7.5%	3.4%-6%
Data Source: Welch (2001), Table 5				

Table 8

Table 8 shows that there may be a "lemming" effect, especially among economists who are not directly involved in the ERP question. Stated differently, all the academic and popular press, together with the prior Welch survey[48] could condition the non-expert, the "less involved", that the expected ERP was lower than historic levels.

The Behavioral Approach

Benartzi and Thaler (1995) analyze the equity risk premium puzzle from the point of view of prospect theory (Kahneman and Tversky; 1979). Prospect theory[49] has "loss aversion", the fact that individuals are more sensitive to potential loss than gain, as one of its central tenets. Once an asymmetry in risk aversion is introduced into the model of the rational representative investor or agent, the unusual risk aversion problem raised initially by Mehra and Prescott (1985) can be "explained" within this behavioral model of decision-making under uncertainty. Stated differently, given the historical ERP series, there exists a model of investor behavior that can produce those or similar results. Benartzi and Thaler combine loss aversion with "mental accounting", the behavioral process people use to evaluate their status relative to gains and losses compared to expectations, utility and wealth, to get "myopic loss aversion". In particular, mental

[47] For the Ibbotson 1926-2002 data, the arithmetic return is about 190 basis points higher than the geometric return rather than the inferred 90 basis points. This suggests the participant's beliefs may not be internally consistent.

[48] The prior Welch survey in 1998 had a consensus ERP of about 7%.

[49] A current survey of the applications of prospect theory to finance can be found in Benartzi et al. (2001).

accounting for a portfolio needs to take place infrequently because of loss aversion, in order to reduce the chances of observing loss versus gain. The authors concede that there is a puzzle with the standard expected utility-maximizing paradigm but that the myopic loss aversion view may resolve the puzzle. The authors' views are not free of controversy; any progress along those lines is sure to match the advance of behavioral economics in the large.

The adoption of other behavioral aspects of investing may also provide support for the historical patterns of ERPs we see from 1802-2002. For example, as the true nature of risk and rewards has been uncovered by the virtual army of 20th century researchers, and as institutional investors held sway in the latter fifty years of the century, the demand for higher rewards seen in the later historical data may be a natural and rational response to the new and expanded information set. Dimson et al. (2002, Figure 4-6) displays increasing real US equity returns of 6.7, 7.4, 8.2 and 10.2 for periods of 101, 75, 50 and 25 years ending in 2001 consistent with this "risk-learning" view.

Next Ten Years

The "next ten years" is an issue that experts reviewing Social Security assumptions and Campbell and Shiller address either explicitly or implicitly. Experts evaluating Social Security's proposals predicted that the "next ten years", indicating a period beginning around 2000, of returns were likely to be below the historical return. However, a historical return was recommended as appropriate for the remaining 65 of the 75 years to be projected. For Campbell and Shiller (2001), the period they discuss is approximately 2000-2010. Based on the current state of valuation ratios, they predict lower stock market returns over "the next ten years". These expert predictions, and other pessimistic low estimates, have already come to fruition as market results 2000 through 2002.[50] The US equities market has decreased 37.6% since 1999, or an annual decrease of 14.6%. Although these forecasts have proved to be accurate in the short term, for future long-run projections, the market is not at the same valuation today as it was when these conditional estimates were originally given. Therefore, actuaries should be wary of using the low long-run estimates made prior to the large market correction of 2000-2002.

Treasury Inflation Protection Securities (TIPS)

Several of the ERP researchers refer to TIPS when considering the real risk free rates. Historically, they adjust Treasury yields downward to a real rate by an estimate of inflation, presumably for the term of the Treasury security. As Table 3 shows using the Siegel data, the modern era data show a low real long-term risk-free rate of return (2.2%). This contrasts with the initial[51] TIPS issue yields of 3.375%. Some researchers use those TIPS yields as (market) forecasts of real risk-free returns for intermediate and long-horizon, together with reduced (real) equity returns to produce low estimates of ex ante ERPs. None consider the volatility of TIPS as indicative of the accuracy of their ERP estimate.

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[50] The Social Security Advisory Board will revisit the seventy five year rate of return assumption during 2003, Social Security Advisory Board (2002).

[51] TIPS were introduced by the Treasury in 1996 with the first issue in January, 1997.

Table 9 shows a recent market valuation of ten and thirty year TIPS issued in 1998-2002.

Inflation-Indexed Treasury Securities		
Maturity	Coupon Issue Rate	Yield to Maturity
1/11	3.500	1.763
1/12	3.375	1.831
7/12	3.000	1.878
4/28	3.625	2.498
4/29	3.875	2.490
4/32	3.375	2.408
Source: WSJ 1 2/24/2003		

Table 9

Note the large 90-180 basis point decrease in the current "real" yields from the issue yields as recent as ten months ago. While there can be several explanations for the change (revaluation of the inflation option, flight to Treasury quality, paucity of 30 year Treasuries), the use of these current "real" risk free yields, with fixed expected returns, would raise ERPs by at least one percent.

Conclusion

This paper has sought to bring the essence of recent research on the equity risk premium to practicing actuaries. The researchers covered here face the same ubiquitous problems that actuaries face daily: Do I rely on past data to forecast the future (costs, premiums, investments) or do I analyze the past and apply informed judgment as to future differences, if any, to arrive at actuarially fair forecasts? Most of the ERP estimates lower than the unconditional historical estimate have an undue reliance on recent lower dividend yields (without a recognition of capital gains[52]) and/or on data prior to 1926.

Despite a spate of research suggesting ex ante ERPs lower than recent realized ERPs, actuaries should be aware of the range of estimates covered here (Appendix B); be aware of the underlying assumptions, data and terminology; and be aware that their independent analysis is required before adopting an estimate other than the historical average. We believe that the Ibbotson-Chen (2003) layout, reproduced here as Appendix D, offers the actuary both an understanding of the fundamental components of the historical ERP and the opportunity to change the estimates based upon good judgment and supportable beliefs. We believe that reliance solely on "expert" survey averages, whether of financial analysts, academic economists, or CFOs, is fraught with risks of statistical bias to fair estimates of the forward ERP.

[52] Under the current US tax code, capital gains are tax-advantaged relative to dividend income for the vast majority of equity holders (households and mutual funds are 55% of the total equity holders, Federal Flow of Funds, 2002 Q3, Table L-213). Curiously, the reverse is true for property-liability insurers because of the 70% stock dividend exclusion afforded insurers.

It is dangerous for actuaries to engage in simplistic analyses of historical ERPs to generate ex ante forecasts that differ from the realized mean.[53] The research we have catalogued in Appendix B, the common level ERPs estimated in Appendix C, and the building block (historical) approach of Ibboston and Chen in Appendix D all discuss important concepts related to both ex post and ex ante ERPs and cannot be ignored in reaching an informed estimate. For example, Richard Wendt, writing in a 2002 issue of Risks and Rewards, a newsletter of the Society of Actuaries, concludes that a linear relationship is a better predictor of future returns than a "constant" ERP based on the average historical return. He arrives at this conclusion by estimating a regression equation[54] relating long bond yields with 15-year geometric mean market returns starting monthly in 1960. First, there is no significant relationship between short, intermediate or long-term income returns over 1926-2002 (or 1960-2002) and ERPs, as evidenced by simple regressions using Ibbotson data.[55] Second, if the linear structural equation indeed held, there would be no need for an ERP since the (15-year) return could be predicted within small error bars. Third, there is always a negative bias introduced when geometric averages are used as dependent variables (Brennan and Schwartz, 1985). Finally, the results are likely to be spurious due to the high autocorrelations of the target and independent variables; an autocorrelation correction would eliminate any significant relationship of long-yields to the ERP.

Actuaries should also be aware of the variability of both the ERP and risk-free rate estimates discussed in this paper (see Tables 4 and 9). All too often, return estimates are made without noting the error bars and that can lead to unexpected "surprises". As one example, recent research by Francis Longstaff (2002), proposes that a 1991-2001 "flight to quality" has created a valuation premium (and lowered yields) in the entire yield curve of Treasuries. He finds a 10 to 16 basis point liquidity premium throughout the zero coupon Treasury yield curve. He translates that into a 10% to 15% pricing difference at the long end. This would imply a simple CAPM market estimate for the long horizon might be biased low.

Finally, actuaries should know that the research catalogued in Appendix B is not definitive. No simple model of ERP estimation has been universally accepted. Undoubtedly, there will be still more empirical and theoretical research into this data rich financial topic. We await the potential advances in understanding the return process that the behavioral view may uncover.

Post Script: Appendices A-D

We provide four appendices that catalogue the ERP approaches and estimates discussed in the paper. Actuaries, in particular, should find the numerical values, and descriptions of assumptions underlying those values, helpful for valuation work that

[53] ERPs are derived from historical or expected after corporate tax returns. Pre-tax returns depend uniquely on the tax schedule for the differing sources of income.

[54] 15-year mean returns = 2.032 (Long Government Bond Yield) – 0.0242, $R^2 = 0.882$.

[55] The p-values on the yield-variables in an ERP/Yield regression using 1926-2002 annual data are 0.1324, 0.2246, and 0.3604 for short, intermediate and long term yields respectively with adjusted r square virtually zero.

adjusts for risk. Appendix A provides the annual Ibbotson data from 1926 through 2002 from Ibbotson Associates referred to throughout this paper. The equity risk-premium shown is a simple difference of the arithmetic stock returns and the arithmetic U.S. Treasury Bills total returns. Appendix B is a compilation of articles and books related to the equity risk premium. The puzzle research section contains the articles and books that were most related to addressing the equity risk premium puzzle. Page 1 of Appendix B gives each source, along with risk-free rate and equity risk premium estimates. Then, each source's estimate is classified by type (indicated with an X for the appropriate type). Page 2 of Appendix B shows further details collected from each source. This page adds the data period used, if applicable, and the projection period. We also list the general methodology used in the reference. The final three pages of Appendix B provide the footnotes which give additional details on the sources' intent.

Appendix C adjusts all the equity risk premium estimates to a short-horizon, arithmetic, unconditional ERP estimate. We begin with the authors' estimates for a stock return (the risk-free rate plus the ERP estimate). Next, we make adjustments if the ERP "type" given by the author(s) is not given in this format. For example, to adjust from a geometric to an arithmetic ERP estimate, we adjust upwards by the 1926-2002 historical difference in the arithmetic large company stocks' total return and the geometric large company stocks' total return of 2%. Next, if the estimate is given in real instead of nominal terms, we adjust the stock return estimate upwards by 3.1%, the 1926-2002 historical return for inflation.

We make an approximate adjustment to move the estimate from a conditional to unconditional estimate based on Fama and French (2002). Using the results for the 1951-2000 period shown in Table 4 of their paper and the standard deviations provided in Table 1, we have four adjustments based on their data. For the 1951-2000 period, Fama and French use an adjustment of 1.28% for the dividend growth model and 0.46% for the earnings growth model. Following a similar calculation, the 1872-2000 period would require a 0.82% adjustment using a dividend growth model; the 1872-1950 period would require a 0.54% adjustment using a dividend growth model. Earnings growth models were used by Fama and French only for the 1951-2000 data period. Therefore, we selected the lowest adjustment (0.46%) as a minimum adjustment from a conditional estimate to an unconditional estimate. Finally, we subtract the 1926-2002 historical U.S. Treasury Bills' total return to arrive at an adjusted equity risk premium.

These adjustments are only approximations because the various sources rely on different underlying data, but the changes in the ERP estimate should reflect the underlying concept that different "types" of ERPs cannot be directly compared and require some attempt to normalize the various estimates.

Page 1 of Appendix D is a table from Ibbotson and Chen which breaks down historical returns using various methods that correspond to their 2003 paper (reprinted with permission of Ibbotson Associates). The bottom portion provides forward-looking estimates. Page 2 of Appendix D is provided to show the formulas that Ibbotson and Chen develop within their paper.

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Appendix A
Ibbotson Market Data 1926-2002*

Year	Common Stocks Total Annual Returns	U. S. Treasury Bills Total Annual Returns	Arithmetic Short-Horizon Equity Risk Premia
1926	11.62%	3.27%	8.35%
1927	37.49%	3.12%	34.37%
1928	43.61%	3.56%	40.05%
1929	- 8.42%	4.75%	-13.17%
1930	-24.90%	2.41%	-27.31%
1931	-43.34%	1.07%	-44.41%
1932	- 8.19%	0.96%	- 9.15%
1933	53.99%	0.30%	53.69%
1934	- 1.44%	0.16%	- 1.60%
1935	47.67%	0.17%	47.50%
1936	33.92%	0.18%	33.74%
1937	-35.03%	0.31%	-35.34%
1938	31.12%	- 0.02%	31.14%
1939	- 0.41%	0.02%	- 0.43%
1940	- 9.78%	0.00%	- 9.78%
1941	-11.59%	0.06%	-11.65%
1942	20.34%	0.27%	20.07%
1943	25.90%	0.35%	25.55%
1944	19.75%	0.33%	19.42%
1945	36.44%	0.33%	36.11%
1946	- 8.07%	0.35%	- 8.42%
1947	5.71%	0.50%	5.21%
1948	5.50%	0.81%	4.69%
1949	18.79%	1.10%	17.69%
1950	31.71%	1.20%	30.51%
1951	24.02%	1.49%	22.53%
1952	18.37%	1.66%	16.71%
1953	- 0.99%	1.82%	- 2.81%
1954	52.62%	0.86%	51.76%
1955	31.56%	1.57%	29.99%
1956	6.56%	2.46%	4.10%

Appendix A
Ibbotson Market Data 1926-2002*

	Common Stocks	U. S. Treasury Bills	Arithmetic Short-Horizon
Year	Total Annual Returns	Total Annual Returns	Equity Risk Premia
1957	-10.78%	3.14%	-13.92%
1958	43.36%	1.54%	41.82%
1959	11.96%	2.95%	9.01%
1960	0.47%	2.66%	- 2.19%
1961	26.89%	2.13%	24.76%
1962	- 8.73%	2.73%	-11.46%
1963	22.80%	3.12%	19.68%
1964	16.48%	3.54%	12.94%
1965	12.45%	3.93%	8.52%
1966	-10.06%	4.76%	-14.82%
1967	23.98%	4.21%	19.77%
1968	11.06%	5.21%	5.85%
1969	- 8.50%	6.58%	-15.08%
1970	4.01%	6.52%	- 2.51%
1971	14.31%	4.39%	9.92%
1972	18.98%	3.84%	15.14%
1973	-14.66%	6.93%	-21.59%
1974	-26.47%	8.00%	-34.47%
1975	37.20%	5.80%	31.40%
1976	23.84%	5.08%	18.76%
1977	- 7.18%	5.12%	-12.30%
1978	6.56%	7.18%	- 0.62%
1979	18.44%	10.38%	8.06%
1980	32.42%	11.24%	21.18%
1981	- 4.91%	14.71%	-19.62%
1982	21.41%	10.54%	10.87%
1983	22.51%	8.80%	13.71%
1984	6.27%	9.85%	- 3.58%
1985	32.16%	7.72%	24.44%
1986	18.47%	6.16%	12.31%
1987	5.23%	5.47%	- 0.24%
1988	16.81%	6.35%	10.46%
1989	31.49%	8.37%	23.12%

Appendix A
Ibbotson Market Data 1926-2002*

Year	Common Stocks Total Annual Returns	U. S. Treasury Bills Total Annual Returns	Arithmetic Short-Horizon Equity Risk Premia
1990	- 3.17%	7.81%	-10.98%
1991	30.55%	5.60%	24.95%
1992	7.67%	3.51%	4.16%
1993	9.99%	2.90%	7.09%
1994	1.31%	3.90%	- 2.59%
1995	37.43%	5.60%	31.83%
1996	23.07%	5.21%	17.86%
1997	33.36%	5.26%	28.10%
1998	28.58%	4.86%	23.72%
1999	21.04%	4.68%	16.36%
2000	- 9.11%	5.89%	-15.00%
2001	-11.88%	3.83%	-15.71%
2002	-22.10%	1.65%	-23.75%
mean=	**12.20%**	**3.83%**	**8.37%**
Standard Dev=	**20.49%**	**3.15%**	**20.78%**

* *2003 SBBI Yearbook pages 38 and 39*

Appendix B

Source	Risk-free-Rate	ERP Estimate	Real risk-free rate	Nominal risk-free rate	Geometric	Arithmetic	Long-horizon	Short-horizon	Short-run expectation	Long-run expectation	Conditional	Unconditional
Historical												
Ibbotson Associates	3.8% [7]	8.4% [31]		X		X		X		X		X
Social Security												
Office of the Chief Actuary [1]	2.3%,3.0% [8]	4.7%,4.0% [32]	X		X		X			X		X
John Campbell [2]	3% to 3.5% [9]	1.5-2.5%, 3-4% [33]	X		X	X	X	X		X	X	
Peter Diamond	2.2% [10]	<4.8% [34]	X		X		X			X	X	
Peter Diamond [3]	3.0% [11]	3.0% to 3.5% [35]	X		X		X			X	X	
John Shoven [4]	3.0%,3.5% [12]	3.0% to 3.5% [36]	X		X		X			X	X	
Puzzle Research												
Robert Arnott and Peter Bernstein	3.7% [13]	2.4% [37]	X		X		X			X	X	
Robert Arnott and Ronald Ryan	4.1% [14]	-0.9% [38]	X		X		X			X	X	
John Campbell and Robert Shiller	N/A	Negative [39]	X		?		?		X		X	
James Claus and Jacob Thomas	7.64% [15]	3.39% or less [40]		X		X	X			X	X	
George Constantinides	2.0% [16]	6.9% [41]	X			X		X		X		X
Bradford Cornell	5.6%, 3.8% [17]	3.5-5.5%, 5-7% [42]		X		X	X	X		X	X	
Dimson, Marsh, & Staunton	1.0% [18]	5.4% [43]	X			X		X		X	X	
Eugene Fama and Kenneth French	3.24% [19]	3.83% & 4.78% [44]	X			X		X		X		X
Robert Harris and Felicia Marston	8.53% [20]	7.14% [45]		X		X	X		X		X	
Roger Ibbotson and Peng Chen	2.05% [21]	4% and 6% [46]	X		X	X	X			X		X
Jeremy Siegel	4.0% [22]	-0.9% to -0.3% [47]	X		X		X			X	X	
Jeremy Siegel	3.5% [23]	2-3% [48]	X		X		X			?	X	
Surveys												
John Graham and Campbell Harvey	? by survey [24]	3-4.7% [49]		X		?	X		X		X	
Ivo Welch	N/A [25]	7% [50]		X		X		X		X	X	
Ivo Welch [5]	5% [26]	5.0% to 5.5% [51]		X		X		X		X	X	
Misc.												
Barclays Global Investors	5% [27]	2.5%, 3.25% [52]		X	X		X		X		X	
Richard Brealey and Stewart Myers	N/A [28]	6 to 8.5% [53]		X		X		X		X		X
Burton Malkiel	5.25% [29]	2.75% [54]		X	X		X			X	X	
Richard Wendt [6]	5.5% [30]	3.3% [55]		X		X	X			X	X	

Long-run expectation considered to be a forecast of more than 10 years.
Short-run expectation considered to be a forecast of 10 years or less.

Source	Risk-free Rate	ERP Estimate	Data Period	Methodology
Historical				
Ibbotson Associates	3.8% [7]	8.4% [31]	1926-2002	Historical
Social Security				
Office of the Chief Actuary [1]	2.3%, 3.0% [8]	4.7%, 4.0% [32]	1900-1995, Projecting out 75 years	Historical
John Campbell [2]	3% to 3.5% [9]	1.5-2.5%, 3-4% [33]	Projecting out 75 years	Historical & Ratios (Div/Price & Earn Gr)
Peter Diamond	2.2% [10]	<4.8% [34]	Last 200 yrs for eq/ 75 for bonds, Proj 75 yrs	Fundamentals: Div Yld, GDP Gr
Peter Diamond [3]	3.0% [11]	3.0% to 3.5% [35]	Projecting out 75 years	Fundamentals: Div/Price
John Shoven [4]	3.0%, 3.5% [12]	3.0% to 3.5% [36]	Projecting out 75 years	Fundamentals: P/E, GDP Gr
Puzzle Research				
Robert Arnott and Peter Bernstein	3.7% [13]	2.4% [37]	1802 to 2001, normal	Fundamentals: Div Yld & Gr
Robert Arnott and Ronald Ryan	4.1% [14]	-0.9% [38]	Past 74 years, 74 year projection [56]	Fundamentals: Div Yld & Gr
John Campbell and Robert Shiller	N/A	Negative [39]	1871 to 2000, ten-year projection	Ratios: P/E and Div/Price
James Claus and Jacob Thomas	7.64% [15]	3.39% or less [40]	1985-1998, long-term	Abnormal Earnings model
George Constantinides	2.0% [16]	6.9% [41]	1872 to 2000, long-term	Hist. and Fund.: Price/Div & P/E
Bradford Cornell	5.6%, 3.8% [17]	3.5-5.5%, 5-7% [42]	1926-1997, long run forward-looking	Weighing theoretical and empirical evid
Dimson, Marsh, & Staunton	1.0% [18]	5.4% [43]	1900-2000, prospective	Adj hist ret, Var of Gordon gr model
Eugene Fama and Kenneth French	3.24% [19]	3.83% & 4.78% [44]	Estimate for 1951-2000, long-term	Fundamentals: Dividends and Earnings
Robert Harris and Felicia Marston	8.53% [20]	7.14% [45]	1982-1998, expectational	Fin analysts' est, div gr model
Roger Ibbotson and Peng Chen	2.05% [21]	4% and 6% [46]	1926-2000, long-term	Historical and supply side approaches
Jeremy Siegel	4.0% [22]	-0.9% to -0.3% [47]	1871 to 1998, forward-looking	Fundamentals: P/E, Div Yld, Div Gr
Jeremy Siegel	3.5% [23]	2-3% [48]	1802-2001, forward-looking	Earnings yield
Surveys				
John Graham and Campbell Harvey	? by survey [24]	3-4.7% [49]	2Q 2000 thru 3Q 2002, 1 & 10 year proj	Survey of CFO's
Ivo Welch	N/A [25]	7% [50]	30-Year forecast, surveys in 97/98 & 99	Survey of financial economists
Ivo Welch [5]	5% [26]	5.0% to 5.5% [51]	30-Year forecast, survey around August 2001	Survey of financial economists
Misc.				
Barclays Global Investors	5% [27]	2.5%, 3.25% [52]	Long-run (10-year) expected return	Fundamentals: Inc, Earn Gr, & Repricing
Richard Brealey and Stewart Myers	N/A [28]	6 to 8.5% [53]	1926-1997	Predominantly Historical
Burton Malkiel	5.25% [29]	2.75% [54]	1926 to 1997, estimate millennium [57]	Fundamentals: Div Yld, Earn Gr
Richard Wendt [6]	5.5% [30]	3.3% [55]	1960-2000, estimate for 2001-2015 period	Linear regression model

Footnotes:

[1] Social Security Administration.

[2] Presented to the Social Security Advisory Board.

[3] Presented to the Social Security Advisory Board. Update of 1999 article.

[4] Presented to the Social Security Advisory Board.

[5] Update to Welch 2000.

[6] Newsletter of the Investment Section of the Society of Actuaries.

[7] Arithmetic mean of U.S. Treasury bills annual total returns from 1926-2002.

[8] 2.3% Long-run real yield on Treasury bonds; used for Advisory Council proposals. 3.0% Long-term real yield on Treasury bonds; used in 1999 Social Security Trustees Report.

