EJM - 1

EXHIBIT EJM-1

Statisticians' Report

BELLSOUTH

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Victoria K. McHenry
General Counsel – LA

February 29, 2000

VIA FEDERAL EXPRESS

Ms. Susan Cowart
Louisiana Public Service Commission
Suite 1630
One American Place
Baton Rouge, LA 70825

RE: LPSC Docket No. U-22252-C
Louisiana Public Service Commission, ex parte
In re: BellSouth Telecommunications, Inc.
Service Quality Performance Measurements

Dear Ms. Cowart:

Enclosed are the original and one (1) copy each of the following documents to be filed into the record of the referenced matter:

1. Updated BellSouth SQM Report

2. Updated Statistician's Report

These items were not specifically included in the Commission's most recently issued Notice so I am unsure when they are due. In any event, we are providing them as soon as possible.

Sincerely,

Victoria K. McHenry

Victoria K. McHenry

VKM/as
Encs.

cc: Official Service List (w/enc.)(via email)

199343

DRAFT

Statistical Techniques For The Analysis And Comparison Of Performance Measurement Data

Submitted to Louisiana Public Service Commission (LPSC) Docket U-22252 Subdocket C

Revised February 28, 2000

I. Introduction and Scope

The Louisiana Public Service Commission (LPSC) staff has requested Drs. S. Hinkins, E. Mulrow, and F. Scheuren[1] of Ernst & Young LLP (consultants for BellSouth Telecommunications), and Dr. C. Mallows of AT&T Labs-Research to set out their views on the application of a statistical analysis to performance measurement data. The present report is intended to provide a detailed statistical report on appropriate methodology.

The setting for the analysis is crucial to the interpretation of any statistical significance that might be found. There is no doubt that, to quote the Commission staff, "statistical analysis can help reveal the likelihood that reported differences in an ILECs performance toward its retail customers and CLECs are due to underlying differences in behavior rather than random chance" (Staff Final Recommendation, LPSC Docket No. U-22252 - Subdocket C, dated August 12, 1998, pages 15 - 16).

To frame our presentation the next paragraph from the LPSC Docket U-22252 is quoted in its entirety.

> **"Statistical tests are effective in identifying those measurements where differences in performance exist. The tests themselves cannot identify the cause of the apparent differences. The differences may be due to a variety of reasons, including: 1) when the ILEC and CLEC processes being measured are actually different and should not be expected to produce the same result, 2) when the ILEC is employing discriminatory practices, or 3) when assumptions necessary for the statistical test to be valid are not being met."** (Ibid., page 16)

Apparent statistically significant differences in BellSouth and CLEC performance can arise when

- the ILEC and CLEC processes being measured are actually different and should not be expected to produce the same result
- the ILEC is employing discriminatory practices, or
- assumptions necessary for the statistical test to be valid are not being met.

[1] Dr. Scheuren is now a Senior Fellow at the Urban Institute.

To meet the Louisiana Commission's purpose, we will recommend techniques that are robust in the presence of possible assumption failure, carefully examine BellSouth Telecommunications (BST) and CLEC performance so "like" is compared only to "like," and are still able, in a highly efficient manner, to detect differences. Upon investigation any differences detected might lead to concerns about possible discriminatory practices.

The LPSC staff also states "that a uniform methodology which identifies those items which need to be measured, how they are to be measured, and how the results are to be reported is also desirable and would be beneficial to all parties" (Ibid., page 16). We agree with this goal as well, stipulating only that the use of a single method may not be desirable while a single methodology (or a set of methods) could be.

The statistical process for testing if CLEC and ILEC customers are being treated equally involves more than just a mathematical formula. Three key elements need to be considered before an appropriate decision process can be developed. These are

- the type of data,
- the type of comparison, and
- the type of performance measure.

When examining the various combinations of these elements, we find that there is a set of testing principles that can be applied uniformly. However, the statistical formulae that need to be used change as the situation changes.

To be responsive to the Commission, we have divided our discussion into four sections and five appendices. The contents of each of these are briefly mentioned below -- first for the main report and then for the extensive supporting appendix materials.

For the main report, this section (Section I) introduces our work and sets out the required scope. The next two sections (Sections II and III) discuss the type of comparisons that need to be identified, and the appropriate testing principles. The final section (Section IV) provides an overview of appropriate testing methodologies, based on what we have learned from our examination of BellSouth's performance measure data in Louisiana.

The five appendices provide technical details on the statistical calculations involved in the Truncated Z statistic (Appendix A), the implementation of the methodology for the trunk blocking performance measure (Appendix B), the calculations involved in computing the balancing critical value of a test (Appendix C), examples of ways to present the results using detailed statistical displays so that results can be audited (Appendix D), and the technical details involved in data trimming (Appendix E).

2. Data Considerations, Comparisons, and Measurement Types

This section makes general distinctions which apply to the performance measures. These distinctions will be important in the determination of appropriate methodologies.

Data Set Types. The type of statistical methodology used depends on the form of the data available. In general, there are two ways to classify the data used for performance measure comparisons. These are:

- transaction level data, and
- aggregated summaries.

Records in a transaction level data set represent a single transaction, e.g. an individual customer order, or the record of a specific trouble reported by a customer. This type of data set allows for deep like-to-like comparisons, and may also allow one to identify the root cause of a problem. A testing methodology needs to be carefully chosen so that it incorporates the comparison levels and does not cover up problem areas.

Records in an aggregated summary data set are typically summaries of related transactions For example, the total number of blocked calls in a trunk group during the noon hour of a day is a summary statistic. This type of data set may not contain as much information as a transaction level data set, and it therefore needs to be treated differently. While a general methodology may be determined for a transaction level data set, it may not be possible to do so for aggregated summaries. Testing methodology needs to be developed on a case-by-case basis.

Comparison Types. An ILEC's performance in providing services to CLEC customers is tested in one of two ways:

- by comparing CLEC performance to ILEC performance when a retail analog exists, or
- by comparing CLEC performance to a benchmark.

The testing methodologies for these two situations will have similarities, but there are differences that need to be understood.

Table 1 categorizes those performance measures that E&Y has examined by data type and comparison type. The table shows that five performance measures with retail analogs have transaction level data, while three others with retail analogs only have summary level data. No performance measures using benchmarks have been studied.

Table 1. Classification of Performance Measures by Data and Comparison Type (only measures previously examined by E&Y are included)

Level of Data	Comparison Type	
	Retail Analog	**Benchmark**
Transaction Level	Order Completion Interval Maintenance Average Duration % Missed Installations % Missed Repair Trouble Report Rate	No Measures Examined
Summary Level	Billing Timeliness OSS Response Interval Trunk Blocking	No Measures Examined

Measurement Types. The performance measures that will undergo testing are of four types: means, proportions (an average of a measure that takes on only the values of 0 or 1), rates, and ratios.

While all four have similar characteristics, proportions and rates are derived from count data while means and ratios are derived from interval measurements. Table 2 classifies the performance measures by the type of measurement.

Table 2: Classification of Performance Measures by Measurement Type

Mean	Proportion	Rate	Ratio
Order Completion Interval Maint. Ave. Duration OSS Response Interval	Percent Missed Installations Percent Missed Repairs Billing Timeliness Trunk Blocking	Trouble Report Rate	Billing Accuracy

3. Testing Principles

This section describes five general principles which the final methodology should satisfy:

1. *When possible, data should be compared at appropriate levels, e.g. wire center, time of month, dispatched, residential, new orders.*
2. *Each performance measure of interest should be summarized by one overall test statistic giving the decision maker a rule that determines whether a statistically significant difference exists.*
3. *The decision system must be developed so that it does not require intermediate manual intervention.*
4. *The testing methodology should balance Type I and Type II Error probabilities.*
5. *Trimming of extreme observations from BellSouth and CLEC distributions is needed in order to ensure that a fair comparison is made between performance measures.*

Like-to-Like Comparisons. *When possible, data should be compared at appropriate levels, e.g. wire center, time of month, dispatched, residential, new orders.*

In particular, to meet this goal the testing process should:

- Identify variables that may affect the performance measure.
- Record important confounding covariates.
- Adjust for the observed covariates in order to remove potential biases and to make the CLEC and the ILEC units as comparable as possible.

It is a well known principle that comparisons should be made on equal footing: apples-to-apples, oranges-to-oranges. Statistical techniques that are addressed in most text books usually assume that this is the case beforehand. Some higher level books address the issue of "designed experiments" and discuss appropriate ways to structure the data collection method so that the text books' formulae can be used in analyzing the data.

Performance measure testing does not involve data from a designed experiment. Rather, the data is obtained from an observational study. That being the case, one must impose a structure on the data after it is gathered in order to assure that fair comparisons are being made. For example, it is important to disaggregate the data to a fine level so that appropriate like-to-like comparisons of CLEC and ILEC data can be made. Any statistical methodology that ignores important confounding variables can produce biased results.

