REQUEST:

[Rate of Return] - Please provide a copy of the testimony of Dr. Vander Weide in Microsoft Word format.

RESPONSE:

Please see Attachment 1.

ATTACHMENT:

ATTACHMENT 1 - Atmos Energy Corporation, Kentucky Direct Testimony of Dr. James Vander Weide, 97 Pages.

Respondent: Dr. James Vander Weide

ATMOS ENERGY CORPORATION

DOCKET NO.

PREPARED DIRECT TESTIMONY OF

JAMES H. VANDER WEIDE, PH.D.

RATE OF RETURN

BEFORE THE PUBLIC SERVICE COMMISSION

COMMONWEALTH OF KENTUCKY

IN THE MATTER OF	)	
	)	
RATE APPLICATION BY	)	**Case No. 2009-00354**
	)	
ATMOS ENERGY CORPORATION	)	

<u>TESTIMONY OF JAMES H. VANDER WEIDE, PH.D</u>

ATMOS ENERGY CORPORATION
RATE OF RETURN

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ATMOS ENERGY CORPORATION

A. Introduction and Summary

Q. 1 Please state your name, title, and business address for the record.

A. 1 My name is James H. Vander Weide. I am Research Professor of Finance and Economics at Duke University, The Fuqua School of Business. I am also President of Financial Strategy Associates, a firm that provides strategic and financial consulting services to business clients. My business address is 3606 Stoneybrook Drive, Durham, North Carolina 27705.

Q. 2 Please summarize your qualifications.

A. 2 I received a Bachelor's Degree in Economics from Cornell University and a Ph.D. in Finance from Northwestern University. After joining the faculty of the School of Business at Duke University, I was named Assistant Professor, Associate Professor, and then Professor. I have published research in the areas of finance and economics, taught courses in these fields at Duke over the last 35 years, and taught in numerous executive programs at Duke. I am now retired from my teaching duties at Duke.

Q. 3 Have you previously testified on financial or economic issues?

A. 3 Yes. As an expert on financial and economic theory and practice, I have participated in more than 400 regulatory and legal proceedings before the U.S. Congress, the Canadian Radio-Television and Telecommunications Commission, the Federal Communications Commission, the National Telecommunications and Information Administration, the Federal Energy Regulatory Commission, the National Energy Board (Canada), the Alberta Utilities Board (Canada), the public service commissions of 43 states, the insurance commissions of five states, the Iowa State Board of Tax Review, the National Association of Securities Dealers, and the North Carolina Property Tax Commission. In addition, I have prepared expert testimony in proceedings before the U.S. District Court for the District of Nebraska; the U.S. District Court for the District of New Hampshire; U.S. District Court for the District of Northern Illinois; the U.S. District Court for the Eastern District of North

Carolina; Montana Second Judicial District Court, Silver Bow County; the U.S. District Court for the Northern District of California; the Superior Court, North Carolina; the U.S. Bankruptcy Court for the Southern District of West Virginia; and the U. S. District Court for the Eastern District of Michigan. My resume is shown in Appendix 1.

Q. 4 What is the purpose of your testimony?

A. 4 I have been asked by Atmos Energy Corporation ("Atmos Energy" or "Company") to prepare an independent appraisal of Atmos Energy's cost of equity and to recommend a rate of return on equity that is fair, that allows the Company to attract capital on reasonable terms, and that allows the Company to maintain its financial integrity.

Q. 5 How do you estimate Atmos Energy's cost of equity?

A. 5 I estimate Atmos Energy's cost of equity by applying several standard cost of equity methods, including the discounted cash flow ("DCF"), risk premium, and capital asset pricing model ("CAPM") to a group of comparable companies.

Q. 6 Why do you apply your cost of equity methods to a group of comparable risk companies rather than solely to Atmos Energy?

A. 6 I apply my cost of equity methods to a group of comparable risk companies because standard cost of equity methodologies such as the DCF, risk premium, and CAPM require inputs of quantities that are not easily measured. Since these inputs can only be estimated, there is naturally some degree of uncertainty surrounding the estimate of the cost of equity for each company. However, the uncertainty in the estimate of the cost of equity for an individual company can be greatly reduced by applying cost of equity methodologies to a sample of comparable companies. Intuitively, unusually high estimates for some individual companies are offset by unusually low estimates for other individual companies. Thus, financial economists invariably apply cost of equity methodologies to a group of comparable companies. In utility regulation, the practice of using a group of comparable companies, called the comparable company approach, is further supported by the United States Supreme Court standard that the utility should be allowed to earn a return on its investment that is commensurate with returns being earned on other investments of the same risk.[1]

Q. 7 What cost of equity do you find for your comparable companies in this proceeding?

A. 7 On the basis of my studies, I find that the cost of equity for my comparable companies is in the range 10.2 percent to 11.9 percent (see Table 1), with an average result of 11.0 percent.

TABLE 1
COST OF EQUITY MODEL RESULTS

Method	Model Result
Discounted Cash Flow	11.9%
Ex Ante Risk Premium	10.9%
Ex Post Risk Premium	10.6%
Historical CAPM	10.2%
DCF CAPM	11.5%
Average	11.0%

Q. 8 What is your recommendation regarding Atmos Energy's allowed rate of return on equity?

A. 8 I conservatively recommend that Atmos Energy be allowed a rate of return on equity equal to 11.0 percent.

Q. 9 Why is your recommended return on equity conservative?

A. 9 My recommended return on equity is conservative because the financial risk of my comparable companies, which is based on the equity ratio resulting from the market values of their equity and debt, is less than the financial risk implied by the lower equity ratio in Atmos Energy's ratemaking capital structure, which is based on its book values of equity and debt. In addition, my recommendation does not reflect: (1) the observation that forecasted yields on both A-rated utility bonds and Treasury bonds are significantly higher than the current yields on these securities; (2) the small size premium for small market capitalization companies such as those in my proxy group of natural gas companies; and

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1 *See Bluefield Water Works and Improvement Co. v. Public Service Comm'n.* 262 U.S. 679 (1923) and *Hope Natural Gas Co.*, 320 U.S. 591 (1944).

(3) the evidence that the CAPM underestimates the cost of equity for companies with betas less than 1.0.

Q. 10 Do you have exhibits accompanying your testimony?

A. 10 Yes. I have exhibits consisting of eight schedules and five appendices that were prepared by me or under my direction and supervision.

B. Economic and Legal Principles

Q. 11 What is the economic definition of the required rate of return, or cost of capital, associated with particular investment decisions, such as the decision to invest in natural gas distribution facilities?

A. 11 The cost of capital is the return investors expect to receive on alternative investments of comparable risk.

Q. 12 How does the cost of capital affect a firm's investment decisions?

A. 12 A central goal of a firm is to maximize the value of the firm. This goal can be accomplished by accepting all investments in plant and equipment with an expected rate of return greater than the cost of capital. Thus, from an economic perspective, a firm should continue to invest in plant and equipment only so long as the return on its investment is greater than or equal to its cost of capital.

Q. 13 How does the cost of capital affect investors' willingness to invest in a company?

A. 13 The cost of capital measures the return investors can expect on investments of comparable risk. The cost of capital also measures the investor's required rate of return on investment because rational investors will not invest in a particular investment opportunity if the expected return on that opportunity is less than the cost of capital. Thus, the cost of capital is a hurdle rate for both investors and the firm.

Q. 14 Do all investors have the same position in the firm?

A. 14 No. Bond investors have a fixed claim on a firm's assets and income that must be paid prior to any payment to the firm's equity investors. Since the firm's equity investors have a residual claim on the firm's assets and income, equity investments are riskier than bond investments. Thus, the cost of equity exceeds the cost of debt.

Q. 15 What is the overall or average cost of capital?

A. 15 The overall or average cost of capital is a weighted average of the cost of debt and cost of equity, where the weights are the percentages of debt and equity in a firm's capital structure.

Q. 16 Can you illustrate the calculation of the overall or weighted average cost of capital?

A. 16 Yes. Assume that the cost of debt is 7 percent, the cost of equity is 13 percent, and the percentages of debt and equity in the firm's capital structure are 50 percent and 50 percent, respectively. Then the weighted average cost of capital is expressed by .50 times 7 percent plus .50 times 13 percent, or 10.0 percent.

Q. 17 What is the economic definition of the cost of equity?

A. 17 The cost of equity is the return investors expect to receive on alternative equity investments of comparable risk. Since the return on an equity investment of comparable risk is not a contractual return, the cost of equity is more difficult to measure than the cost of debt. However, as I have already noted, the cost of equity is greater than the cost of debt. The cost of equity, like the cost of debt, is both forward looking and market based.

Q. 18 What is the correct economic measure of the percentages of debt and equity in a firm's capital structure?

A. 18 The percentages of debt and equity in a firm's capital structure are measured by first calculating the market value of the firm's debt and the market value of its equity. The percentage of debt is then calculated by the ratio of the market value of debt to the combined market value of debt and equity, and the percentage of equity by the ratio of the market value of equity to the combined market values of debt and equity. For example, if a firm's debt has a market value of $25 million and its equity has a market value of $75 million, then its total market capitalization is $100 million, and its capital structure contains 25% debt and 75% equity.

Q. 19 Why is a firm's capital structure correctly measured in terms of the market values of its debt and equity?

A. 19 A firm's capital structure is correctly measured in terms of the market values of its debt and equity because: (1) the weighted average cost of capital is defined as the return investors expect to earn on a portfolio of the company's debt and equity securities; (2) investors measure the expected return and risk on their portfolios using market value weights, not book value weights; and (3) market values are the best measures of the amounts of debt and equity investors have invested in the company on a going forward basis.

Q. 20 Why do investors measure the return on their investment portfolios using market value weights rather than book value weights?

A. 20 Investors measure the return on their investment portfolios using market value weights because market value weights are the best measure of the amounts the investors currently have invested in each security in the portfolio. From the point of view of investors, the historical cost or book value of their investment is entirely irrelevant to the current risk and return on their portfolios because if they were to sell their investments, they would receive market value, not historical cost. Thus, the return can only be measured in terms of market values.

Q. 21 Is the economic definition of the weighted average cost of capital consistent with regulators' traditional definition of the weighted average cost of capital?

A. 21 No. The economic definition of the weighted average cost of capital is based on the market costs of debt and equity, the market value percentages of debt and equity in a company's capital structure, and the future expected risk of investing in the company. In contrast, regulators have traditionally defined the weighted average cost of capital using the embedded cost of debt and the book values of debt and equity in a company's capital structure.

Q. 22 Does the required rate of return on an investment vary with the risk of that investment?

A. 22 Yes. Since investors are averse to risk, they require a higher rate of return on investments with greater risk.

Q. 23 Do investors consider future industry changes when they estimate the risk of a particular investment?

A. 23 Yes. Investors consider all the risks that a firm might incur over the future life of the company.

Q. 24 Are these economic principles regarding the fair return for capital recognized in any United States Supreme Court cases?

A. 24 Yes. These economic principles, relating to the supply of and demand for capital, are recognized in two United States Supreme Court cases: (1) *Bluefield Water Works and Improvement Co. v. Public Service Commission*; and (2) *Federal Power Commission v. Hope Natural Gas Co.* In the *Bluefield Water Works* case, the Court states:

> A public utility is entitled to such rates as will permit it to earn a return upon the value of the property which it employs for the convenience of the public equal to that generally being made at the same time and in the same general part of the country on investments in other business undertakings which are attended by corresponding risks and uncertainties; but it has no constitutional right to profits such as are realized or anticipated in highly profitable enterprises or speculative ventures. The return should be reasonably sufficient to assure confidence in the financial soundness of the utility, and should be adequate, under efficient and economical management, to maintain and support its credit, and enable it to raise the money necessary for the proper discharge of its public duties. [*Bluefield Water Works and Improvement Co. v. Public Service Comm'n.* 262 U.S. 679, 692 (1923)].

The Court clearly recognizes here that: (1) a regulated firm cannot remain financially sound unless the return it is allowed to earn on the value of its property is at least equal to the cost of capital (the principle relating to the demand for capital); and (2) a regulated firm will not be able to attract capital if it does not offer investors an opportunity to earn a return on their investment equal to the return they expect to earn on other investments of the same risk (the principle relating to the supply of capital).

In the *Hope Natural Gas* case, the Court reiterates the financial soundness and capital attraction principles of the *Bluefield* case:

> From the investor or company point of view it is important that there be enough revenue not only for operating expenses but also for the capital costs of the business. These include service on the debt and dividends on the stock... By that standard the return to the equity owner should be

commensurate with returns on investments in other enterprises having corresponding risks. That return, moreover, should be sufficient to assure confidence in the financial integrity of the enterprise, so as to maintain its credit and to attract capital. [*Federal Power Comm'n v. Hope Natural Gas Co.*, 320 U.S. 591, 603 (1944)].