[9] Estimate for safe real interest rates in the future based on yield of long-term inflation-indexed Treasury securities of 3.5% and short-term real interest rates recently averaging about 3%.

[10] Real long-term bond yield using 75 year historical average.

[11] Real yield on long-term Treasuries (assumption by OCACT).

[12] 3.0% is the OCACT assumption. 3.5% is the real return on long-run (30-year) inflation-indexed Treasury securities.

[13] Long-term expected real geometric bond return (10 year-horizon).

[14] The yield on US government inflation-indexed bonds (starting bond real yield in Jan 2000).

[15] Average 10-year Government T-bond yield between 1985 and 1998 (yield of 11.43% in 1985 to 5.64% in 1998. The mean 30-year risk-free rate for each year of the U.S. sample period is 31 basis points higher than the mean 10-year risk-free rate.

[16] Rolled-over real arithmetic return of three-month Treasury bills and certificates.

[17] Historical 20-year Treasury bond return of 5.6%. Yield on 20-year Treasury bonds in 1998 was approximately 6%. Historical 1-month Treasury bill return of 3.8%. Yield on 1-month Treasury bills in 1998 was approximately 4%.

[18] United States historical arithmetic real Treasury bill return over 1900-2000 period. 0.9% geometric Treasury bill return.

[19] Average real return on six-month commercial paper (proxy for risk-free interest rate). Substituting the one-month Treasury bill rate for the six-month commercial paper rate causes estimates of the annual equity premium for 1951-2000 to rise by about 1.00%.

[20] Average yield to maturity on long-term U.S. government bonds, 1982-1998.

[21] Real, geometric risk-free rate. Geometric risk-free rate with inflation (nominal) 5.13%.
Nominal yield equivalent to historical geometric long-term government bond income return for 1926-2000.

[22] The ten- and thirty-year TIPS bond yielded 4.0% in August 1999.

[23] Return on inflation-indexed securities.

[24] Current 10-year Treasury bond yield. Survey administered from June 6, 2000 to June 4, 2002. The rate on the 10-year Treasury bond changes in each survey. For example, in the Dec. 1, 2000 survey, the current annual yield on the 10-year Treasury bond was 5.5%. For the June 6, 2001 survey, the current annual yield on the 10-year Treasury bond was 5.3%.

[25] Arithmetic per-annum average return on rolled-over 30-day T-bills.

[26] Average forecast of arithmetic risk-free rate of about 5% by deducting ERP from market return.

[27] Current nominal 10-year bond yield.

[28] Return on Treasury bills. Treasury bills yield of about 5 percent in mid-1998. Average historical return on Treasury bills 3.8 percent.

[29] Good quality corporate bonds will earn approximately 6.5% to 7%. Long-term zero-coupon Treasury bonds will earn about 5.25%. Long-term TIPS will earn a real return of 3.75%.

[30] 1/1/01 Long T-Bond yield; uses initial bond yields in predictive model.

[31] Arithmetic short-horizon expected equity risk premium. Arithmetic intermediate-horizon expected equity risk premium 7.4%. Arithmetic long-horizon expected equity risk premium 7.0%. Geometric short-horizon expected equity risk premium 6.4%.

[32] Geometric equity premium over long-term Treasury securities. OCACT assumes a constant geometric real 7.0% stock return.

[33] Long-run average equity premium of 1.5% to 2.5% in geometric terms and 3% to 4% in arithmetic terms.

[34] Lower return over the next decade, followed by a geometric, real 7.0% stock return for remaining 65 years or lower rate of return for entire 75-year period (obscures pattern of returns).

[35] Most likely poor return over the next decade followed by a return to historic yields. Working from OCACT stock return assumption, he gives a single rate of return on equities for projection purposes of *6.0 to 6.5% (geometric, real)*.

[36] Geometric real stock return over the geometric real return on long-term government bonds.

[37] Expected geometric return over long-term government bonds. Their current risk premium is approximately zero, and their recommended expectation for the future real return for both stocks and bonds is 2-4 percent. The "normal" level of the risk premium is modest (2.4 percent or quite possibly less).

[38] Geometric real returns on stocks are likely to be in the 3%-4% range for the foreseeable future (10-20 years).

[39] Substantial declines in real stock prices, and real stock returns below zero, over the next ten years (2001-2010).

[40] The equity premium for each year between 1985 and 1998 in the United States. Similar results for five other markets.

[41] Unconditional, arithmetic mean aggregate equity premium over the 1872-2000 period. Over the period 1951 to 2000, the adjusted estimate of the unconditional mean premium is 6.0%. The corresponding estimate over the 1926 to 2000 period is 8.0%. Sharp distinction between conditional, short-term forecasts of the mean equity return and premium and estimates of the unconditional mean.

[42] Long run arithmetic future ERP of 3.5% to 5.5% over Treasury bonds and 5% to 7% over Treasury bills. Compares estimates to historical returns of 7.4% for bond premium and 9.2% for bill premium.

[43] 5.4% United States arithmetic expected future ERP relative to bills. 4.0% World (16 countries) arithmetic expected future ERP relative to bills. 4.1% United States geometric expected future ERP relative to bills. 3.0% World (16 countries) geometric expected future ERP relative to bills.

[44] 3.83% unconditional expected annual simple equity premium return (referred to as the annual-bias adjusted estimate of the annual equity premium) using dividend growth model. 4.78% unconditional expected annual simple equity premium return (referred to as the annual-bias adjusted estimate of the annual equity premium) using earnings growth model. Compares these results against historical real equity risk premium of 7.43% for 1951-2000.

[45] Average expectational risk premium. Because of the possible bias of analysts' optimism, the estimates are interpreted as "upper bounds" for the market premium. The average expectational risk premium is approximately equal to the arithmetic (7.5%) long-term differential between returns on stocks and long-term government bonds.

[46] 4% geometric (real) and 6% arithmetic (real). Forward looking long-horizon sustainable equity risk premium.

[47] Using the dividend discount model, the forward-looking real long-term geometric return on equity is 3.3%. Based on the earnings yield, the forward-looking real long-term geometric return on equity is between 3.1% and 3.7%.

[48] Future geometric equity premium. Future real return on equities of about 6%.

[49] The 10-year premium. The one-year risk premium averages between 0.4 and 5.2% depending on the quarter surveyed.

[50] Arithmetic 30-year forecast relative to short-term bills; 10-year same estimate. Second survey 6.8% for 30 and 10-year estimate. 1-year horizon between 0.5% and 1.5% lower. Geometric 30-year forecast around 5.2% (50% responded to this question).

[51] Arithmetic 30-year equity premium (relative to short-term T-bills). Geometric about 50 basis points below arithmetic. Arithmetic 1-year equity premium 3 to 3.5%.

[52] 2.5% current (conditional) geometric equity risk premium. 3.25% long-run, geometric normal or equilibrium equity risk premium.

[53] Extra arithmetic return versus Treasury bills. "Brealey and Myers have no official position on the exact market risk premium, but we believe a range of 6 to 8.5 percent is reasonable for the United States. We are most comfortable with figures towards the upper end of the range."

[54] The projected geometric (nominal) total return for the S&P 500 is 8 percent per year.

[55] Arithmetic mean 15 year horizon.

[56] 74 years since Dec 1925 and 74 years starting Jan 2000.

[57] Estimate the early decades of the twenty-first century.

Appendix C
Estimating a Short-Horizon Arithmetic Unconditional Equity Risk Premium

Source	Risk-free Rate	ERP Estimate	Stock Return Estimate	Geometric to arithmetic	Real to nominal	Conditional to unconditional [60]	Fixed short-horizon RFR	Short-horizon arithmetic unconditional ERP estimate
	I	II	III	IV	V	VI	VII	VIII
Historical								
Ibbotson Associates	3.8% [7]	8.4% [31]	12.2%	0.0%	0.0%	0.00%	3.8%	8.4%
Social Security								
Office of the Chief Actuary [1]	2.3%,3.0% [8]	4.7%,4.0% [32]	7.0%	2.0%	3.1%	0.00%	3.8%	8.3%
John Campbell [2]	3% to 3.5% [9]	1.5-2.5%, 3-4% [33]	6.0%-7.5%	0.0%	3.1%	0.46%	3.8%	5.8%-7.3%
Peter Diamond	2.2% [10]	<4.8% [34]	<7.0%	2.0%	3.1%	0.46%	3.8%	<8.8%
Peter Diamond [3]	3.0% [11]	3.0% to 3.5% [35]	6.0%-6.5%	2.0%	3.1%	0.46%	3.8%	7.8%-8.3%
John Shoven [4]	3.0%,3.5% [12]	3.0% to 3.5% [36]	6.0%-7.0%	2.0%	3.1%	0.46%	3.8%	7.8%-8.8%
Puzzle Research								
Robert Arnott and Peter Bernstein	3.7% [13]	2.4% [37]	6.1%	2.0%	3.1%	0.46%	3.8%	7.9%
Robert Arnott and Ronald Ryan	4.1% [14]	-0.9% [38]	3.2%	2.0%	3.1%	0.46%	3.8%	5.0%
John Campbell and Robert Shiller	N/A	Negative [39]	Negative	N/A	N/A	N/A	N/A	N/A
James Claus and Jacob Thomas	7.64% [15]	3.39% or less [40]	11.03%	0.0%	0.0%	0.46%	3.8%	7.69%
George Constantinides	2.0% [16]	6.9% [41]	8.9%	0.0%	3.1%	0.00%	3.8%	8.2%
Bradford Cornell	5.6%, 3.8% [17]	3.5-5.5%, 5-7% [42]	8.8%-10.8%	0.0%	0.0%	0.46%	3.8%	5.5%-7.5%
Dimson, Marsh, & Staunton	1.0% [18]	5.4% [43]	6.4% [58]	0.0%	3.1%	0.46%	3.8%	6.2% [61]
Eugene Fama and Kenneth French	3.24% [19]	3.83% & 4.78% [44]	7.07%-8.02%	0.0%	3.1%	0.00%	3.8%	6.37%-7.32%
Robert Harris and Felicia Marston	8.53% [20]	7.14% [45]	12.34% [59]	0.0%	0.0%	0.46%	3.8%	9.00%
Roger Ibbotson and Peng Chen	2.05% [21]	4% and 6% [46]	8.05%	0.0%	3.1%	0.00%	3.8%	7.35%
Jeremy Siegel	4.0% [22]	-0.9% to -0.3% [47]	3.1%-3.7%	2.0%	3.1%	0.46%	3.8%	4.9%-5.5%
Jeremy Siegel	3.5% [23]	2-3% [48]	5.5%-6.5%	2.0%	3.1%	0.46%	3.8%	7.3%-8.3%
Surveys								
John Graham and Campbell Harvey	? by survey [24]	3-4.7% [49]	8.3%-10.2%	N/A	0.0%	0.46%	3.8%	5.0%-6.9%
Ivo Welch	N/A [25]	7% [50]	N/A	0.0%	0.0%	0.46%	0.0%	7.5%
Ivo Welch [5]	5% [26]	5.0% to 5.5% [51]	10.0%-10.5%	0.0%	0.0%	0.46%	3.8%	6.7%-7.2%
Misc.								
Barclays Global Investors	5% [27]	2.5%, 3.25% [52]	7.5%,8.25%	2.0%	0.0%	0.46%	3.8%	6.16%-6.91%
Richard Brealey and Stewart Myers	N/A [28]	6 to 8.5% [53]	N/A	0.0%	0.0%	0.00%	0.0%	6.0%-8.5%
Burton Malkiel	5.25% [29]	2.75% [54]	8.0%	2.0%	0.0%	0.46%	3.8%	6.7%
Richard Wendt [6]	5.5% [30]	3.3% [55]	8.8%	0.0%	0.0%	0.46%	3.8%	5.5%

Column formulas:
III = I + II
VIII = III + IV + V + VI – VII

Source for adjustments:
2003 Ibbotson Yearbook Table 2-1 page 33
Fama French 2002 (see footnote 60)

Footnotes (1-57 from Appendix B):
[58] World estimate of 5.0%.
[59] Long risk-free of 5.2% plus 7.14%.
[60] For the 1951-2000 period, Fama and French (2002) adjust the conditional dividend growth model estimate upwards by 1.28% for an unconditional estimate, and they make a 0.46% upwards adjustment to the earnings growth model. We select the smaller of the two as an approximate minimum adjustment. For the longer period of 1872-2000, a comparable adjustment would be 0.82% for the dividend growth model and 0.54% for the 1872-1950 period using a dividend growth model. Earnings growth rates are shown by Fama and French only for the 1951-2000 period.
[61] World estimate of 4.8%.

Appendix D

Historical and Forecasted Equity Returns- All Ibbotson and Chen Models (Percent).

Method/ Model	Sum	Inflation	Real Risk-Free Rate	Equity Risk Premium	Real Capital Gain	g(Real EPS)	g(Real Div)	- g (Pay out Ratio)	g (BV)	g (ROE)	g P/E)	g(Real GDP/ POP)	g(FS-GDP/ POP)	Income Return	Re-Investment + Interaction	Additional Growth	Forecast Earnings Growth
Column #	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	XV	XVI	XVII
Historical																	
Method 1	10.70	3.08	2.05	5.24											0.33		
Method 2	10.70	3.08			3.02									4.28	0.32		
Method 3	10.70	3.08				1.75					1.25			4.28	0.34		
Method 4	10.70	3.08					1.23	0.51			1.25			4.28	0.35		
Method 5	10.70	3.08							1.46	0.31	1.25			4.28	0.31		
Method 6	10.70	3.08										2.04	0.96	4.28	0.32		
Forecast with Historical Dividend Yield																	
Model 3F	9.37	3.08				1.75								4.28	0.26		
Model 3F (ERP)	9.37	3.08	2.05	3.97											0.27		
Forecast with Current Dividend Yield																	
Model 4F	5.44	3.08					1.23							1.10 [a]	0.03		
Model 4F (ERP)	5.44	3.08	2.05	0.24											0.07		
Model $4F_2$	9.37	3.08					1.23	0.51						2.05 [b]	0.21	2.28	
Model $4F_2$ (FG)	9.37	3.08												1.10 [a]	0.21		4.98

Source: The data and format was made available by Ibbotson/Chen and is reprinted with permission by Ibbotson Associates. Corresponds to Ibbotson/Chen Table 2 Exhibit; column numbers have been added.

[a] 2000 dividend yield

[b] Assuming the historical average dividend-payout ratio, the 2000 dividend yield is adjusted up 0.95 pps.

	Formula*	Description of Method
Historical		
Method 1	I=(1+II)*(1+III)*(1+IV)-1	Building Blocks Method: inflation, real risk-free rate, and equity risk premium.
Method 2	I=[(1+II)*(1+V)-1]+XIV+XV	Capital Gain and Income Method: inflation, real capital gain, and income return.
Method 3	I=[(1+II)*(1+VI)*(1+XI)-1]+XIV+XV	Earnings Model: inflation, growth in earnings per share, growth in price to earnings ratio, and income return.
Method 4	I=[(1+II)*(1+XI)*(1+VII)/(1-VIII)-1]+XIV+XV	Dividends Model: inflation, growth rate of price earnings ratio, growth rate of the dollar amount of dividends after inflation, growth rate of payout ratio, and dividend yield (income return).
Method 5	I=[(1+II)*(1+XI)*(1+IX)*(1+X)-1]+XIV+XV	Return on Book Equity Model: inflation, growth rate of price earnings ratio, growth rate of book value, growth rate of ROE, and income return.
Method 6	I=[(1+II)*(1+XII)*(1+XIII)-1]+XIV+XV	GDP Per Capita Model: inflation, real growth rate of the overall economic productivity (GDP per capita), increase of the equity market relative to the overall economic productivity, and income return.
Forecast with Historical Dividend Yield		
Model 3F	I=[(1+II)*(1+VI)-1]+XIV+XV	Forward-looking Earnings Model: inflation, growth in real earnings per share, and income return.
Model 3F (ERP)	IV=(1+I)/[(1+II)*(1+III)]-1	Using Model 3F result to calculate ERP.
Forecast with Current Dividend Yield		
Model 4F	I=[(1+II)*(1+VII)-1]+XIV+XV	Forward-looking Dividends Model: inflation, growth in real dividend, and dividend yield (income return); also referred to as Gordon model.
Model 4F (ERP)	IV=(1+I)/[(1+II)*(1+III)]-1	Using Model 4F result to calculate ERP.
Model $4F_2$	I=[(1+II)*(1+VII)*(1+VIII)-1]+XIV+XV+XVI	Attempt to reconcile Model 4F and Model 3F.
Model $4F_2$ (FG)	XVII=[(1+I)/(1+II)-1]-XIV-XV	Using Method $4F_2$ result to calculate forecasted earnings.

Explanation of Ibbotson/Chen Table 2 Exhibit; using column numbers to represent formula.

Kentucky-American Water Company
Case No. 2004-00103
Information Request Response to KAWC
Respondent: OAG Witness Dr. J. Randall Woolridge
Set II

KAWC-II-4. Footnote 2, p. 6. Please provide a copy of the cited reference.

Response:

The requested document is included at attachment KAWC-II-4A.

Greenspan Speech, October 14, 1999

There can be little doubt that the dramatic improvements in information technology in recent years have altered our approach to risk. Some analysts perceive that information technology has permanently lowered equity premiums and, hence, permanently raised the prices of the collateral that underlies all financial assets.

The reason, of course, is that information is critical to the evaluation of risk. The less that is known about the current state of a market or a venture, the less the ability to project future outcomes and, hence, the more those potential outcomes will be discounted.

The rise in the availability of real-time information has reduced the uncertainties and thereby lowered the variances that we employ to guide portfolio decisions. At least part of the observed fall in equity premiums in our economy and others over the past five years does not appear to be the result of ephemeral changes in perceptions. It is presumably the result of a permanent technology-driven increase in information availability, which by definition reduces uncertainty and therefore risk premiums. This decline is most evident in equity risk premiums. It is less clear in the corporate bond market, where relative supplies of corporate and Treasury bonds and other factors we cannot easily identify have outweighed the effects of more readily available information about borrowers.

The marked increase over this decade in the projected slope of technology advance, of course, has also augmented expectations of earnings growth, as evidenced by the dramatic increase since 1995 in security analysts' projections of long-term earnings. While it may be that the expectations of higher earnings embodied in equity values have had a spillover effect on discount factors, the latter remain essentially independent of the earnings expectations themselves.

That equity premiums have generally declined during the past decade is not in dispute. What is at issue is how much of the decline reflects new, irreversible technologies, and what part is a consequence of a prolonged business expansion without a significant period of adjustment. The business expansion is, of course, reversible, whereas the technological advancements presumably are not.

Kentucky-American Water Company
Case No. 2004-00103
Information Request Response to KAWC
Respondent: OAG Witness Dr. J. Randall Woolridge
Set II

KAWC-II-5. Footnote 3, p. 12. Please provide a copy of the cited reference.

Response:

The requested document was provided in response to PSC-I-14 as attachment PSC-I-14A.

Kentucky-American Water Company
Case No. 2004-00103
Information Request Response to KAWC
Respondent: OAG Witness Dr. J. Randall Woolridge
Set II

KAWC-II-6. Page 52. Please provide C. A. Turner Utility Reports August 2004 that verify the percent of revenues from gas for Dr. Vander Weide's LDC group.

Response:

The requested document is included at attachment KAWC-II-6A.

NATURAL GAS DISTRIBUTION & INTEGRATED NAT. GAS COMPANIES

COMPANY	OPER REV $ MILL (1)	% GAS REV	NET PLANT $ MILL	NET PLANT PER $ REV (1)	S&P BOND RATING	MOODY'S BOND RATING	COMMON EQUITY RATIO (3)	% RETURN ON BOOK VALUE: COMMON EQUITY (4)	% RETURN ON BOOK VALUE: TOTAL CAPITAL	REGULATION: ALLOWED ROE	REGULATION: ORDER DATE
AGL Resources Inc. (NYSE-ATG)	[redacted]	78	[redacted]	1.85	[redacted]	A3	[redacted]	15.0	[redacted]	10.99	[redacted]
Atmos Energy Corporation (NYSE-ATO)	[redacted]	46	[redacted]	0.47	[redacted]	A3	[redacted]	10.4	[redacted]	12.10	[redacted]
Cascade Natural Gas Corporation (NYSE-CGC)	[redacted]	100	[redacted]	1.08	[redacted]	Baa1	[redacted]	9.7	[redacted]	11.75	[redacted]
Chesapeake Utilities Corporation (NYSE-CPK)	[redacted]	67	[redacted]	1.00	[redacted]	NR	[redacted]	12.1	[redacted]	-	[redacted]
Delta Natural Gas Company (NDQ-DGAS)	[redacted]	63	[redacted]	1.49	[redacted]	NR	[redacted]	7.5	[redacted]	-	[redacted]
El Paso Corporation (NYSE-EP)	[redacted]	26	[redacted]	2.30	[redacted]	B1	[redacted]	NM	[redacted]		[redacted]
Energen Corporation (NYSE-EGN)	[redacted]	63	[redacted]	1.63	[redacted]	A1	[redacted]	17.1	[redacted]	13.40	[redacted]
Energy West Incorporated (NDQ-EWST)	[redacted]	59	[redacted]	0.55	[redacted]	NR	[redacted]	NM	[redacted]	12.63	[redacted]
EnergySouth, Inc. (NDQ-ENSI)	[redacted]	97	[redacted]	1.80	[redacted]	NR	[redacted]	14.3	[redacted]	13.60	[redacted]
Equitable Resources, Inc. (NYSE-EQT)	[redacted]	60	[redacted]	1.61	[redacted]	A2	[redacted]	19.2	[redacted]	11.00	[redacted]
KeySpan Corp. (NYSE-KSE)	[redacted]	61	[redacted]	1.24	[redacted]	A2	[redacted]	11.4	[redacted]	10.20	[redacted]
Kinder Morgan, Inc. (NYSE-KMI)	[redacted]	32	[redacted]	5.37	[redacted]	Baa2	[redacted]	15.3	[redacted]	-	[redacted]
Laclede Group, Inc. (NYSE-LG)	[redacted]	74	[redacted]	0.55	[redacted]	A3	[redacted]	11.3	[redacted]	-	[redacted]
National Fuel Gas Company (NYSE-NFG)	[redacted]	56	[redacted]	1.45	[redacted]	A3	[redacted]	15.9	[redacted]	11.50	[redacted]
New Jersey Resources Corp. (NYSE-NJR)	[redacted]	35	[redacted]	0.36	[redacted]	Aa3	[redacted]	16.7	[redacted]	11.50	[redacted]
NICOR Inc. (NYSE-GAS)	[redacted]	88	[redacted]	0.95	[redacted]	Aa3	[redacted]	10.4	[redacted]	-	[redacted]
Northwest Natural Gas Co. (NYSE-NWN)	[redacted]	99	[redacted]	1.85	[redacted]	A2	[redacted]	10.1	[redacted]	10.20	[redacted]
NUI Corporation (NYSE-NUI)	[redacted]	98	[redacted]	1.53	[redacted]	Ba3	[redacted]	NM	[redacted]	10.60	[redacted]
ONEOK, Inc. (NYSE-OKE)	[redacted]	61	[redacted]	1.33	[redacted]	Baa1	[redacted]	13.7	[redacted]	-	[redacted]
Peoples Energy Corporation (NYSE-PGL)	[redacted]	69	[redacted]	0.85	[redacted]	Aa3	[redacted]	10.8	[redacted]	11.20	[redacted]
Piedmont Natural Gas Co., Inc. (NYSE-PNY)	[redacted]	77	[redacted]	1.28	[redacted]	A3	[redacted]	12.9	[redacted]	11.30	[redacted]
Questar Corporation (NYSE-STR)	[redacted]	47	[redacted]	1.78	[redacted]	A2	[redacted]	14.8	[redacted]	11.20	[redacted]
RGC Resources, Inc. (NDQ-RGCO)	[redacted]	76	[redacted]	0.61	[redacted]	NR	[redacted]	13.0	[redacted]	9.85	[redacted]
SEMCO Energy, Inc. (NYSE-SEN)	[redacted]	85	[redacted]	1.00	[redacted]	Ba2	[redacted]	NM	[redacted]	11.98	[redacted]
South Jersey Industries, Inc. (NYSE-SJI)	[redacted]	65	[redacted]	1.04	[redacted]	Baa1	[redacted]	12.4	[redacted]	10.00	[redacted]
Southern Union Company (NYSE-SUG)	[redacted]	76	[redacted]	1.85	[redacted]	Baa3	[redacted]	8.9	[redacted]	10.93	[redacted]
Southwest Gas Corporation (NYSE-SWX)	[redacted]	85	[redacted]	1.69	[redacted]	Baa2	[redacted]	8.4	[redacted]	10.69	[redacted]
Southwestern Energy Company (NYSE-SWN)	[redacted]	80	[redacted]	2.30	[redacted]	Ba2	[redacted]	16.3	[redacted]	11.30	[redacted]
UGI Corporation (NYSE-UGI)	[redacted]	16	[redacted]	0.56	[redacted]	A3	[redacted]	13.4	[redacted]	-	[redacted]
WGL Holdings, Inc. (NYSE-WGL)	[redacted]	62	[redacted]	0.91	[redacted]	A2	[redacted]	11.0	[redacted]	10.95	[redacted]
Williams Companies, Inc. (NYSE-WMB)	[redacted]	8	[redacted]	0.79	[redacted]	B1	[redacted]	0.7	[redacted]		[redacted]
AVERAGE	[redacted]		[redacted]		[redacted]		[redacted]	12.4	[redacted]	11.31	[redacted]

KAWC-II-7. Footnote 20, p. 54. Please provide a copy of the cited reference.