Aggregate Level Test Statistic. *Each performance measure of interest should be summarized by one overall test statistic giving the decision maker a rule that determines whether a statistically significant difference exists.*

To achieve this goal, the aggregate test statistic should have the following properties:

- The method should provide a single overall index, on a standard scale.
- If entries in comparison cells are exactly proportional over a covariate, the aggregated index should be very nearly the same as if comparisons on the covariate had not been done.
- The contribution of each comparison cell should depend on the number of observations in the cell.
- Cancellation between comparison cells should be limited, i.e., positive outcomes should not be allowed to cancel negative ones.
- The index should be a continuous function of the observations.

Since the data are being disaggregated to a very deep level, thousands of like-to-like comparison cells are created. An aggregate summary statistic is needed in order to make an overall judgment.

The aggregate level statistic should be insensitive to small changes in cells values, and its value should not be affected if some of the disaggregation for like-to-like cells is truly unnecessary. Furthermore, individual cell results should be weighted so that those cells with more transactions have larger effects on the overall result.

Production Mode Process. *The decision system must be developed so that it does not require intermediate manual intervention.*

Two statistical paradigms are possible for examining performance measure data. In the exploratory paradigm, data are examined and methodology is developed that is consistent with what is found. In a production paradigm a methodology is decided upon before data exploration. For the production paradigm to succeed

- Calculations should be well defined for possible eventualities.
- The decision process should be based on an algorithm that needs no manual intervention.
- Results should be arrived at in a timely manner.
- The system must recognize that resources are needed for other performance measure-related processes that also must be run in a timely manner.
- The system should be both auditable and adjustable over time.

While the exploratory paradigm provides protection against using erroneous data, it requires a great deal of lead time and is unsuitable for timely monthly performance measure testing. A production paradigm will not only promptly produce overall test results but will also provide documentation that can be used to explore the data after the test results are released.

Error Probability Balancing. *The testing methodology should balance Type I and Type II Error probabilities.*

Specifically. what is required to achieve this goal is

- The probability of a Type I error should equal the probability of a Type II error for well-defined null and alternative hypotheses.
- The formula for a test's balancing critical value should be simple enough to calculate using standard mathematical functions, i.e. one should avoid methods that require computationally intensive techniques.
- Little to no information beyond the null hypothesis, the alternative hypothesis, and the number of observations should be required for calculating the balancing critical value.

The objective of a statistical test is to test a hypothesis concerning the values of one or more population parameters. Usually an inquiry into whether or not there is evidence to support a hypothesis, called the *alternative hypothesis*, is conducted by seeking statistical evidence that the converse of the alternative, the *null hypothesis*, is most likely false. If there is not sufficient evidence to reject the null hypothesis, then a case for accepting the alternative has not been made.

Two types of errors are possible in any decision-making process. These have been summarized in Table 3.

Table 3: Statistical Testing Errors

Decision Error	General Description	In terms of Performance Measure Testing
Type I	Rejecting the null hypothesis (accepting the alternative) when the null is true.	Deciding that BST favors its own customers when it does not.
Type II	Accepting the null hypothesis when the alternative is true.	Deciding that BST does not favor its own customers when it does.

In a controlled experimental study where the sample sizes are relatively small, it is generally desirable to control the Type I error closely to avoid making a conclusion that there is a difference when, in fact, there is none. The probability of a Type II error is not directly controlled but is determined by the sample size and the distance between the null and the alternative hypotheses.

If a standard of materiality is set by stating a specific alternative for the test, and the distribution of the test statistic under both the null and alternative hypotheses is understood, then a critical value can be determined so that the two error probabilities are equal.

Trimming. *Trimming of extreme observations from BellSouth and CLEC distributions is needed in order to ensure that a fair comparison is made between performance measures.*

Three conditions are needed to accomplish this goal. These are:

- Trimming should be based on a general rule that can be used in a production setting.
- Trimmed observations should not simply be discarded; they need to be examined and possibly used in the final decision making process.
- Trimming should only be used on performance measures that are sensitive to "outliers."

For the purpose of performance measure testing, trimming refers to removing transactions that significantly distort the performance measure statistic for the set of transactions under consideration. For example, the arithmetic average (or mean) is extremely sensitive to "outliers" since a single large value can significantly distort the average.

The term "outliers" refers to:

1) extreme data values that may be valid, but since they are rare measurements, they may be considered to be statistically unique; or
2) large values that should not be in the analysis data set because of errors in the measurement or in selecting the data.

Trimming is beneficial since it puts both ILEC and CLEC transactions on equal footing with respect to the largest value in each set. Note, though, that it is only needed for performance measures that are distorted by outliers. Of the three types of measures defined in Section 2, only mean (average) measures require trimming. Appendix E sets forth a trimming plan for mean performance measures.

4. Testing Methodology

This section details the testing methodology that is most appropriate for the various types of performance measures. First, transaction level testing will be discussed when there is a retail analog. Next, transaction level testing against a benchmark. Then, testing when only aggregated summaries are available.

Transaction Level - Retail Analog: The Truncated Z Statistic. When a retail analog is available CLEC performance can be directly compared with ILEC performance. Over the last year, for transaction level data, many test statistics have been examined. We now believe that the "Truncated Z" test statistic provides the best compromise with respect to possessing the desired qualities outlined in Section 3, above.

The Truncated Z is fully described in Appendix A, and formulae for calculation of a balancing critical value are found in Appendix C. The main features of this statistic are:

- A basic test statistic is calculated within each comparison cell.
- The value of a cell's result is left "as is" if the result suggests that "favoritism" may be taking place. Otherwise, the result is set to zero. This is called the truncation step.
- Weights that depend on the volume of both ILEC and CLEC transactions within the cell are determined, and a weighted sum of the "truncated" cell results is calculated.
- The weighted sum is theoretically corrected to account for the truncation, and a final overall statistic is determined.
- This overall test value is compared to a balancing critical value to determine if favoritism is likely.

The test statistic itself is based on like-to-like comparisons, and it possesses all five of the properties of an aggregate test statistic (Section 3). While the test requires a large amount of calculations, our studies of the process on some of BellSouth's performance measure data indicate that the calculations can be completed in a reasonable amount of time. Therefore, the process can be put into production mode. Finally, since a balancing critical value can be calculated, it is possible to balance the error probabilities.

<u>Transaction Level - Benchmark</u>. When a benchmark is used, CLEC performance is not compared with ILEC performance. Like-to-like comparison cells are not needed, thus greatly simplifying the testing process. Statistical testing can be done using a probability model, or non-statistical testing can be done using a deterministic model. No data for this data/comparison class has been studied at this point in time.

<u>Aggregated Summary - Retail Analog or Benchmark</u>. We cannot provide any one single set of rules for the analysis of data in this class. Data that is an aggregated summary of transactions may or may not present problems. For example, BellSouth's trunk blocking data is saved as summaries by hour of the day. Collectively, the summaries do provide sufficient information to proceed with the Truncated Z methodology.

On the other hand, our examination of the data for the OSS response interval revealed that information necessary for computing a Truncated Z was not available. In this case, however, we were able to construct a satisfactory time series method to analyze the measure.

Each measure falling into this class needs to be handled on a case-by-case basis. If sufficient information is available to use the Truncated Z method, then we feel it should be used. When the Truncated Z cannot be used, a testing methodology that adheres closely to the principles outlined in Section 3 should be determined and followed.

Appendix A. The Truncated Z Statistic

The Truncated Z test statistic was developed by Dr. Mallows in order to have an aggregate level test when transaction level data are available that

- provides a single overall index on a standard scale;
- will not change the outcome if the disaggregation is unnecessary,
- incorporates the number of observations in a cell into the determination of the weight for the contribution of each comparison cell,
- limits the amount of "neutralization" between comparison cells, and
- is a continuous function of the observations.

The Ernst & Young statistical team and Dr. Mallows have studied the implementation of the statistic using some of BellSouth's performance measure data. This has resulted in an overall process for comparing CLEC and ILEC performance such that the following principles hold:

1) Like-to-Like Comparisons are made. (See Appendix B for an example based on the trunk blocking measure.)
2) Error probabilities are balanced. (See Appendix C)
3) Extreme values are trimmed from the data sets when they significantly distort the performance measure statistic. (See Appendix E)
4) The testing process is an automated production system. (Discussed here. See Appendix D for reporting guidelines.)
5) The determination of ILEC favoritism is based on a single aggregate level test statistic. (Discussed here.)

This appendix provides the details behind computing the Truncated Z test statistic so that principles 4 and 5 hold. We start by assuming that any necessary trimming of the data is complete, and that the data are disaggregated so that comparisons are made within appropriate classes or adjustment cells that define "like" observations.

Notation and Exact Testing Distributions

Below, we have detailed the basic notation for the construction of the truncated z statistic. In what follows the word "cell" should be taken to mean a like-to-like comparison cell that has both one (or more) ILEC observation and one (or more) CLEC observation.