C. Business and Financial Risks in Natural Gas Distribution Business

Q. 25 What are the major factors that affect business risk in the natural gas distribution business?

A. 25 Business risk in the natural gas distribution business is generally affected by the following economic factors:

1. High Operating Leverage. The natural gas distribution business is a business that requires a large commitment to fixed costs in relation to variable costs, a situation called high operating leverage. The relatively high degree of fixed costs in the natural gas distribution industry arises because of the average natural gas company's large investment in fixed distribution and peaking facilities. High operating leverage causes the average natural gas company's net income to be highly sensitive to sales fluctuations.
2. Demand Uncertainty. The business risk of the natural gas distribution business is increased by the high degree of demand uncertainty in the industry. Demand uncertainty is caused by: (a) the strong dependence of natural gas demand on the state of the economy and the weather; (b) the ability of customers to switch to alternative sources of energy in response to relative price differentials in these sources of energy; (c) the ability of some retail customers to purchase natural gas from competitive suppliers; and (d) rapidly changing prices for natural gas and alternate sources of energy.
3. Investment Uncertainty. The natural gas distribution business requires large investments in long-lived gas distribution and peaking facilities that are largely sunk once the investment is made. Future amounts of required investment in these facilities are highly uncertain as a result of the inherent uncertainty in forecasting energy requirements for many years into the

future, high volatility in fuel prices, and uncertainty in environmental regulations.

4. Peak Demand. The need to invest substantial sums in expensive fixed plant is further exacerbated by the peak nature of natural gas demand. The peak demand for natural gas is unusually high relative to average sales in non-peak periods.

D. Cost of Equity Estimation Methods

Q. 26 What methods do you use to estimate the cost of common equity capital for Atmos Energy?

A. 26 I use three generally accepted methods for estimating Atmos Energy's cost of common equity. These are the DCF model, the risk premium approach, and the CAPM. The DCF model assumes that the current market price of a firm's stock is equal to the discounted value of all expected future cash flows. The risk premium approach assumes that investors' required return on an equity investment is equal to the interest rate on a long-term bond plus an additional equity risk premium to compensate the investor for the risks of investing in common equities compared to bonds. The CAPM assumes that the investors' required rate of return is equal to a risk-free rate of interest plus the product of a company-specific risk factor, beta, and the expected risk premium on the market portfolio.

E. Discounted Cash Flow (DCF) Method

Q. 27 Please describe the DCF model.

A. 27 The DCF model is based on the assumption that investors value an asset on the basis of the future cash flows they expect to receive from owning the asset. Thus, investors value an investment in a bond because they expect to receive a sequence of semi-annual coupon payments over the life of the bond and a terminal payment equal to the bond's face value at the time the bond matures. Likewise, investors value an investment in a firm's stock because they expect to receive a sequence of dividend payments and, perhaps, expect to sell the stock at a higher price sometime in the future.

A second fundamental principle of the DCF method is that investors value a dollar received in the future less than a dollar received today. A future dollar is valued less than a current dollar because investors could invest a current dollar in an interest earning account and increase their wealth. This principle is called the time value of money.

Applying the two fundamental DCF principles noted above to an investment in a bond leads to the conclusion that investors value their investment in the bond on the basis of the present value of the bond's future cash flows. Thus, the price of the bond should be equal to:

EQUATION 1

$$P_B = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \ldots + \frac{C+F}{(1+i)^n}$$

where:

P_B	=	Bond price;
C	=	Cash value of the coupon payment (assumed for notational convenience to occur annually rather than semi-annually);
F	=	Face value of the bond;
i	=	The rate of interest the investor could earn by investing his money in an alternative bond of equal risk; and
n	=	The number of periods before the bond matures.

Applying these same principles to an investment in a firm's stock suggests that the price of the stock should be equal to:

EQUATION 2

$$P_s = \frac{D_1}{(1+k)} + \frac{D_2}{(1+k)^2} + \cdots + \frac{D_n + P_n}{(1+k)^n}$$

where:

P_S	=	Current price of the firm's stock;
$D_1, D_2 ... D_n$	=	Expected annual dividend per share on the firm's stock;
P_n	=	Price per share of stock at the time the investor expects to sell the stock; and
k	=	Return the investor expects to earn on alternative investments of the same risk, i.e., the investor's required rate of return.

Equation (2) is frequently called the annual discounted cash flow model of stock valuation. Assuming that dividends grow at a constant annual rate, g, this equation can be solved for k, the cost of equity. The resulting cost of equity equation is $k = D_1/P_s + g$, where k is the cost of equity, D_1 is the expected next period annual dividend, P_s is the current price of the stock, and g is the constant annual growth rate in earnings, dividends, and book value per share. The term D_1/P_s is called the dividend yield component of the annual DCF model, and the term g is called the growth component of the annual DCF model.

Q. 28 Are you recommending that the annual DCF model be used to estimate Atmos Energy's cost of equity?

A. 28 No. The DCF model assumes that a company's stock price is equal to the present discounted value of all expected future dividends. The annual DCF model is only a correct expression for the present discounted value of future dividends if dividends are paid annually at the end of each year. Since the companies in my proxy group all pay dividends quarterly, the current market price that investors are willing to pay reflects the expected quarterly receipt of dividends. Therefore, a quarterly DCF model must be used to estimate the cost of equity for these firms. The quarterly DCF model differs from the annual DCF model in that it expresses a company's price as the present discounted value of a quarterly stream of dividend payments. A complete analysis of the implications of the quarterly payment of dividends on the DCF model is provided in Appendix 1. For the reasons cited there, I employed the quarterly DCF model throughout my calculations.

Q. 29 Please describe the quarterly DCF model you use.

A. 29 The quarterly DCF model I use is described on Schedule 1 and in Appendix 2. The quarterly DCF equation shows that the cost of equity is: the sum of the future expected dividend yield and the growth rate, where the dividend in the dividend yield is the equivalent future value of the four quarterly dividends at the end of the year, and the growth rate is the expected growth in dividends or earnings per share.

Q. 30 How do you estimate the quarterly dividend payments in your quarterly DCF model?

A. 30 The quarterly DCF model requires an estimate of the dividends, d_1, d_2, d_3, and d_4, investors expect to receive over the next four quarters. I estimate the next four quarterly dividends by multiplying the previous four quarterly dividends by the factor, (1 + the growth rate, g).

Q. 31 Can you illustrate how you estimated the next four quarterly dividends with data for a specific company?

A. 31 Yes. In the case of AGL Resources, for example, the last four quarterly dividends are equal to .42, .42, .43, and .43. Thus dividends, d_1, d_2, d_3, and d_4 are equal to .438, .438, .448 and .448 [.42 x (1 + .0425)] = .438 and [.43 x (1 + .0425) = .448.]. (As noted previously, the logic underlying this procedure is described in Appendix 2.)

Q. 32 In Appendix 2, you demonstrate that the quarterly DCF model provides the theoretically correct valuation of stocks when dividends are paid quarterly. Do investors, in practice, recognize the actual timing and magnitude of cash flows when they value stocks and other securities?

A. 32 Yes. In valuing long-term government or corporate bonds, investors recognize that interest is paid semi-annually. Thus, the price of a long-term government or corporate bond is simply the present value of the semi-annual interest and principal payments on these bonds. Likewise, in valuing mortgages, investors recognize that interest is paid monthly. Thus, the value of a mortgage loan is simply the present value of the monthly interest and principal payments on the loan. In valuing stock investments, stock investors correctly recognize that dividends are paid quarterly. Thus, a firm's stock price is the present value of the stream of quarterly dividends expected from owning the stock.

Q. 33 When valuing bonds, mortgages, or stocks, would investors assume that cash flows are received only at the end of the year, when, in fact, the cash flows are received semi-annually, quarterly, or monthly?

A. 33 No. Assuming that cash flows are received at the end of the year when they are received semi-annually, quarterly, or monthly would lead investors to make serious mistakes in valuing investment opportunities. No rational investor would make the mistake of assuming that dividends or other cash flows are paid annually when, in fact, they are paid more frequently.

Q. 34 How do you estimate the growth component of the quarterly DCF model?

A. 34 I use the analysts' estimates of future earnings per share (EPS) growth reported by I/B/E/S Thomson Reuters.

Q. 35 What are the analysts' estimates of future EPS growth?

A. 35 As part of their research, financial analysts working at Wall Street firms periodically estimate EPS growth for each firm they follow. The EPS forecasts for each firm are then published. Investors who are contemplating purchasing or selling shares in individual companies review the forecasts. These estimates represent five-year forecasts of EPS growth.

Q. 36 What is I/B/E/S?

A. 36 I/B/E/S is a firm (now owned by Thomson Reuters) that reports analysts' EPS growth forecasts for a broad group of companies. The forecasts are expressed in terms of a mean forecast and a standard deviation of forecast for each firm. Investors use the mean forecast as a consensus estimate of future firm performance.

Q. 37 Why do you use the I/B/E/S growth estimates?

A. 37 The I/B/E/S growth rates: (1) are widely circulated in the financial community, (2) include the projections of multiple reputable financial analysts who develop estimates of future EPS growth, (3) are reported on a timely basis to investors, and (4) are widely used by institutional and other investors.

Q. 38 Why do you rely on analysts' projections of future EPS growth in estimating the investors' expected growth rate rather than looking at past historical growth rates?

A. 38 I rely on analysts' projections of future EPS growth because I believe that investors use analysts' forecasts to estimate future earnings growth. As discussed below, my research supports my belief.

Q. 39 Have you performed any studies concerning the use of analysts' forecasts as an estimate of investors' expected growth rate, g?

A. 39 Yes, I prepared a study in conjunction with Willard T. Carleton, Professor of Finance Emeritus at the University of Arizona, on why analysts' forecasts are the best estimate of investors' expectation of future long-term growth. This study is described in a paper entitled "Investor Growth Expectations and Stock Prices: Analysts vs. History," published in the Spring 1988 edition of *The Journal of Portfolio Management.*

Q. 40 Please summarize the results of your study.

A. 40 First, we performed a correlation analysis to identify the historically oriented growth rates which best described a firm's stock price. Then we did a regression study comparing the historical growth rates with the consensus analysts' forecasts. In every case, the regression equations containing the average of analysts' forecasts statistically outperformed the regression equations containing the historical growth estimates. These results are consistent with those found by Cragg and Malkiel, the early major research in this area (John G. Cragg and Burton G. Malkiel, *Expectations and the Structure of Share Prices*, University of Chicago Press, 1982). These results are also consistent with the hypothesis that investors use analysts' forecasts, rather than historically oriented growth calculations, in making stock buy and sell decisions. They provide overwhelming evidence that the analysts' forecasts of future growth are superior to historically oriented growth measures in predicting a firm's stock price.

Q 41 Has your study been updated?

A 41 Yes. Researchers at State Street Financial Advisors updated my study using data through year-end 2003. Their results continue to confirm that analysts' growth forecasts are superior to historically-oriented growth measures in predicting a firm's stock price.

Q. 42 What price do you use in your DCF model?

A. 42 I use a simple average of the monthly high and low stock prices for each firm for the three-month period ending July 2009. These high and low stock prices were obtained from Thomson Reuters.

Q. 43 Why do you use the three-month average stock price in applying the DCF method?

A. 43 I use a three-month average stock price in applying the DCF method because stock prices fluctuate daily, while financial analysts' forecasts for a given company are generally changed less frequently, often on a quarterly basis. Thus, to match the stock price with an earnings forecast, it is appropriate to average stock prices over a three-month period.

Q. 44 Do you include an allowance for flotation costs in your DCF analysis?

A. 44 Yes. I include a five percent allowance for flotation costs in my DCF calculations.

Q. 45 Please explain your inclusion of flotation costs.

A. 45 All firms that have sold securities in the capital markets have incurred some level of flotation costs, including underwriters' commissions, legal fees, printing expense, etc. These costs are withheld from the proceeds of the stock sale or are paid separately, and must be recovered over the life of the equity issue. Costs vary depending upon the size of the issue, the type of registration method used and other factors, but in general these costs range between three and five percent of the proceeds from the issue.[2] In addition to these costs, for large equity issues (in relation to outstanding equity shares), there is likely to be a decline in price associated with the sale of shares to the public. On average, the decline due to market pressure has been estimated at two to three percent.[3] Thus, the

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2 *See* Lee, Inmoo, Scott Lochhead, Jay Ritter, and Quanshui Zhao, "The Costs of Raising Capital," *The Journal of Financial Research*, Vol. XIX No 1 (Spring 1996), 59-74, and Clifford W. Smith, "Alternative Methods for Raising Capital," *Journal of Financial Economics* 5 (1977) 273-307.

3 *See* Richard H. Pettway, "The Effects of New Equity Sales Upon Utility Share Prices," *Public Utilities Fortnightly*, May 10, 1984, 35—39.

total flotation cost, including both issuance expense and market pressure, could range anywhere from five to eight percent of the proceeds of an equity issue. I believe a combined five percent allowance for flotation costs is a conservative estimate that should be used in applying the DCF model in this proceeding.

Q. 46 Is there any evidence that Atmos Energy, in fact, incurs flotation costs equal to approximately five percent of its stock price when it issues new equity securities

A. 46 Yes. In the Company's most recent equity offering, December 7, 2006, Atmos Energy's stock price just prior to the offering was $32.07 per share, and the net proceeds to the Company were $30.3975 per share. The difference between the pre-offering stock price and the proceeds to the Company represent a 5.21 percent discount to the recent market price. The difference between the recent market price and the net proceeds per share reflects both the issuance expenses and market pressure, as explained in Appendix 3 of my direct testimony. Additional information on Atmos Energy's three most recent stock issuances are contained in the prospectuses for these issuances. (For ease of reference, the cover page of each of Atmos Energy's three most recent public offerings are shown in Schedule 2.)