Response:

The requested document was provided in response to PSC-I-30 as attachment PSC-I-30A.

KAWC-II-8. Page 56. Please provide a copy of the source documents underlying the data displayed in the graph titled, "Analysts Forecasted 5-Year EPS Growth for the S&P 500, 1985 – 2003.

Response:

The requested document is included at attachment KAWC-II-8A. The data is estimated from long-term consensus expected earnings growth rate for the S&P 500 as found in Figure 7 on page 22.

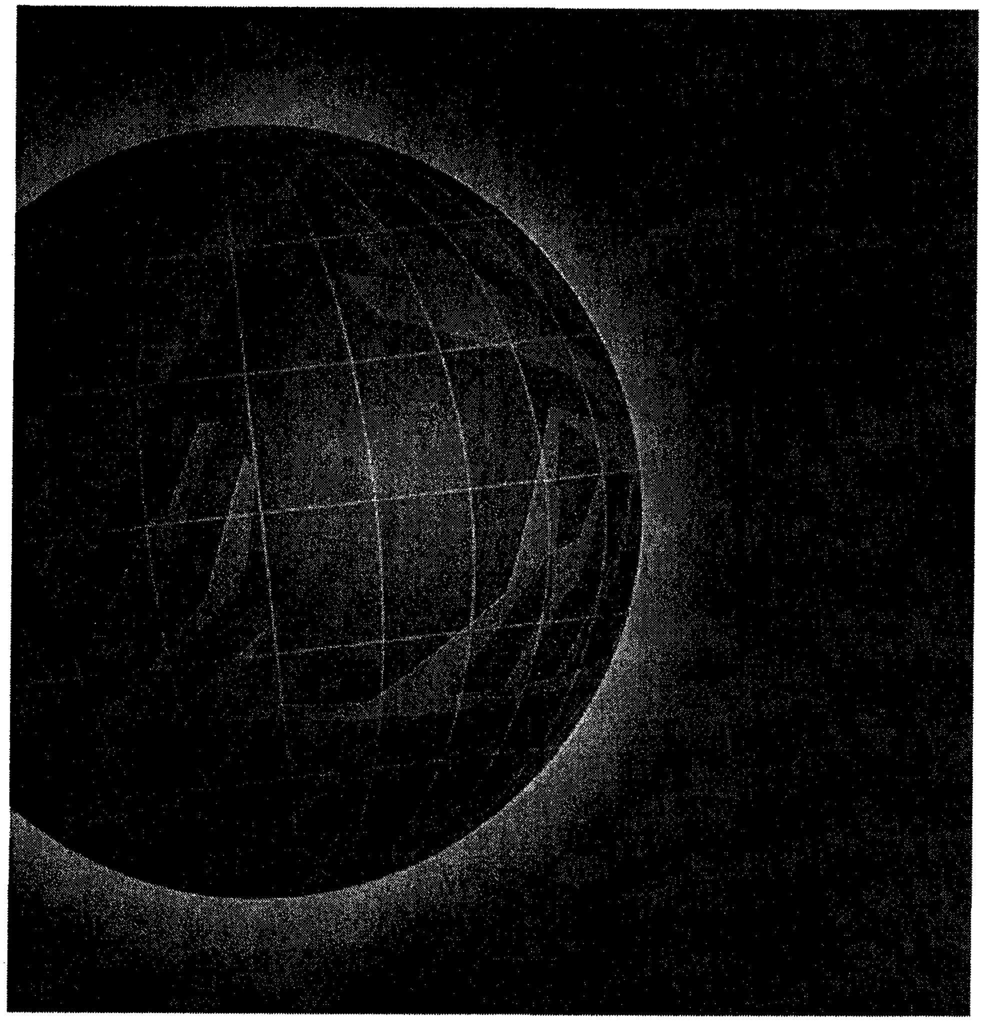
Prudential Financial

RESEARCH

January 6, 2003

STOCK VALUATION MODELS (4.1)

Topical Study #58



All disclosures can be found on the back page.

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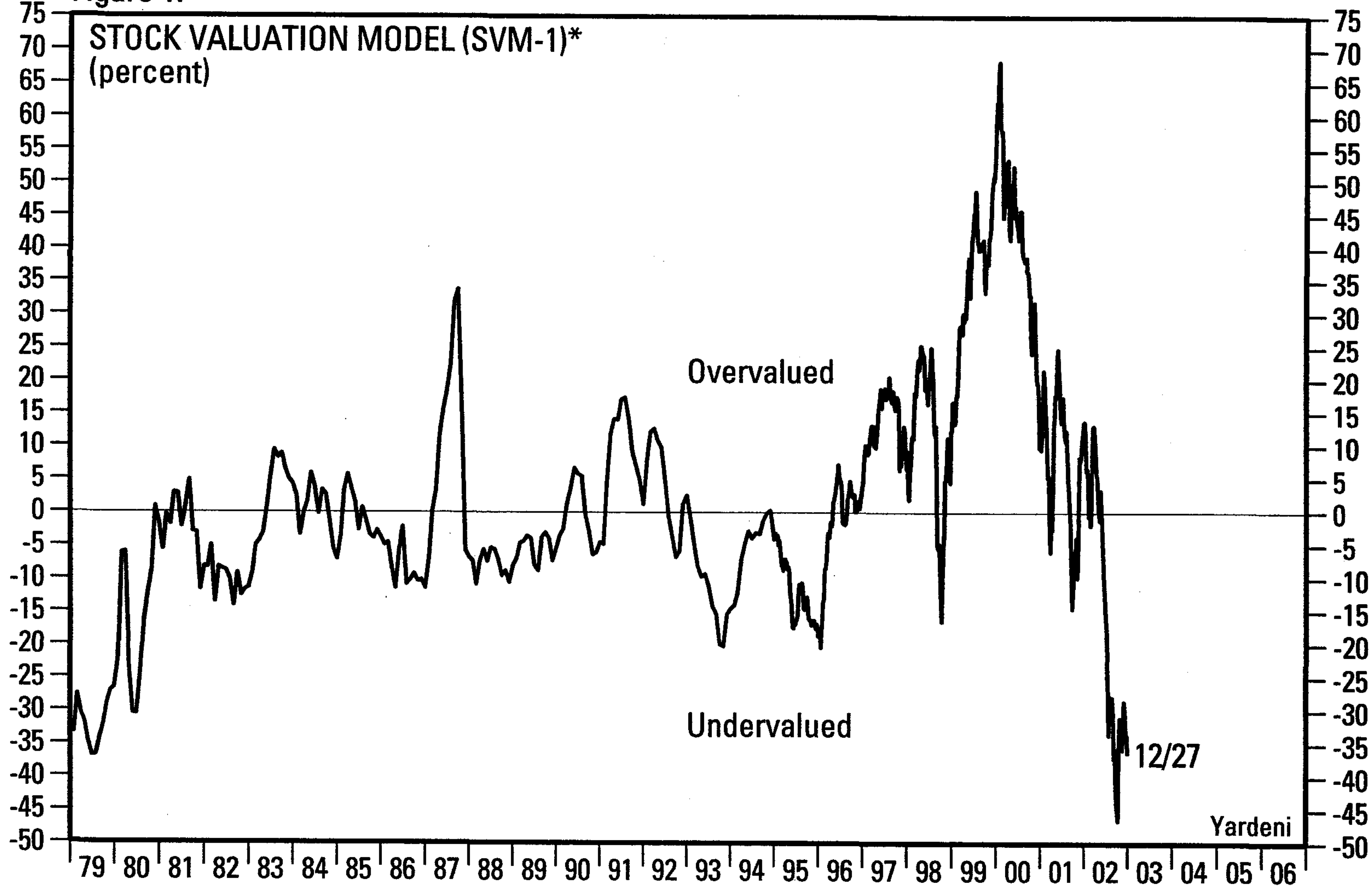
Figure 1.



* Ratio of S&P 500 index to its fair value (i.e. 52-week forward consensus expected S&P 500 operating earnings per share divided by the 10-year U.S. Treasury bond yield) minus 100. Monthly through March 1994, weekly after.
Source: Thomson Financial.

I. The Art Of Valuation

Since the summer of 1997, I have written three major studies on stock valuation and numerous commentaries on the subject.[1] This is the fourth edition of this ongoing research. More so in the past than in the present, it was common for authors of investment treatises to publish several editions to update and refine their thoughts. My work on valuation has been acclaimed, misunderstood, and criticized. In this latest edition, I hope to clear up the misunderstandings and address some of the criticisms.

I do not claim to have invented a scientific method for determining the one and only way to judge whether the stock market is overvalued or undervalued. Rather, my goal is to provide variations of a stock valuation model that can generate useful monthly and even weekly guidelines for judging the valuation of the stock market. Nevertheless, I believe valuation is a subjective art much more than it is a mathematically precise objective science.

In my earlier work, I focused on developing empirical methods for valuing the overall stock market, not individual stocks. Valuation is a relative exercise. We value things relative to other things or relative to a standard of value, like a unit of paper money (e.g., one dollar) or an ounce of gold. Stocks as an asset class are valued relative to other asset classes, like Treasury bills ("cash"), bonds, real estate, and commodities. In my valuation work, I focus primarily on the valuation of stocks relative to bonds. This means that the models can also be useful in assessing the relative value of bonds.

This fourth edition incorporates most of my analysis and conclusions from my previous research, which was based on 12-month forward consensus expected earnings for the S&P 500. The data are available both on a weekly and monthly basis. It is widely recognized that stock prices should be equivalent to the present discounted value of *expected* earnings, not trailing earnings. Yet a few widely respected investment analysts base their valuation work on trailing earnings and often derive conclusions that are quite different from the models based on forward expected earnings. As discussed below in Section V, I do monitor the backward-looking models, but I don't think they are especially helpful in explaining the valuation of expected earnings. The advocates of trailing earnings models do have the choice of using either reported earnings or operating earnings, i.e., excluding one-time writeoffs. Of course, the more pessimistically inclined analysts focus on reported earnings, the lower of the two measures. In either case, the data are available only on a quarterly basis with a lag of several weeks.

A similar data delay is experienced by analysts who believe that valuation should be based on quarterly dividends rather than forward earnings. I have added Section IV, which discusses the importance of dividends in assessing stock market valuation. I am amazed that critics of models based on forward earnings claim that they didn't work prior to 1979, which happens to be the first year that such data became available! As I will explain below, there is at least one good

[1] More information is available in *Topical Study #56*, "Stock Valuation Models," August 8, 2002, *Topical Study #44*, "New, Improved Stock Valuation Model," July 26, 1999 and *Topical Study #38*, "Fed's Stock Valuation Model Finds Overvaluation," August 25, 1997.

reason to believe that dividends mattered more than earnings prior to the 1980s. Dividends may matter more again if the double taxation of dividends is either eliminated or reduced.

So how can we judge whether stock prices are too high, too low, or just right? Investment strategists are fond of using stock valuation models to do so. Some of these are simple. Some are complex. Data on earnings, dividends, interest rates, and risk are all thrown into these black boxes to derive a "fair value" for the stock market. If the stock market's price index exceeds this number, then the market is overvalued. If it is below fair value, then stocks are undervalued. Presumably, investors should buy when stocks are undervalued, and sell when they are overvalued.

Previously, I examined a simple stock valuation model, which has been quite useful (Figure 1). I started to study the model after reading about it in the Federal Reserve Board's *Monetary Policy Report to the Congress* of July 1997. I dubbed it the "Fed's Stock Valuation Model (FSVM)," though no one at the Fed ever officially endorsed it. To avoid any confusion that this is an official model, in my recent research reports I have renamed it "Stock Valuation Model #1 (SVM-1)." This nomenclature is also meant to indicate that there are plenty of alternative SVMs as discussed in Section V.

Barron's frequently mentions SVM-1, especially since 9/11. The cover page of the September 24, 2001, issue observed that the stock market was "the biggest bargain in years." The bullish article, titled "Buyers' Market" and written by Michael Santoli, was entirely based on the SVM-1, which showed that stocks were extremely undervalued when the New York Stock Exchange reopened for trading on September 17, 2001.

A model can help us to assess value. But any model is just an attempt to simplify reality, which is always a great deal more complex, random, and unpredictable. Valuation is ultimately a judgment call. Like beauty, it is in the eyes of the beholder. It is also a relative concept. There are no absolutes. Stocks are cheap or dear relative to other investment and spending alternatives. A model can always be constructed to explain nearly 100% of what happened in the past. "Dummy variables" can be added to account for one-time unpredictable events or shocks in the past. However, the future is always full of surprises that create "outliers," e.g., valuations that can't be explained by the model. For investors, these anomalies present both the greatest risks and the greatest rewards.

More specifically, most valuation models went on red alert in 1999 and 2000. Stocks were grossly overvalued. With the benefit of hindsight, it was one of the greatest stock market bubbles ever. Investors simply chose to believe that the models were wrong. The pressure to go with the flow of consensus sentiment was so great that some strategists reengineered their models to show that stocks were still relatively attractive. One widely followed pundit simply replaced the bond yield variable with the lower inflation rate variable in his model to accomplish the alchemy of transforming an overvalued market into an undervalued one.

During the summer of 1999, I did fiddle with the simple model to find out whether it was missing something, as stocks soared well above earnings. I devised a second version of the model, SVM-2. It convinced me that stocks were priced for perfection, as investors seemed increasingly to accept the increasing optimism of Wall Street's industry analysts about the long-term prospects for earnings growth. The improved model also demonstrated that investors were giving more weight to these increasingly irrational expectations for earnings in the valuation of stocks! As I will show, analysts have been slashing their long-term earnings growth forecasts since early 2000, and investors are once again giving very little weight to earnings projections beyond the next 12 months.[2]

The question during the fall of 2002 was whether investor sentiment had swung too far from greed to fear. According to SVM-1, stocks were 49% undervalued in early October. This was the most extreme such reading on the record since 1979. Despite an impressive jump in stock prices at the end of October and through November, SVM-1 has become quite controversial. The bears contend that the model is flawed. Stocks are not undervalued at all, in their opinion. They believe stocks are still overvalued and may fall much lower in 2003. Ironically, not too long ago, it was the bulls who declared that stocks were not overvalued, and offered lots of reasons to ignore SVM-1.

I believe that the model is still useful and should not be ignored. Nevertheless, it should be only one of several inputs investors use to assess whether it is a good or bad time to buy stocks. For example, while SVM-1 indicated that I should increase my recommended exposure to equities in June and July of 2002, I went the other way: I lowered my exposure from 30/70 bonds/stocks to 35/65 for a Moderately Aggressive investor. For a Moderate investor I changed my recommended cash/bonds/stocks allocation from 10/40/50 to 10/50/40. I did so because I concluded that investors might continue to worry about the quality of earnings after WorldCom disclosed on June 26, 2002, that the company's earnings for the past several quarters were overstated as a result of fraudulent accounting.

I have one more warning before proceeding: Neither SVM-1 nor SVM-2 is likely to work if deflation becomes a more serious problem for the economy and earnings. According to SVM-1, the fair-value P/E is equal to the reciprocal of the Treasury bond yield. So the P/E should be 25 now with the bond yield at 4%. But why would investors be willing to pay such a high multiple for the lackluster earnings environment implied by such a low bond yield? I believe we have a better chance of seeing a 20 multiple if the bond yield rises to 5% and stays there than if the bond yield remains at 4%. If instead, the bond yield continues to fall, suggesting that deflation is proliferating, then the valuation multiple might actually fall, too.

[2] In my *Topical Study #44*, "New, Improved Stock Valuation Model," dated July 26, 1999, I wrote, "My analysis will demonstrate that the market's assumptions about risk, and especially about long-term earnings growth may be unrealistically optimistic, leaving it vulnerable to a big fall….The stock market is clearly priced for perfection. If perpetual prosperity continues uninterrupted, then perhaps the market's exuberant expectations will be realized. I, however, see more potential for disappointment, given the extreme optimism about long-term earnings growth embedded in current market prices."

II. SVM-1

After Fed Chairman Alan Greenspan famously worried out loud for the first time about "irrational exuberance" on December 5, 1996, his staff apparently examined stock market valuation models to help him evaluate the extent of the market's exuberance. One such model was made public, though buried, in the Fed's *Monetary Policy Report to the Congress*, which accompanied Mr. Greenspan's Humphrey-Hawkins testimony on July 22, 1997.[3] Twice a year, in February and July, the Chairman of the Federal Reserve delivers a monetary policy report to Congress. The Chairman's testimony is widely followed and analyzed. Virtually no one reads the actual policy report, which accompanies the testimony. I regularly read these reports.

The model was summed up in its July 22, 1997, report, in one paragraph and one chart on page 24 of the 25-page report (Figure A). The chart showed a strong correlation between the 10-year Treasury bond yield (TBY) and the S&P 500 current earnings yield (CEY)—i.e., the ratio of 12-month forward consensus expected operating earnings (E) to the price index for the S&P 500 companies (P). SVM-1 is based on this relationship.

Figure A: Excerpt from Fed's July 1997 Monetary Policy Report

> The run-up in stock prices in the spring was bolstered by unexpectedly strong corporate profits for the first quarter. Still, the ratio of prices in the S&P 500 to consensus estimates of earnings over the coming twelve months has risen further from levels that were already unusually high. Changes in this ratio have often been inversely related to changes in long-term Treasury yields, but this year's stock price gains were not matched by a significant net decline in interest rates. As a result, the yield on ten-year Treasury notes now exceeds the ratio of twelve-month-ahead earnings to prices by the largest amount since 1991, when earnings were depressed by the economic slowdown. One important factor behind the increase in stock prices this year appears to be a further rise in analysts' reported expectations of earnings growth over the next three to five years. The average of these expectations has risen fairly steadily since early 1995 and currently stands at a level not seen since the steep recession of the early 1980s, when earnings were expected to bounce back from levels that were quite low.

Source: Federal Reserve Board, *Monetary Policy Report to the Congress*.

[3] More information is available at http://www.federalreserve.gov/boarddocs/hh/1997/july/ReportSection2.htm

It is relatively easy to calculate 12-month forward earnings for the S&P 500. It is simply a time-weighted average of the current and next years' consensus estimates produced by Wall Street's industry analysts. Every month, Thomson Financial surveys these folks and compiles monthly consensus earnings estimates for the current and coming year. The consensus data for the S&P 500 companies are aggregated on a market-capitalization-weighted basis. To calculate the 12-month forward earnings series for the S&P 500, we need 24 months of data for each year. For example, during January of the current year, 12-month forward earnings are identical to January's expectations for the current year. One month later, in February of the current year, forward earnings are equal to 11/12 of February's estimate for the current year plus 1/12 of February's estimates for earnings in the next year (Figure B).

Figure B: Weights Used to Derive 12-Month Forward Earnings

	Current Calendar Year	Next Calendar Year
January	12/12	0/12
February	11/12	1/12
March	10/12	2/12
April	9/12	3/12
May	8/12	4/12
June	7/12	5/12
July	6/12	6/12
August	5/12	7/12
September	4/12	8/12
October	3/12	9/12
November	2/12	10/12
December	1/12	11/12

Source: Thomson Financial.

This method of calculating forward earnings doesn't exactly jibe with actual expectations for the coming 12 months. For example, half of forward earnings in July reflects half of the earnings expected for the current year, which is already half over. Furthermore, in this case, the other half of forward earnings reflects half of earnings expectations for *all* of next year. The problem is that there are no data available from analysts for the next 12 months. We can come close using quarterly earnings forecasts, which are also available from Thomson Financial. This is unnecessary, in my opinion. The method used by Thomson Financial should be a good enough approximation. The data start in September 1978 on a monthly basis (Figures 2 and 3). Weekly data are also available since 1994.

Because write-offs are one-shot events, analysts can't model them in their spread sheets. In other words, forward earnings are essentially projections of operating earnings. I use forward earnings, rather than either reported or operating trailing earnings, in most of my analyses because market prices reflect future earnings expectations. The past is relevant, but only to the extent that it is influencing the formation of current expectations about the future outlook for earnings.

Again, I believe the close relationship between the 10-year Treasury bond yield and the current earnings yield of stocks is impressive. The intuitive interpretation is that when Treasury bonds yield more than the earnings yield on the stock market, which is riskier than bonds, stocks are an unattractive investment. The average spread between CEY and TBY is only 26 basis points since 1979 (Figure 4). This suggests that the stock market is fairly valued when:

(1) CEY = TBY

It is undervalued (overvalued) when CEY is greater (less) than TBY. Another way to see this is to take the reciprocal of both variables in the equation above. In the investment community, we tend to follow the price-to-earnings (P/E) ratio more than the earnings yield. The ratio of the S&P 500 price index to forward earnings is highly correlated with the reciprocal of the 10-year bond yield, and on average the two have been nearly identical (Figure 5). This suggests that the "fair value" of the valuation multiple, using forward earnings, is simply one divided by the Treasury bond yield. For example, when the Treasury yield is 5%, the fair value P/E is 20. So in the Fed's valuation model, the "fair-value" price for the S&P 500 (FVP) is equal to expected earnings divided by the bond yield and the fair-value P/E is the reciprocal of the Treasury bond yield:

(2) FVP = E / TBY or,

(3) FVP / E = 1 / TBY

The ratio of the actual S&P 500 price index to the fair-value price shows the degree of overvaluation or undervaluation (Figure 1). History shows that markets can stay overvalued and become even more overvalued for a while. But eventually, overvaluation can be corrected in three ways: 1) interest rates can fall, 2) earnings expectations can rise, and of course, 3) stock prices can drop—the old-fashioned way to decrease values. Undervaluation can be corrected by rising yields, lower earnings expectations, and higher stock prices.