L = the total number of occupied cells

j = $1,\ldots,L$; an index for the cells

n_{1j} = the number of ILEC transactions in cell j

n_{2j} = the number of CLEC transactions in cell j

n_j = the total number transactions in cell j; $n_{1j} + n_{2j}$

X_{1jk} = individual ILEC transactions in cell j; $k = 1,\ldots, n_{1j}$

X_{2jk} = individual CLEC transactions in cell j; $k = 1,\ldots, n_{2j}$

Y_{jk} = individual transaction (both ILEC and CLEC) in cell j

$$= \begin{cases} X_{1jk} & k = 1,\ldots, n_{1j} \\ X_{2jk} & k = n_{1j}+1,\ldots, n_j \end{cases}$$

$\Phi^{-1}(\cdot)$ = the inverse of the cumulative standard normal distribution function

For Mean Performance Measures the following additional notation is needed.

$\overline{X}_{1j}$ = the ILEC sample mean of cell j

$\overline{X}_{2j}$ = the CLEC sample mean of cell j

s_{1j}^2 = the ILEC sample variance in cell j

s_{2j}^2 = the CLEC sample variance in cell j

$\{y_{jk}\}$ = a random sample of size n_{2j} from the set of $Y_{j1},\ldots, Y_{jn_j}$; $k = 1,\ldots,n_{2j}$

M_j = the total number of distinct pairs of samples of size n_{1j} and n_{2j};

$$= \binom{n_j}{n_{1j}}$$

The exact parity test is the permutation test based on the "modified Z" statistic. For large samples, we can avoid permutation calculations since this statistic will be normal (or Student's t) to a good approximation. For small samples, where we cannot avoid permutation calculations, we have found that the difference between "modified Z" and the textbook "pooled Z" is negligible. We therefore propose to use the permutation test based on pooled Z for small samples. This decision speeds up the permutation computations considerably, because for each permutation we need only compute the sum of the CLEC sample values, and not the pooled statistic itself.

A permutation probability mass function distribution for cell j, based on the "pooled Z" can be written as

$$PM(t) = P(\sum_k y_{jk} = t) = \frac{\textit{the number of samples that sum to } t}{M_j},$$

and the corresponding cumulative permutation distribution is

$$CPM(t) = P(\sum_k y_{jk} \le t) = \frac{\textit{the number of samples with sum } \le t}{M_j}.$$

For Proportion Performance Measures the following notation is defined

a_{1j} = the number of ILEC cases possessing an attribute of interest in cell j

a_{2j} = the number of CLEC cases possessing an attribute of interest in cell j

a_j = the number of cases possessing an attribute of interest in cell j; $a_{1j}+ a_{2j}$

The exact distribution for a parity test is the hypergeometric distribution. The hypergeometric probability mass function distribution for cell j is

$$HG(h) = P(H = h) = \begin{cases} \dfrac{\binom{n_{1j}}{h}\binom{n_{2j}}{a_j - h}}{\binom{n_j}{a_j}}, & \max(0, a_j - n_{2j}) \le h \le \min(a_j, n_{1j}) \\ 0 & \text{otherwise} \end{cases},$$

and the cumulative hypergeometric distribution is

$$CHG(x) = P(H \le x) = \begin{cases} 0 & x < \max(0, a_j - n_{2j}) \\ \sum_{h=\max(0,a_j-n_{1j})}^{x} HG(h), & \max(0, a_j - n_{2j}) \le x \le \min(a_j, n_{1j}) \\ 1 & x > \min(a_j, n_{1j}) \end{cases}.$$

For Rate Measures, the notation needed is defined as

b_{1j} = the number of ILEC base elements in cell j

b_{2j} = the number of CLEC base elements in cell j

b_j = the total number of base elements in cell j; $b_{1j}+ b_{2j}$

r_{1j} = the ILEC sample rate of cell j; n_{1j}/b_{1j}

r_{2j} = the CLEC sample rate of cell j; n_{2j}/b_{2j}

q_j = the relative proportion of ILEC elements for cell j; b_{1j}/b_j

The exact distribution for a parity test is the binomial distribution. The binomial probability mass function distribution for cell j is

$$BN(k) = P(B = k) = \begin{cases} \binom{n_j}{k} q_j^k (1-q_j)^{n_j-k}, & 0 \le k \le n_j \\ 0 & \text{otherwise} \end{cases},$$

and the cumulative binomial distribution is

$$CBN(x) = P(B \le x) = \begin{cases} 0 & x < 0 \\ \sum_{k=0}^{x} BN(k), & 0 \le x \le n_j . \\ 1 & x > n_j \end{cases}$$

For Ratio Performance Measures the following additional notation is needed.

U_{1jk} = additional quantity of interest of an individual ILEC transaction in cell j; $k = 1,\ldots, n_{1j}$

U_{2jk} = additional quantity of interest of an individual CLEC transaction in cell j; $k = 1,\ldots, n_{2j}$

$\hat{R}_{ij}$ = the ILEC ($i = 1$) or CLEC ($i = 2$) ratio of the total additional quantity of interest to the base transaction total in cell j, i.e., $\sum_k U_{ijk} \Big/ \sum_k X_{ijk}$

Calculating the Truncated Z

The general methodology for calculating an aggregate level test statistic is outlined below.

1. **Calculate cell weights**, W_j. A weight based on the number of transactions is used so that a cell which has a larger number of transactions has a larger weight. The actual weight formulae will depend on the type of measure.

Mean or Ratio Measure

$$W_j = \sqrt{\frac{n_{1j} n_{2j}}{n_j}}$$

Proportion Measure

$$W_j = \sqrt{\frac{n_{2j} n_{1j}}{n_j} \cdot \frac{a_j}{n_j} \cdot \left(1 - \frac{a_j}{n_j}\right)}$$

Rate Measure

$$W_j = \sqrt{\frac{b_{1j} b_{2j}}{b_j} \cdot \frac{n_j}{b_j}}$$

2. **In each cell, calculate a Z value,** Z_j. A Z statistic with mean 0 and variance 1 is needed for each cell.

- If $W_j = 0$, set $Z_j = 0$.
- Otherwise, the actual Z statistic calculation depends on the type of performance measure.

Mean Measure

$$Z_j = \Phi^{-1}(\alpha)$$

where α is determine by the following algorithm.

If $\min(n_{1j}, n_{2j}) > 6$, then determine α as

$$\alpha = P(t_{n_{1j}-1} \le T_j),$$

that is, α is the probability that a t random variable with n_{1j} - 1 degrees of freedom, is less than

$$T_j = t_j + \frac{g}{6}\left(\frac{n_{1j} + 2n_{2j}}{\sqrt{n_{1j}\, n_{2j}(n_{1j} + n_{2j})}}\right)\left(t_j^2 + \frac{n_{2j} - n_{1j}}{2n_{1j} + n_{2j}}\right),$$

where

$$t_j = \frac{\overline{X}_{1j} - \overline{X}_{2j}}{s_{1j}\sqrt{\frac{1}{n_{1j}} + \frac{1}{n_{2j}}}}$$

and the coefficient g is an estimate of the skewness of the parent population, which we assume is the same in all cells. It can be estimated from the ILEC values in the largest cells. This needs to be done only once for each measure. We have found that attempting to estimate this skewness parameter for each cell separately leads to excessive variability in the "adjusted" t. We therefore use a single compromise value in all cells.

Note, that t_j is the "modified Z" statistic. The statistic T_j is a "modified Z" corrected for the skewness of the ILEC data.

If $\min(n_{1j}, n_{2j}) \le 6$, and

a) $M_j \le 1{,}000$ (the total number of distinct pairs of samples of size n_{1j} and n_{2j} is 1,000 or less).

- Calculate the sample sum for all possible samples of size n_{2j}.
- Rank the sample sums from smallest to largest. Ties are dealt by using average ranks.
- Let R_0 be the rank of the observed sample sum with respect all the sample sums.

$$\alpha = 1 - \frac{R_0 - 0.5}{M_j}$$

b) $M_j > 1{,}000$

- Draw a random sample of 1,000 sample sums from the permutation distribution.
- Add the observed sample sum to the list. There is a total of 1001 sample sums. Rank the sample sums from smallest to largest. Ties are dealt by using average ranks.
- Let R_0 be the rank of the observed sample sum with respect all the sample sums.

$$\alpha = 1 - \frac{R_0 - 0.5}{1001}.$$

Proportion Measure

$$Z_j = \frac{n_j\, a_{1j} - n_{1j}\, a_j}{\sqrt{\dfrac{n_{1j}\, n_{2j}\, a_j\, (n_j - a_j)}{n_j - 1}}}.$$

Rate Measure

$$Z_j = \frac{n_{1j} - n_j\, q_j}{\sqrt{n_j\, q_j (1 - q_j)}}$$

Ratio Measure

$$Z_j = \frac{\hat{R}_{1j} - \hat{R}_{2j}}{\sqrt{V(\hat{R}_{1j})\left(\frac{1}{n_{1j}} + \frac{1}{n_{2j}}\right)}}$$

$$V(\hat{R}_{1j}) = \frac{\sum_k \left(U_{1jk} - \hat{R}_{1j}X_{1jk}\right)^2}{\bar{X}_{1j}^2(n_{1j}-1)} = \frac{\sum_k U_{1jk}^2 - 2\hat{R}_{1j}\sum_k \left(U_{1jk}X_{1jk}\right) + \hat{R}_{1j}^2 \sum_k X_{1jk}^2}{\bar{X}_{1j}^2(n_{1j}-1)}$$

3. **Obtain a truncated Z value for each cell,** Z_j^*. To limit the amount of cancellation that takes place between cell results during aggregation, cells whose results suggest possible favoritism are left alone. Otherwise the cell statistic is set to zero. This means that positive equivalent Z values are set to 0, and negative values are left alone. Mathematically, this is written as

$$Z_j^* = \min(0, Z_j).$$

4. **Calculate the theoretical mean and variance of the truncated statistic under the null hypothesis of parity,** $E(Z_j^*|H_0)$ and $Var(Z_j^*|H_0)$. In order to compensate for the truncation in step 3, an aggregated, weighted sum of the Z_j^* will need to be centered and scaled properly so that the final aggregate statistic follows a standard normal distribution.