Q. 47 Is a flotation cost adjustment only appropriate if a company issues stock during the last year?

A. 47 As described in Appendix 3, a flotation cost adjustment is required whether or not a company issued new stock during the last year. Previously incurred flotation costs have not been recovered in previous rate cases; rather, they are a permanent cost associated with past issues of common stock. Just as an adjustment is made to the embedded cost of debt to reflect previously incurred debt issuance costs (regardless of whether additional bond issuances were made in the test year), so should an adjustment be made to the cost of equity regardless of whether additional stock was issued during the last year.

Q. 48 Does an allowance for recovery of flotation costs associated with stock sales in prior years constitute retroactive rate-making?

A. 48 No. An adjustment for flotation costs on equity is not meant to recover any cost that is properly assigned to prior years. In fact, the adjustment allows Atmos Energy to recover only the current carrying costs associated with flotation expenses incurred at the time stock sales were made. The original flotation costs themselves will never be recovered, because the stock is assumed to have an infinite life.

Q. 49 How do you apply the DCF approach to obtain the cost of equity capital for Atmos Energy?

A. 49 I apply the DCF approach to the Value Line natural gas companies shown in Schedule 1.

Q. 50 How do you select your proxy group of natural gas companies?

A. 50 I select all the companies in Value Line's groups of natural gas companies that provide local distribution service and: (1) paid dividends during every quarter of the last two years; (2) did not decrease dividends during any quarter of the past two years; (3) have at least two analysts included in the I/B/E/S mean growth forecast; (4) have an investment grade bond rating and a Value Line Safety Rank of 1, 2, or 3; and (5) have not announced a merger.

Q. 51 Why do you eliminate companies that have either decreased or eliminated their dividend in the past two years?

A. 51 The DCF model requires the assumption that dividends will grow at a constant rate into the indefinite future. If a company has either decreased or eliminated its dividend in recent years, an assumption that the company's dividend will grow at the same rate into the indefinite future is questionable.

Q. 52 Why do you eliminate companies that have fewer than two analysts included in the I/B/E/S mean forecasts?

A. 52 The DCF model also requires a reliable estimate of a company's expected future growth. For most companies, the I/B/E/S mean growth forecast is the best available estimate of the growth term in the DCF model. However, the I/B/E/S estimate may be less reliable if the mean estimate is based on the inputs of very few analysts. On the basis of my professional judgment, I normally specify that the I/B/E/S long-term earnings growth forecast must include the forecasts of at least three analysts. However, in August 2009 there are only five natural gas companies with growth forecasts from at least three analysts. In this study, therefore, I also include results for companies that had growth forecasts based on two analysts' growth forecasts.

Q. 53 Why do you eliminate companies that have announced mergers that are not yet completed?

A. 53 A merger announcement can sometimes have a significant impact on a company's stock price because of anticipated merger-related cost savings and new market opportunities. Analysts' growth forecasts, on the other hand, are necessarily related to companies as they currently exist, and do not reflect investors' views of the potential cost savings and new market opportunities associated with mergers. The use of a stock price that includes the value of potential mergers in conjunction with growth forecasts that do not include the growth enhancing prospects of potential mergers produces DCF results that tend to distort a company's cost of equity.

Q. 54 Is your natural gas company group a reasonable risk proxy for Atmos Energy?

A. 54 Yes. Many investors use the Value Line Safety Rank as a measure of equity risk. The average Value Line Safety Rank for my proxy group of natural gas companies is approximately 2 on a simple average basis and 2.5 on a market-weighted basis, on a scale where 1 is the most safe and 5 is the least safe, compared to a Value Line Safety Rank of 2 for Atmos Energy. The average S&P bond rating of the natural gas companies in my proxy group is approximately A- to BBB+. The S&P bond rating for Atmos Energy is BBB+. (See Schedule 1.)

Q. 55 Please summarize the results of your application of the DCF model to your natural gas company proxy group.

A. 55 I obtain a DCF result of 11.9 percent (see Schedule 1).

F. Risk Premium Method

Q. 56 Please describe the risk premium method of estimating Atmos Energy's cost of equity.

A. 56 The risk premium method is based on the principle that investors expect to earn a return on an equity investment in Atmos Energy that reflects a "premium" over and above the return they expect to earn on an investment in a portfolio of bonds. This equity risk premium compensates equity investors for the additional risk they bear in making equity investments versus bond investments.

Q. 57 Does the risk premium approach specify what debt instrument should be used to estimate the interest rate component in the methodology?

A. 57 No. The risk premium approach can be implemented using virtually any debt instrument. However, the risk premium approach does require that the debt instrument used to estimate the risk premium be the same as the debt instrument used to calculate the interest rate component of the risk premium approach. For example, if the risk premium on equity is calculated by comparing the returns on stocks and the returns on A-rated utility bonds, then the interest rate on A-rated utility bonds must be used to estimate the interest rate component of the risk premium approach.

Q. 58 Does the risk premium approach require that the same companies be used to estimate the stock return as are used to estimate the bond return?

A. 58 No. For example, many analysts apply the risk premium approach by comparing the return on a portfolio of stocks to the return on Treasury securities such as long-term Treasury bonds. Clearly, in this widely-accepted application of the risk premium approach, the same companies are not used to estimate the stock return as are used to estimate the bond return, since the U.S. government is not a company.

Q. 59 How do you measure the required risk premium on an equity investment in Atmos Energy?

A. 59 I use two methods to estimate the required risk premium on an equity investment in Atmos Energy. The first is called the ex ante risk premium method and the second is called the ex post risk premium method.

1. Ex Ante Risk Premium Method

Q. 60 Please describe your ex ante risk premium method of measuring the required risk premium on an equity investment in Atmos Energy.

A. 60 My ex ante risk premium method is based on studies of the DCF expected return on my comparable group of natural gas companies compared to the interest rate on Moody's A-rated utility bonds. Specifically, for each month in my study period, I calculate the risk premium using the equation,

$$RP_{PROXY} = DCF_{PROXY} - I_A$$

where:

RP_{PROXY}	=	the required risk premium on an equity investment in the proxy group of companies,
DCF_{PROXY}	=	average DCF estimated cost of equity on a portfolio of proxy companies; and
I_A	=	the yield to maturity on an investment in A-rated utility bonds.

I then perform a regression analysis to determine if there is a relationship between the calculated risk premium and interest rates. I use the results of the regression analysis to estimate the investors' required risk premium. To estimate the cost of equity, I then add the required risk premium to the current yield on A-rated utility bonds. A detailed description of my ex ante risk premium studies is contained in Appendix 4, and the underlying DCF results and interest rates are displayed in Schedule 3.

Q. 61 Why do you add the required risk premium to the current yield to maturity on A-rated utility bonds rather than the forecasted yield to maturity?

A. 61 Although it is appropriate in theory to add the required risk premium to the forecasted yield to maturity on A-rated utility bonds, I did not have information on the forecasted yield to maturity on A-rated utility bonds at the time Atmos Energy needed my cost of equity input for their cost of service studies. I have recently obtained interest rate forecasts from *Blue Chip Financial Forecasts* that indicates that the forecasted yield to maturity on A-rated utility bonds exceeds the current interest rate used in my studies by approximately 100 basis points.[4] Given the positive spread between forecasted interest rates and current interest rates, my cost of equity estimates based on the current interest rates are conservative.

Q. 62 What cost of equity do you obtain from your ex ante risk premium method?

A. 62 As described above, to estimate the cost of equity using the ex ante risk premium method, one may add the estimated risk premium over the yield on A-rated utility bonds to the yield to maturity on A-rated utility bonds.[5] The average yield to maturity on Moody's A-rated utility bonds at July 2009 is 5.97 percent. My analyses produce an estimated risk premium over the yield on A-rated utility bonds equal to 4.94 percent. Adding an estimated risk premium of 4.94 percent to the 5.97 percent average yield to maturity on A-rated utility bonds produces a cost of equity estimate of 10.9 percent using the ex ante risk premium method.

2. Ex Post Risk Premium Method

Q. 63 Please describe your ex post risk premium method for measuring the required risk premium on an equity investment in Atmos Energy.

A. 63 I first perform a study of the comparable returns received by bond and stock investors over the last 72 years. I estimate the returns on stock and bond portfolios, using stock price and dividend yield data on the S&P 500 and bond yield data on Moody's A-rated Utility Bonds. My study consists of making an investment of one dollar in the S&P 500 and Moody's A-rated Utility Bonds at the beginning of 1937, and reinvesting the principal plus return each year to 2009. The return associated with each stock portfolio is the sum of the annual dividend yield and capital gain (or loss) which accrued to this portfolio during

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4 Blue Chip does not provide a forecast for A-rated utility bond yields. I estimate the forecasted yield on A-rated utility bonds using Blue Chip forecasts for Baa-rated corporate bonds plus the current difference between A-rated utility and Baa-rated corporate bonds.

5 As noted above, one could use the yield to maturity on other debt investments to measure the interest rate component of the risk premium approach as long as one uses the yield on the same debt investment to measure the expected risk premium component of the risk premium approach. I chose to use the yield on A-rated utility bonds because it is a frequently used benchmark for utility bond yields.

the year(s) in which it was held. The return associated with the bond portfolio, on the other hand, is the sum of the annual coupon yield and capital gain (or loss) which accrued to the bond portfolio during the year(s) in which it was held. The resulting annual returns on the stock and bond portfolios purchased in each year between 1937 and 2009 are shown on Schedule 4. The average annual return on an investment in the S&P 500 stock portfolio is 10.8 percent, while the average annual return on an investment in the Moody's A-rated utility bond portfolio is 6.3 percent. Thus, the risk premium on the S&P 500 stock portfolio is 4.5 percent.

I also conduct a second study using stock data on the S&P Utilities rather than the S&P 500. As shown on Schedule 5, the S&P utilities stock portfolio showed an average annual return of 10.5 percent per year. Thus, the return on the S&P utilities stock portfolio exceeds the return on the Moody's A-rated utility bond portfolio by 4.2 percent.

Q. 64 Why is it appropriate to perform your ex post risk premium analysis using both the S&P 500 and the S&P Utilities stock indices?

A. 64 I perform my ex post risk premium analysis on both the S&P 500 and the S&P Utilities because I believe utilities today face risks that are somewhere in between the average risk of the S&P Utilities and the S&P 500 over the years 1937 to 2009. Thus, I use the average of the two historically-based risk premiums as my estimate of the required risk premium in my ex post risk premium method. I note that the spread between the average risk premium on the S&P 500 and the average risk premium on the S&P Utilities is just 30 basis points.

Q. 65 Why do you analyze investors' experiences over such a long time frame?

A. 65 Because day-to-day stock price movements can be somewhat random, it is inappropriate to rely on short-run movements in stock prices in order to derive a reliable risk premium. Rather than buying and selling frequently in anticipation of highly volatile price movements, most investors employ a strategy of buying and holding a diversified portfolio of stocks. This buy-and-hold strategy will allow an investor to achieve a much more predictable long-run return on stock investments and at the same time will minimize transaction costs. The situation is very similar to the problem of predicting the results of coin tosses. I cannot predict with any reasonable degree of accuracy the result of a single, or even a few, flips of a balanced coin; but I can predict with a good deal of confidence that approximately 50 heads will appear in 100 tosses of this coin. Under these circumstances, it is most appropriate to estimate future experience from long-run evidence of investment performance.

Q. 66 Would your study provide a different risk premium if you started with a different time period?

A. 66 Yes. The risk premium results do vary somewhat depending on the historical time period chosen. My policy was to go back as far in history as I could get reliable data. I thought it would be most meaningful to begin after the passage and implementation of the Public Utility Holding Company Act of 1935. This Act significantly changed the structure of the public utility industry. Since the Public Utility Holding Company Act of 1935 was not implemented until the beginning of 1937, I felt that numbers taken from before this date would not be comparable to those taken after. (The repeal of the 1935 Act has not materially impacted the structure of the public utility industry; thus, the Act's repeal does not have any impact on my choice of time period.)

Q. 67 Why is it necessary to examine the yield from debt investments in order to determine the investors' required rate of return on equity capital?

A. 67 As previously explained, investors expect to earn a return on their equity investment that exceeds currently available bond yields. This is because the return on equity, being a residual return, is less certain than the yield on bonds and investors must be compensated for this uncertainty. Second, the investors' current expectations concerning the amount by which the return on equity will exceed the bond yield will be influenced by historical differences in returns to bond and stock investors. For these reasons, we can estimate investors' current expected returns from an equity investment from knowledge of current bond yields and past differences between returns on stocks and bonds.

Q. 68 Has there been any significant trend in the equity risk premium over the 1937 to 2009 time period of your risk premium study?

A. 68 No. Statisticians test for trends in data series by regressing the data observations against time. I have performed such a time series regression on my two data sets of historical risk premiums. As shown below, there is no statistically significant trend in my risk premium data. Indeed, the coefficient on the time variable is insignificantly different from zero (if there were a trend, the coefficient on the time variable should be significantly different from zero).