SVM-1 has worked quite well in the past, in my view. It identified when stock prices were excessively overvalued or undervalued, and likely to fall or rise:

1) The market was extremely undervalued from 1979 through 1982, setting the stage for a powerful rally that lasted through the summer of 1987.

2) Stock prices crashed after the market rose to an at-the-time record 34% overvaluation peak during September 1987.

3) Then the market was undervalued in the late 1980s, and stock prices rose.

4) In the early 1990s, it was moderately overvalued, and stock values advanced at a lackluster pace.

5) Stock prices were mostly undervalued during the mid-1990s, and a great bull market started in late 1994.

6) Ironically, the market was actually fairly valued during December 1996 when the Fed Chairman worried out loud about irrational exuberance, and stock prices continued to advance.

7) During both the summers of 1997 and 1998, overvaluation conditions were corrected by a sharp drop in stock prices.

8) Then a two-month undervaluation condition during September and October 1998 was quickly reversed as stock prices soared to a remarkable record 70% overvaluation reading during January 2000. This bubble was led by the Nasdaq and technology stocks, which crashed over the rest of the year, bringing the market closer to fair value in late 2000 through early 2002.

9) As noted above, the model suggested that stock prices were significantly undervalued immediately after the 9/11 attacks in 2001. As a result of the subsequent rally, they were fairly valued again by early 2002. But concerns about the quality of corporate earnings and the economic outlook drove stock prices back down through early October, when SVM-1 was undervalued by a record 49%. Then the market rallied.

According to Ned Davis Research, when the model has shown stocks to be more than 5% undervalued since 1980, the average one-year gain in the S&P 500 has been 31.7%. When the model has been more than 15% overvalued, the market has dropped 8.7%, on average, in the following year.[4]

III. SVM-2

The stock market is a very efficient market. In efficient markets, all available information is fully discounted in prices. In other words, efficient markets should always be "correctly" valued, at least in theory (i.e., the so-called Efficient Markets Hypothesis). All buyers and all sellers have access to exactly the same information. They are completely free to act upon this information by buying or selling stocks as they choose. So the market price is always at the correct price, reflecting all available information. In his June 17, 1999, congressional testimony, Federal Reserve Chairman Alan Greenspan soliloquized about valuation:

[4] See "Good-Looking Models," by Michael Santoli in *Barron's*, August 5, 2002.

> The 1990s have witnessed one of the great bull stock markets in American history. Whether that means an unstable bubble has developed in its wake is difficult to assess. A large number of analysts have judged the level of equity prices to be excessive, even taking into account the rise in "fair value" resulting from the acceleration of productivity and the associated long-term corporate earnings outlook. But bubbles generally are perceptible only after the fact. To spot a bubble in advance requires a judgment that hundreds of thousands of informed investors have it all wrong. Betting against markets is usually precarious at best.[5]

This is another one of the chairman's ambiguous insights, which may have contributed to the very bubble he was worrying about. He seems to be saying that the stock market might be a bubble, but since the market efficiently reflects the expectations of "thousands of informed investors," maybe the market is right because all those people can't be wrong. They *were* wrong, and so was the Fed chairman, about the judgment of all those folks. However, at the time, the available information obviously convinced the crowd that stocks were worth buying. The crowd didn't realize that it was a bubble until it burst. In other words, efficient markets can experience bubbles when investors irrationally buy into unrealistically bullish assumptions about the future prospects of stocks.[6]

Of course, individually, we can all have our own opinions about whether stocks are cheap or expensive at the going market price. Perhaps we should consider replacing the terms "undervalued" and "overvalued" with "underpriced" and "overpriced," respectively. I think in this way, we acknowledge that the stock market is efficient and that the market price should usually be the *objective* fair value. At the same time, the new terminology allows us to devise valuation models to formulate *subjective* opinions about market prices. If my model shows that the market is overpriced, I am simply stating that I disagree with the weight of opinion that has lifted the market price above my own assessment of the right price.

Now let's formulate a new, "improved model" (SVM-2) that more explicitly identifies the variables that together determine the value of the stock market. If, for example, SVM-1 shows that stocks are 50% overvalued, we need to add variables that can explain why the aggregate of all buyers and sellers believe that the price is right. Once we agree on what is "in" the market, we can each make our own pro or con case, and invest accordingly.

SVM-1 is missing some variables, which might explain why the current earnings yield diverges from the Treasury yield. We clearly need to account for variables that differentiate stocks from bonds. If the government guarantees that stock earnings will be fixed for the next 10 years, then the price of the S&P 500 would be at a level that nearly equates the current earnings yield to the 10-year Treasury bond yield. But there is no such guarantee for stocks. Earnings can go down. Companies can lose money. They can also go out of business. Earnings can also go up. We need variables to capture:

1) Business risk to earnings.
2) Earnings expectations beyond the next 12 months.

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[5] More information is available at http://www.bog.frb.fed.us/BOARDDOCS/TESTIMONY/1999/19990617.htm
[6] Perhaps the simplest and best explanation for bubbles is that they occur when we all foolishly invest in assets we know are overvalued, but we just can't stand the mental anguish of seeing our friends and relatives getting rich.

The new, "improved" valuation model reflecting these variables (i.e., SVM-2), should have the following structure:

(4) $\mathbf{CEY = a + b \cdot TBY + c \cdot RP - d \cdot LTEG}$

CEY is the current earnings yield defined as 12-month forward earnings of the S&P 500, divided by the S&P 500 price index. TBY is the 10-year Treasury bond yield. The two new additional variables are the risk premium (RP) and long-term expected earnings growth beyond the next 12 months (LTEG). My assumption is that the current earnings yield ("the dependent variable") is a linear function of the three independent variables on the right of the equation above. Obviously, there are several other ways to specify the model. But this should do for now.

How should we measure risk in the model? An obvious choice is to use the spread between corporate bond yields and Treasury bond yields.[7] This spread measures the market's assessment of the risk that some corporations might be forced to default on their bonds. Of course, such events are very unusual, especially for companies included in the S&P 500.

However, the spread is only likely to widen during periods of economic distress, when bond investors tend to worry that profits won't be sufficient to meet the debt-servicing obligations of some companies. Most companies won't have this problem, but their earnings would most likely be depressed during such periods. So the new, "improved" model can be represented as follows:

(5) $\mathbf{CEY = a + b \cdot TBY + c \cdot (CBY - TBY) - d \cdot LTEG}$

CBY is the corporate bond yield. Which corporate bond yield should we use in the model? We can try Moody's composites of the yields on corporate bonds rated "Aaa," "Aa," "A," or "Baa." I found that the spread between the A-rated corporate composite yield and the Treasury bond yield fits quite well. This spread averaged 159 basis points since 1979. It tends to widen most during "flight-to-quality" credit crunches, when Treasury bond yields tend to fall fastest (Figure 6).

The final variable included in SVM-2 is one for expected earnings growth beyond the next 12 months. Thomson Financial compiles data on consensus long-term earnings growth for the S&P 500 (Figure 7). The monthly data start in 1985 and are based on industry analysts' projections for the next three to five years (Figure B).

In equation (5) above, my presumption is that a=0 and b=c=1. So,

(6) $\mathbf{CEY = CBY - d \cdot LTEG}$ or,

(7) $\mathbf{CEY = TBY + RP - d \cdot LTEG}$

[7] My models do not include the so-called equity risk premium, which is a fuzzy concept, in my opinion, and difficult to measure.

In other words, in this version of SVM-2, investors demand that the current earnings yield fully reflects the Treasury bond yield and the default risk premium in bonds, less some fraction of long-term expected earnings growth. In this model, the market is always fairly valued; the only question is whether the implied value of "d" and the consensus expectations for long-term earnings growth are too pessimistic (excessively cautious), too optimistic (irrationally exuberant), or just about right (rational).

We can derive "d" from equation (5) as follows:

(8) d = (CBY – CEY) / LTEG

Plugging in the available data since 1985, "d" has ranged between -.0027 and +0.33, and averaged 0.13 (Figure 8). This means that on average investors assign a weight of 0.13 to LTEG. They don't give it much weight because historically it has been biased upward (Figure B). They also don't give it much weight because long-term earnings are harder to forecast than earnings over the coming 12 months.

Notice that in 1999 and early 2000, investors effectively gave LTEG a weight of 0.23, or nearly twice as much as the historical average. Actually, up until 1999, "d" averaged only 0.10. This supports my observation at the beginning of this study that investors were irrationally giving more weight to irrationally high long-term earnings expectations in the late 1990s. At the end of last year, "d" was back down around 0.05, near the bottom of its range.

We can derive fair-value time series for the S&P 500 and for the valuation multiple for different values of "d" using the following formula:

(9) FVP = E / (CBY– d · LTEG)

(10) FVP / E = 1 / (CBY– d · LTEG)

Obviously, to avoid nonsensical results like a negative fair-value price or an infinite P/E, CBY > d · LTEG. We can draw fair-value price series for the S&P 500 using equation (9). We have data for all the variables except the "d" coefficient. Nevertheless, we can proceed by plotting a series for various plausible fixed values of "d". Based on the analysis above, I've chosen the following values: 0.10, 0.20, and 0.25. Now we can compare the matrix of the three resulting FVP series to the actual S&P 500. During December 2002, the latest fair value, using d = 0.10, was 989. The S&P 500 was 9.1% below this level (Figures 9 and 10).

Figure C: Long-Term Consensus Expected Earnings Growth

In the long-run, profits don't, and can't grow faster than GDP. Historically, this growth rate has averaged about 7% annually. So, why do Wall Street's industry analysts collectively and consistently predict that corporate earnings will grow much faster than 7%? From the start of the data in 1985 through 1995, analysts estimated that S&P 500 earnings will grow between 10.8% and 12.1% (Figure 7). This range well exceeds 7%. The collective forecast of industry analysts for long-term earnings growth is obviously biased to the upside. Wall Street's analysts are extrapolating the earnings growth potential for their companies, in their industries. It is unlikely that most analysts will have the interest and staying power to cover companies and industries they believe are likely to be underperformers for the next several years. So naturally, their long-term outlook is likely to be relatively rosy. This bias is best revealed when the consensus data are compiled and compared to reality.

If the projected earnings growth overshoot is constant over time, then investors can make an adjustment for the overly optimistic bias of analysts, and invest accordingly. This is harder to do during a speculative bubble, when even the best analysts can get sucked into the mania. As stock prices soared during the second half of the 1990s, analysts became more bullish on the outlook for their companies. As they became more bullish, so did investors and speculators. Analysts increasingly justified high stock prices and lofty valuation multiples by raising their estimates for the long-term potential earnings growth rates of their companies.

Long-term earnings growth expectations for the S&P 500 companies started to rise steadily after 1995 up to 14.9% by the end of 1998. Then they soared through 2000, peaking at 18.7% during August of that year. Analysts, investors, and speculators ignored the natural speed limits imposed by the natural growth of the economy and earnings. They forgot that nothing on our small Planet Earth can compound at such extraordinary rates without eventually consuming all the oxygen in the atmosphere.

Once the speculative bubble began to burst in March 2000, analysts scrambled to reassess their wildly optimistic projections. Consensus long-term earnings growth expectations plunged to 12.8% for the S&P 500 by the end of 2002 from the all-time 18.7% peak the year before. The reversal for the technology sector of the S&P 500 was even more dramatic with growth expectations dropping to 16% at the end of 2002 from the 2000 peak rate of 28.7%.

Source: Dr. Edward Yardeni, Prudential Securities.

Notice that equations (9) and (10) describing the same SVM-2 both morph into SVM-1 when RP—the corporate bond's default risk premium—is equal to the long-term earnings growth term d · LTEG. Historically, on average, this is the case, which is why the simple version of the model has worked surprisingly well.[8]

In my *Topical Study #45*, "Earnings: The Phantom Menace (Episode I)" dated August 16, 1999, I observed that according to SVM-1, "...the market is extremely overpriced and vulnerable to a significant fall." I also explained that the model uses the market's earnings expectations, not mine. I argued again that the market's expectations were unrealistically optimistic and that earnings were inflated by phantom revenues and unexpensed stock options:

> A related problem is that many companies are overstating their earnings by using questionable accounting and financial practices. Some are significantly overstating their profits, and they tend to have the highest valuation multiples in the stock market. This suggests that investors are not aware that the quality of earnings may be relatively low among some of the companies reporting the fastest earnings growth.

This suggests an interesting twist on the valuation model. Let's assume that the stock market is always fairly valued, i.e., the P/E is always equal to the reciprocal of the 10-year Treasury bond yield. Using SVM-1, we can easily calculate the market's estimate of forward earnings (E) by multiplying the level of the S&P 500 (P) by the 10-year bond yield (E/P). Currently, with the S&P 500 closing price at 909 on January 2 and the yield at 4%, the market's assessment is that earnings are actually $37.00 per share, or 32.5% below the analysts' consensus forecast (Figure 11).

Again, from this perspective, the market isn't a screaming buy as suggested by SVM-1. Rather, over the past few months, it has adjusted to a lower and more realistic level of earnings. If this is correct, then the good news is that any downward adjustments made by companies and analysts may already be largely discounted.

The model can be used to assess several major overseas stock markets for which forward earnings data are available since 1989 (Figures 12 and 13). Not surprisingly, there is a high degree of correlation between the SVM-1 results for the United States and Canada (0.47), the United Kingdom (0.33), Germany (0.40), and France (0.53). The correlation is low with Japan (-0.36). The model doesn't work for Japan because deflationary forces have pushed the 10-year bond yield to under 1.5% in recent years, which implies a nonsensical valuation multiple.

IV. Discounting Dividends

My focus until now has been entirely on earnings. Don't dividends matter? They did prior to 1982, but seemed to matter less and less after that year. If the Bush administration succeeds in convincing Congress to eliminate the double taxation of dividends, then dividends should matter more again.

[8] Since 1985, RP and d · LTEG have averaged 161 and 181 basis points, respectively—not an exact match, but close enough.

My views on this subject were heavily influenced by an excellent speech on "Corporate Governance," presented by Federal Reserve Chairman Alan Greenspan on March 26, 2002, at New York University. Mr. Greenspan observed that shareholders' obsession with earnings is a relatively new phenomenon:

> Prior to the past several decades, earnings forecasts were not nearly so important a factor in assessing the value of corporations. In fact, I do not recall price-to-earnings ratios as a prominent statistic in the 1950s. Instead, investors tended to value stocks on the basis of their dividend yields.

Everything changed in 1982, according to the Fed chairman. That year, a simple regulatory move combined with the different tax rates on dividend income and capital gains—the marginal individual tax rate on dividends, with rare exceptions, has always exceeded the marginal tax rate on capital gains—put us on the path to the recent upheaval in the corporate world. In 1982, the Securities and Exchange Commission (SEC) gave companies a safe harbor to conduct share repurchases without risk of investigation. Repurchases raise per-share earnings through share reduction. Before then, companies that repurchased their shares risked an SEC investigation for price manipulation. "This action prompted a marked shift toward repurchases in lieu of dividends to avail shareholders of a lower tax rate on their cash receipts," said the Fed chairman.

As a result, "The sharp fall in dividend payout ratios and yields has dramatically shifted the focus of stock price evaluation toward earnings." The dividend payout ratios, which in decades past averaged about 55%, in recent years fell on average to about 35%. Dividend yields—the ratio of dividends per share to a company's share price—fell even faster than the payout ratio, as stock prices soared over the past two decades. Fifty years ago, dividend yields on stocks typically averaged 6%. Today, such yields are barely above 1%. Contributing to the drop in both ratios has been the sharp drop in the percent of S&P 500 companies paying dividends from 87% during 1982 to 73% in 2001 (Figures 14 and 15).

Mr. Greenspan observed that earnings accounting is much more subjective than cash dividends, "whose value is unambiguous." More specifically, "Although most pretax profits reflect cash receipts less out-of-pocket cash costs, a significant part results from changes in balance-sheet valuations. The values of almost all assets are based on the assets' ability to produce future income. But an appropriate judgment of that asset value depends critically on a forecast of forthcoming events, which by their nature are uncertain." So, for example, depreciation expenses are based on book values, but are very crude approximations of the actual reduction in the economic value of physical plant and equipment. "The actual deterioration will not be known until the asset is retired or sold." Mr. Greenspan also takes a swipe at corporate pension plan accounting: "And projections of future investment returns on defined-benefit pension plans markedly affect corporate pension contributions and, hence, pretax profits."

Because earnings are "ambiguous," they are prone to manipulation and to hype. During a period of rapid technological change, innovative companies are likely to be especially profitable over the short-run. But, this tends to increase the incentive for competitors to enter the market and reduce profitability in the long run. Mr. Greenspan noted, "Not surprisingly then, with the longer-term outlook increasingly amorphous, the level and recent growth of short-term earnings have taken on especial significance in stock price evaluation, with quarterly earnings reports subject to anticipation, rumor, and 'spin.' Such tactics, presumably, attempt to induce investors to extrapolate short-term trends into a favorable long-term view that would raise the current stock price." This has led to the current sorry state of corporate affairs, according to him:

> CEOs, under increasing pressure from the investment community to meet short-term elevated expectations, in too many instances have been drawn to accounting devices whose sole purpose is arguably to obscure potential adverse results. Outside auditors, on several well-publicized occasions, have sanctioned such devices, allegedly for fear of losing valued corporate clients. Thus, *it is not surprising that* since 1998 earnings restatements have proliferated. This situation is a far cry from earlier decades when, if my recollection serves me correctly, firms competed on the basis of which one had the most conservative set of books. Short-term stock price values then seemed less of a focus than maintaining unquestioned credit worthiness.

Mr. Greenspan concluded his speech on an optimistic note, seeing signs that the market is already fixing the problem as the sharp decline in stock and bond prices following Enron's collapse punished many of the companies that used questionable accounting practices. "Markets are evidently beginning to put a price-earnings premium on reported earnings that appear free of spin." In other words, market discipline is already raising corporate accounting and governance standards. The Fed chairman endorsed any legislative and regulatory initiatives that provide incentives for corporate officers to act in the best interests of their shareholders. He warned against excessive regulation, which "has, over the years, proven only partially successful in dissuading individuals from playing with the rules of accounting."

In my opinion, eliminating the taxation of dividend income should be a very effective way to fix most of the problems with the current system that the Fed chairman identified so brilliantly in his speech. Shareholders should be encouraged to act as owners of the corporations in which they invest. Managers should be encouraged to treat them as owners, too. It is the owners of the corporation who pay taxes on profits. Why should they be taxed again on their dividend income? I think this double taxation creates a tremendous incentive for management to retain rather than distribute earnings. It has given management a convincing story to tell shareholders: "Instead of paying you dividends, we will invest retained earnings on your behalf to grow our business even faster, and we will also buy back our stock to boost earnings per share."

This system gives too much power to management and tends to effectively disenfranchise the shareholder, in my opinion. In other words, this system is prone to be abused and corrupted, as occurred during the previous decade. Without the discipline of dividend payments, management may have a great incentive to use every trick in the rule book and every conceivable accounting gimmick to boost earnings. Investors are forced to value stocks on easily manipulated and inflated earnings, rather than on the cold, hard cash of dividends.

If, instead, dividends were exempt from the personal income tax, then investors would tend to favor companies that pay dividends and have established a record of steadily raising their payouts to shareholders. Shareholders could then decide for themselves whether to reinvest their dividend income in the corporation based on the ability of management to grow dividend payments, rather than earnings. Obviously, dividends would grow at the same rate as earnings, assuming a fixed payout ratio. But dividends would discipline the accounting for earnings. Management can't pay cash to shareholders unless the cash actually is earned.

V. Other Models

SVM-1 is a very simple stock valuation model. It should be used along with other stock valuation tools, including SVM-2. Of course, there are numerous other more sophisticated and complex models. The SVM models are not market-timing tools. As noted above, an overvalued (undervalued) market can become even more overvalued (undervalued). However, SVM-1 does have a good track record of showing whether stocks are cheap or expensive. Investors are likely to earn below (above) average returns over the next 12 to 24 months when the market is overvalued (undervalued).

Both SVM-1 and SVM-2 are alternative versions of the Gordon discounted cash flow stock market valuation model.[9] This model has long been used by many investors to determine valuation. The Association of Investment Management and Research—the organization that conducts the Certified Financial Analysts (CFA) program—recently published an authoritative and comprehensive text titled "Analysis of Equity Investments: Valuation." The Gordon growth model is discussed in 20 pages of the book. The Dividend Yield is discussed in five pages. SVM-1 is briefly mentioned on pages 202 and 203 and is called the Fed Stock Valuation Model. SVM-2 is briefly mentioned on pages 203 and 204 and is called the Yardeni Model.

Tobin's q model is not mentioned at all in the CFA book. I studied under the late Professor James Tobin of Yale University. He was the chairman of my Ph.D. committee. In his model, q is the ratio of the market value of a corporation to its replacement cost. When q is greater than one, it makes more sense to rebuild it at cost than to buy it in the market. When q is less than one, it is cheaper to buy the corporation in the market than to build it from scratch. The model

[9] Myron J. Gordon, *The Investment, Financing, and Valuation of the Corporation*. Irwin (1962).

appears logical, but empirically very questionable, since it requires data on the replacement cost of companies. While this exercise may be doable for an individual company, it also seems very questionable whether a realistic and accurate time series can be constructed for all the companies in the S&P 500.

Nevertheless, the credibility of this model received a big boost after the publication in March 2000 of *Valuing Wall Street* by Andrew Smithers and Stephen Wright. According to the book's Web site: "The U.S. stock market is massively overvalued. As a result, the Dow could easily plummet to 4,000—or lower—losing more than 50% of its value wiping out nest eggs for millions of investors...Using the *q* ratio developed by Nobel Laureate James Tobin of Yale University, Smithers & Wright present a convincing argument that shows the Dow plummeting from recent peaks to lows not seen in a decade."[10]

A Fed staff economist, Michael Kiley, wrote a research paper in January 2000 titled "Stock Prices and Fundamentals in a Production Economy." Based on a model that is more like Tobin's than Gordon's, he concluded that "the skyrocketing market value of firms in the second half of the 1990s may reflect a degree of irrational exuberance."[11] That was exactly the same conclusion that was suggested by both SVM-1 and SVM-2, which showed that the S&P 500 was overvalued by nearly 70% and 57%, respectively, at the time. The two models currently show that stocks are undervalued. Tobin's *q* is back down below one for the first time since 1994 (Figure 16).