   - If $W_j = 0$, then no evidence of favoritism is contained in the cell. The formulae for calculating $E(Z_j^* \mid H_0)$ and $Var(Z_j^* \mid H_0)$ cannot be used. Set both equal to 0.

   - If $\min(n_{1j}, n_{2j}) > 6$ for a mean measure, $\min\left\{a_{1j}\left(1-\frac{a_{1j}}{n_{1j}}\right), a_{2j}\left(1-\frac{a_{2j}}{n_{2j}}\right)\right\} > 9$ for a proportion measure, $\min\left(n_{1j}, n_{2j}\right) > 15$ and $n_j q_j(1-q_j) > 9$ for a rate measure, or n_{1j} and n_{2j} are large for a ratio measure then

$$E(Z_j^* \mid H_0) = -\frac{1}{\sqrt{2\pi}}, \text{ and}$$

$$Var(Z_j^* \mid H_0) = \frac{1}{2} - \frac{1}{2\pi}.$$

- Otherwise, determine the total number of values for Z_j^*. Let z_{ji} and θ_{ji}, denote the values of Z_j^* and the probabilities of observing each value, respectively.

$$E(Z_j^* \mid H_0) = \sum_i \theta_{ji} z_{ji} \text{ ,and}$$

$$Var(Z_j^* \mid H_0) = \sum_i \theta_{ji} z_{ji}^2 - \left[E(Z_j^* \mid H_0)\right]^2.$$

The actual values of the z's and θ's depends on the type of measure.

Mean Measure

$$N_j = \min(M_j, 1{,}000), \quad i = 1, K, N_j$$

$$z_{ji} = \min\left\{0, \Phi^{-1}\left(1 - \tfrac{R_i - 0.5}{N_j}\right)\right\} \quad \text{where } R_i \text{ is the rank of sample sum i}$$

$$\theta_j = \frac{1}{N_j}$$

Proportion Measure

$$z_{ji} = \min\left\{0, \frac{n_j\, i - n_{1j}\, a_j}{\sqrt{\dfrac{n_{1j}\, n_{2j}\, a_j\, (n_j - a_j)}{n_j - 1}}}\right\}, \quad i = \max(0, a_j - n_{2j}), K, \min(a_j, n_{1j})$$

$$\theta_{ji} = HG(i)$$

Rate Measure

$$z_{ji} = \min\left\{0, \frac{i - n_j\, q_j}{\sqrt{n_j\, q_j (1 - q_j)}}\right\}, \quad i = 0, K, n_j$$

$$\theta_{ji} = BN(i)$$

Ratio Measure

The performance measure that is in this class is billing accuracy. The sample sizes for this measure are quite large, so there is no need for a small sample technique. If one does need a small sample technique, then a resampling method can be used.

5. **Calculate the aggregate test statistic,** Z^T.

$$Z^T = \frac{\sum_j W_j Z_j^* - \sum_j W_j E(Z_j^* | H_0)}{\sqrt{\sum_j W_j^2 \mathrm{Var}(Z_j^* | H_0)}}$$

Decision Process

Once Z^T has been calculated, it is compared to a critical value to determine if the ILEC is favoring its own customers over a CLEC's customers. The derivation of the critical value is found in Appendix C.

This critical value changes as the ILEC and CLEC transaction volume change. One way to make this transparent to the decision maker, is to report the difference between the test statistic and the critical value, $diff = Z^T - c_B$. If favoritism is concluded when $Z^T < c_B$, then the $diff < 0$ indicates favoritism.

This make it very easy to determine favoritism: a positive *diff* suggests no favoritism, and a negative *diff* suggests favoritism. Appendix D provides an example of how this information can be reported for each month.

Appendix B. Trunk Blocking

This Appendix provides an example of how the trunk blocking data can be processed to apply the Truncated Z Statistic. Trunk blocking is defined as the proportion of blocked calls a trunk group experiences in a time interval. It is a ratio of two numbers—blocked and attempted calls, both of which can vary over time and across trunk groups. Since the measure is a proportion where the numerator is a subset of the denominator, the truncated Z statistic, modified for proportions, can be applied here (see Appendix A).

As with other performance measures, data are first assigned to like-to-like cells, and the Z statistic is then computed within each cell. For trunk blocking, cells are defined by three variables: hour, day, and trunk group size or capacity. The next sections will describe the data and the data processing steps in greater detail.

The approach used in this example needs to be reviewed by subject matter expert to determine if it proper to use for trunk blocking.

Data Sources

Two data files are processed for the trunk blocking measure. One is the Trunk Group Data File that contains the Trunk Group Serial Number (TGSN), Common Language Location Identifier (CLLI) , and other characteristics needed to categorize trunk groups and to identify them as BellSouth or CLEC.

The other file is the Blocking Data File (BDF), which contains the actual 24 hour blocking ratios for each weekday. There are 4 or 5 weeks in a monthly report cycle. The current system, however, allows the storage of daily blocking data by hour for a week only. Therefore, the data elements necessary to compute the Truncated Z must be extracted each week.

Two important data fields of interest on the Blocking Data File are the Blocking Ratio and Offered Load. The basic definition of Blocking Ratio is the proportion of all attempted calls that were blocked. For the simplest case of one way trunk groups, this is computed by dividing the number of blocked calls by the total call attempts, given that the data are valid. If they are not valid (e.g., actual usage exceeds capacity), blocking is estimated via the Neal Wilkinson algorithm.

Although the raw data--blocked calls (overflow) and peg counts (total call attempts)--are available, the calculation of the Blocking Ratio may be complicated for two-way trunk groups and trunk groups with invalid data. For this reason, we use the blocking ratios from the BDF instead of computing the ratios from the raw data. In order to reflect different call volumes processed through each trunk group, however, the blocking ratios need to be either weighted by call volume or converted to blocked and attempted calls before they are aggregated.

The measure of call traffic volume recommended for weighting is Offered Load. Offered Load is different from call counts in that it incorporates call duration as well. Since it is not just the number of calls but the total usage—number of calls multiplied by average call duration--that determines the occurrence of any blocking, this pseudo measure, Offered Load, appears to be the best indicator of call volume.

Cells or comparison classes are determined by three factors—hour, day, and trunk group capacity (number of trunks in service). The first two factors represent natural classes because trunk blocking changes over time. The third factor is based on our finding that high blocking tends to occur in small trunk groups. A pattern was found not only in the magnitude of blocking but also in its variability. Both the magnitude and variability of blocking decrease as trunk group capacity increases. Additional work is needed to establish the appropriate number of capacity levels and the proper location of boundaries.

Data Processing

The data are processed using the five steps below:

1. Merge the two files by TGSN and select only trunk groups listed in both files.
2. Reset the blocking of all high use trunk groups to zero[1].
3. Assign trunk group categories to CLEC and BellSouth: Categories 1, 3, 4, 5, 10, and 16 for CLEC and 9 for BellSouth[2]. The categories used here for comparison are:

Category	Administrator	Point A	Point B
1	BellSouth	BellSouth End Office	BellSouth Access Tandem
3	BellSouth	BellSouth End Office	CLEC Switch
4	BellSouth	BellSouth Local Tandem	CLEC Switch
5	BellSouth	BellSouth Access Tandem	CLEC Switch
9	BellSouth	BellSouth End Office	BellSouth End Office
10	BellSouth	BellSouth End Office	BellSouth Local Tandem
16	BellSouth	BellSouth Tandem	BellSouth Tandem

4. Recode the missing data. The Blocking Data File assigns all missing data (no valid measurement data) zero blocking. To differentiate true zero blocking from zeroes due to missing data, invalid records were identified and the ratios reset to missing. The blocking value was invalid if both the number of Loaded Days and the Offered Load were 0 for a given hourly period.
5. Form comparison classes based either on the data (i.e., quartiles) or on a predetermined set of values.