TABLE 2

REGRESSION OUTPUT FOR RISK PREMIUM ON S&P 500

LINE NO.		INTERCEPT	TIME	ADJUSTED R SQUARE	F
1	Coefficient	3.096	(0.002)	0.023	2.66
2	T Statistic	1.654	(1.630)		

TABLE 3

REGRESSION OUTPUT FOR RISK PREMIUM ON S&P UTILITIES

LINE NO.		INTERCEPT	TIME	ADJUSTED R SQUARE	F
1	Coefficient	1.383	-0.001	-0.006	0.56
2	T Statistic	0.776	-0.751		

Q. 69 Is your conclusion that there is no significant trend in the equity risk premium supported in the financial literature?

A. 69 Yes. The *Stocks, Bonds, Bills, and Inflation® 2009 Valuation Edition Yearbook* ("Ibbotson® SBBI®") published by Morningstar, Inc., contains an analysis of "trends" in historical risk premium data. Ibbotson® SBBI® uses correlation analysis to determine if there is any pattern or "trend" in risk premiums over time. This analysis also demonstrates that there are no trends in risk premiums over time.

Q. 70 Why is it significant that historical risk premiums have no trend or other statistical pattern over time?

A. 70 The significance of this evidence is that the average historical risk premium is a reasonable estimate of the future expected risk premium. As noted in Ibbotson® SBBI®:

The significance of this evidence is that the realized equity risk premium next year will not be dependent on the realized equity risk premium from this year. That is, there is no discernable pattern in the realized equity risk premium—it is virtually impossible to forecast next year's realized risk premium based on the premium of the previous year. For example, if this year's difference between the riskless rate and the return on the stock market is higher than last year's, that does not imply that next year's will be higher than this year's. It is as likely to be higher as it is lower. The best estimate of the expected value of a variable that has behaved randomly in the past is the average (or arithmetic mean) of its past values. [Ibbotson® SBBI®, page 61.]

Q. 71 What conclusions do you draw from your ex post risk premium analyses about the required return on an equity investment in Atmos Energy?

A. 71 My studies provide strong evidence that investors today require an equity return of approximately 4.2 to 4.5 percentage points above the expected yield on A-rated utility bonds. The average yield on A-rated utility bonds at July 2009 is 5.97 percent. Adding a 4.2 to 4.5 percentage point risk premium to a yield of 5.97 percent on A-rated utility bonds, I obtain an expected return on equity from the ex post risk premium method in the range 10.2 percent to 10.4 percent, with a midpoint of 10.3 percent. Because the ex post methodology does not reflect flotation costs, I add a 27 basis-point allowance for flotation costs, which I determine by calculating the difference in my DCF results with and without a flotation cost allowance. Adding a 27 basis-point allowance for flotation costs, I obtain an estimate of 10.6 percent as the cost of equity for Atmos Energy using the ex post risk premium method.[6]

G. Capital Asset Pricing Model (CAPM)

Q. 72 What is the CAPM?

A. 72 The CAPM is an equilibrium model of the security markets in which the expected or required return on a given security is equal to the risk-free rate of interest, plus the company equity "beta," times the market risk premium:

Cost of equity = Risk-free rate + Equity beta x Market risk premium

6 This estimate, which is based on current interest rates rather than forecasted rates, is conservative. If I were to use the forecasted interest rate on A-rated utility bonds, my ex post risk premium estimate of the cost of equity would be approximately 100 basis points higher. (See Question and Answer 61 above.)

The risk-free rate in this equation is the expected rate of return on a risk-free government security, the equity beta is a measure of the company's risk relative to the market as a whole, and the market risk premium is the premium investors require to invest in the market basket of all securities compared to the risk-free security.

Q. 73 How do you use the CAPM to estimate the cost of equity for your proxy companies?

A. 73 The CAPM requires an estimate of the risk-free rate, the company-specific risk factor or beta, and the expected return on the market portfolio. For my estimate of the risk-free rate, I use the average yield to maturity on 20-year Treasury bonds at July 2009, 4.38 percent. For my estimate of the company-specific risk, or beta, I use the average Value Line beta of 0.85 for my proxy companies. For my estimate of the expected risk premium on the market portfolio, I use two approaches. First, I use the Ibbotson® SBBI® 6.5 percent risk premium on the market portfolio, which is measured from the difference between the arithmetic mean return on the S&P 500 (11.7 percent) and the income return on 20-year Treasury bonds (5.2 percent), as reported by Ibbotson® SBBI® (11.7 – 5.2 = 6.5). Second, I estimate the risk premium on the market portfolio from the difference between the DCF cost of equity for the S&P 500 (12.7 percent) and the yield to maturity on 20-year Treasury bonds, (4.38 percent). My second approach produces a risk premium equal to 8.3 percent (12.7 - 4.38 = 8.3).

Q. 74 Why do you recommend that the risk premium on the market portfolio be estimated using the difference between the arithmetic mean return on the S&P 500?

A. 74 As explained in Ibbotson® SBBI®, the arithmetic mean return is the best approach for calculating the return investors expect to receive in the future:

> The equity risk premium data presented in this book are arithmetic average risk premia as opposed to geometric average risk premia. The arithmetic average equity risk premium can be demonstrated to be most appropriate when discounting future cash flows. For use as the expected equity risk premium in either the CAPM or the building block approach, the arithmetic mean or the simple difference of the arithmetic means of stock market returns and riskless rates is the relevant number. This is because both the CAPM and the building block approach are additive models, in

> which the cost of capital is the sum of its parts. The geometric average is more appropriate for reporting past performance, since it represents the compound average return. [SBBI, p. 59.]

A discussion of the importance of using arithmetic mean returns in the context of CAPM or risk premium studies is contained in Schedule 6.

Q. 75 Why do you recommend that the risk premium on the market portfolio be estimated using the income return on 20-year Treasury bonds rather than the total return on these bonds?

A. 75 As discussed above, the CAPM requires an estimate of the risk-free rate of interest. When Treasury bonds are issued, the income return on the bond is risk free, but the total return, which includes both an income and capital gains or losses, is not. Thus, the income return should be used in the CAPM because it is only the income return that is risk free.

Q. 76 What CAPM result do you obtain when you estimate the expected return on the market portfolio from the arithmetic mean difference between the return on the market and the yield on 20-year Treasury bonds?

A. 76 I obtain a CAPM estimate of 10.2 percent [see Schedule 7].

Q. 77 What CAPM result do you obtain when you estimate the risk premium on the market portfolio by applying the DCF model to the S&P 500?

A. 77 I obtain a CAPM result of 11.5 percent [see Schedule 8].

Q. 78 Can a reasonable application of the CAPM produce higher cost of equity results than you have just reported?

A. 78 Yes. The CAPM tends to underestimate the cost of equity for small market capitalization companies such as my natural gas proxy companies.[7]

Q. 79 Does the finance literature support an adjustment to the CAPM equation to account for a company's size as measured by market capitalization supported in the finance literature?

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7 In addition, as discussed above, these estimates based on current interest rates rather than forecasted rates is conservative. If I were to use the forecasted interest rate on Treasury bonds, my historical CAPM estimate of the cost of equity would be approximately 60 basis points higher and my DCF-based CAPM estimate would be approximately 10 basis points higher.

A. 79 Yes. For example, Ibbotson® SBBI® supports such an adjustment. Their estimates of the size premium required to be added to the basic CAPM cost of equity are shown below in Table 4.

TABLE 4

IBBOTSON® ESTIMATES OF PREMIUMS FOR COMPANY SIZE[8]

SIZE	SMALLEST MKT. CAP. ($MILLIONS)	PREMIUM
Large-Cap (No Adjustment)	>7,360.271	--
Mid-Cap	1,849.950	0.94%
Low-Cap	453.398	1.74%
Micro-Cap	1.575	3.74%

Q. 80 Are there other reasons to believe that the CAPM may produce cost of equity estimates at this time that are unreasonably low?

A. 80 Yes. There is considerable evidence in the finance literature that the CAPM tends to underestimate the cost of equity for companies whose equity beta is less than 1.0 and to overestimate the cost of equity for companies whose equity beta is greater than 1.0.[9]

Q. 81 Can you briefly summarize the evidence that the CAPM underestimates the required returns for securities or portfolios with betas less than 1.0 and overestimates required returns for securities or portfolios with betas greater than 1.0?

A. 81 Yes. The CAPM conjectures that security returns increase with increases in security betas in line with the equation

$$ER_i = R_f + \beta_i \lfloor ER_m - R_f \rfloor,$$

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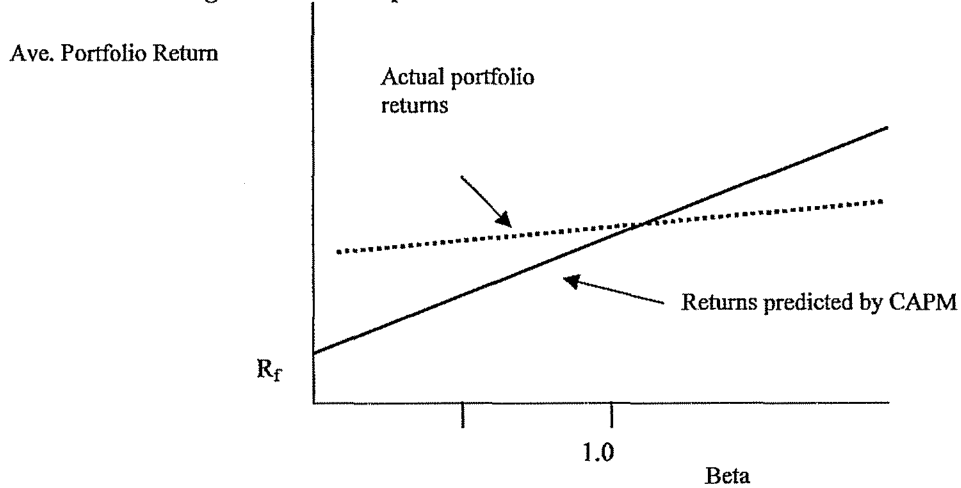
8 Ibbotson® SBBI® 2009 Valuation Yearbook.

9 *See*, for example, Fischer Black, Michael C. Jensen, and Myron Scholes, "The Capital Asset Pricing Model: Some Empirical Tests," in *Studies in the Theory of Capital Markets*, M. Jensen, ed. New York: Praeger, 1972; Eugene Fama and James MacBeth, "Risk, Return, and Equilibrium: Empirical Tests," *Journal of Political Economy* 81 (1973), pp. 607-36; Robert Litzenberger and Krishna Ramaswamy, "The Effect of Personal Taxes and Dividends on Capital Asset Prices: Theory and Empirical Evidence," *Journal of Financial Economics* 7 (1979), pp. 163-95.; Rolf Banz, "The Relationship between Return and Market Value of Common Stocks," *Journal of Financial Economics* (March 1981), pp. 3-18; and Eugene Fama and Kenneth French, "The Cross-Section of Expected Returns," *Journal of Finance* (June 1992), pp. 427-465.

where ER_i is the expected return on security or portfolio *i*, R_f is the risk-free rate, $ER_m - R_f$ is the expected risk premium on the market portfolio, and β_i is a measure of the risk of investing in security or portfolio *i*. If the CAPM correctly predicts the relationship between risk and return in the marketplace, then the realized returns on portfolios of securities and the corresponding portfolio betas should lie on the solid straight line with intercept R_f and slope $[R_m - R_f]$ shown below.

Figure 1
Average Returns Compared to Beta for Portfolios Formed on Prior Beta



Financial scholars have found that the relationship between realized returns and betas is inconsistent with the relationship posited by the CAPM. As described in Fama and French (1992) and Fama and French (2004), the actual relationship between portfolio betas and returns is shown by the dotted line in the figure above. Although financial scholars disagree on the reasons why the return/beta relationship looks more like the dotted line in the figure than the solid line, they generally agree that the dotted line lies above the solid line for portfolios with betas less than 1.0 and below the solid line for portfolios with betas greater than 1.0. Thus, in practice, scholars generally agree that the CAPM underestimates portfolio returns for companies with betas less than 1.0, and overestimates portfolio returns for portfolios with betas greater than 1.0.

Q. 82 What conclusions do you reach from your review of the literature on the CAPM to predict the relationship between risk and return in the marketplace?

A. 82 I conclude that the financial literature strongly supports the proposition that the CAPM underestimates the cost of equity for companies such as public utilities with betas less than 1.0.

H. Fair Rate of Return on Equity

Q. 83 Based on your analyses, what is your conclusion regarding your proxy companies' cost of equity?

A. 83 Based on my analyses, which included the application of several cost of equity methods to my proxy companies, I conclude that my proxy companies' cost of equity is in the range 10.2 percent to 11.9 percent, with an average cost of equity equal to 11.0 percent.