Kiley's goal was to demonstrate that some of the more bullish prognosticators in the late 1990s based their conclusions on exuberant versions of the Gordon model. He specifically mentions *Dow 36,000* by James K. Glassman and Kevin A. Hassett (Times Books, 1999) who argued that stocks are much less risky than widely believed. So a lower equity risk premium justified higher P/Es. Kiley also mentions work by Jeremy J. Siegel. In the second edition (1998) of his widely read book, *Stocks For The Long Run*, the dust jacket claims that "when long-term purchasing power is considered, stocks are actually **safer than bank deposits!**" [12]

One of the most popular and simplest tools for gauging valuation is simply to compare the market's P/E to its historical average. These crude "reversion-to-the-mean" models are worth tracking, in my view, but they ignore how changes in interest rates, inflation, and technologies might impact valuation both on a short-term and long-term basis (Figure 17). Of course, Robert J. Shiller earned much fame and fortune with his 2000 book, *Irrational Exuberance*, in which he argued that the market's P/E was too high by historical standards. [13]

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[10] More information is available at http://www.valuingwallstreet.com/
[11] More information is available at http://www.federalreserve.gov/pubs/feds/2000/200005/200005pap.pdf
[12] Jeremy J. Siegel, *Stocks For The Long Run*, McGraw-Hill (1998)
[13] Robert J. Shiller, *Irrational Exuberance*, Princeton University Press (2000)

VI. Greenspan On Valuation

Fed Chairman Alan Greenspan delivered his latest thoughts on the stock market, asset bubbles, and valuation on August 30, 2002.[14] Much of the discussion of valuation seems to be based on a model that is very similar to SVM-2. In footnote 3 of his speech, Mr. Greenspan writes:

> For continuous discounting over an infinite horizon, k (E/P) = r + b - g, where k equals the current, and assumed future, dividend payout ratio, E current earnings, P the current stock price, r the riskless interest rate, b the equity premium, and g the growth rate of earnings.

In my SVM-2 model, k = 1 because I believe that the market discounts earnings, not dividends. Furthermore, r = the 10-year Treasury bond yield, b = the default risk premium in corporate bonds, and g = long-term expected earnings growth.

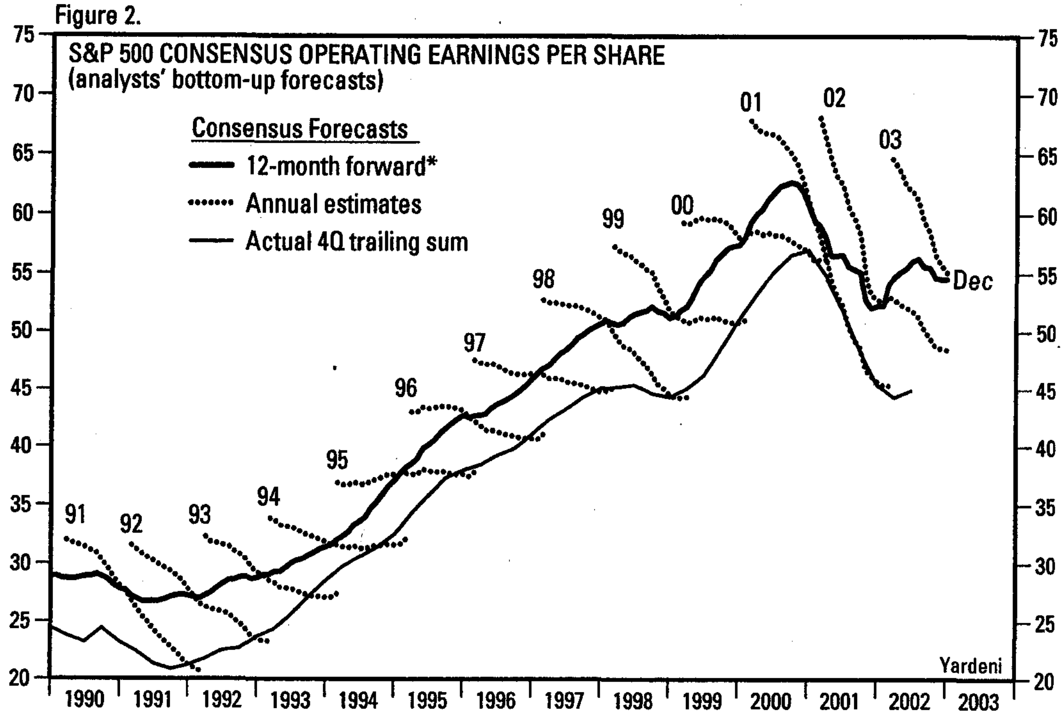
According to the speech, Mr. Greenspan has concluded that the Fed has no unambiguous tools to gauge whether stocks are overvalued or undervalued. Therefore, he believes that the Fed could do nothing about stock market bubbles, other than to wait to see if they burst! Unlike the Fed chairman, most investors must rely on valuation models to provide some guidance to their decision-making process. The models are not full proof and they are not great market-timing tools. However, they are useful, especially if used with other investment tools. For example, in my *Stock Market Cycles*, I present numerous charts relating key economic and financial indicators to stock price cycles.[15] I found that consumer sentiment indicators are especially good at confirming major market bottoms (Figures 18 and 19). I am also fond of using technical indicators to supplement the insights from stock valuation models (Figure 20). In other words, the best approach for investing in the stock market is to use a number of disciplines.

* * *

[14] More information is available at http://www.federalreserve.gov/boarddocs/speeches/2002/20020830/default.htm
[15] More information is available at http://www.prudential-yardeni.com/public/cycle.pdf

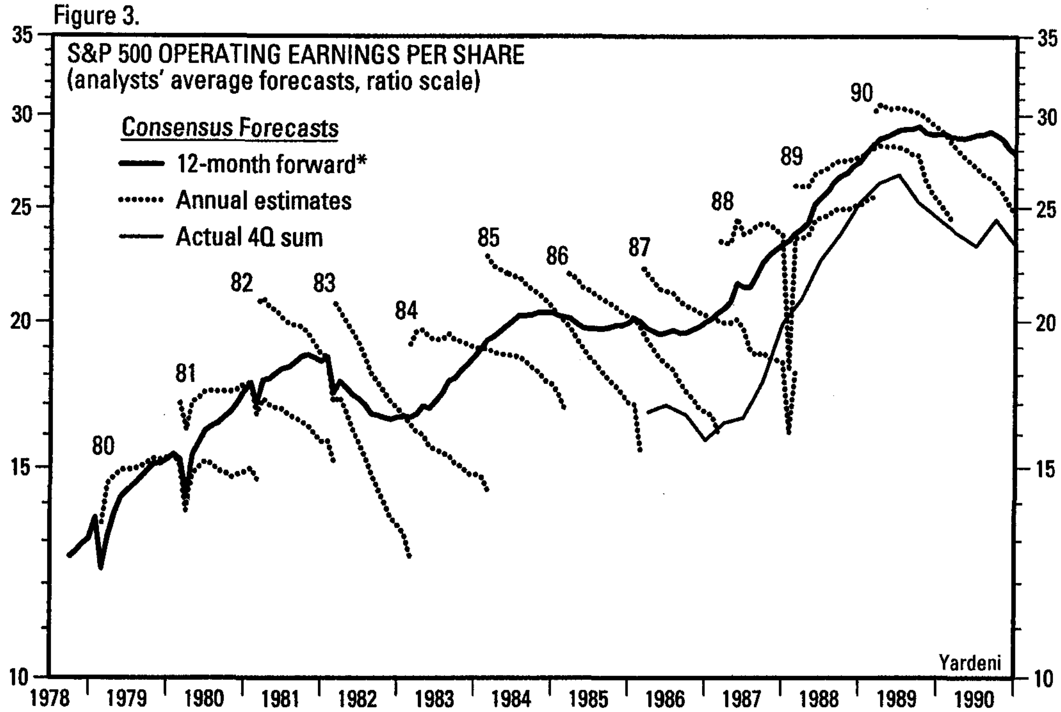
Analysts tend to be too optimistic about the outlook for earnings in any one year. The stock market tends to discount forward earnings, the time-weighted average of the current and coming years' consensus expected earnings.

Figure 2.



* Time-weighted average of current and next years' consensus earnings estimates.
Source: Thomson Financial.

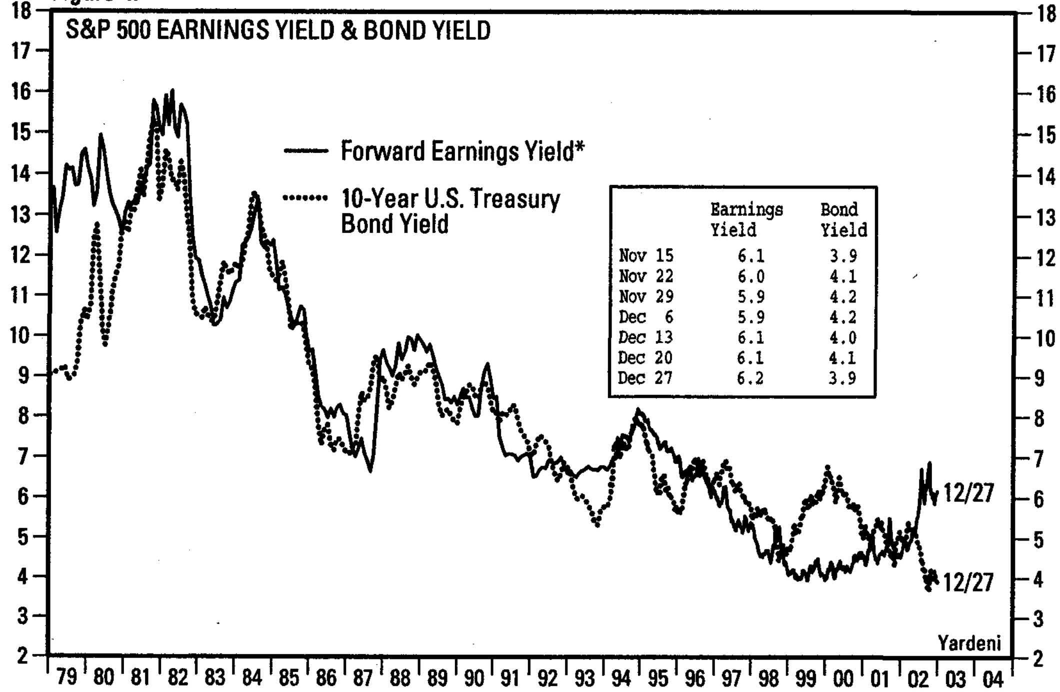
Figure 3.



* Time-weighted average of current and next years' consensus earnings estimates.
Source: Thomson Financial.

Since 1979, when forward earnings data first became available, the foward earnings yield has tracked the 10-year bond yield very closely. Since 1998, the two series have diverged more.

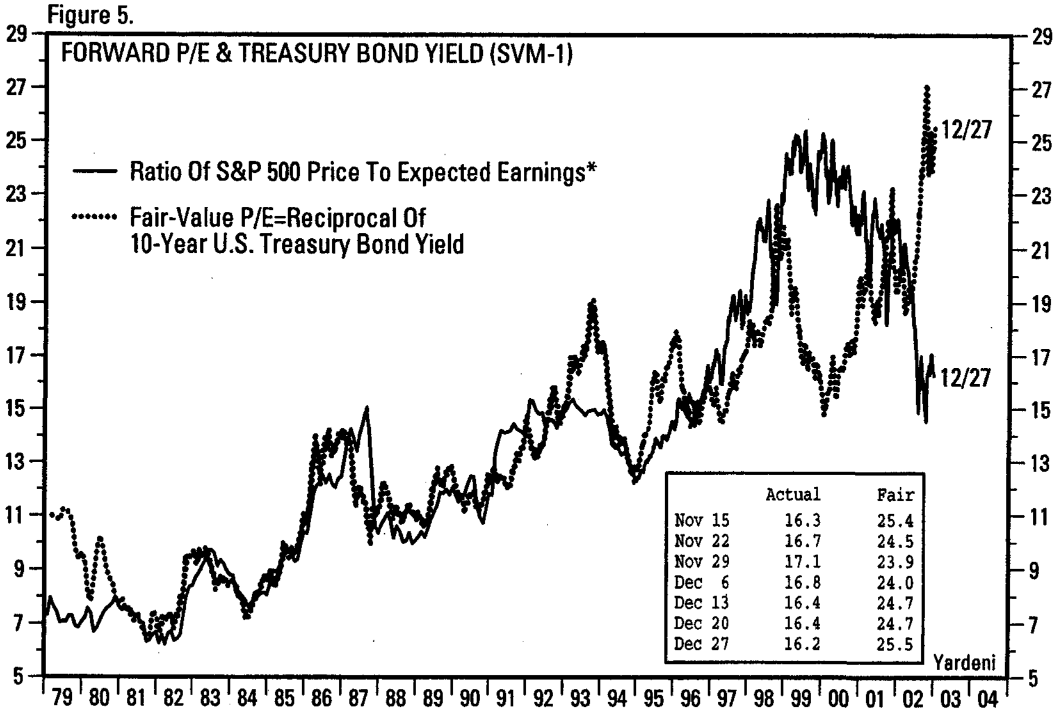
Figure 4.



* 52-week forward consensus expected S&P 500 operating earnings per share divided by S&P 500 Index. Monthly through March 1994, weekly after.
Source: Thomson Financial.

SVM-1 shows that the reciprocal of the 10-year bond yield is a useful measure of the fair-value P/E.

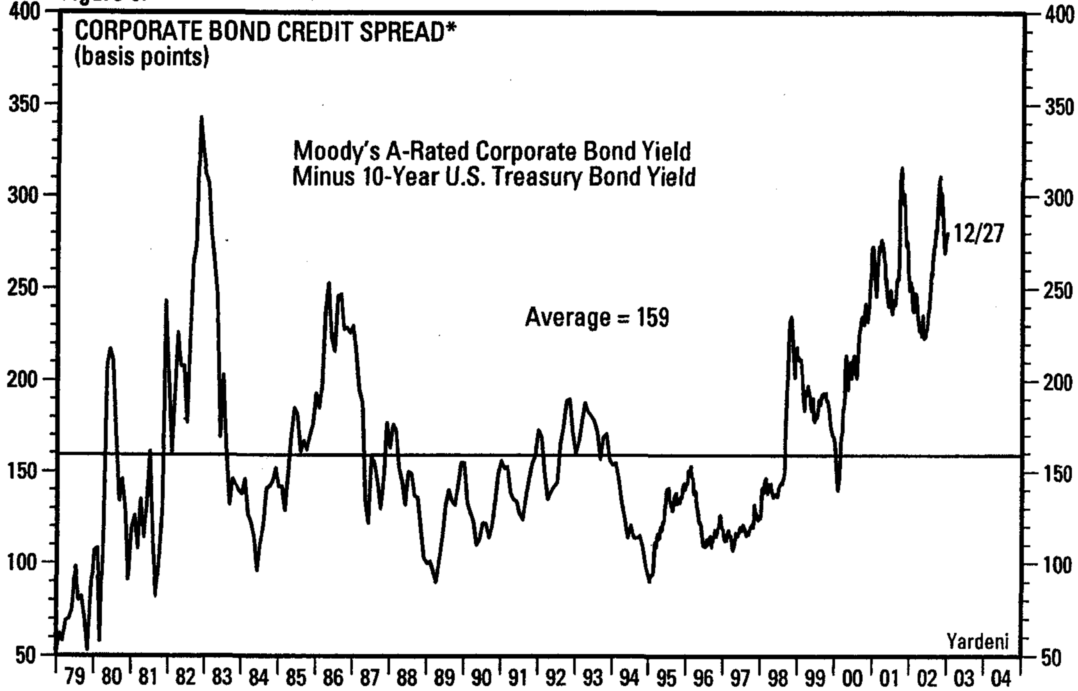
Figure 5.



* 52-week forward consensus expected S&P 500 operating earnings per share. Monthly through March 1994, weekly after.
Source: Thomson Financial.

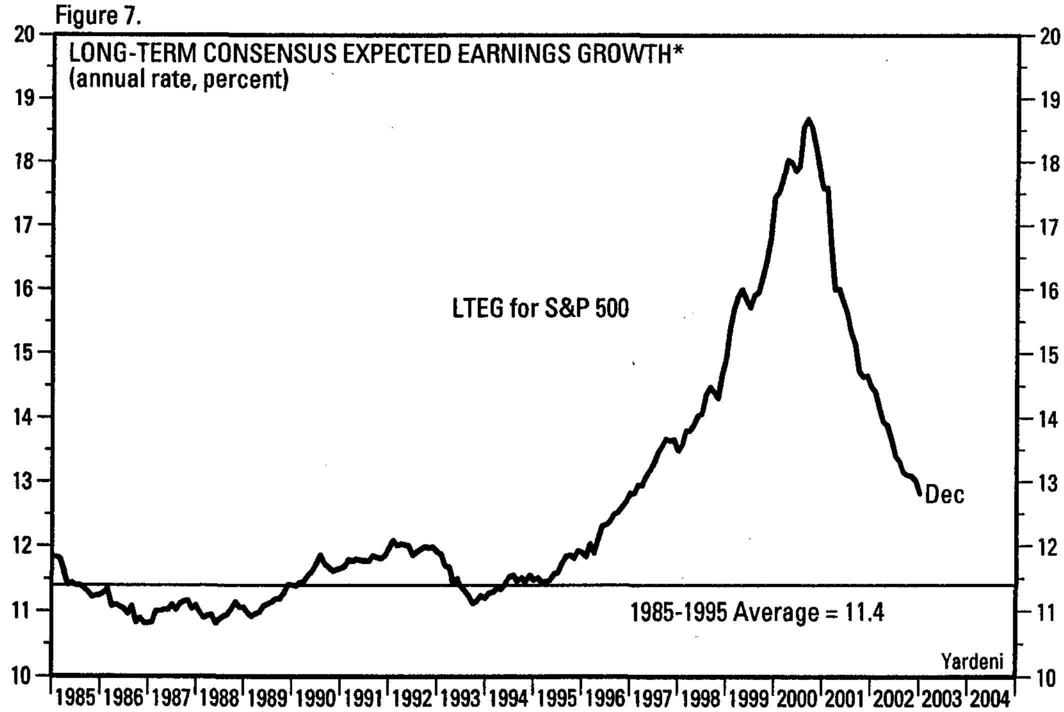
SVM-2 includes the corporate bond credit quality spread and long-term consensus expected earnings growth (LTEG). The spread remains wide, and LTEG is still falling back to the 1985-1995 level.

Figure 6.



* Monthly through 1994, weekly thereafter.
Source: Board of Governors of the Federal Reserve System and Moody's Investors Service.

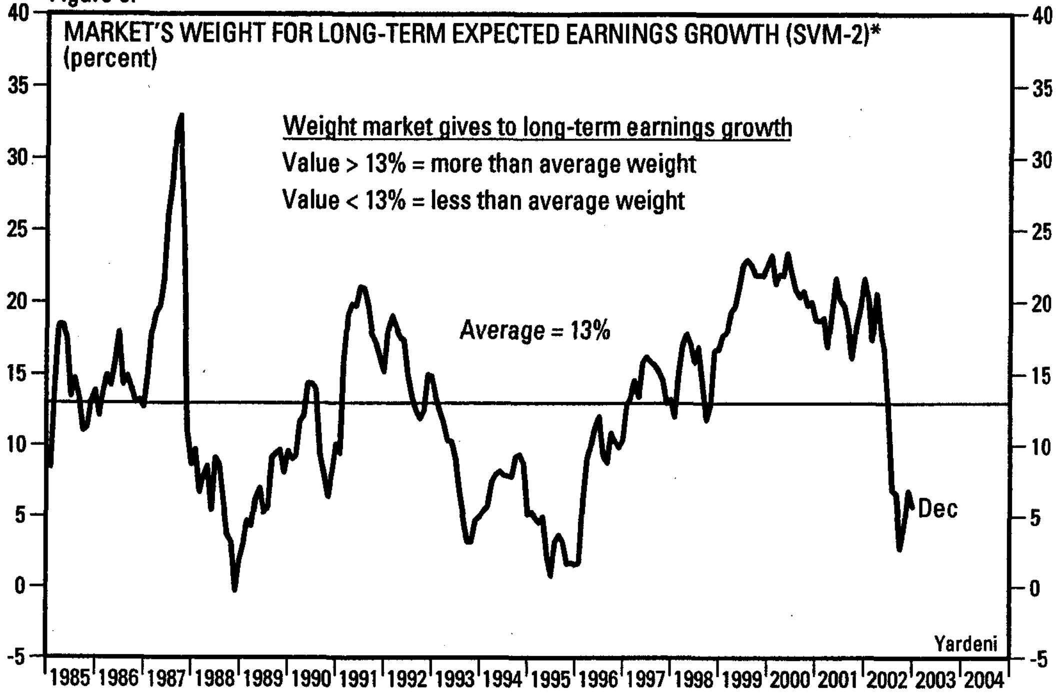
Figure 7.



* 5-year forward consensus expected S&P 500 earnings growth.
Source: Thomson Financial.

The stock market is giving much less weight to LTEG, now that it is falling, than during the 1999-2000 Bubble, when it soared to record highs.

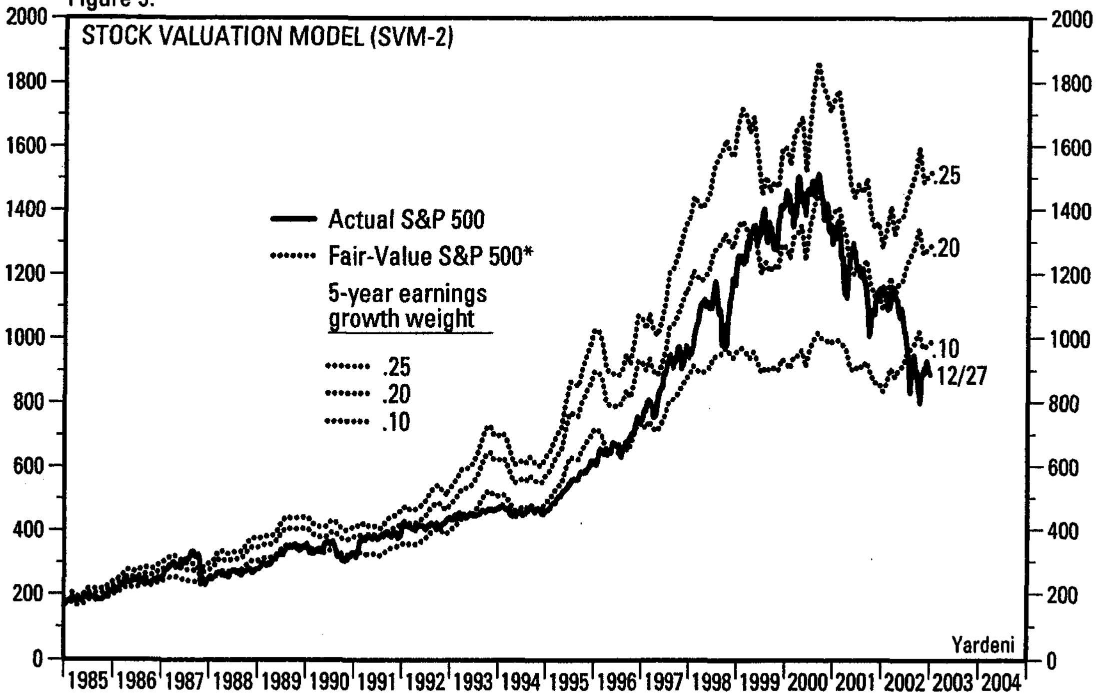
Figure 8.



* Moody's A-rated corporate bond yield less earnings yield divided by 5-year consensus expected earnings growth.
Source: Standard and Poor's Corporation, Thomson Financial and Moody's Investors Service.

During the Bubble, investors doubled the weight they gave LTEG, which soared to irrationally exuberant new highs.