---

[1] The high use trunk groups cannot have any blocking. These are set up such that all overflow calls are automatically routed to other trunk groups instead of being physically blocked.

[2] More detailed information on all categories is described in a report 'Trunk Performance Report Generation' by Ernst & Young (March 1999).

Calculation of the Proportion of Blocked Calls

Each cell is determined by day of the month, hour of the day, and trunk group capacity. To use the Truncated Z method, we generate summary information, to include the total number of blocked calls and the total number of attempted calls, for each cell.

For the details of each calculation step, the following notation is used. For a given hour of a day, let $\overline{X}_{1ij}$ be the proportion of BellSouth blocked calls for trunk group i in cell j and $\overline{X}_{2ij}$ be the corresponding proportion for CLEC. Then $\overline{X}_{ij} = X_{1ij} / n_{1ij}$ where X_{1ij} denotes the number of BellSouth blocked calls and n_{1ij} denotes the number of BellSouth total call attempts (indicated by Offered Load) for trunk group i in cell j. Likewise, $\overline{X}_{2ij} = X_{2ij} / n_{2ij}$. For the steps outlined below, only the CLEC notation is provided.

1. Compute the number of blocked calls for trunk group i: $X_{2ij} = \overline{X}_{2ij} * n_{2ij}$
2. Compute total call attempts for all trunk groups in the cell: $n_{2j} = \sum_i n_{2ij}$
3. Compute mean blocking proportion for cell j: $\overline{X}_{2j} = \sum_i X_{2ij} \Big/ \sum_i n_{2ij}$
4. Compute the total number of BellSouth and CLEC blocked calls in cell j: $t_j = \sum_i X_{1ij} + \sum_i X_{2ij}$
5. Apply the Truncated Z Statistic for Proportion measures presented in Appendix A.

Appendix C
Balancing the Type I and Type II Error Probabilities of the Truncated Z Test Statistic

This appendix describes a the methodology for balancing the error probabilities when the Truncated Z statistic, described in Appendix A, is used for performance measure parity testing. There are four key elements of the statistical testing process:

1. the null hypothesis, H_0, that parity exists between ILEC and CLEC services
2. the alternative hypothesis, H_a, that the ILEC is giving better service to its own customers
3. the Truncated Z test statistic, Z^T, and
4. a critical value, c

The decision rule[1] is

- If $Z^T < c$ then accept H_a.
- If $Z^T \geq c$ then accept H_0.

There are two types of error possible when using such a decision rule:

Type I Error: Deciding favoritism exists when there is, in fact, no favoritism.
Type II Error: Deciding parity exists when there is, in fact, favoritism.

The probabilities of each type of each are:

Type I Error: $\alpha = P(Z^T < c \mid H_0)$.
Type II Error: $\beta = P(Z^T \geq c \mid H_a)$.

In what follows, we show how to find a balancing critical value, c_B, so that $\alpha = \beta$.

General Methodology

The general form of the test statistic that is being used is

[1] This decision rule assumes that a negative test statistic indicates poor service for the CLEC customer. If the opposite is true, then reverse the decision rule.

$$z_0 = \frac{\hat{T} - E(\hat{T}|H_0)}{SE(\hat{T}|H_0)}, \tag{C.1}$$

where

$\hat{T}$ is an estimator that is (approximately) normally distributed,

$E(\hat{T} \mid H_0)$ is the expected value (mean) of $\hat{T}$ under the null hypothesis, and

$SE(\hat{T} \mid H_0)$ is the standard error of $\hat{T}$ under the null hypothesis.

Thus, under the null hypothesis, z_0 follows a standard normal distribution. However, this is not true under the alternative hypothesis. In this case,

$$z_a = \frac{\hat{T} - E(\hat{T}|H_a)}{SE(\hat{T}|H_a)}$$

has a standard normal distribution. Here

$E(\hat{T} \mid H_a)$ is the expected value (mean) of $\hat{T}$ under the alternative hypothesis, and

$SE(\hat{T} \mid H_a)$ is the standard error of $\hat{T}$ under the alternative hypothesis.

Notice that

$$\begin{aligned} \beta &= P(z_0 > c \mid H_a) \\ &= P\left(z_a > \frac{c SE(\hat{T} \mid H_0) + E(\hat{T} \mid H_0) - E(\hat{T} \mid H_a)}{SE(\hat{T} \mid H_a)} \right) \end{aligned} \tag{C.2}$$

and recall that for a standard normal random variable z and a constant b, $P(z < b) = P(z > -b)$. Thus,

$$\alpha = P(z_0 < c) = P(z_0 > -c) \tag{C.3}$$

Since we want $\alpha = \beta$, the right hand sides of (C.2) and (C.3) represent the same area under the standard normal density. Therefore, it must be the case that

$$-c = \frac{c SE(\hat{T} \mid H_0) + E(\hat{T} \mid H_0) - E(\hat{T} \mid H_a)}{SE(\hat{T} \mid H_a)}.$$

Solving this for c gives the general formula for a balancing critical value:

$$c_B = \frac{E(\hat{T} \mid H_a) - E(\hat{T} \mid H_0)}{SE(\hat{T} \mid H_a) + SE(\hat{T} \mid H_0)} \qquad (C.4)$$

The Balancing Critical Value of the Truncated Z

In Appendix A, the Truncated Z statistic is defined as

$$Z^T = \frac{\sum_j W_j Z_j^* - \sum_j W_j E(Z_j^* \mid H_0)}{\sqrt{\sum_j W_j^2 Var(Z_j^* \mid H_0)}}$$

In terms of equation (C.1) we have

$$\hat{T} = \sum_j W_j Z_j^*$$

$$E(\hat{T} \mid H_0) = \sum_j W_j E(Z_j^* \mid H_0)$$

$$SE(\hat{T} \mid H_0) = \sqrt{\sum_j W_j^2 Var(Z_j^* \mid H_0)}$$

To compute the balancing critical value (C.4), we also need $E(\hat{T} \mid H_a)$ and $SE(\hat{T} \mid H_a)$. These values are determined by

$$E(\hat{T} \mid H_a) = \sum_j W_j E(Z_j^* \mid H_a), \text{ and}$$

$$SE(\hat{T} \mid H_a) = \sqrt{\sum_j W_j^2 \, var(Z_j^* \mid H_a)} .$$

In which case equation (C.4) gives

$$c_B = \frac{\sum_j W_j E(Z_j^* \mid H_a) - \sum_j W_j E(Z_j^* \mid H_0)}{\sqrt{\sum_j W_j^2 \, var(Z_j^* \mid H_a)} + \sqrt{\sum_j W_j^2 \, var(Z_j^* \mid H_0)}} . \qquad (C.5)$$

Thus, we need to determine how to calculate $E(Z_j^* \mid H_0)$, $Var(Z_j^* \mid H_0)$, $E(Z_j^* \mid H_a)$, and $Var(Z_j^* \mid H_a)$.

If Z_j has a normal distribution with mean μ and standard error σ, then the mean of the distribution truncated at 0 is

$$M(\mu,\sigma) = \int_{-\infty}^{0} \frac{x}{\sqrt{2\pi}\sigma} \exp\left(-\tfrac{1}{2}\left(\tfrac{x-\mu}{\sigma}\right)^2\right) dx,$$

and the variance is

$$V(\mu,\sigma) = \int_{-\infty}^{0} \frac{x^2}{\sqrt{2\pi}\sigma} \exp\left(-\tfrac{1}{2}\left(\tfrac{x-\mu}{\sigma}\right)^2\right) dx - M(\mu,\sigma)^2$$

It can be shown that

$$M(\mu,\sigma) = \mu\,\Phi(\tfrac{-\mu}{\sigma}) - \sigma\,\phi(\tfrac{-\mu}{\sigma})$$

and

$$V(\mu,\sigma) = (\mu^2 + \sigma^2)\Phi(\tfrac{-\mu}{\sigma}) - \mu\,\sigma\,\phi(\tfrac{-\mu}{\sigma}) - M(\mu,\sigma)^2$$

where $\Phi(\cdot)$ is the cumulative standard normal distribution function, and $\phi(\cdot)$ is the standard normal density function.

The cell test statistic, Z_j, is constructed so that it has mean 0 and standard deviation 1 under the null hypothesis. Thus,

$$E(Z_j^* | H_0) = M(0,1) = -\frac{1}{\sqrt{2\pi}}, \text{ and}$$

$$\text{var}(Z_j^* | H_0) = V(0,1) = \frac{1}{2} - \frac{1}{2\pi}.$$

The mean and standard error of Z_j under the alternative hypothesis depends on the type of measure and the form of the alternative. These are discussed below. For now, denote the mean and standard error of Z_j under the alternative by m_j and se_j respectively. Thus,

$$E(Z_j^* | H_a) = M(m_j, se_j), \text{ and}$$

$$SE(Z_j^* | H_a) = V(m_j, se_j).$$

Using the above notation, and equation (C.5), we get the formula for the balancing critical of Z^{T}.

$$c_B = \frac{\sum_j W_j M(m_j, se_j) - \sum_j W_j \frac{-1}{\sqrt{2\pi}}}{\sqrt{\sum_j W_j^2 V(m_j, se_j)} + \sqrt{\sum_j W_j^2 \left(\frac{1}{2} - \frac{1}{2\pi}\right)}}. \qquad (C.6)$$

This formula assumes that Z_j is approximately normally distributed within cell j. When the cell sample sizes, n_{1j} and n_{2j}, are small this may not be true. It is possible to determine the cell mean and variance under the null hypothesis when the cell sample sizes are small. It is much more difficult to determine these values under the alternative hypothesis. Since the cell weight, W_j will also be small (see Appendix A) for a cell with small volume, the cell mean and variance will not contribute much to the weighted sum. Therefore, formula (C.6) provides a reasonable approximation to the balancing critical value.