Q. 84 Does the cost of equity for Atmos Energy depend on its ratemaking capital structure?

A. 84 Yes. My analyses are based on the average market value capital structure of my proxy companies, which has more than 58 percent equity on a composite basis or more than 63 percent equity on a simple average basis. If Atmos Energy's ratemaking, or book value capital structure, is used to set rates, the cost of equity for Atmos Energy will necessarily be higher than the cost of equity for the proxy group because the financial risk associated with Atmos Energy's book value capital structure is significantly higher than the financial risk reflected in the cost of equity estimate for my proxy companies.

Q. 85 What ROE do you recommend for Atmos Energy?

A. 85 I recommend an ROE of 11.0 percent for Atmos Energy. My recommendation takes into consideration Atmos Energy's policy decision to moderate the impact of its rate request on ratepayers. My recommended return on equity is conservative in that it does not reflect: (1) the higher financial risk implicit in the book value capital structure of Atmos Energy, which will be used to set rates in this proceeding; (2) the observation that forecasted yields on both A-rated utility bonds and Treasury bonds are significantly higher than the current yields on these securities; (3) the small size premium for small market capitalization companies such as those in my proxy group of natural gas companies; and (4) the evidence that the CAPM underestimates the cost of equity for companies with betas less than 1.0.

I. Allowed Rate of Return on Total Capital

Q. 86 What is Atmos Energy's recommended capital structure and debt cost rate?

A. 86 As discussed in the testimony of Company Witness Laurie M. Sherwood, Atmos Energy is recommending a capital structure containing 48.6 percent long-term debt and 51.4 percent equity. The cost rate for long-term debt 6.87 percent.

Q. 87 What allowed rate of return on total capital is derived using this capital structure, the long-term debt cost rate of 6.87 percent, and the 11.0 percent cost of equity you find for your proxy group?

A. 87 Using a capital structure containing 48.6 percent long-term debt and 51.4 percent equity and cost rates of 6.87 percent and 11.0 percent, respectively, produces an overall rate of return equal to 9.00 percent for the purpose of setting Atmos Energy's rates in this case, as shown below in Table 5.

TABLE 5
WEIGHTED AVERAGE COST OF CAPITAL

SOURCE OF CAPITAL	% OF TOTAL	COST RATE	WEIGHTED COST
Long-term Debt	48.6%	6.87%	3.34%
Common Equity	51.4%	11.00%	5.66%
Total	100.0%		9.00%

Q. 88 Does this conclude your testimony?

A. 88 Yes, it does.

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ATMOS ENERGY
SCHEDULE 1
SUMMARY OF DISCOUNTED CASH FLOW ANALYSIS
FOR NATURAL GAS COMPANIES

LINE NO.	COMPANY	D_0	D_0	P_0	GROWTH	COST OF EQUITY
1	AGL Resources	0.430	1.72	31.017	4.25%	10.5%
2	Atmos Energy	0.330	1.32	25.230	5.00%	11.0%
3	EQT Corp.	0.220	0.88	35.962	9.00%	11.9%
4	National Fuel Gas	0.325	1.34	35.078	8.50%	12.9%
5	Nicor Inc.	0.465	1.86	33.610	4.33%	10.6%
6	NiSource Inc.	0.230	0.92	11.570	3.00%	12.0%
7	Northwest Nat. Gas	0.395	1.58	43.398	4.75%	8.9%
8	ONEOK Inc.	0.400	1.68	29.035	7.25%	13.8%
9	Piedmont Natural Gas	0.270	1.08	23.733	6.93%	12.2%
10	South Jersey Inds.	0.298	1.19	34.848	9.67%	13.7%
11	Southwest Gas	0.238	0.95	21.663	6.00%	10.9%
12	Market-Weighted Average					11.9%

Notes:

d_0 = Most recent quarterly dividend.

d_1, d_2, d_3, d_4 = Next four quarterly dividends, calculated by multiplying the last four quarterly dividends per Value Line, by the factor (1 + g).

P_0 = Average of the monthly high and low stock prices during the three months ending July 2009 per Thomson Reuters.

FC = Flotation costs expressed as a percent of gross proceeds (5%).

g = I/B/E/S forecast of future earnings growth July 2009.

k = Cost of equity using the quarterly version of the DCF model.

$$k = \frac{d_1(1+k)^{.75} + d_2(1+k)^{.50} + d_3(1+k)^{.25} + d_4}{P_0(1-FC)} + g$$

ATMOS ENERGY
SCHEDULE 1 (continued)
VALUE LINE SAFETY RANKS AND STANDARD & POOR'S BOND RATINGS FOR PROXY GAS COMPANIES

LINE NO.	COMPANY	SAFETY RANK	S&P BOND RATING	S&P BOND RATING (NUMERICAL)
1	AGL Resources	2	A-	5
2	Atmos Energy	2	BBB+	6
3	EQT Corp.	3	BBB	7
4	National Fuel Gas	2	BBB	7
5	Nicor Inc.	3	AA	1
6	NiSource Inc.	3	BBB-	8
7	Northwest Nat. Gas	1	AA-	2
8	ONEOK Inc.	3	BBB	7
9	Piedmont Natural Gas	2	A	4
10	South Jersey Inds.	2	BBB+	6
11	Southwest Gas	3	BBB	7
12	Market-Weighted Average	2.5	BBB+	6.0
13	Simple Average	2.4	A- to BBB+	5.5

Source of data: Standard & Poor's, August 2009; The Value Line Investment Analyzer August 2009.

PROSPECTUS SUPPLEMENT
(To Prospectus dated January 30, 2002)

8,650,000 Shares

ATMOS energy

Atmos Energy Corporation

Common Stock

Atmos Energy Corporation is selling all of the shares.

The shares trade on the New York Stock Exchange under the symbol "ATO." On July 13, 2004, the last sale price of the shares as reported on the New York Stock Exchange was $24.91 per share.

Investing in our common stock involves risks that are described in the "Risk Factors" section beginning on page S-7 of this prospectus supplement.

	Per Share	Total
Public offering price	$24.75	$214,087,500
Underwriting discount	$.99	$8,563,500
Proceeds, before expenses, to Atmos	$23.76	$205,524,000

The underwriters may also purchase up to an additional 1,289,393 shares at the public offering price, less the underwriting discount, within 30 days from the date of this prospectus supplement to cover overallotments.

Neither the Securities and Exchange Commission nor any state securities commission has approved or disapproved of these securities or determined if this prospectus supplement or the accompanying prospectus is truthful or complete. Any representation to the contrary is a criminal offense.

The shares will be ready for delivery on or about July 19, 2004.

Merrill Lynch & Co.
JPMorgan
Lehman Brothers
UBS Investment Bank
A.G. Edwards
Edward Jones

The date of this prospectus supplement is July 13, 2004.

PROSPECTUS SUPPLEMENT
(To prospectus dated September 15, 2004)

14,000,000 Shares

ATMOS energy

Atmos Energy Corporation

Common Stock

Atmos Energy Corporation is selling all of the shares.

The shares trade on the New York Stock Exchange under the symbol "ATO." On October 21, 2004, the last sale price of the shares as reported on the New York Stock Exchange was $25.20 per share.

Investing in our common stock involves risks. See the "Risk Factors" section beginning on page S-11 of this prospectus supplement.

	Per Share	Total
Public offering price	$24.75	$346,500,000
Underwriting discount	$.99	$13,860,000
Proceeds, before expenses, to Atmos	$23.76	$332,640,000

The underwriters may also purchase up to an additional 2,100,000 shares at the public offering price, less the underwriting discount, within 30 days from the date of this prospectus supplement to cover overallotments.

Neither the Securities and Exchange Commission nor any state securities commission has approved or disapproved of these securities or determined if this prospectus supplement or the accompanying prospectus is truthful or complete. Any representation to the contrary is a criminal offense.

The shares will be ready for delivery on or about October 27, 2004.

Merrill Lynch & Co.
Banc of America Securities LLC
JPMorgan
SunTrust Robinson Humphrey
Wachovia Securities

The date of this prospectus supplement is October 21, 2004.

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PROSPECTUS SUPPLEMENT
(To Prospectus dated December 4, 2006)

5,500,000 Shares

ATMOS energy

Common Stock

This is an offering of 5,500,000 shares of the common stock of Atmos Energy Corporation.

Our common stock is listed on the New York Stock Exchange under the symbol "ATO." The last reported sales price of our common stock on December 7, 2006 was $32.07.

Investing in our common stock involves risks. See "Risk Factors" beginning on page 1 of the accompanying prospectus.

	Per Share	Total
Price to the public	$31.5000	$173,250,000
Underwriting discounts and commissions	$ 1.1025	$ 6,063,750
Proceeds to Atmos Energy Corporation (before expenses)	$30.3975	$167,186,250

We have granted to the underwriters the option to purchase up to 825,000 additional shares of common stock on the same terms and conditions set forth above if the underwriters sell more than 5,500,000 shares of common stock in this offering.

Neither the Securities and Exchange Commission nor any state securities commission has approved or disapproved of these securities or passed on the adequacy or accuracy of this prospectus supplement. Any representation to the contrary is a criminal offense.

Lehman Brothers and Goldman, Sachs & Co., on behalf of the underwriters, expect to deliver the shares on or about December 13, 2006.

Joint Book-Running Managers

LEHMAN BROTHERS GOLDMAN, SACHS & CO.

BANC OF AMERICA SECURITIES LLC
JPMORGAN
MERRILL LYNCH & CO.
SUNTRUST ROBINSON HUMPHREY
WACHOVIA SECURITIES

December 7, 2006

ATMOS ENERGY
SCHEDULE 3
COMPARISON OF DCF EXPECTED RETURN ON AN INVESTMENT IN NATURAL GAS COMPANIES TO THE INTEREST RATE ON MOODY'S A-RATED UTILITY BONDS

LINE NO.	DATE	DCF	BOND YIELD	RISK PREMIUM
1	Jun-98	0.1154	0.0703	0.0451
2	Jul-98	0.1186	0.0703	0.0483
3	Aug-98	0.1234	0.0700	0.0534
4	Sep-98	0.1273	0.0693	0.0580
5	Oct-98	0.1260	0.0696	0.0564
6	Nov-98	0.1211	0.0703	0.0508
7	Dec-98	0.1185	0.0691	0.0494
8	Jan-99	0.1195	0.0697	0.0498
9	Feb-99	0.1243	0.0709	0.0534
10	Mar-99	0.1257	0.0726	0.0531
11	Apr-99	0.1260	0.0722	0.0538
12	May-99	0.1221	0.0747	0.0474
13	Jun-99	0.1208	0.0774	0.0434
14	Jul-99	0.1222	0.0771	0.0451
15	Aug-99	0.1220	0.0791	0.0429
16	Sep-99	0.1226	0.0793	0.0433
17	Oct-99	0.1233	0.0806	0.0427
18	Nov-99	0.1240	0.0794	0.0446
19	Dec-99	0.1280	0.0814	0.0466
20	Jan-00	0.1301	0.0835	0.0466
21	Feb-00	0.1344	0.0825	0.0519
22	Mar-00	0.1344	0.0828	0.0516
23	Apr-00	0.1316	0.0829	0.0487
24	May-00	0.1292	0.0870	0.0422
25	Jun-00	0.1295	0.0836	0.0459
26	Jul-00	0.1317	0.0825	0.0492
27	Aug-00	0.1290	0.0813	0.0477
28	Sep-00	0.1257	0.0823	0.0434
29	Oct-00	0.1260	0.0814	0.0446
30	Nov-00	0.1251	0.0811	0.0440
31	Dec-00	0.1239	0.0784	0.0455
32	Jan-01	0.1261	0.0780	0.0481
33	Feb-01	0.1261	0.0774	0.0487
34	Mar-01	0.1275	0.0768	0.0507
35	Apr-01	0.1227	0.0794	0.0433
36	May-01	0.1302	0.0799	0.0503
37	Jun-01	0.1304	0.0785	0.0519
38	Jul-01	0.1338	0.0778	0.0560
39	Aug-01	0.1327	0.0759	0.0568

LINE NO.	DATE	DCF	BOND YIELD	RISK PREMIUM
40	Sep-01	0.1268	0.0775	0.0493
41	Oct-01	0.1268	0.0763	0.0505
42	Nov-01	0.1268	0.0757	0.0511
43	Dec-01	0.1254	0.0783	0.0471
44	Jan-02	0.1236	0.0766	0.0470
45	Feb-02	0.1241	0.0754	0.0487
46	Mar-02	0.1189	0.0776	0.0413
47	Apr-02	0.1159	0.0757	0.0402
48	May-02	0.1162	0.0752	0.0410
49	Jun-02	0.1170	0.0741	0.0429
50	Jul-02	0.1242	0.0731	0.0511
51	Aug-02	0.1234	0.0717	0.0517
52	Sep-02	0.1260	0.0708	0.0552
53	Oct-02	0.1250	0.0723	0.0527
54	Nov-02	0.1221	0.0714	0.0507
55	Dec-02	0.1216	0.0707	0.0509
56	Jan-03	0.1219	0.0706	0.0513
57	Feb-03	0.1232	0.0693	0.0539
58	Mar-03	0.1195	0.0679	0.0516
59	Apr-03	0.1162	0.0664	0.0498
60	May-03	0.1126	0.0636	0.0490
61	Jun-03	0.1114	0.0621	0.0493
62	Jul-03	0.1127	0.0657	0.0470
63	Aug-03	0.1139	0.0678	0.0461
64	Sep-03	0.1127	0.0656	0.0471
65	Oct-03	0.1123	0.0643	0.0480
66	Nov-03	0.1089	0.0637	0.0452
67	Dec-03	0.1071	0.0627	0.0444
68	Jan-04	0.1059	0.0615	0.0444
69	Feb-04	0.1039	0.0615	0.0424
70	Mar-04	0.1037	0.0597	0.0440
71	Apr-04	0.1041	0.0635	0.0406
72	May-04	0.1045	0.0662	0.0383
73	Jun-04	0.1036	0.0646	0.0390
74	Jul-04	0.1011	0.0627	0.0384
75	Aug-04	0.1008	0.0614	0.0394
76	Sep-04	0.0976	0.0598	0.0378
77	Oct-04	0.0974	0.0594	0.0380
78	Nov-04	0.0962	0.0597	0.0365
79	Dec-04	0.0970	0.0592	0.0378
80	Jan-05	0.0990	0.0578	0.0412
81	Feb-05	0.0979	0.0561	0.0418
82	Mar-05	0.0979	0.0583	0.0396
83	Apr-05	0.0988	0.0564	0.0424
84	May-05	0.0981	0.0553	0.0427
85	Jun-05	0.0976	0.0540	0.0436