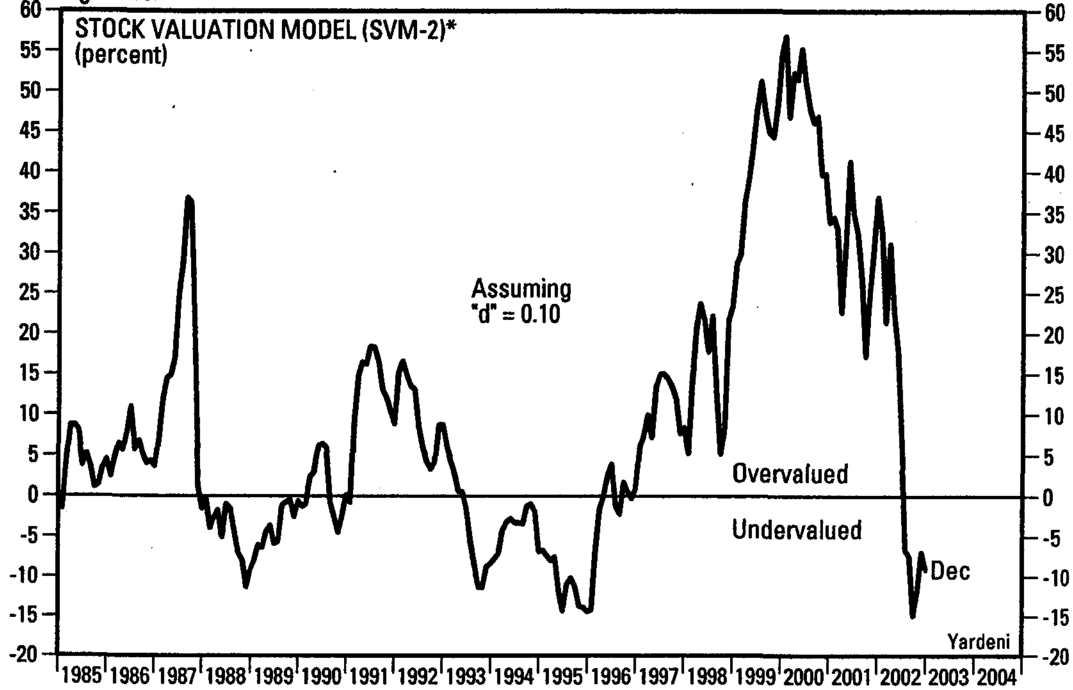
Figure 9.



* Fair value is 12-month forward consensus expected S&P 500 operating earnings per share divided by the difference between Moody's A-rated corporate bond yield less the fraction (as shown above) of 5-year consensus expected earnings growth.
Source: Thomson Financial.

According to SVM-2, stocks were 9.1% undervalued during December.

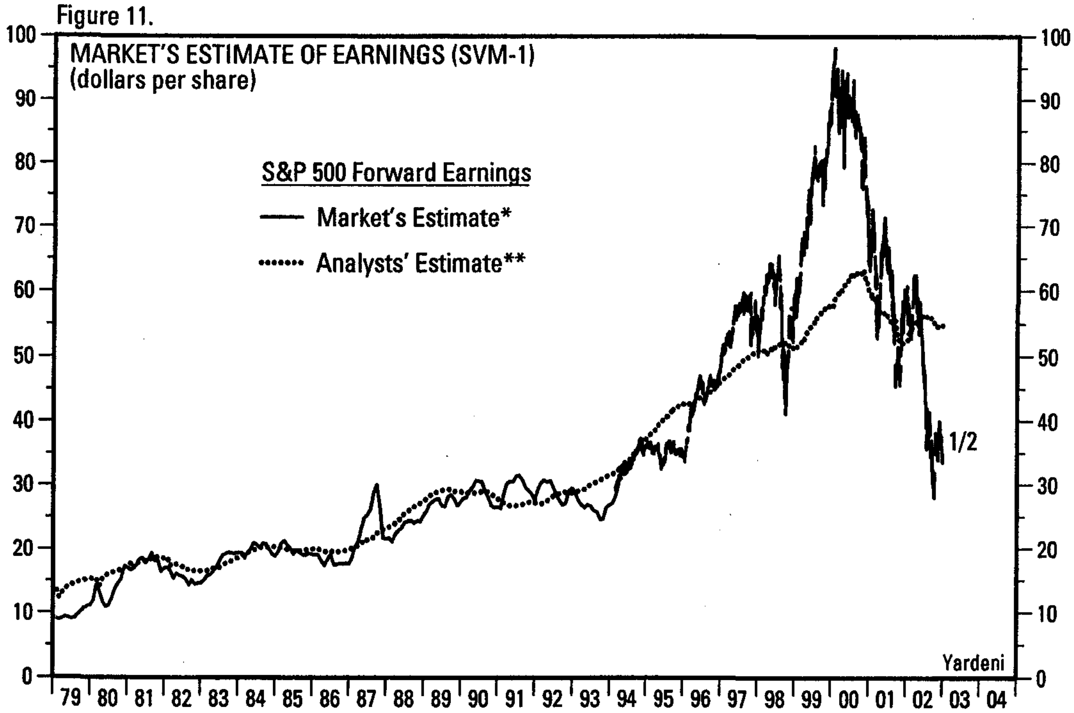
Figure 10.



* Ratio of S&P 500 index to its fair value--i.e., 12-month forward consensus expected S&P 500 operating earnings per share divided by difference between Moody's A-rated corporate bond yield less fraction (0.10) of 5-year consensus expected earnings growth.
Source: Thomson Financial.

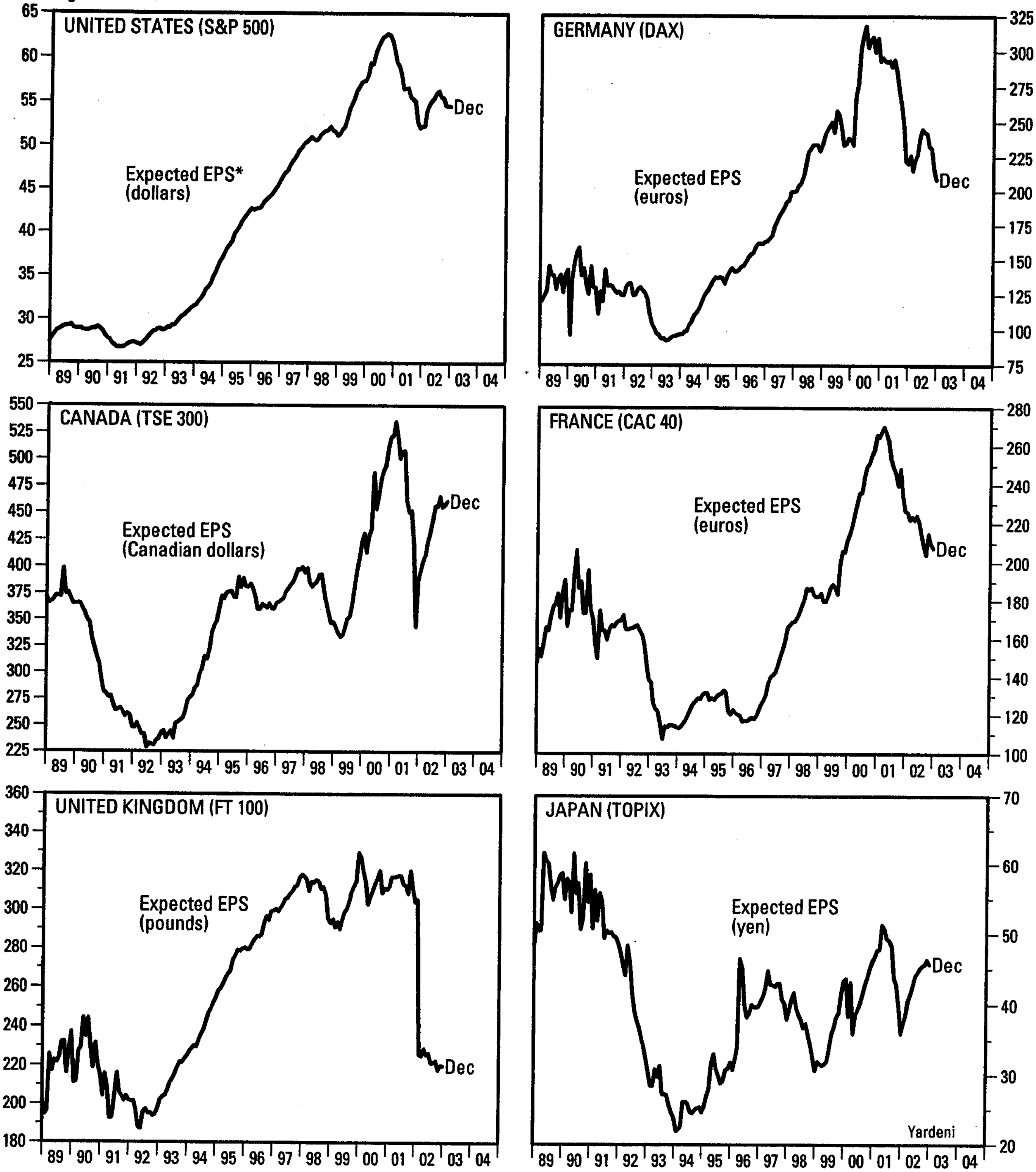
If stocks are always fairly valued, then the market's earnings estimate is currently 32.5% below analysts' consensus.

Figure 11.



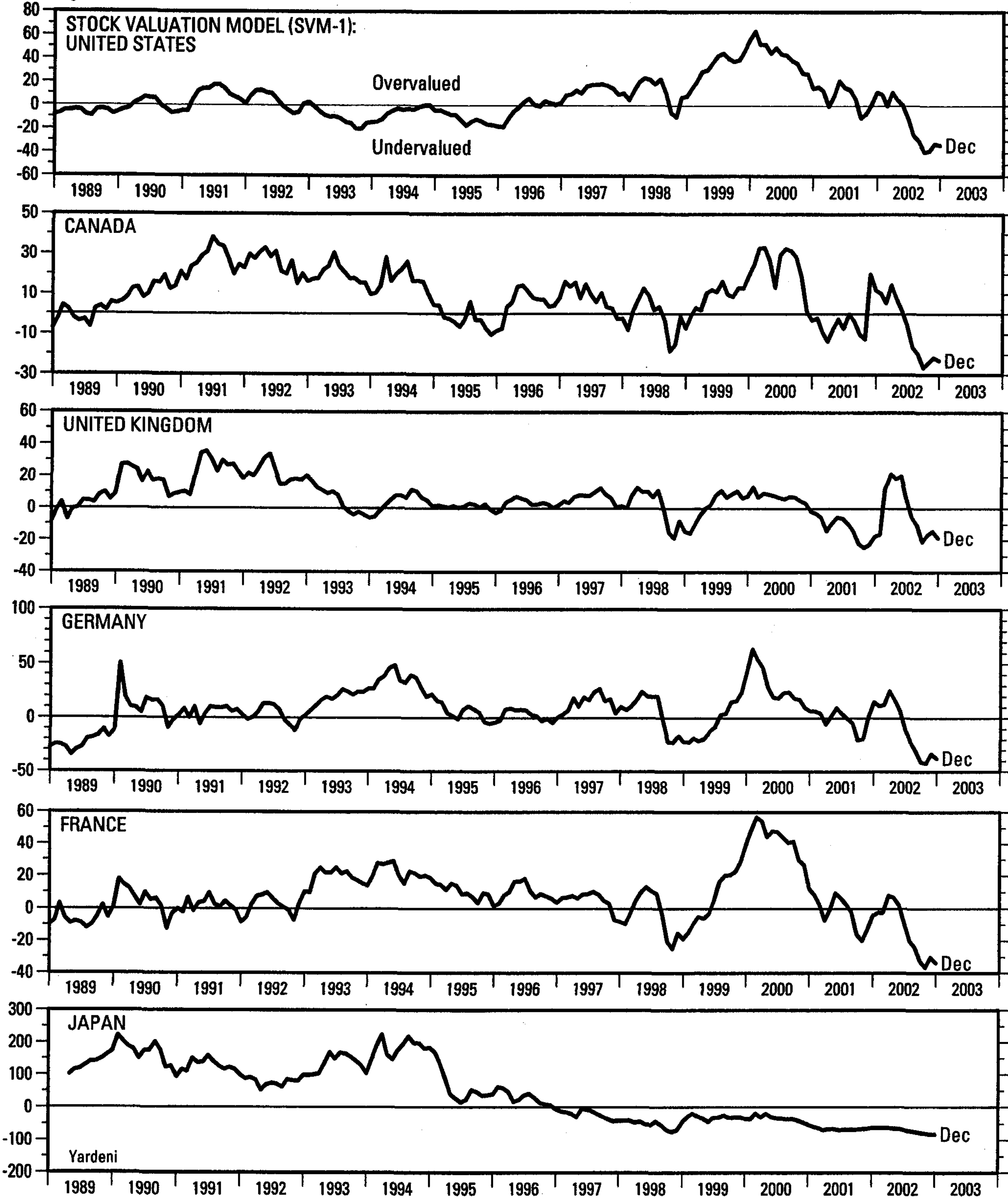
* S&P 500 index multiplied by ten-year government bond yield. Monthly through March 1994, weekly after.
** 12-month forward consensus expected S&P 500 operating earnings per share. Monthly through March 1994, weekly after.
Source: Standard & Poor's Corporation and Thomson Financial.

Figure 12.



* 12-month forward consensus expected operating earnings per share.
Source: Thomson Financial.

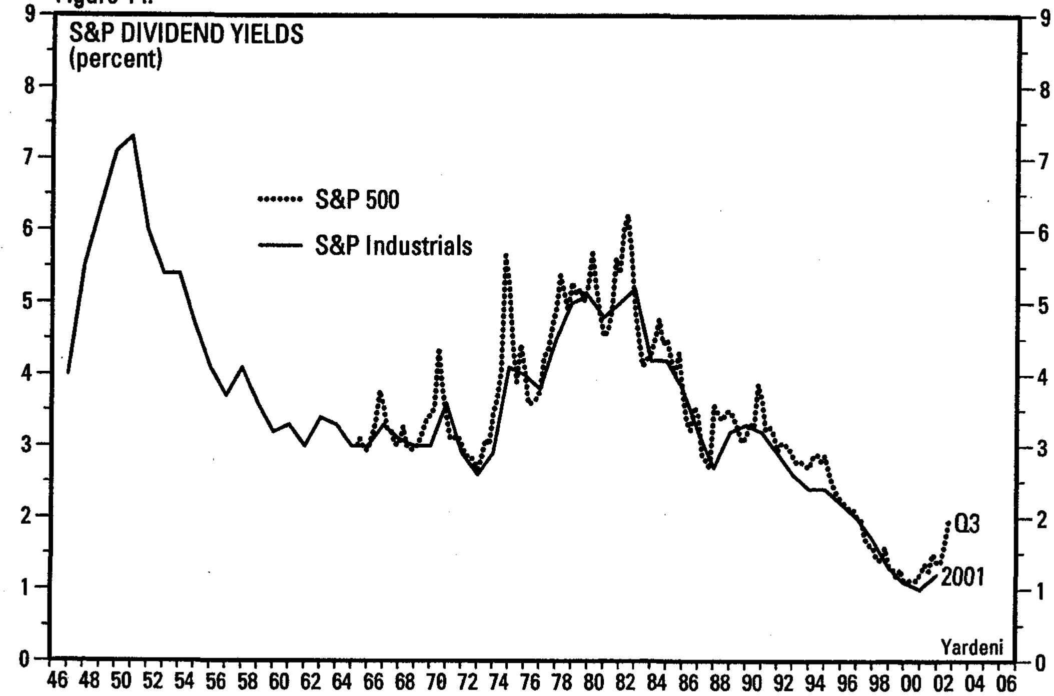
Figure 13.



Source: Thomson Financial.

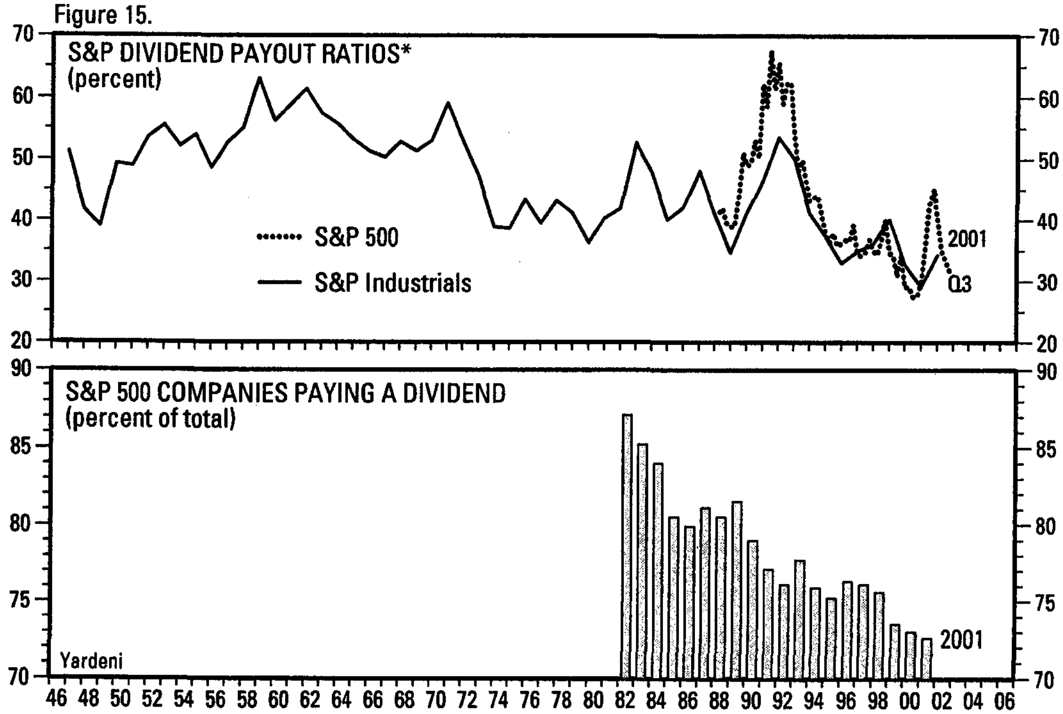
Dividend yield and dividend payout ratio fell sharply since the early 1980s partly because the percentage of S&P 500 companies paying any dividends at all dropped from 87% in 1982 to 73% in 2001.

Figure 14.



Source: Standard & Poor's Corporation.

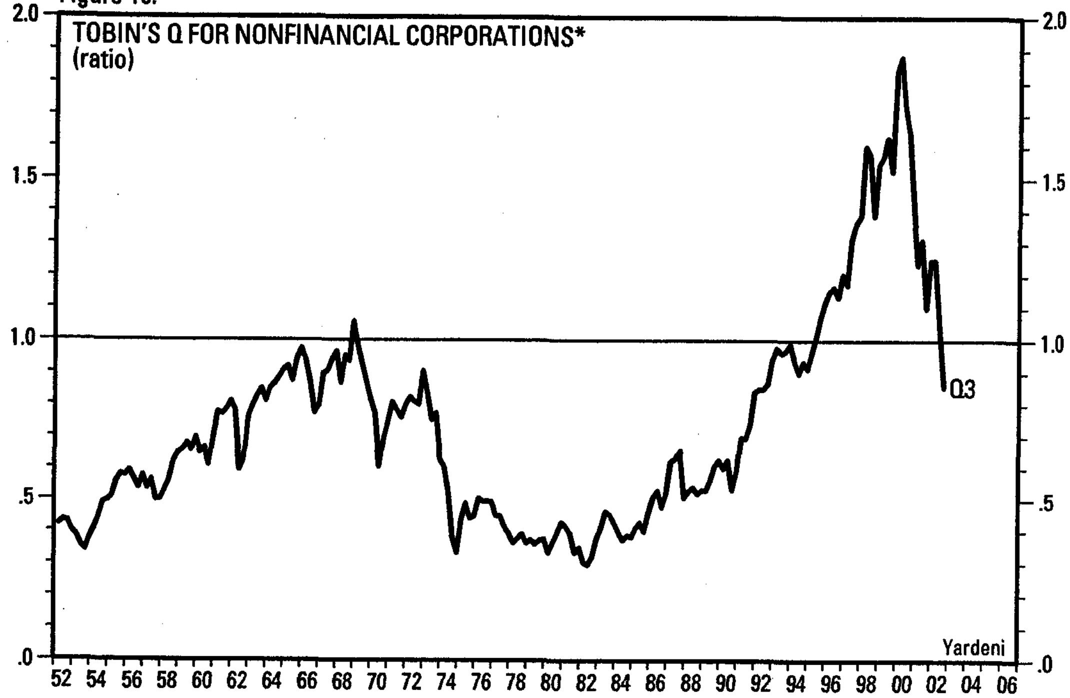
Figure 15.



* Total dividends divided by total earnings.
Source: Standard & Poor's Corporation and FactSet.

Tobin's Q has limited value as a stock valuation model, though it did indicate significant overvaluation during late 1990s, as did SVM-1 and SVM-2.

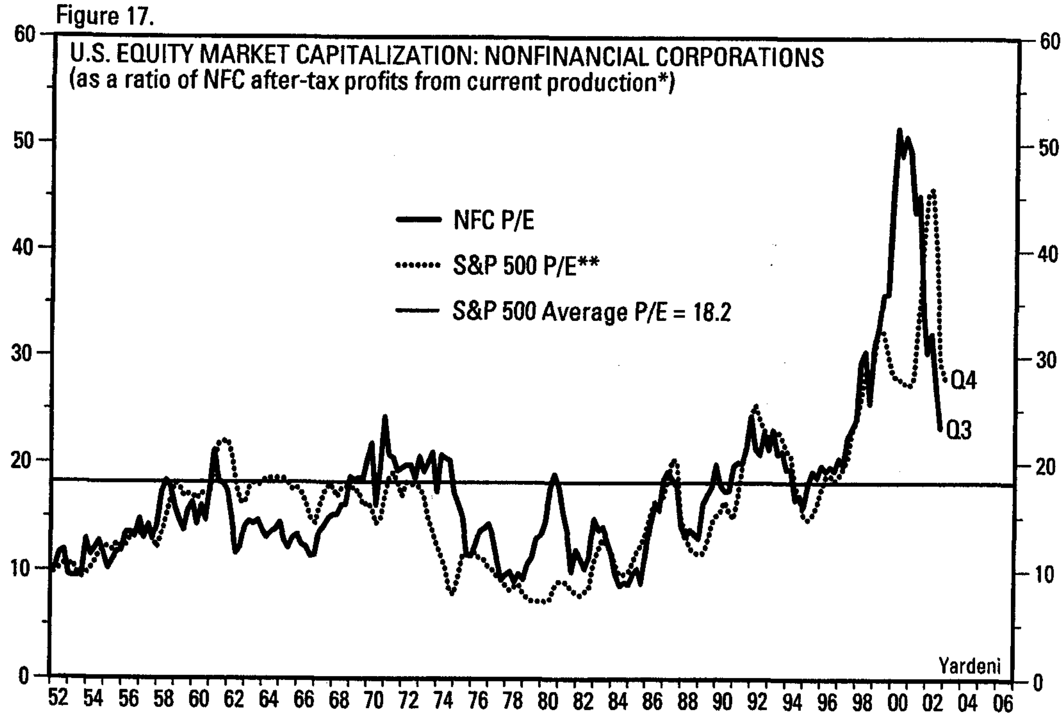
Figure 16.



* Ratio of market value of equities to net worth at market value, which includes real estate at market value and equipment and software and inventories at replacement cost.
Source: Federal Reserve Board, Flow of Funds Accounts.