Alternative Hypotheses

Mean Measure

For mean measures, one is concerned with two parameters in each cell, namely, the mean and variance. A possible lack of parity may be due to a difference in cell means, and/or a difference in cell variances. One possible set of hypotheses that capture this notion, and take into account the assumption that transaction are identically distributed within cells is:

H_0: $\mu_{1j} = \mu_{2j}$, $\sigma_{1j}^2 = \sigma_{2j}^2$

H_a: $\mu_{2j} = \mu_{1j} + \delta_j \cdot \sigma_{1j}$, $\sigma_{2j}^2 = \lambda_j \cdot \sigma_{1j}^2$ $\qquad$ $\delta_j > 0$, $\lambda_j \geq 1$ and $j = 1, \ldots, L$.

Under this form of alternative hypothesis, the cell test statistic Z_j has mean and standard error given by

$$m_j = \frac{-\delta_j}{\sqrt{\frac{1}{n_{1j}} + \frac{1}{n_{2j}}}}, \text{ and}$$

$$se_j = \sqrt{\frac{\lambda_j n_{1j} + n_{2j}}{n_{1j} + n_{2j}}}$$

Proportion Measure

For a proportion measure there is only one parameter of interest in each cell, the proportion of transaction possessing an attribute of interest. A possible lack of parity may be due to a difference in cell proportions. A set of hypotheses that take into account the assumption that transaction are identically distributed within cells while allowing for an analytically tractable solution is:

$$H_0: \frac{p_{2j}(1-p_{1j})}{(1-p_{2j})p_{1j}} = 1$$

$$H_a: \frac{p_{2j}(1-p_{1j})}{(1-p_{2j})p_{1j}} = \psi_j \qquad \psi_j > 1 \text{ and } j = 1,\ldots,L.$$

These hypotheses are based on the "odds ratio." If the transaction attribute of interest is a missed trouble repair, then an interpretation of the alternative hypothesis is that a CLEC trouble is ψ_j times more likely to be missed than an ILEC trouble.

Under this form of alternative hypothesis, the within cell asymptotic mean and variance of a_{1j} are given by[2]

$$E(a_{1j}) = n_j \pi_j^{(1)}$$

$$\operatorname{var}(a_{1j}) = \frac{n_j}{\frac{1}{\pi_j^{(1)}} + \frac{1}{\pi_j^{(2)}} + \frac{1}{\pi_j^{(3)}} + \frac{1}{\pi_j^{(4)}}} \qquad \text{(C.7)}$$

where

$$\pi_j^{(1)} = f_j^{(1)}\left(n_j^2 + f_j^{(2)} + f_j^{(3)} - f_j^{(4)}\right)$$

$$\pi_j^{(2)} = f_j^{(1)}\left(-n_j^2 - f_j^{(2)} + f_j^{(3)} + f_j^{(4)}\right)$$

$$\pi_j^{(3)} = f_j^{(1)}\left(-n_j^2 + f_j^{(2)} - f_j^{(3)} + f_j^{(4)}\right)$$

$$\pi_j^{(4)} = f_j^{(1)}\left(n_j^2\left(\tfrac{2}{\psi_j} - 1\right) - f_j^{(2)} - f_j^{(3)} - f_j^{(4)}\right)$$

$$f_j^{(1)} = \frac{1}{2n_j^2\left(\frac{1}{\psi_j} - 1\right)}$$

$$f_j^{(2)} = n_j n_{1j}\left(\tfrac{1}{\psi_j} - 1\right)$$

$$f_j^{(3)} = n_j a_j\left(\tfrac{1}{\psi_j} - 1\right)$$

$$f_j^{(4)} = \sqrt{n_j^2\left[4n_{1j}\left(n_j - a_j\right)\left(\tfrac{1}{\psi_j} - 1\right) + \left(n_j + \left(a_j - n_{1j}\right)\left(\tfrac{1}{\psi_j} - 1\right)\right)^2\right]}$$

[2] Stevens, W. L. (1951) Mean and Variance of an entry in a Contingency Table. *Biometrica*, **38**, 468-470.

Recall that the cell test statistic is given by

$$Z_j = \frac{n_j a_{1j} - n_{1j} a_j}{\sqrt{\frac{n_{1j} n_{2j} a_j (n_j - a_j)}{n_j - 1}}}.$$

Using the equations in (C.7), we see that Z_j has mean and standard error given by

$$m_j = \frac{n_j^2 \pi_j^{(1)} - n_{1j} a_j}{\sqrt{\frac{n_{1j} n_{2j} a_j (n_j - a_j)}{n_j - 1}}}, \text{ and}$$

$$se_j = \sqrt{\frac{n_j^3 (n_j - 1)}{n_{1j} n_{2j} a_j (n_j - a_j)\left(\frac{1}{\pi_j^{(1)}} + \frac{1}{\pi_j^{(2)}} + \frac{1}{\pi_j^{(3)}} + \frac{1}{\pi_j^{(4)}}\right)}}.$$

Rate Measure

A rate measure also has only one parameter of interest in each cell, the rate at which a phenomenon is observed relative to a base unit, e.g. the number of troubles per available line. A possible lack of parity may be due to a difference in cell rates. A set of hypotheses that take into account the assumption that transaction are identically distributed within cells is:

H_0: $r_{1j} = r_{2j}$

H_a: $r_{2j} = \varepsilon_j r_{1j}$ $\qquad$ $\varepsilon_j > 1$ and $j = 1,\ldots,L$.

Given the total number of ILEC and CLEC transactions in a cell, n_j, and the number of base elements, b_{1j} and b_{2j}, the number of ILEC transaction, n_{1j}, has a binomial distribution from n_j trials and a probability of

$$q_j^* = \frac{r_{1j} b_{1j}}{r_{1j} b_{1j} + r_{2j} b_{2j}}.$$

Therefore, the mean and variance of n_{1j}, are given by

$$\begin{aligned} E(n_{1j}) &= n_j q_j^* \\ var(n_{1j}) &= n_j q_j^* (1 - q_j^*) \end{aligned} \qquad (C.8)$$

Under the null hypothesis

$$q_j^* = q_j = \frac{b_{1j}}{b_j},$$

but under the alternative hypothesis

$$q_j^* = q_j^a = \frac{b_{1j}}{b_{1j} + \varepsilon_j b_{2j}}. \tag{C.9}$$

Recall that the cell test statistic is given by

$$Z_j = \frac{n_{1j} - n_j\, q_j}{\sqrt{n_j\, q_j(1-q_j)}}.$$

Using (C.8) and (C.9), we see that Z_j has mean and standard error given by

$$m_j = \frac{n_j\left(q_j^a - q_j\right)}{\sqrt{n_j q_j(1-q_j)}} = (1-\varepsilon_j)\sqrt{\frac{n_j b_{1j} b_{2j}}{b_{1j} + \varepsilon_j b_{2j}}}, \text{ and}$$

$$se_j = \sqrt{\frac{q_j^a(1-q_j^a)}{q_j(1-q_j)}} = \sqrt{\varepsilon_j}\,\frac{b_j}{b_{1j} + \varepsilon_j b_{2j}}.$$

Ratio Measure

As with mean measures, one is concerned with two parameters in each cell, the mean and variance, when testing for parity of ratio measures. As long as sample sizes are large, as in the case of billing accuracy, the same method for finding m_j and se_j that is used for mean measures can be used for ratio measures.

Determining the Parameters of the Alternative Hypothesis

In this appendix we have indexed the alternative hypothesis of mean measures by two sets of parameters, λ_j and δ_j. Proportion and rate measures have been indexed by one set of parameters each, ψ_j and ε_j respectively. A major difficulty with this approach is that more than one alternative will be of interest; for example we may consider one alternative in which all the δ_j are set to a common non-zero value, and another set of alternatives in each of which just one δ_j is non-zero, while all the rest are zero. There are very many other possibilities. Each possibility leads to a single value for the balancing critical value; and each possible critical value corresponds to many sets of alternative hypotheses, for each of which it constitutes the correct balancing value.