LINE NO.	DATE	DCF	BOND YIELD	RISK PREMIUM
86	Jul-05	0.0966	0.0551	0.0415
87	Aug-05	0.0969	0.0550	0.0419
88	Sep-05	0.0980	0.0552	0.0428
89	Oct-05	0.0990	0.0579	0.0411
90	Nov-05	0.1049	0.0588	0.0461
91	Dec-05	0.1045	0.0580	0.0465
92	Jan-06	0.0982	0.0575	0.0407
93	Feb-06	0.1124	0.0582	0.0542
94	Mar-06	0.1127	0.0598	0.0529
95	Apr-06	0.1100	0.0629	0.0471
96	May-06	0.1056	0.0642	0.0414
97	Jun-06	0.1049	0.0640	0.0409
98	Jul-06	0.1087	0.0637	0.0450
99	Aug-06	0.1041	0.0620	0.0421
100	Sep-06	0.1053	0.0600	0.0453
101	Oct-06	0.1030	0.0598	0.0432
102	Nov-06	0.1033	0.0580	0.0453
103	Dec-06	0.1035	0.0581	0.0454
104	Jan-07	0.1013	0.0596	0.0417
105	Feb-07	0.1018	0.0590	0.0428
106	Mar-07	0.1018	0.0585	0.0433
107	Apr-07	0.1007	0.0597	0.0410
108	May-07	0.0967	0.0599	0.0368
109	Jun-07	0.0970	0.0630	0.0340
110	Jul-07	0.1006	0.0625	0.0381
111	Aug-07	0.1021	0.0624	0.0397
112	Sep-07	0.1014	0.0618	0.0396
113	Oct-07	0.1080	0.0611	0.0469
114	Nov-07	0.1083	0.0597	0.0486
115	Dec-07	0.1084	0.0616	0.0468
116	Jan-08	0.1113	0.0602	0.0511
117	Feb-08	0.1139	0.0621	0.0518
118	Mar-08	0.1147	0.0621	0.0526
119	Apr-08	0.1167	0.0629	0.0538
120	May-08	0.1069	0.0627	0.0442
121	Jun-08	0.1062	0.0638	0.0424
122	Jul-08	0.1086	0.0640	0.0446
123	Aug-08	0.1123	0.0637	0.0486
124	Sep-08	0.1130	0.0649	0.0481
125	Oct-08	0.1213	0.0756	0.0457
126	Nov-08	0.1221	0.0760	0.0461
127	Dec-08	0.1162	0.0654	0.0508
128	Jan-09	0.1131	0.0639	0.0492
129	Feb-09	0.1155	0.0630	0.0524
130	Mar-09	0.1198	0.0642	0.0556
131	Apr-09	0.1146	0.0648	0.0498

LINE NO.	DATE	DCF	BOND YIELD	RISK PREMIUM
132	May-09	0.1225	0.0649	0.0576
133	Jun-09	0.1208	0.0620	0.0588
134	Jul-09	0.1166	0.0597	0.0569
135	Average	0.1145	0.0679	0.0466

Notes: Utility bond yield information from *Mergent Bond Record* (formerly Moody's). See Appendix 4 for a description of the ex ante risk premium methodology. DCF results are calculated using a quarterly DCF model as follows:

D_0 = Latest quarterly dividend per Value Line
P_0 = Average of the monthly high and low stock prices for each month per Thomson Reuters.
FC = Flotation costs expressed as a percent of gross proceeds.
g = I/B/E/S forecast of future earnings growth for each month.
k = Cost of equity using the quarterly version of the DCF model.

$$k = \left[\frac{d_0 (1+g)^{\frac{1}{4}}}{P_0 (1-FC)} + (1+g)^{\frac{1}{4}} \right]^4 - 1$$

ATMOS ENERGY
SCHEDULE 4
COMPARATIVE RETURNS ON S&P 500 STOCK INDEX AND MOODY'S A-RATED BONDS 1937—2009

Line No.	Year	S&P 500 Stock Price	Stock Dividend Yield	Stock Return	A-rated Bond Price	Bond Return
1	2009	865.58	0.0310		$68.43	
2	2008	1,380.33	0.0211	-35.19%	$72.25	0.24%
3	2007	1,424.16	0.0181	-1.27%	$72.91	4.59%
4	2006	1,278.72	0.0183	13.20%	$75.25	2.20%
5	2005	1,181.41	0.0177	10.01%	$74.91	5.80%
6	2004	1,132.52	0.0162	5.94%	$70.87	11.34%
7	2003	895.84	0.0180	28.22%	$62.26	20.27%
8	2002	1,140.21	0.0138	-20.05%	$57.44	15.35%
9	2001	1,335.63	0.0116	-13.47%	$56.40	8.93%
10	2000	1,425.59	0.0118	-5.13%	$52.60	14.82%
11	1999	1,248.77	0.0130	15.46%	$63.03	-10.20%
12	1998	963.35	0.0162	31.25%	$62.43	7.38%
13	1997	766.22	0.0195	27.68%	$56.62	17.32%
14	1996	614.42	0.0231	27.02%	$60.91	-0.48%
15	1995	465.25	0.0287	34.93%	$50.22	29.26%
16	1994	472.99	0.0269	1.05%	$60.01	-9.65%
17	1993	435.23	0.0288	11.56%	$53.13	20.48%
18	1992	416.08	0.0290	7.50%	$49.56	15.27%
19	1991	325.49	0.0382	31.65%	$44.84	19.44%
20	1990	339.97	0.0341	-0.85%	$45.60	7.11%
21	1989	285.41	0.0364	22.76%	$43.06	15.18%
22	1988	250.48	0.0366	17.61%	$40.10	17.36%
23	1987	264.51	0.0317	-2.13%	$48.92	-9.84%
24	1986	208.19	0.0390	30.95%	$39.98	32.36%
25	1985	171.61	0.0451	25.83%	$32.57	35.05%
26	1984	166.39	0.0427	7.41%	$31.49	16.12%
27	1983	144.27	0.0479	20.12%	$29.41	20.65%
28	1982	117.28	0.0595	28.96%	$24.48	36.48%
29	1981	132.97	0.0480	-7.00%	$29.37	-3.01%
30	1980	110.87	0.0541	25.34%	$34.69	-3.81%
31	1979	99.71	0.0533	16.52%	$43.91	-11.89%
32	1978	90.25	0.0532	15.80%	$49.09	-2.40%
33	1977	103.80	0.0399	-9.06%	$50.95	4.20%
34	1976	96.86	0.0380	10.96%	$43.91	25.13%
35	1975	72.56	0.0507	38.56%	$41.76	14.75%
36	1974	96.11	0.0364	-20.86%	$52.54	-12.91%
37	1973	118.40	0.0269	-16.14%	$58.51	-3.37%
38	1972	103.30	0.0296	17.58%	$56.47	10.69%
39	1971	93.49	0.0332	13.81%	$53.93	12.13%
40	1970	90.31	0.0356	7.08%	$50.46	14.81%

Line No.	Year	S&P 500 Stock Price	Stock Dividend Yield	Stock Return	A-rated Bond Price	Bond Return
41	1969	102.00	0.0306	-8.40%	$62.43	-12.76%
42	1968	95.04	0.0313	10.45%	$66.97	-0.81%
43	1967	84.45	0.0351	16.05%	$78.69	-9.81%
44	1966	93.32	0.0302	-6.48%	$86.57	-4.48%
45	1965	86.12	0.0299	11.35%	$91.40	-0.91%
46	1964	76.45	0.0305	15.70%	$92.01	3.68%
47	1963	65.06	0.0331	20.82%	$93.56	2.61%
48	1962	69.07	0.0297	-2.84%	$89.60	8.89%
49	1961	59.72	0.0328	18.94%	$89.74	4.29%
50	1960	58.03	0.0327	6.18%	$84.36	11.13%
51	1959	55.62	0.0324	7.57%	$91.55	-3.49%
52	1958	41.12	0.0448	39.74%	$101.22	-5.60%
53	1957	45.43	0.0431	-5.18%	$100.70	4.49%
54	1956	44.15	0.0424	7.14%	$113.00	-7.35%
55	1955	35.60	0.0438	28.40%	$116.77	0.20%
56	1954	25.46	0.0569	45.52%	$112.79	7.07%
57	1953	26.18	0.0545	2.70%	$114.24	2.24%
58	1952	24.19	0.0582	14.05%	$113.41	4.26%
59	1951	21.21	0.0634	20.39%	$123.44	-4.89%
60	1950	16.88	0.0665	32.30%	$125.08	1.89%
61	1949	15.36	0.0620	16.10%	$119.82	7.72%
62	1948	14.83	0.0571	9.28%	$118.50	4.49%
63	1947	15.21	0.0449	1.99%	$126.02	-2.79%
64	1946	18.02	0.0356	-12.03%	$126.74	2.59%
65	1945	13.49	0.0460	38.18%	$119.82	9.11%
66	1944	11.85	0.0495	18.79%	$119.82	3.34%
67	1943	10.09	0.0554	22.98%	$118.50	4.49%
68	1942	8.93	0.0788	20.87%	$117.63	4.14%
69	1941	10.55	0.0638	-8.98%	$116.34	4.55%
70	1940	12.30	0.0458	-9.65%	$112.39	7.08%
71	1939	12.50	0.0349	1.89%	$105.75	10.05%
72	1938	11.31	0.0784	18.36%	$99.83	9.94%
73	1937	17.59	0.0434	-31.36%	$103.18	0.63%
74	S&P 500 Return 1937--2009		10.8%			
75	A-rated Utility Bond Return		6.3%			
76	Risk Premium		4.5%			

Note: See Appendix 5 for an explanation of how stock and bond returns are derived and the source of the data presented.

ATMOS ENERGY
SCHEDULE 5
COMPARATIVE RETURNS ON S&P UTILITY STOCK INDEX AND MOODY'S A-RATED BONDS 1937—2009

Line No.	Year	S&P Utility Stock Price	Stock Dividend Yield	Stock Return	A-rated Bond Yield	Bond Return
1	2009				$68.43	
2	2008			-25.90%	$72.25	0.24%
3	2007			16.56%	$72.91	4.59%
4	2006			20.76%	$75.25	2.20%
5	2005			16.05%	$74.91	5.80%
6	2004			22.84%	$70.87	11.34%
7	2003			23.48%	$62.26	20.27%
8	2002			-14.73%	$57.44	15.35%
9						
10	2002	243.79	0.0362		$57.44	
11	2001	307.70	0.0287	-17.90%	$56.40	8.93%
12	2000	239.17	0.0413	32.78%	$52.60	14.82%
13	1999	253.52	0.0394	-1.72%	$63.03	-10.20%
14	1998	228.61	0.0457	15.47%	$62.43	7.38%
15	1997	201.14	0.0492	18.58%	$56.62	17.32%
16	1996	202.57	0.0454	3.83%	$60.91	-0.48%
17	1995	153.87	0.0584	37.49%	$50.22	29.26%
18	1994	168.70	0.0496	-3.83%	$60.01	-9.65%
19	1993	159.79	0.0537	10.95%	$53.13	20.48%
20	1992	149.70	0.0572	12.46%	$49.56	15.27%
21	1991	138.38	0.0607	14.25%	$44.84	19.44%
22	1990	146.04	0.0558	0.33%	$45.60	7.11%
23	1989	114.37	0.0699	34.68%	$43.06	15.18%
24	1988	106.13	0.0704	14.80%	$40.10	17.36%
25	1987	120.09	0.0588	-5.74%	$48.92	-9.84%
26	1986	92.06	0.0742	37.87%	$39.98	32.36%
27	1985	75.83	0.0860	30.00%	$32.57	35.05%
28	1984	68.50	0.0925	19.95%	$31.49	16.12%
29	1983	61.89	0.0948	20.16%	$29.41	20.65%
30	1982	51.81	0.1074	30.20%	$24.48	36.48%
31	1981	52.01	0.0978	9.40%	$29.37	-3.01%
32	1980	50.26	0.0953	13.01%	$34.69	-3.81%
33	1979	50.33	0.0893	8.79%	$43.91	-11.89%
34	1978	52.40	0.0791	3.96%	$49.09	-2.40%
35	1977	54.01	0.0714	4.16%	$50.95	4.20%
36	1976	46.99	0.0776	22.70%	$43.91	25.13%
37	1975	38.19	0.0920	32.24%	$41.76	14.75%
38	1974	48.60	0.0713	-14.29%	$52.54	-12.91%
39	1973	60.01	0.0556	-13.45%	$58.51	-3.37%
40	1972	60.19	0.0542	5.12%	$56.47	10.69%
41	1971	63.43	0.0504	-0.07%	$53.93	12.13%