Reversion-to-the-mean models shouldn't be ignored.

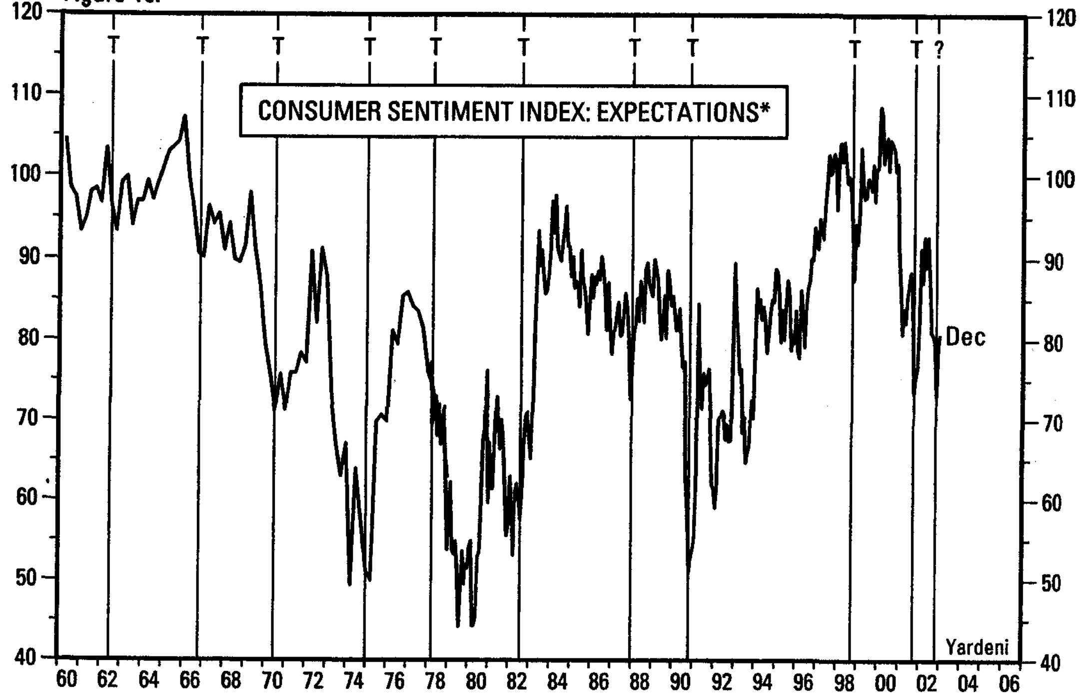
Figure 17.



* Including IVA and CCadj. These two adjustments restate the historical-cost basis used in profits tax accounting for inventory withdrawals and depreciation to the current-cost measures used in GDP.
** Using four-quarter trailing reported earnings.
Source: Federal Reserve Board, Flow of Funds Accounts and Standard & Poor's Corporation.

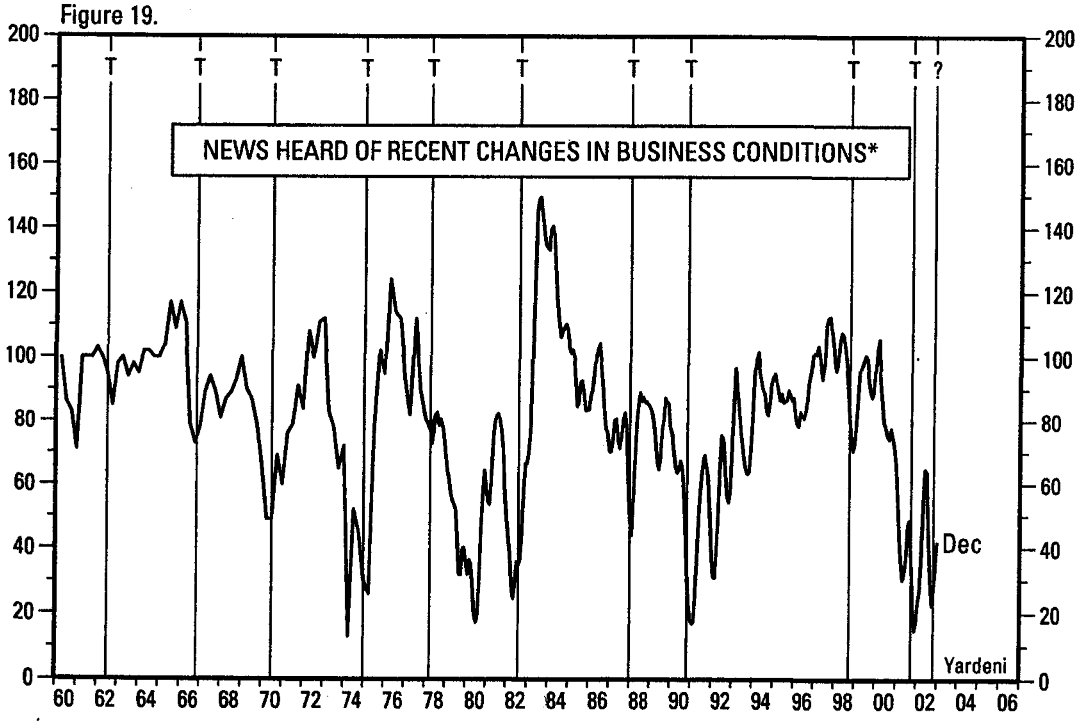
These two measures of consumer sentiment are especially good for confirming major market bottoms. Expectations are most depressed and news heard is most pessimistic at bottoms.

Figure 18.



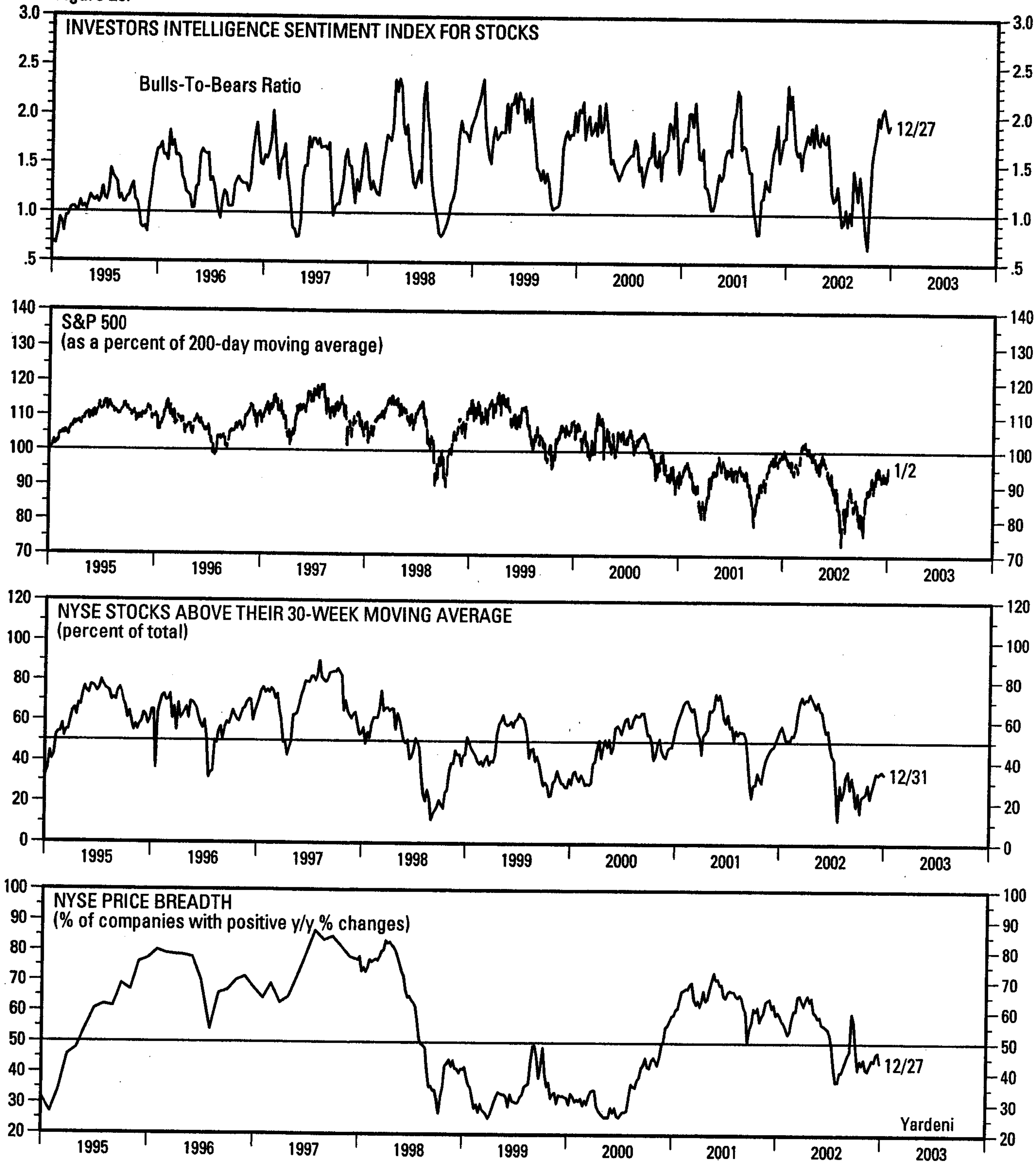
T = S&P 500 major cyclical trough.
* Quarterly through 1978, monthly therafter.
Source: Survey Research Center, University of Michigan.

Figure 19.



T = S&P 500 major cyclical trough.
* Favorable minus unfavorable plus 100. Quarterly through 1977, 3-month moving average thereafter.
Source: Survey Research Center, University of Michigan.

Figure 20.



Source: Standard & Poor's Corporation, Investors Intelligence, and FactSet.

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Additional information on the securities discussed herein is available upon request.

January 6, 2003

Kentucky-American Water Company
Case No. 2004-00103
Information Request Response to KAWC
Respondent: OAG Witness Dr. J. Randall Woolridge
Set II

KAWC-II-9. Footnote 24, page 68. Please provide a copy of the cited reference.

Response:

The requested document is included at attachment KAWC-II-9A.

Journal of Financial Economics 12 (1983) 371–386. North-Holland

ON COMPUTING MEAN RETURNS AND THE SMALL FIRM PREMIUM

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The mean return computational method has a substantial effect on the estimated small firm premium. The buy-and-hold method, which best mimics actual investment experience, produces an estimated small-firm premium only one-half as large as the arithmetic and re-balanced methods which are often used in empirical studies. Similar biases can be expected in mean returns when securities are classified by any variable related to trading volume.

1. Introduction

There is a potentially serious problem in estimating expected return differences between small and large firms. Even with exactly the same sample observations, the method used to compute sample mean returns can have a substantial effect on the estimates.

With an arithmetic computational method, daily returns on individual stocks are averaged across both firms and days to obtain the mean daily return on an equally-weighted portfolio; then the portfolio's mean daily return is compounded to obtain an estimate of the expected return over a longer interval. With a buy-and-hold method, individual stock returns are first obtained for the longer interval by linking together the daily individual returns; then an equally-weighted portfolio's mean return is computed by averaging the longer-term (individual) returns.

Defining a 'longer interval' as one year, the arithmetic method produces an average annual return difference of 14.9 percent between AMEX and NYSE stocks[1] over the 19 complete calendar years, 1963–1981 inclusive. The buy-and-hold method gives an annual return difference of only 7.45 percent. Assuming that annual returns are statistically independent, the arithmetic

*Comments and suggestions by Gordon Alexander, Kenneth French, Stephen Ross and the referee, Allan Kleidon, are gratefully acknowledged.

[1]The effect of smallness can be measured by the difference in returns of stock listed on the American Exchange (AMEX) and the New York Exchange (NYSE) because AMEX issues are, on average, much smaller than NYSE issues. Most of the results presented here are based on the AMEX–NYSE differential because it is convenient and easy to use. Some confirmatory results based directly on measured size will also be presented.

method's return differential had an associated t-statistic of 3.07 while the buy-and-hold method yielded a t-statistic of 1.53.

Speculation on possible causes of the small firm premium has occupied the attention of many finance theorists over the past few years; but perhaps this attention has been premature. If the estimated small firm premium can be cut in half simply by compounding individual returns before averaging them, some consideration should be given to whether the magnitude of the true premium is really all that large. The various explanations for the premium offered so far would become more plausible if the premium is actually smaller than has been previously reported.

This paper investigates why the mean return computational method can be such a significant choice in some empirical research. The reason seems to be that individual asset returns are not as well-behaved as we might like. Individual assets do not trade continuously and there are significant trading costs. In some empirical studies, the effect of these factors might be safely ignored; but when the object of investigation is related to trading volume (and thus to trading frequency and trading costs), there can be measurement problems. Firm size is related to trading volume and it is used as an example throughout the paper. Other variables related to size and to trading, such as dividend yield, price/earnings ratio, and beta, could also present similar empirical difficulties. Section 2 gives a brief theoretical discussion of mean return computational methods and section 3 presents details of the empirical results for small firm premia.

2. Compounding and the bias in mean return calculation

2.1. Formulae for computing mean returns

To elucidate the differences in mean return computation and explain why they might produce different results, consider a sample of N securities, each having returns observed for T periods. Let R_{it} be the value relative (1 + return), of security i in period t. Suppose also that investment results are reviewed every τ periods. For example, if data were available daily but returns were to be reviewed every month, we would have $\tau \cong 21$ since there are usually about 21 trading days per month.

Two alternative methods of computing the mean equally-weighted return over the review period can be written algebraically as

$$\bar{R}_{AR} = \left[\frac{1}{N \cdot \tau} \sum_i \sum_t R_{it}\right]^{\tau}, \tag{1}$$

$$\bar{R}_{BH} = \frac{1}{N} \sum_i \left[\prod_t R_{it}\right], \tag{2}$$

where the subscripts 'AR' and 'BH' denote 'arithmetic' and 'buy-and-hold', respectively. These labels are intended to portray the sense of the computation method. The first method (1) is simply an arithmetic mean raised to the τth power while the second method gives the actual investment results an investor would achieve from buying equal dollar amounts of N securities and holding the shares for τ periods.

There is also a third possible definition of mean return,

$$\bar{R}_{RB}=\prod_t\left[\frac{1}{N}\sum_i R_{it}\right], \tag{3}$$

where the subscript 'RB' stands for 'rebalanced'. This would be the actual investment return (ignoring transactions costs) on a portfolio which begins with equal investments in the N securities and *maintains* equal investments by rebalancing at the end of each period, $t=1,\ldots,\tau$.

To compare results over different review periods, we must choose some typical and familiar calendar interval, say a year, and express the results as percentage returns over that common calendar interval. In the tables below, annualization is accomplished and reported for 'linked' returns; the review period returns within each calendar year are simply multiplied together (or linked) in order to obtain an annual return.[2] Linked annualization includes *every* daily observation in some review period during the year. This assures that in any comparison of the results across review periods, the observed differences are due to review period alone and cannot be ascribed to slightly different sample observations.

The next two subsections investigate some properties of these sample mean returns. Subsection 2.2 derives their expected values under the assumption of temporally independent individual asset returns. Subsection 2.3 then examines the effect of intertemporal dependence.

[2]The exact formulae for linked returns can be written as follows. Let $\bar{R}_m(y,\tau)$ denote the mean annualized linked return for year y ($y=1,\ldots,Y$) using a review period whose length is τ trading days and using method ($m=$ BH, AR, RB), to compute the review period returns. Then,

$$\bar{R}_{BH}(y,\tau)=\prod_{j=(y-1)k_\tau+1}^{y\cdot k_\tau}\left[\frac{1}{N}\sum_i\prod_{t=(j-1)\tau+1}^{j\cdot\tau}(R_{it})\right],$$

$$\bar{R}_{AR}(y,\tau)=\prod_{j=(y-1)k_\tau+1}^{y\cdot k_\tau}\left[\frac{1}{N\cdot\tau}\sum_i\sum_{t=(j-1)\tau+1}^{j\cdot\tau}R_{it}\right]^\tau,$$

$$\bar{R}_{RB}(y,\tau)=\prod_{j=(y-1)k_\tau+1}^{y\cdot k_\tau}\left\{\prod_{t=(j-1)\tau+1}^{j\cdot\tau}\left[\frac{1}{N}\sum_i R_{it}\right]\right\},$$

where $k_\tau=T/(Y\cdot\tau)$ is the number of review periods per year and T is the total number of trading days in the entire sample. When returns are reviewed in natural calendar intervals such as months, the review period cannot be a fixed number of trading days and thus τ in the formulae above varies slightly with the actual number of trading days.

2.3. Sample mean return biases with temporal independence

Following Blume (1974), assume that each individual asset return is drawn from a stationary distribution with temporally independent disturbances; that is,

$$\tilde{R}_{it} = \mu_i + \tilde{\varepsilon}_{it}, \quad \forall i, \tag{4}$$

with $E(\tilde{R}_{it}) = \mu_i$, a constant for all t, and where the unexpected return, $\tilde{\varepsilon}_{it}$, satisfies $\text{cov}(\tilde{\varepsilon}_{i,t}, \tilde{\varepsilon}_{i,t-j}) = 0$ for $j \neq 0$.

The expected value of the arithmetic mean (1) can be expressed as

$$E(\bar{R}_{AR}) = E\left[\left(\frac{1}{N}\sum_i \mu_i + \bar{h}\right)^{\tau}\right], \tag{5}$$

where

$$\bar{h} \equiv \frac{1}{N \cdot \tau} \sum_i \sum_t \tilde{\varepsilon}_{it}$$

is the average disturbance on the equally-weighted portfolio over the sample review period τ.

The expected value of the buy-and-hold mean (2) is

$$E(\bar{R}_{BH}) = \frac{1}{N}\sum_i \left[E \prod_t (\mu_i + \tilde{\varepsilon}_{it})\right] = \frac{1}{N}\sum_i (\mu_i^{\tau}). \tag{6}$$

This follows since the expectation can be taken inside the product with independent returns and since $E(\tilde{\varepsilon}) = 0$, by definition.

The rebalancing method (3) produces a mean return whose expectation is

$$E(\bar{R}_{RB}) = \prod_t \left[\frac{1}{N}\sum_i \mu_i\right] = \left(\frac{1}{N}\sum_i \mu_i\right)^{\tau}, \tag{7}$$

where, again, the expectation can be taken inside the product because of time independence.

Expressions (5), (6) and (7) imply that the three different mean return definitions do not produce the same results. By Jensen's inequality,

$$E(\bar{R}_{AR}) \geqq E(\bar{R}_{RB}),$$ [3]

[3] Jensen's inequality for a random variable $\tilde{x}$ and a convex function $f(x)$ is $E[f(\tilde{x})] \geqq f[E(x)]$. Let $\tilde{x} \equiv (1/N)\sum_i \mu_i + \bar{h}$, then $f(\tilde{x}) = \tilde{x}^{\tau}$ is convex since $\tau > 1$. $E(\bar{R}_{AR}) > E(\bar{R}_{RB})$ follows immediately from (5) and (7) since $E(\bar{h}) = 0$.

with strict inequality if $\operatorname{var}(\tilde{h}) > 0$, and

$$\mathrm{E}(\bar{R}_{BH}) \geqq \mathrm{E}(\bar{R}_{RB}),^4$$

with strict inequality if $N > 1$ and at least two assets have different returns. Since we generally have some randomness $[\operatorname{var}(\tilde{h}) > 0]$, and many securities, $(N > 1)$, the rebalanced method generally should produce lower mean returns than either the arithmetic or the buy-and-hold method, provided that returns are temporally independent.

The relation between the buy-and-hold and arithmetic means is more complex; and, indeed, neither is invariably smaller than the other. The larger the cross-sectional dispersion of individual expected returns, the larger $\mathrm{E}(\bar{R}_{BH})$ relative to $\mathrm{E}(\bar{R}_{AR})$. But there is an offsetting influence: the larger the intertemporal dispersion of unexpected returns $(\tilde{h})$, the larger $\mathrm{E}(\bar{R}_{AR})$ relative to $\mathrm{E}(\bar{R}_{BH})$.[5] Their relation in a given sample depends, therefore, on the characteristics of the underlying individual returns.

2.3. *Time series dependence and its effect on estimated expected returns*

The effect of serial dependence is seen most easily by examining expected mean returns when the review period is doubled, say from daily to bi-daily or from bi-weekly to monthly. Assume first that returns are collected for the shorter review period and then let $\tau = 2$ (a doubling of the period). Over the doubled review period, the three mean returns are

$$\bar{R}_{AR} = \left[\frac{1}{N}\sum_i\left(\mu_i + \frac{\varepsilon_{i1} + \varepsilon_{i2}}{2}\right)\right]^2, \tag{8}$$

[4] Define $f(\mu_i) = \mu_i^\tau$, a convex function for $\tau > 1$. With $1/N$ used as a (pseudo) probability, $\mathrm{E}(\bar{R}_{BH}) \geqq \mathrm{E}(\bar{R}_{RB})$ follows immediately from (6) and (7). (Cf. footnote 3.) Strict inequality holds if at least two μ_i's are different. [This result was noted by Cheng and Deets in (1971).]

The inequality above grows with the cross-sectional dispersion in μ_i, ceteris paribus. To prove this, expand μ_i^τ in a Taylor series about $\bar{\mu} \equiv (1/N)\sum_i \mu_i$; the second-order term is a positive function of the cross-sectional variance in μ_i. If μ_i were cross-sectionally normally distributed, the variance *alone* would determine the size of the inequality.

[5] This can be confirmed by using a Taylor series expansion of $\mathrm{E}(\bar{R}_{AR})$. Define $\bar{\mu} = (1/N)\sum_i \mu_i$; then

$$\mathrm{E}(\bar{R}_{AR}) = \bar{\mu}^\tau \mathrm{E}\left[1 + \frac{\tilde{h}^2}{2}(\tau)(\tau-1)\bar{\mu}^{-2} + \frac{\tilde{h}^3}{3!}(\tau)(\tau-1)(\tau-2)\bar{\mu}^{-3} + \ldots + \tilde{h}^\tau \bar{\mu}^{-\tau}\right].$$

Jensen's inequality (see footnote 4 above), implies that $\mathrm{E}(\bar{R}_{BH}) > \bar{\mu}^\tau$ with the inequality being larger the larger the cross-sectional variance in μ_i. But the term in brackets just above shows that $\mathrm{E}(\bar{R}_{AR})$ increases with the higher moments of $\tilde{h}$ (since $\bar{\mu}$ is strictly positive). For example, the second term in brackets involves the variance of $\tilde{h}$. Conceivably, this term could more than offset the cross-sectional variance in μ_i. If the unexpected arithmetic portfolio return h happens to be normally-distributed, the expression above simplifies to $\mathrm{E}(\bar{R}_{AR}) = \bar{\mu}^\tau[1 + k \cdot \operatorname{var}(\tilde{h})]$ with the constant $k > 0$. In this case, there is a simple and direct tradeoff between the cross-sectional variance in expected return, μ_i, and the variance of the unexpected portfolio return, $\tilde{h}$.

$$\bar{R}_{BH}=\frac{1}{N}\sum_i[(\mu_i+\varepsilon_{i1})(\mu_i+\varepsilon_{i2})], \tag{9}$$

$$\bar{R}_{RB}=\left[\frac{1}{N}\sum_i(\mu_i+\varepsilon_{i1})\right]\left[\frac{1}{N}\sum_i(\mu_i+\varepsilon_{i2})\right], \tag{10}$$

where $R_{it}=\mu_i+\varepsilon_{it}$ is the observed return on individual stock i ($i=1,\ldots,N$) in period t, and μ_i is i's single-period (i.e., shorter review period) expected return.