The formulas we have presented can be used to evaluate the impact of different choices of the overall critical value. For each putative choice, we can evaluate the set of alternatives for which this is the correct balancing value. While statistical science can be used to evaluate the impact of different choices of these parameters, there is not much that an appeal to statistical principles can offer in directing specific choices. Specific choices are best left to telephony experts. Still, it is possible to comment on some aspects of these choices:

- Parameter Choices for λ_j. The set of parameters λ_j index alternatives to the null hypothesis that arise because there might be greater unpredictability or variability in the delivery of service to a CLEC customer over that which would be achieved for an otherwise comparable ILEC customer. While concerns about differences in the variability of service are important, it turns out that the truncated Z testing which is being recommended here is relatively insensitive to all but very large values of the λ_j. Put another way, reasonable differences in the values chosen here could make very little difference in the balancing points chosen.

- Parameter Choices for δ_j. The set of parameters δ_j are much more important in the choice of the balancing point than was true for the λ_j. The reason for this is that they directly index differences in average service. The truncated Z test is very sensitive to any such differences; hence, even small disagreements among experts in the choice of the δ_j could be very important. Sample size matters here too. For example, setting all the δ_j to a single value – $\delta_j = \delta$ – might be fine for tests across individual CLECs where currently in Louisiana the CLEC customer bases are not too different. Using the same value of δ for the overall state testing does not seem sensible. At the state level we are aggregating over CLECs, so using the same δ as for an individual CLEC would be saying that a "meaningful" degree of disparity is one where the violation is the same (δ) for each CLEC. But the detection of disparity for any component CLEC is important, so the relevant "overall" δ should be smaller.

- Parameter Choices for ψ_j or ε_j. The set of parameters ψ_j or ε_j are also important in the choice of the balancing point for tests of their respective measures. The reason for this is that they directly index increases in the proportion or rate of service performance. The truncated Z test is sensitive to such increases; but not as sensitive as the case of δ for mean measures. Sample size matters here too. As with mean measures, using the same value of ψ or ε for the overall state testing does not seem sensible.

The three parameters are related however. If a decision is made on the value of δ, it is possible to determine equivalent values of ψ and ε. The following equations, in conjunction with the definitions of ψ and ε, show the relationship with delta.

$$\delta = 2 \cdot \arcsin(\sqrt{\hat{p}_2}) - 2 \cdot \arcsin(\sqrt{\hat{p}_1})$$
$$\delta = 2\sqrt{\hat{r}_2} - 2\sqrt{\hat{r}_1}$$

The bottom line here is that beyond a few general considerations, like those given above, a principled approach to the choice of the alternative hypotheses to guard against must come from elsewhere.

Appendix D: Examples of Statistical Reports

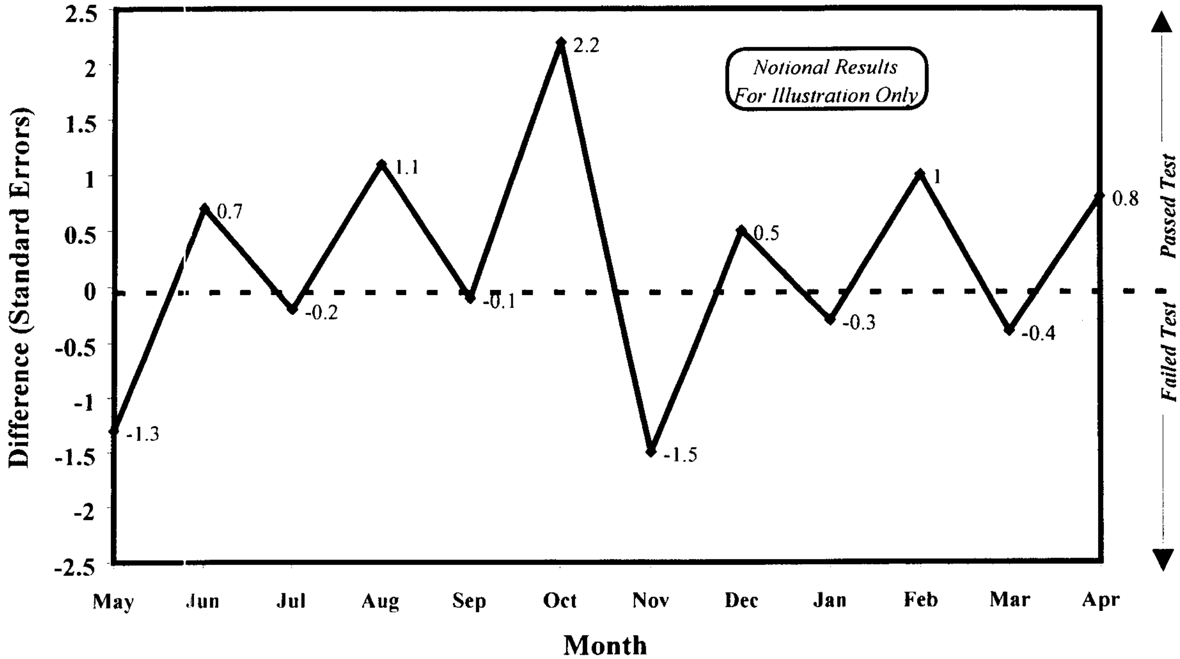
The general structure for reporting statistical results in a production environment will be the same for the different measures and we suggest that it consist of at least three components. For each measure present, (1) the monthly test statistics over a period of time, (2) the results for the current month, with summary statistics, test statistics, and descriptive graphs, and (3) a summary of any adjustments to the data made in the process of running the tests, including a description of how many records were excluded from analysis and the reason for the exclusion (i.e., excluded due to business rules, or due to statistical/methodological rules pertaining to the measure). The last component is important to assure that the reported results can be audited.

Selected components of the reporting structure are illustrated in the samples that follow. An outline of the report is shown below. Monthly results will be presented for each level of aggregation required.

- I. Test Statistics Over Time
- II. Monthly Results
  - A. Summary Statistics
  - B. Test Statistics
  - C. Descriptive Graphs (Frequency Distributions, etc.)
- III. Adjustments to Data
  - A. Records Excluded Due to Business Rules
  - B. Records Excluded Due to Statistical Rules

<u>Test Statistic Over Time</u>. The first component of the reporting structure is an illustration of the trend of the particular performance measure over time together with a tabular summary of results for the current month. We will show at a glance whether the tests consistently return non-statistically significant results; consistently indicate disparity (be that in favor of BellSouth or in favor of the CLECs); or vary month by month in their results. An example of this component follows.

Notional Performance Measure
Through April XXXX
Differences Between Test Statistic and Balancing Critical Value

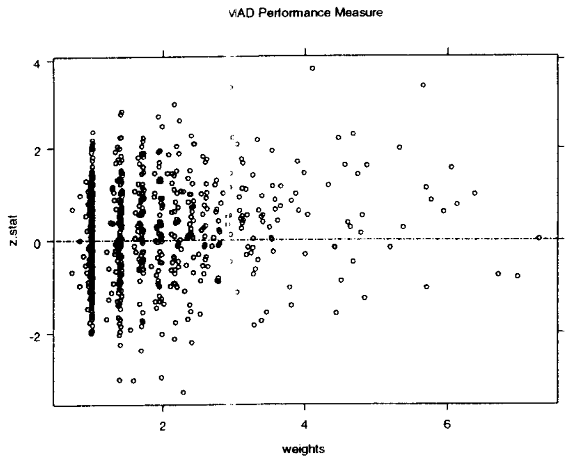


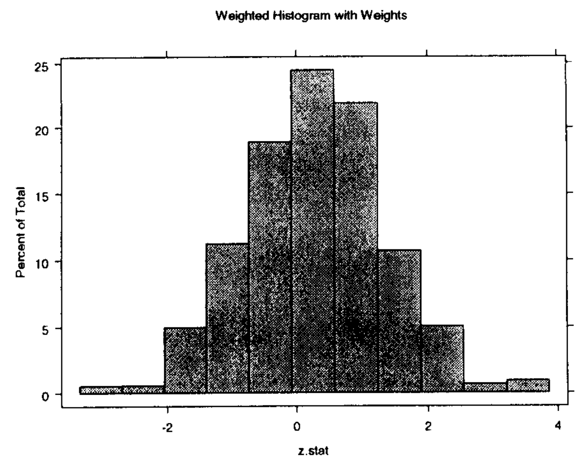
Result for Current Month	
Test Statistic	-0.410
Balancing Critical Value	-1.210
Difference	0.800

<u>Monthly Results</u>. The most important component of the reporting structure is the part which presents results of the monthly statistical tests on the given performance measure. The essential aspects included in this component are the summary statistics; the test statistics and results; and descriptive graphs of the results.

It is important to present basic summary statistics to complete the comparison between BellSouth and the CLECs. At a minimum, these statistics will include the means, standard deviations, and population sizes. In addition to basic descriptive statistics, we also present the test statistic results. Examples of ways we have presented these statistics in the past can be found in BellSouth's February 25, 1999 filing before the Louisiana Public Service Commission.