Line No.	Year	S&P Utility Stock Price	Stock Dividend Yield	Stock Return	A-rated Bond Yield	Bond Return
42	1970	55.72	0.0561	19.45%	$50.46	14.81%
43	1969	68.65	0.0445	-14.38%	$62.43	-12.76%
44	1968	68.02	0.0435	5.28%	$66.97	-0.81%
45	1967	70.63	0.0392	0.22%	$78.69	-9.81%
46	1966	74.50	0.0347	-1.72%	$86.57	-4.48%
47	1965	75.87	0.0315	1.34%	$91.40	-0.91%
48	1964	67.26	0.0331	16.11%	$92.01	3.68%
49	1963	63.35	0.0330	9.47%	$93.56	2.61%
50	1962	62.69	0.0320	4.25%	$89.60	8.89%
51	1961	52.73	0.0358	22.47%	$89.74	4.29%
52	1960	44.50	0.0403	22.52%	$84.36	11.13%
53	1959	43.96	0.0377	5.00%	$91.55	-3.49%
54	1958	33.30	0.0487	36.88%	$101.22	-5.60%
55	1957	32.32	0.0487	7.90%	$100.70	4.49%
56	1956	31.55	0.0472	7.16%	$113.00	-7.35%
57	1955	29.89	0.0461	10.16%	$116.77	0.20%
58	1954	25.51	0.0520	22.37%	$112.79	7.07%
59	1953	24.41	0.0511	9.62%	$114.24	2.24%
60	1952	22.22	0.0550	15.36%	$113.41	4.26%
61	1951	20.01	0.0606	17.10%	$123.44	-4.89%
62	1950	20.20	0.0554	4.60%	$125.08	1.89%
63	1949	16.54	0.0570	27.83%	$119.82	7.72%
64	1948	16.53	0.0535	5.41%	$118.50	4.49%
65	1947	19.21	0.0354	-10.41%	$126.02	-2.79%
66	1946	21.34	0.0298	-7.00%	$126.74	2.59%
67	1945	13.91	0.0448	57.89%	$119.82	9.11%
68	1944	12.10	0.0569	20.65%	$119.82	3.34%
69	1943	9.22	0.0621	37.45%	$118.50	4.49%
70	1942	8.54	0.0940	17.36%	$117.63	4.14%
71	1941	13.25	0.0717	-28.38%	$116.34	4.55%
72	1940	16.97	0.0540	-16.52%	$112.39	7.08%
73	1939	16.05	0.0553	11.26%	$105.75	10.05%
74	1938	14.30	0.0730	19.54%	$99.83	9.94%
75	1937	24.34	0.0432	-36.93%	$103.18	0.63%
76	Return 1937—2009	Stocks	10.5%			
77		Bonds	6.3%			
78	Risk Premium		4.2%			

See Appendix 5 for an explanation of how stock and bond returns are derived and the source of the data presented. Standard & Poor's discontinued its S&P Utilities Index in December 2001 and replaced its utilities stock index with separate indices for electric and natural gas utilities. In this study, the stock returns beginning in 2002 are based on the total returns for the EEI Index of U.S. shareholder-owned electric utilities, as reported by EEI on its website.
http://www.eei.org/industry_issues/finance_and_accounting/finance/research_and_analysis/EEI_Stock_Index

ATMOS ENERGY
SCHEDULE 6
USING THE ARITHMETIC MEAN
TO ESTIMATE THE COST OF EQUITY CAPITAL

Consider an investment that in a given year generates a return of 30 percent with probability equal to .5 and a return of -10 percent with a probability equal to .5. For each one dollar invested, the possible outcomes of this investment at the end of year one are:

Ending Wealth	Probability
$1.30	0.50
$0.90	0.50

At the end of year two, the possible outcomes are:

Ending Wealth			Probability	Value x Probability
(1.30) (1.30)	=	$1.69	0.25	0.4225
(1.30) (.9)	=	$1.17	0.50	0.5850
(.9) (.9)	=	$0.81	0.25	0.2025
Expected Wealth	=			$1.21

The expected value of this investment at the end of year two is $1.21. In a competitive capital market, the cost of equity is equal to the expected rate of return on an investment. In the above example, the cost of equity is that rate of return which will make the initial investment of one dollar grow to the expected value of $1.21 at the end of two years. Thus, the cost of equity is the solution to the equation:

$$1(1+k)^2 = 1.21 \text{ or}$$

$$k = (1.21/1)^{.5} - 1 = 10\%.$$

The arithmetic mean of this investment is:

$$(30\%)\,(.5) + (-10\%)\,(.5) = 10\%.$$

Thus, the arithmetic mean is equal to the cost of equity capital.

The geometric mean of this investment is:

$$[(1.3)\,(.9)]^{.5} - 1 = .082 = 8.2\%.$$

Thus, the geometric mean is not equal to the cost of equity capital.

The lesson is obvious: for an investment with an uncertain outcome, the arithmetic mean is the best measure of the cost of equity capital.

ATMOS ENERGY
SCHEDULE 7
CALCULATION OF CAPITAL ASSET PRICING MODEL COST OF EQUITY USING IBBOTSON® SBBI® 6.5 PERCENT RISK PREMIUM

Line			
1	Risk-free Rate	4.38%	Long-term (20-year) Treasury bond yield[10]
2	Beta	0.85	Average Beta Proxy Companies
3	Risk Premium	6.50%	Long-horizon Ibbotson risk premium
4	Beta x Risk Premium	5.53%	
5	Flotation Cost	0.27%	
6	CAPM cost of equity	10.2%	

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10 Average 20-year Treasury bond yield July 2009 as reported by the Federal Reserve.

ATMOS ENERGY
SCHEDULE 7 (continued)
PROXY COMPANY VALUE LINE BETAS

LINE NO.	COMPANY	BETA	MARKET CAP $ (MIL)
1	AGL Resources	0.75	2,598
2	Atmos Energy	0.65	2,499
3	EQT Corp.	1.15	5,024
4	National Fuel Gas	0.90	3,227
5	Nicor Inc.	0.75	1,648
6	NiSource Inc.	0.85	3,539
7	Northwest Nat. Gas	0.60	1,183
8	ONEOK Inc.	0.95	3,485
9	Piedmont Natural Gas	0.65	1,796
10	South Jersey Inds.	0.65	1,099
11	Southwest Gas	0.75	1,083
12	Market-Weighted Average	0.85	

Betas from The Value Line Investment Analyzer August 2009

ATMOS ENERGY
SCHEDULE 8
CALCULATION OF CAPITAL ASSET PRICING MODEL COST OF EQUITY USING DCF ESTIMATE OF THE EXPECTED RATE OF RETURN ON THE MARKET PORTFOLIO

Line			
1	Risk-free rate	4.38%	Long-term (20-year) Treasury bond yield[11]
2	Beta	0.85	Average Beta Proxy Companies
3	DCF S&P 500	12.7%	DCF Cost of Equity S&P 500 (see following)
4	Risk Premium	8.4%	
5	Beta x Risk Premium	7.1%	
6	CAPM cost of equity	11.5%	

11 Average 20-year Treasury bond yield August 2008 as reported by the Federal Reserve.

ATMOS ENERGY
SCHEDULE 8 (continued)
CALCULATION OF CAPITAL ASSET PRICING MODEL COST OF EQUITY USING DCF ESTIMATE OF THE EXPECTED RATE OF RETURN ON THE MARKET PORTFOLIO
SUMMARY OF DISCOUNTED CASH FLOW ANALYSIS FOR S&P 500 COMPANIES

COMPANY	P_0	D_0	GROWTH	COST OF EQUITY
AMERISOURCEBERGEN	18.38	0.20	11.57%	12.9%
AETNA	25.61	0.04	12.60%	12.8%
ALLERGAN	47.14	0.20	13.28%	13.8%
ASSURANT	24.26	0.60	8.75%	11.6%
ALLSTATE	25.15	0.80	9.20%	12.9%
APPLIED MATS.	11.75	0.24	8.71%	11.1%
ABERCROMBIE & FITCH	27.61	0.70	10.98%	14.0%
AON	37.40	0.60	12.35%	14.3%
AMERICAN EXPRESS	25.55	0.72	10.00%	13.3%
BOEING	43.97	1.68	8.29%	12.7%
BECTON DICKINSON	67.82	1.32	11.72%	14.0%
FRANKLIN RESOURCES	70.83	0.84	10.00%	11.4%
BROWN-FORMAN 'B'	44.95	1.15	8.10%	11.0%
BANK OF NEW YORK MELLON	28.69	0.36	11.43%	12.9%
BEMIS	25.01	0.90	8.00%	12.1%
BRISTOL MYERS SQUIBB	20.23	1.24	7.04%	14.1%
CA	18.01	0.16	9.60%	10.6%
CATERPILLAR	36.63	1.68	9.00%	14.4%
CHUBB	40.82	1.40	8.50%	12.5%
COCA COLA ENTS.	17.31	0.32	9.20%	11.3%
COLGATE-PALM.	68.42	1.76	9.75%	12.8%
CLOROX	55.64	2.00	9.67%	13.9%
COMCAST 'A'	14.45	0.27	11.25%	13.5%
CME GROUP	291.33	4.60	10.92%	12.8%
CUMMINS	34.44	0.70	10.33%	12.7%
CMS ENERGY	11.92	0.50	6.75%	11.5%
CONSOL EN.	35.90	0.40	12.03%	13.3%
COSTCO WHOLESALE	47.29	0.72	11.54%	13.3%
CAMPBELL SOUP	28.57	1.00	8.43%	12.5%
CSX	33.21	0.88	9.88%	13.0%
CINTAS	23.53	0.47	11.75%	14.1%
CVS CAREMARK	31.75	0.30	13.05%	14.2%
DOMINION RES.	32.50	1.75	6.36%	12.5%
DEERE	42.30	1.12	7.60%	10.6%
QUEST DIAGNOSTICS	53.12	0.40	12.39%	13.3%
DUKE ENERGY	14.38	0.96	3.50%	11.0%
ESTEE LAUDER COS.'A'	33.17	0.55	12.00%	14.0%
EATON	45.95	2.00	7.25%	12.2%
ENTERGY	74.35	3.00	9.02%	13.7%
FAMILY DOLLAR STORES	30.50	0.54	12.15%	14.3%
FIRSTENERGY	39.49	2.20	6.67%	13.1%
FEDERATED INVRS.'B'	24.16	0.96	9.00%	13.6%
FLUOR	47.91	0.50	12.40%	13.6%
FORTUNE BRANDS	36.46	0.76	8.23%	10.6%

COMPANY	P_0	D_0	GROWTH	COST OF EQUITY
FPL GROUP	56.43	1.89	9.59%	13.5%
GENERAL DYNAMICS	55.12	1.52	8.86%	12.1%
GENERAL ELECTRIC	12.66	0.40	9.07%	12.7%
GENUINE PARTS	33.66	1.60	6.00%	11.4%
GAP	16.37	0.34	10.00%	12.4%
GOLDMAN SACHS GP.	143.65	1.40	12.40%	13.6%
WW GRAINGER	81.86	1.84	11.26%	13.9%
HASBRO	25.19	0.80	9.00%	12.7%
HOME DEPOT	24.20	0.90	9.88%	14.2%
HARTFORD FINL.SVS.GP.	13.78	0.20	9.33%	11.0%
HARLEY-DAVIDSON	18.41	0.40	9.50%	12.0%
HONEYWELL INTL.	32.88	1.21	9.38%	13.7%
HEWLETT-PACKARD	37.47	0.32	10.07%	11.1%
HARRIS	29.42	0.76	11.00%	14.0%
INTERNATIONAL BUS.MCHS.	106.61	2.20	9.92%	12.3%
INTL.GAME TECH.	16.02	0.24	12.50%	14.3%
INTEL	16.61	0.56	10.00%	14.0%
ITT	43.96	0.85	8.50%	10.7%
PENNEY JC	28.39	0.80	10.27%	13.6%
JOHNSON & JOHNSON	56.35	1.96	8.13%	12.1%
JANUS CAPITAL GP.	11.11	0.04	10.67%	11.1%
JP MORGAN CHASE & CO.	35.33	0.20	12.00%	12.7%
NORDSTROM	21.78	0.64	10.00%	13.4%
KELLOGG	45.48	1.50	9.84%	13.7%
KB HOME	15.03	0.25	10.50%	12.4%
KRAFT FOODS	26.03	1.16	8.47%	13.6%
LENNAR 'A'	9.43	0.16	8.67%	10.6%
L3 COMMUNICATIONS	72.36	1.40	10.66%	12.9%
LOCKHEED MARTIN	80.81	2.28	10.56%	13.9%
LINCOLN NAT.	16.66	0.04	11.45%	11.7%
LOWE'S COMPANIES	20.03	0.36	11.75%	13.9%
SOUTHWEST AIRLINES	6.99	0.02	12.67%	13.0%
MCDONALDS	57.06	2.00	8.99%	13.1%
MCKESSON	43.02	0.48	11.27%	12.6%
MOODY'S	27.52	0.40	9.00%	10.7%
MEDTRONIC	33.68	0.82	10.54%	13.4%
3M	60.46	2.04	10.13%	14.1%
MORGAN STANLEY	27.72	0.20	11.60%	12.4%
MICROSOFT	22.15	0.52	10.17%	12.9%
M&T BK.	51.92	2.80	4.72%	10.8%
NISOURCE	11.57	0.92	3.00%	11.9%
NIKE 'B'	54.06	1.00	12.11%	14.3%
NORTHEAST UTILITIES	21.59	0.95	8.33%	13.4%
NEWELL RUBBERMAID	11.08	0.20	9.80%	11.9%
OMNICOM GP.	31.94	0.60	11.63%	13.9%
PEOPLES UNITED FINANCIAL	15.78	0.61	9.33%	13.8%
PACCAR	32.16	0.36	10.25%	11.6%
PG&E	37.52	1.68	7.07%	12.2%
PROCTER & GAMBLE	52.00	1.76	9.50%	13.5%
PROGRESS ENERGY	36.58	2.48	5.36%	13.1%
PARKER-HANNIFIN	44.24	1.00	10.00%	12.6%
PERKINELMER	17.12	0.28	11.75%	13.7%