For notational convenience, define the cross-sectional averages

$$\bar{\mu}=\frac{1}{N}\sum_i\mu_i \quad \text{and} \quad \bar{\varepsilon}_t=\frac{1}{N}\sum_i\varepsilon_{it}.$$

Then the three mean returns have expected values,

$$\mathrm{E}(\bar{R}_{AR})=\bar{\mu}^2+\tfrac{1}{2}(\sigma^2_{\bar{\varepsilon}}+\sigma_{\bar{\varepsilon}_1,\bar{\varepsilon}_2}), \tag{11}$$

$$\mathrm{E}(\bar{R}_{BH})=\frac{1}{N}\sum_i\mu_i^2+\frac{1}{N}\sum_i\sigma_{\varepsilon_{i1},\varepsilon_{i2}}, \tag{12}$$

$$\mathrm{E}(\bar{R}_{RB})=\bar{\mu}^2+\sigma_{\bar{\varepsilon}_1,\bar{\varepsilon}_2}, \tag{13}$$

where σ^2_x is the variance of x and $\sigma_{x,y}$ is the covariance of x and y.

Even with serial dependence, the expected arithmetic mean still exceeds the expected rebalanced mean in all circumstances since,

$$\mathrm{E}(\bar{R}_{AR}-\bar{R}_{RB})=\tfrac{1}{2}(\sigma^2_{\bar{\varepsilon}}-\sigma_{\bar{\varepsilon}_1,\bar{\varepsilon}_2})>0. \tag{14}$$

Comparing the buy-and-hold means and the rebalanced means, we have

$$\mathrm{E}(\bar{R}_{BH}-\bar{R}_{RB})=\sigma^2_{\mu_i}+\left(\frac{1}{N}\sum_i\sigma_{\varepsilon_{i1},\varepsilon_{i2}}-\sigma_{\bar{\varepsilon}_1,\bar{\varepsilon}_2}\right).$$

With no serial dependence in the ε's, the term in parentheses is zero and the BH mean would exceed the RB mean by the cross-sectional variance in expected individual returns.

However, with negative serial dependence in unexpected individual returns (ε_{i1} and ε_{i2}) or positive dependence in portfolio returns ($\bar{\varepsilon}_1$ and $\bar{\varepsilon}_2$), the rebalanced mean would become larger; enough such dependence could conceivably render it larger that the buy-and-hold mean. Since the expected arithmetic mean exceeds the expected rebalanced mean, it too could be larger than the BH mean with enough serial dependence of the right type.

There is some reason to anticipate just this type of serial dependence because of the intertemporal characteristics of individual returns. Scholes and Williams (1977, pp. 313–314) explain that because of non-synchronous trading individual assets display first-order *negative* serial dependence while diversified portfolios display *positive* dependence. A difference in the sign of serial dependence between individual assets and portfolios is relevant here because buy-and-hold (BH) means are mainly affected by individual asset serial dependence [see (12)], while the arithmetic (AR) and rebalanced (RB) means are affected by portfolio serial dependence [see (11) and (13)]. The Scholes/Williams explanation implies that BH means would tend to fall as review period lengthens while the AR and RB means would tend to rise.

There is also negative serial dependence induced in very short-term returns because of the institutional arrangement of trading. Neiderhoffer and Osborne (1966) pointed out that negative serial dependence should be anticipated when a market maker is involved in most transactions (because successive transactions are conducted at either the bid or the asked price).[6]

First-order negative serial dependence in individual returns has the effect of widening the disparity between the buy-and-hold mean and the arithmetic and rebalanced means as the review period lengthens. This follows from the fact that a doubling of the review period introduces serial covariance terms in *addition* to those already present. However, the marginal effect of lengthening the review period should probably diminish as the review period becomes longer; the effect on measured mean return should be greater when changing from, say, a daily to a weekly review period than from a monthly to an annual period. The exact impact of serial dependence can, of course, only be determined empirically and we now turn to an examination of the data.

3. The empirical small firm premium

3.1. Results

In the previous section, we found that the computational formula for sample mean returns can affect the estimated expected return. The buy-and-hold (BH) mean (2) gives an unbiased estimate of the holding period return on a realistic portfolio. The rebalanced (RB) mean (3), gives an unbiased estimate of return for its strategy but it is not realistic if the period is short since rebalancing is so costly. Except under a fortuitous combination of circumstances, the arithmetic (AR) mean (3) gives a biased estimate of *both* the rebalanced and the buy-and-hold investment returns.

[6]A paper by Blume and Stambaugh (1983), which came to my attention after the first version of this paper was written, investigates this explanation for serial dependence in detail. They find empirical results very similar to those reported here. See also Cohen et al. (1979).

Although the arithmetic and rebalanced methods of calculating the mean return probably do not portray realistic investment experience, the small-firm premium is calculated as the *difference* between the two mean returns and one might hope that the improper portrayal in these methods would cancel. Unfortunately, this is not likely for several reasons. The intertemporal variance in the portfolio disturbance, $\bar{\eta}$, and the cross-sectional variance in individual security expected returns, μ_{i}, will not be the same in samples of large and small firms. The disturbance, $\bar{\eta}$, will almost certainly have a larger variance for portfolios of small firms while the cross-sectional variances of μ_i within large- and small-firm portfolios could conceivably differ in either direction. Furthermore, serial dependence has an effect which is stronger for stocks with lower trading volumes and thus with less synchronous trading and with larger bid/ask spreads.

Empirical evidence is reported in table 1. Small Firm Premia (AMEX–NYSE) are given for the 19 complete calendar years, 1963–1981, according to the method of computation and the 'review' period. As explained earlier, the 'review' period refers to the rebalancing interval for buy-and-hold returns. For example, with a monthly review period, an equal allocation is made to stocks listed on the first day of the month and the original positions are held until the end of the month. This is repeated for each calendar month of the sample. The daily rebalancing method uses the same available returns, but it re-initializes equal positions every day during the month. The arithmetic method simply averages the same available returns during the month.

In order to compare results across the different review periods, returns are annualized by linking together the review period returns obtained during the calendar year.[7] Thus, there are 19 annual observations (one for each calendar year, 1963–81), regardless of the review period.[8] Means and t-statistics are calculated from the 19 annual returns differences between exchanges; t-

[7]See footnote 2 for exact computational formulae.

[8]Daily and bi-daily returns are over trading day intervals, while weekly and longer returns use actual calendar intervals. In the weekly case, the first week of the year ends on the same day of the week as the last trading day of the previous year, say Thursday for a given year. Then weekly returns are computed from Thursday to Thursday during that year. If the year does not terminate on a Thursday trading day, the last 'weekly' return of the year is over the remaining fraction of a calendar week. This method of year-end padding was used to ensure that every daily return during a year was included, regardless of the review period. Only the bi-daily, weekly, and bi-weekly returns are subject to such padding because the other intervals are evenly divisible into years.

Weekly returns are not always for five trading day intervals. During 1968, the exchanges were closed on Wednesdays for part of the year so that a week was composed of only four trading days. Holidays are also a problem for weekly returns; if the calendar week ended on a holiday, the return was computed through the next trading day. Then the subsequent week's return covered four trading days. Bi-weekly returns were treated identically to weekly returns with respect to year-end padding, holidays, and exchange closings.

Table 1

The small firm premium as measured by the difference in returns between American Exchange and New York Exchange listed stocks, 1963–1981 (basic data are daily, January 2, 1963 — December 31, 1981).

Review period[a] (number of review periods in sample)	Return computation method[b]		
	Buy-and-hold (BH)	Arithmetic (AR)	Daily rebalancing (RB)
	AMEX–NYSE mean return differential (% per annum)[c]		
Daily (4767)	14.9 (3.16) [7.76]	14.9 (3.16) [7.76]	14.9 (3.16) [7.76]
Bi-daily (2389)	12.3 (2.64) [5.58]	14.9 (3.16) [7.06]	14.8 (3.15) [7.01]
Weekly (992)	9.81 (2.16) [3.35]	14.8 (3.15) [5.64]	14.7 (3.14) [5.62]
Bi-weekly (498)	8.27 (1.84) [2.46]	14.9 (3.14) [5.09]	14.7 (3.13) [5.07]
Monthly (228)	7.06 (1.58) [1.82]	14.9 (3.14) [4.40]	14.7 (3.11) [4.38]
Quarterly (76)	6.42 (1.43) [1.67]	15.0 (3.15) [3.88]	14.8 (3.12) [3.85]
Annual (19)	7.45 (1.53) [1.53]	15.1 (3.10) [3.10]	14.9 (3.07) [3.07]

[a]For the daily and bi-daily cases, one- and two-trading-day intervals were used respectively. For all other cases, actual calendar intervals were used. (In the weekly and bi-weekly cases, a residual interval was necessary to fill out each calendar year). All returns were compounded to an annual basis by linking successive observations within each year (see footnote 2 of the text).

[b]The computation method follows expressions (1), (2) and (3) of the text. For interested readers, the author will gladly supply a mimeographed sheet containing details on the treatment of delisting and listing securities. The main feature of the treatment of new listings and delistings was to assure that all three mean return methods employed exactly the same sample observations.

[c]t-statistics based on the 19 annual (linked) observations are in parentheses; t-statistics based on the review period returns as independent observations are given in brackets. To understand the difference in the two reported t-statistics, consider the example of the daily review period of which there are 4767 in the sample. The t-statistic in brackets is calculated from these 4767 (daily) observations (mean *daily* return divided by standard error of mean daily return). The t-statistic in brackets is calculated from 19 annual observations, each annual observation having been calculated by linking together approximately 250 (4767/19) daily observations observed during that year. In calculating the review-period-based t-statistics for the weekly and bi-weekly cases, ten days were omitted; these ten days were the reminders of partial weeks at year end. It turned out that in 10 years of the 19, the year was exactly 52 weeks plus one trading day long. An earlier version of the paper, available on request, details the effect of omitting these single-day partial weeks. N.B. This is an issue only for the bracketed t-statistics. The linked annual returns include *every* sample day.

statistics are also given based on review period returns taken as independent observations.[9]

The results most like actual investment experience are those in the first column, buy-and-hold returns. Most actual portfolios pursue a buy-and-hold strategy within a given review period with only minor modifications induced by new information about particular individual issues. The results are frequently expressed on an annual percentage basis by comparing wealth levels at the ends of successive years, i.e., after linking sub-year results.

The review period seems to have little effect on the AR and RB means. The annual average difference in returns between AMEX and NYSE issues is about fifteen percent. But for the BH means, the review period has a large impact. Monthly and longer review periods give an AMEX–NYSE return differential of only around seven percent (and the t-statistic does not indicate an overwhelming probability that the differential is even positive). The drop in the BH mean with lengthening review period is statistically significant and so is the difference between the BH and the other means.[10]

[9]Note that the t-statistics in these tables are based on the assumption that the annual returns (t-statistics in parentheses) and review period returns (t-statistics in brackets) are temporally independent. The results indicate that the AR and RB returns are, in fact, close to independent while there is negative serial dependence in the BH returns. This implies that the t-statistics for the BH means are actually *understated*.

[10]A statistical test of the significance of the review period can be conducted by considering each year's mean difference, AMEX–NYSE, as an independent observation. Let $D_{m,y,\tau}$ be the difference for year y, review period τ, and the method m (m=BH, AR, RB). Then the time series mean of $D_{m,y,\tau}-D_{m,y,\tau'}$ ($\tau \neq \tau'$) can be tested for significance under the presumption that the years constitute independent observations. t-statistics for the AR and RB means, for all combinations of τ and τ', never indicated significance. Of the 42 combinations (21 for each mean AR and RB) none exceeded 2.0, five exceeded 1.5, and 28 were less than 1.0. In contrast, the t-statistics for the BH mean comparisons across review periods are given below:

Review period τ'	Review period τ: Daily	Bi-daily	Weekly	Bi-weekly	Monthly	Quarterly
Bi-daily	6.21					
Weekly	6.75	6.82				
Bi-weekly	7.67	8.37	10.8			
Monthly	8.11	8.89	11.3	9.82		
Quarterly	8.10	7.68	8.65	6.49	3.27	
Annual	5.08	4.42	2.81	1.04	−0.532	−1.67

All BH means are significantly different across-review periods except the annual mean versus the bi-weekly, monthly and quarterly means. Note that these table entries are not statistically independent of one another (they were all calculated from the same underlying data).

A similar procedure can be employed to test the statistical significance of mean computational method. The difference $D_{m,y,\tau}-D_{m',y,\tau}$ ($m \neq m'$) forms another time series across years. Based on 19 annual observations, t-statistics for the significance of this difference from zero are as follows:

Given that the BH results in table 1 are most likely to portray actual investment experience, we now turn to the interesting econometric question: What explains the observed pattern of means? To aid in answering this question, the mean returns for each exchange are presented separately in table 2. Notice that the pattern is not predicted by the expected values of the mean returns derived in section 2.2 under the assumption of temporally independent returns. With serial independence, the BH expected mean should be greater than the RB expected mean. The empirical results in table 2 show, however, that serial dependence must be present since $\bar{R}_{BH}$ falls below $\bar{R}_{RB}$ as the review period lengthens.

The arithmetic (AR) mean is larger than the rebalanced (RB) mean as was expected with or without serial dependence. However, these two means are very close and this suggests that serial dependence in *portfolio* returns is not much of an influence [Cf. eq. (14)]. Indeed, the strikingly different behavior of the BH means from the other two means indicates that negative serial dependence in individual securities is the dominant influence on the results.

In order to be certain that the AMEX–NYSE comparison measures the small firm effect properly, table 3 is presented. It contains results for the annual review period and for portfolios classified directly by size. Firm size was calculated as market capitalization (market price times number shares), at the end of each year, 1962–1980. Firms were assigned to fractiles based on market capitalization and their returns were calculated for the following year according to three mean return methods, BH, AR, and RB.

Not surprisingly, the results are consistent with the AMEX corresponding to lower size quintiles and the NYSE to higher quintiles. The overall implication is identical: viz., the estimated small firm premium is much smaller and less significant when mean returns are computed with the buy-

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Review period t	$m = AR$, $m' = BH$	$m = RB$, $m' = BH$	$m = AR$, $m' = RB$
	t-statistic for difference		
Bi-daily	6.82	6.30	1.47
Weekly	7.33	6.80	1.59
Bi-weekly	8.14	7.59	1.74
Monthly	8.44	7.90	2.17
Quarterly	8.21	7.69	2.72
Annual	5.85	5.48	3.16

No statistic was computed in the daily case because all three means are identical by construction in that case. Notice that the BH means are significantly smaller than the other two means for all review periods.

Although the difference between the AR and RB small firm premium is very small (cf. table 1), the AR mean premium is always larger and is significantly larger for monthly, quarterly and annual review periods. This is predicted by eq. (14); the AR mean grows with review period relative to the RB mean.

Table 2
Mean returns on NYSE and AMEX listed securities, 1963–1981.[a]

Review period	Buy-and-hold (BH) NYSE	Buy-and-hold (BH) AMEX	Arithmetic (AR) NYSE	Arithmetic (AR) AMEX	Daily rebalancing (RB) NYSE	Daily rebalancing (RB) AMEX
	Mean returns (% per Annum)					
Daily	17.24 (2.94) [5.09]	32.09 (3.29) [7.72]	17.24 (2.94) [5.09]	32.09 (3.29) [7.72]	17.24 (2.94) [5.09]	32.09 (3.29) [7.72]
Bi-daily	16.93 (2.89) [4.59]	29.23 (3.03) [6.25]	17.53 (2.98) [4.76]	32.42 (3.31) [6.96]	17.24 (2.94) [4.68]	32.09 (3.29) [6.88]
Weekly	16.38 (2.80) [4.47]	26.19 (2.78) [5.32]	17.79 (3.02) [4.81]	32.61 (3.34) [6.44]	17.26 (2.94) [4.68]	31.99 (3.28) [6.32]
Bi-weekly	15.86 (2.72) [4.29]	24.14 (2.58) [4.66]	17.95 (3.05) [4.71]	32.83 (3.36) [5.85]	17.29 (2.95) [4.58]	32.08 (3.28) [5.74]
Monthly	15.34 (2.65) [3.11]	22.39 (2.42) [3.08]	18.07 (3.07) [3.67]	32.96 (3.36) [4.54]	17.34 (2.95) [3.51]	32.08 (3.28) [4.41]
Quarterly	15.01 (2.63) [2.73]	21.42 (2.53) [2.62]	18.17 (3.09) [3.22]	33.17 (3.38) [3.84]	17.38 (2.96) [3.09]	32.19 (3.29) [3.73]
Annual	15.18 (2.69) [2.69]	22.63 (2.39) [2.39]	17.96 (3.11) [3.11]	33.07 (3.36) [3.36]	17.16 (2.98) [2.98]	32.03 (3.27) [3.27]

[a]See footnotes to table 1.

and-hold method than when means are computed with the AR and RB methods.

3.2. Implications for previous research and for the 'risk-adjusted' small firm premium

The implications of these findings for previously-published estimates of the small firm premium are: if the basic data were very short-term and arithmetic or rebalanced means were used, the estimated premium overstates the reward investors can expect from a buy-and-hold position in small firms. Papers by Reinganum (1981a, b, 1982) and Roll (1981) used daily data and arithmetic mean returns. Reinganum's (1982) paper gives monthly and quarterly returns but these were computed with the daily rebalancing method since the author states that '... these holding period returns are created by compounding the daily *portfolio* returns' (p. 34, emphasis added).

Table 3
Mean returns and small firm premia for portfolios classified by size[a] at year-end, 1963–1981, annual review period.

Size quintile	Return computation method[b] Buy-and-hold (BH)	Arithmetic (AR)	Daily rebalancing (RB)
	Mean return (% per annum)[c]		
Smallest	27.9 (2.42)	46.0 (3.68)	44.9 (3.61)
2	21.1 (2.51)	27.6 (3.15)	26.6 (3.04)
3	17.1 (2.41)	20.7 (2.86)	19.7 (2.73)
4	14.6 (2.53)	16.9 (2.89)	16.1 (2.75)
Largest	10.8 (2.50)	12.2 (2.85)	11.5 (2.68)
	Small firm premium, smallest–largest quintile (% per annum)		
	17.1 (1.88)	33.9 (3.47)	33.4 (3.46)
	Small firm premium, smallest–largest decile (% per annum)		
	22.8 (2.07)	49.1 (3.84)	48.3 (3.83)

[a]Firms are included in the kth size fractile if the closing price times the number of outstanding shares is ranked in that fractile among all listed AMEX and NYSE firms.

[b]The computation method follows expressions (1), (2) and (3) of the text. An unpublished appendix (available from the author) contains details on the treatment of listing and delisting.

[c]t-statistics based on 19 annual observations are in parentheses.

Papers with monthly returns are apparently much less subject to mean return estimation problems. Tables 1 and 2 show that there is little additional discrepancy between the BH and other means in going from monthly to annual data. The well-known paper by Banz (1981) used monthly data as did earlier papers on the closely-related stock price effect [Blume and Husic (1973), Bachrach and Galai (1979)]. Thus, it seems unlikely that the results presented in those papers will be much affected by the problem investigated here. In a more recent paper, Reinganum, (1983) used the buy-and-hold method and found results close to those reported above. Reinganum did not, however, contrast the buy-and-hold with other mean returns.

It is important to ascertain whether the *risk-adjusted* small firm premium is attributable solely to econometric problems. Is underestimation of risk for small firms [Roll (1981), Reinganum (1982)], combined with overestimation of expected returns, sufficient to induce the observed risk-adjusted premium; or is the premium really evidence of a misspecified capital asset pricing model (CAPM), perhaps because of omitted factors in the single index CAPM?

This is tantamount to asking whether the implicit CAPM market risk premium $\hat{p}$ ($\hat{p}=\hat{E}(R_{small}-R_{large})/(\hat{\beta}_{small}-\hat{\beta}_{large})$), is in a reasonable range. $\hat{p}$ was computed by Reinganum (1983) as 37.5 percent per annum using (a) buy-and-hold means on the smallest and largest deciles of NYSE and AMEX stocks, (b) Dimson's (1979) aggregated coefficient betas, (c) the value-weighted C.R.S.P. index and (d) daily data for 1963–1980. The return on the value-weighted index during this period was only about 9.5 percent, so $\hat{p}$ is grossly too large, thereby indicating a substantial risk-adjusted small firm premium.

The main problem with such a test was described some time ago [Roll (1977)]. Even if we make the dubious assumption that the value-weighted C.R.S.P. index is ex-ante mean/variance efficient, there is no necessity in the generalized Black (1972) C.A.P.M. that $E(\hat{p})=E(R_M-R_F)$. Instead, the model requires that $E(\hat{p})=E(R_M-R_Z)$ where Z is M's 'zero-beta' portfolio. Depending upon M's position on the efficient frontier, $E(R_Z)$ can be negative and large.

To illustrate the difference in inferences that can be obtained with a different index, I recomputed $\hat{p}$ using (a) buy-and-hold annual means on the smallest and largest deciles of NYSE and AMEX stocks, (b) simple OLS beta coefficients estimated from annual returns,[11] (c) the *equally*-weighted C.R.S.P. index, and (d) annual data for 1963–1981.

The beta estimates (t-statistics) were $\hat{\beta}_{small}=1.78$ (5.59), $\hat{\beta}_{large}=0.598$ (8.60). Using the estimated premium $\hat{E}(R_{small}-R_{large})=22.8\%$ from table 3, we have $\hat{p}=19.3$ percent. The actual ex post return on this market index was 15.3 percent, so $\hat{p}$ is still somewhat too high (thus indicating a risk-adjusted small-firm premium). Nevertheless, the discrepancy between a $\hat{p}$ of 19.3 and a market return of 15.3 is much less aberrant than the difference Reinganum (1983) reports between $\hat{p}=37.5$ and $\bar{R}_M=9.5$ percent.

It still seems that investigation of the observed small firm premium in the context of a more general asset pricing model would be a worthwhile endeavor; but estimation problems in expected returns and in simple risk parameters can explain much of the apparent anomaly.

[11]Instead of the Dimson aggregated coefficient betas, I used betas from annual data because of the now well-documented annual seasonal [Keim (1983), Roll (1983)], which has the potential to induce biases into any betas, including the Dimson type, when they are computed from non-yearly data.

5. Conclusion

Computing mean returns in order to estimate investment experience is not as easy as it sounds. Common stock data have serial dependence which, though seemingly slight, substantially affects the estimates obtained under alternate mean return computational methods. Investment experience is best portrayed by buy-and-hold portfolio returns but scholars often use arithmetic or rebalanced portfolio returns because they are easier to compute.

Perhaps this makes little difference for some studies; but if serial dependence differs systematically with the item being investigated, the computational method can be quite material.

For the small firm premium, as measured by the difference in mean returns of American Exchange and New York Exchange listed stocks, the buy-and-hold mean return difference is only about 7½ percent per annum (for 1963–81) while the rebalanced and arithmetic methods produce annual return differences *with the same stocks and time periods* of over 14 percent. The annual difference in returns between the smallest and largest size quintiles (deciles) is about 34 (49.1) percent using the rebalanced and arithmetic methods and about 17 (22.8) percent using the buy-and-hold method.

The annual small-firm premium is only marginally significant at usual significance levels if mean returns are measured with the buy-and-hold method.

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