Finally, the results will be presented in graphical format. Below is an example of how to graphically present the data behind the Truncated Z statistic. One graph shows a plot of cell Z score versus cell weights. The other is a histogram of the weighted cell Z scores.





Adjustments to Data. The third important component of the reporting structure is information on any adjustments performed on the data. This information is essential in order that the results may be verified and audited. The most prevalent examples of such modifications would be removal of observations and weighting of the data.

Records can be removed from analysis for both business reasons (these will likely be taken into account in the PMAP system) and for statistical reasons. All of the performance measures exclude certain records based on business rules underlying each measure's particular definitions and methodologies. The number of records excluded for each rule will be summarized. In addition, some of the measures will have observations excluded for statistical reasons, particularly in the case of "mean measures" (OCI and MAD); these exclusions will be summarized as well. The tables below show examples of the current method for summarizing this information:

April XXXX
Peromance Measure Filtering Information

This table displays information about the size of the database files and the cases that were removed from the analysis.

		1999	
Unfiltered Total	**28,691**	**Unfiltered Total**	**453,107**
Records Removed for Business Reasons *(e.g. not N, T, C, or P orders, not resale and not UNE)*	**7,242**	**Records Removed for Business Reasons** *(e.g. not N, T, C, or P orders, not retail)*	**78,613**
Total Reported on Web Report	**21,449**	**Total Reported on Web Report**	**374,494**
Additional Records Removed for Business Reasons	**876**	**Additional Records Removed for Business Reasons**	**7,429**
Missing Appointment code is 'S'	844	Missing Appointment code is 'S'	7,172
General Class Service = 'O'	0	General Class Service = 'O'	279
UNE Cases	102		
Records Removed for Statistical Reasons **Extreme Values Removed**	**9**	**Records Removed for Statistical Reasons** **Extreme Values Removed**	**652**
No Matching Classification Removals	**47**	**No Matching Classification Removals**	**21,974**
FILTERED TOTAL	**20,517**	**FILTERED TOTAL**	**344,439**

Appendix E. Trimming Outliers for Mean Measures

The arithmetic average is extremely sensitive to outliers; a single large value, possibly an erroneous value, can significantly distort the mean value. And by inflating the error variance, this also affects conclusions in the test of hypotheses. Extreme data values may be correct, but since they are rare measurements, they may be considered to be statistical outliers. Or they may be values that should not be in the analysis data set because of errors in the measurement or in selecting the data.

At this time, only two mean measures have been analyzed: Order Completion Interval and Maintenance Average Duration. Maintenance Average Duration data are truncated at 240 hours and therefore this measure was not trimmed further. For Order Completion Interval, the underlying distribution of the observations is clearly not normal, but rather skewed with a very long upper-tail.

A useful technique, coming from the field of robust statistical analysis, is to trim a very small proportion from the tails of the distribution before calculating the means. The resulting mean is referred to as a trimmed mean. Trimming is beneficial in that it speeds the convergence of the distribution of the means to a normal distribution. Only extreme values are trimmed, and in many cases the data being trimmed are, in fact, data that might not be used in the analysis on other grounds.

In the first analysis of the verified Order Completion Interval-Provisioning measure, after removing data that were clearly in error or were not applicable, we looked at the cases that represented the largest 0.01% of the BST distribution. In the August data, this corresponded to orders with completion intervals greater than 99 days. All of these were BellSouth orders. In examining the largest 11 individual examples that would be removed from analysis, we found that only 1 of the 11 cases was a valid case where the completion interval was unusually large. The other 10 cases were examples of cases that should not have been included in the analysis. This indicates that at least in preliminary analysis, it is both beneficial to examine the extreme outliers and reasonable to remove them.

A very slight trimming is needed in order to put the central limit theorem argument on firm ground. But finding a robust rule that can be used in a production setting is difficult. Also, any trimming rule should be fully explained and any observations that are trimmed from the data must be fully documented.

When it is determined that a measure should be trimmed, a trimming rule that is easy to implement in a production setting is:

> **Trim the ILEC observations to the largest CLEC value from all CLEC observations in the month under consideration.**

That is, no CLEC values are removed; all ILEC observations greater than the largest CLEC observation are trimmed.

While this method is simple, it does allow for extreme CLEC observations to be part of the analysis. For instance, suppose that the amount of time to complete an order was less than 40 days for all CLEC orders except one. Let's say that this extreme order took 100 days to complete. The trimming rule says that all ILEC orders above 100 days should be trimmed, but a closer look at the data might suggest trimming at 40 days instead.

Since we are operating in a production mode system, it is not possible to explore the data before the trimming takes place. Other automatic trimming rules present other problems, so our solution is to use the simple trimming rule above, and have the system automatically produce a trimming report that can be examined at a later point in time.

The trimming report should include:

- The value of the trim point.
- Summary statistics and graphics of the ILEC observations that were trimmed.
- A listing of the trimmed ILEC transaction for a random sample of 10 trimmed transactions. This listing should not disclose sensitive information.
- A listing of the 10 most extreme CLEC transactions. This listing should not disclose sensitive information.
- The number of ILEC and CLEC observations above some fixed point, so that changes in the upper tail can be better tracked over time.

The trimming report should be part of the overall report discussed in Appendix D. Examples of tables contained within the trimming report are shown below.

April XXXX
Performance Measure Extreme Values

CLEC		BST	
Cutoff	26	Cutoff	26
# of Records	20,573	# of Records	367,065
10 Largest		Extreme Values	652
Minimum	19	Minimum	27
Median	23	Median	32
Maximum	26	Maximum	283
Subtotal	**20,573**	**Subtotal**	**366,413**

April XXXX
Performance Measure Weighting Report

CLEC		BST	
# of Records	20,573	# of Records	366,413
No Matching BST Classification (1)	47	No Matching CLEC Classification (2)	21,974
Subtotal	**20,526**	**Subtotal**	**344,439**

April XXXX
Peromance Measure Filtering Information

This table displays information about the size of the database files and the cases that were removed from the analysis.

1999

CLEC	
Unfiltered Total	**28,691**
Records Removed for Business Reasons *(e.g. not N, T, C, or P orders, not resale and not UNE)*	**7,242**
Total Reported on Web Report	**21,449**
Additional Records Removed for Business Reasons	**876**
Missing Appointment code is 'S'	844
General Class Service = 'O'	0
UNE Cases	102
Records Removed for Statistical Reasons	
Extreme Values Removed	**0**
No Matching Classification Removals	**47**
FILTERED TOTAL	**20,526**

BST	
Unfiltered Total	**453,107**
Records Removed for Business Reasons *(e.g. not N, T, C, or P orders, not retail)*	**78,613**
Total Reported on Web Report	**374,494**
Additional Records Removed for Business Reasons	**7,429**
Missing Appointment code is 'S'	7,172
General Class Service = 'O'	279
Records Removed for Statistical Reasons	
Extreme Values Removed	**652**
No Matching Classification Removals	**21,974**
FILTERED TOTAL	**344,439**

CLEC Extreme Values

Wire Center	Time	Dispatch	Residence	Circuits	Order Type	Order Interval
NWORLAMA	1	1	3	1	N	61
OPLSLATL	1	2	1	1	C	53
NWORLAMA	2	1	3	1	N	44
NWORLAMA	1	1	3	1	N	39
BTRGLAWN	1	1	2	1	C	38
LKCHLADT	1	1	1	1	T	37
NWORLAMA	1	1	3	1	N	32
NWORLAMA	2	1	3	1	N	32
SHPTLACL	1	1	2	1	N	28

Frequency of Extreme Values Removed from BST file (Top 10)

Wire Center	Time	Dispatch	Residence	Circuits	Order Type	Frequency
NWORLAMA	1	1	3	1	N	55
NWORLAMA	2	1	3	1	N	25
BTRGLASB	2	1	3	1	C	23
NWORLAMC	2	1	3	1	C	23
NWORLAMC	1	1	3	1	C	22
NWORLAMA	2	1	3	1	C	18
NWORLAMA	1	1	3	1	C	17
BTRGLASB	1	1	3	1	C	16
LFYTLAMA	1	1	3	1	C	15
NWORLAMA	2	2	3	1	C	14

EJM - 2

EXHIBIT EJM-2

Typographical Corrections

Corrections

LPSC "Statistical Techniques for the Analysis and Comparison of Performance Measure Data",

Appendix A, page A-5

$$T_j = t_j + \frac{g}{6}\left(\frac{n_{1j} + 2n_{2j}}{\sqrt{n_{1j}\, n_{2j}(n_{1j} + n_{2j})}}\right)\left(t_j^2 + \frac{n_{2j} - n_{1j}}{n_{1j} + 2n_{2j}}\right)$$

Appendix C, page C-8, rate measures section for balancing critical value.

$$m_j = \frac{n_j\left(q_j^a - q_j\right)}{\sqrt{n_j q_j (1 - q_j)}} = (1 - \varepsilon_j)\frac{\sqrt{n_j b_{1j} b_{2j}}}{b_{1j} + \varepsilon_j b_{2j}}$$