COMPANY	P_0	D_0	GROWTH	COST OF EQUITY
PINNACLE WEST CAP.	28.90	2.10	5.67%	14.0%
PEPCO HOLDINGS	13.10	1.08	3.67%	13.0%
PRAXAIR	73.12	1.60	9.62%	12.2%
POLO RALPH LAUREN 'A'	54.40	0.20	13.75%	14.2%
ROCKWELL AUTOMATION	33.22	1.16	8.00%	12.0%
RADIOSHACK	13.91	0.25	9.48%	11.6%
RAYTHEON 'B'	45.34	1.24	11.14%	14.4%
SCANA	31.74	1.88	5.34%	12.1%
SCHERING-PLOUGH	24.40	0.26	11.10%	12.4%
SHERWIN-WILLIAMS	54.89	1.42	8.83%	11.8%
SARA LEE	9.49	0.44	8.43%	13.8%
SOUTHERN	30.07	1.75	4.97%	11.6%
STANLEY WORKS	35.98	1.32	8.00%	12.2%
STRYKER	39.44	0.40	12.53%	13.7%
AT&T	24.84	1.64	4.11%	11.5%
MOLSON COORS BREWING 'B'	43.13	0.96	10.82%	13.4%
TIFFANY & CO	27.46	0.68	10.75%	13.7%
TJX COS.	30.80	0.48	12.17%	14.0%
T ROWE PRICE GP.	41.15	1.00	10.75%	13.6%
TOTAL SYSTEM SERVICES	13.49	0.28	9.38%	11.8%
TIME WARNER	24.90	0.75	8.06%	11.5%
TEXTRON	11.10	0.08	11.40%	12.2%
UNITED PARCEL SER.	51.34	1.80	7.65%	11.7%
UNITED TECHNOLOGIES	52.29	1.54	9.00%	12.4%
VERIZON COMMUNICATIONS	30.23	1.84	4.58%	11.4%
WALGREEN	30.32	0.55	12.00%	14.2%
WISCONSIN ENERGY	40.33	1.35	9.03%	12.9%
WELLS FARGO & CO	23.91	0.20	10.75%	11.7%
WINDSTREAM	8.45	1.00	0.82%	14.0%
WESTERN UNION	17.00	0.04	11.64%	11.9%
XCEL ENERGY	18.19	0.98	6.58%	12.8%
DENTSPLY INTL.	30.02	0.20	12.67%	13.5%
XTO EN.	39.15	0.50	11.40%	12.9%
Market-weighted Average				12.7%

Notes: In applying the DCF model to the S&P 500, I include in the DCF analysis only those companies in the S&P 500 group which pay a dividend, have a positive growth rate, and have at least three analysts' long-term growth estimates. I also eliminate those 25% of companies with the highest and lowest DCF results.

D_o	=	Current dividend per Thomson Reuters.
P_0	=	Average of the monthly high and low stock prices during the three months ending July 2009 per Thomson Reuters.
FC	=	Flotation costs expressed as a percent of gross proceeds (5 percent)
g	=	I/B/E/S forecast of future earnings growth July 2009.
k	=	Cost of equity using the quarterly version of the DCF model shown below:

$$k = \left[\frac{d_0 (1+g)^{\frac{1}{4}}}{P_0 (1 - FC)} + (1+g)^{\frac{1}{4}} \right]^4 - 1$$

APPENDIX 1
QUALIFICATIONS OF JAMES H. VANDER WEIDE

JAMES H. VANDER WEIDE, Ph.D.
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James H. Vander Weide is Research Professor of Finance and Economics at Duke University, the Fuqua School of Business. Dr. Vander Weide is also founder and President of Financial Strategy Associates, a consulting firm that provides strategic, financial, and economic consulting services to corporate clients, including cost of capital and valuation studies.

Educational Background and Prior Academic Experience

Dr. Vander Weide holds a Ph.D. in Finance from Northwestern University and a Bachelor of Arts in Economics from Cornell University. He joined the faculty at Duke University and was named Assistant Professor, Associate Professor, Professor, and then Research Professor of Finance and Economics.

Since joining the faculty at Duke, Dr. Vander Weide has taught courses in corporate finance, investment management, and management of financial institutions. He has also taught courses in statistics, economics, and operations research, and a Ph.D. seminar on the theory of public utility pricing. In addition, Dr. Vander Weide has been active in executive education at Duke and Duke Corporate Education, leading executive development seminars on topics including financial analysis, cost of capital, creating shareholder value, mergers and acquisitions, real options, capital budgeting, cash management, measuring corporate performance, valuation, short-run financial planning, depreciation policies, financial strategy, and competitive strategy. Dr. Vander Weide has designed and served as Program Director for several executive education programs, including the Advanced Management Program, Competitive Strategies in Telecommunications, and the Duke Program for Manager Development for managers from the former Soviet Union.

Publications

Dr. Vander Weide has written a book entitled *Managing Corporate Liquidity: An Introduction to Working Capital Management* published by John Wiley and Sons, Inc. He has also written a chapter titled, "Financial Management in the Short Run" for *The Handbook of Modern Finance*;" a chapter for *The Handbook of Portfolio Construction: Contemporary Applications of Markowitz Techniques*, "Principles for Lifetime Portfolio Selection: Lessons from Portfolio Theory," and written research papers on such topics as portfolio management, capital budgeting, investments, the effect of regulation on the performance of public utilities, and cash management. His articles have been published in *American Economic Review, Financial Management, International Journal of Industrial Organization, Journal of Finance, Journal of*

Financial and Quantitative Analysis, Journal of Bank Research, Journal of Portfolio Management, Journal of Accounting Research, Journal of Cash Management, Management Science, Atlantic Economic Journal, Journal of Economics and Business, and *Computers and Operations Research.*

Professional Consulting Experience

Dr. Vander Weide has provided financial and economic consulting services to firms in the electric, gas, insurance, telecommunications, and water industries for more than 25 years. He has testified on the cost of capital, competition, risk, incentive regulation, forward-looking economic cost, economic pricing guidelines, depreciation, accounting, valuation, and other financial and economic issues in more than 400 cases before the United States Congress, the Canadian Radio-Television and Telecommunications Commission, the Federal Communications Commission, the National Energy Board (Canada), the National Telecommunications and Information Administration, the Federal Energy Regulatory Commission, the Alberta Utilities Board (Canada), the public service commissions of 42 states and the District of Columbia, the insurance commissions of five states, the Iowa State Board of Tax Review, the National Association of Securities Dealers, and the North Carolina Property Tax Commission. In addition, he has testified as an expert witness in proceedings before the United States District Court for the District of New Hampshire; United States District Court for the Northern District of California; United States District Court for the Northern District of Illinois, United States District Court for the District of Nebraska; United States District Court for the Eastern District of North Carolina; Superior Court of North Carolina, the United States Bankruptcy Court for the Southern District of West Virginia; and United States District Court for the Eastern District of Michigan. With respect to implementation of the Telecommunications Act of 1996, Dr. Vander Weide has testified in 30 states on issues relating to the pricing of unbundled network elements and universal service cost studies and has consulted with Bell Canada, Deutsche Telekom, and Telefónica on similar issues. He has also provided expert testimony on issues related to electric and natural gas restructuring. He has worked for Bell Canada/Nortel on a special task force to study the effects of vertical integration in the Canadian telephone industry and has worked for Bell Canada as an expert witness on the cost of capital. Dr. Vander Weide has provided consulting and expert witness testimony to the following companies:

Telecommunications Companies

ALLTEL and its subsidiaries	Ameritech (now AT&T new)
AT&T (old)	Verizon (Bell Atlantic) and subsidiaries
Bell Canada/Nortel	BellSouth and its subsidiaries
Centel and its subsidiaries	Cincinnati Bell (Broadwing)
Cisco Systems	Citizens Telephone Company
Concord Telephone Company	Contel and its subsidiaries
Deutsche Telekom	GTE and subsidiaries (now Verizon)
Heins Telephone Company	Lucent Technologies
JDS Uniphase	Tellabs, Inc.
Minnesota Independent Equal Access Corp.	NYNEX and its subsidiaries (Verizon)
Pacific Telesis and its subsidiaries	Phillips County Cooperative Tel. Co.
Pine Drive Cooperative Telephone Co.	Roseville Telephone Company (SureWest)

Siemens
Sherburne Telephone Company
The Stentor Companies
Telefónica
Woodbury Telephone Company
U S West (Qwest)
SBC Communications (now AT&T new)
Southern New England Telephone
Sprint/United and its subsidiaries
Union Telephone Company
United States Telephone Association
Valor Telecommunications (Windstream)

Electric, Gas, and Water Companies
Alcoa Power Generating, Inc.
Alliant Energy
AltaLink, L.P.
Ameren
American Water Works
Atmos Energy
Central Illinois Public Service
Citizens Utilities
Consolidated Natural Gas and its subsidiaries
Dominion Resources
Duke Energy
Empire District Electric Company
EPCOR Distribution & Transmission Inc.
EPCOR Energy Alberta Inc.
FortisAlberta Inc.
Interstate Power Company
Iowa-American Water Company
Iowa-Illinois Gas and Electric
Iowa Southern
Kentucky-American Water Company
Kentucky Power Company
MidAmerican Energy and its subsidiaries
Nevada Power Company
NICOR
North Carolina Natural Gas
Northern Natural Gas Company
NOVA Gas Transmission Ltd.
North Shore Gas
PacifiCorp
PG&E
Peoples Energy and its subsidiaries
The Peoples Gas, Light and Coke Co.
Progress Energy
Public Service Company of North Carolina
PSE&G
Sempra Energy
South Carolina Electric and Gas
Southern Company and subsidiaries
Tennessee-American Water Company
Trans Québec & Maritimes Pipeline Inc.
United Cities Gas Company
Union Gas

Insurance Companies
Allstate
North Carolina Rate Bureau
United Services Automobile Association (USAA)
The Travelers Indemnity Company
Gulf Insurance Company

Other Professional Experience

Dr. Vander Weide conducts in-house seminars and training sessions on topics such as creating shareholder value, financial analysis, competitive strategy, cost of capital, real options, financial strategy, managing growth, mergers and acquisitions, valuation, measuring corporate performance, capital budgeting, cash management, and financial planning. Among the firms for whom he has designed and taught tailored programs and training sessions are ABB Asea Brown Boveri, Accenture, Allstate, Ameritech, AT&T, Bell Atlantic/Verizon, BellSouth, Progress Energy/Carolina Power & Light, Contel, Fisons, GlaxoSmithKline, GTE, Lafarge, MidAmerican Energy, New Century Energies, Norfolk Southern, Pacific Bell Telephone, The Rank Group, Siemens, Southern New England Telephone, TRW, and Wolseley Plc. Dr. Vander Weide has also hosted a nationally prominent conference/workshop on estimating the cost of capital. In 1989, at the request of Mr. Fuqua, Dr. Vander Weide designed the Duke Program for Manager Development for managers from the former Soviet Union, the first in the United States designed exclusively for managers from Russia and the former Soviet republics.

In the 1970's, Dr. Vander Weide helped found University Analytics, Inc., which at that time was one of the fastest growing small firms in the country. As an officer at University Analytics, he designed cash management models, databases, and software packages that are still used by most major U.S. banks in consulting with their corporate clients. Having sold his interest in University Analytics, Dr. Vander Weide now concentrates on strategic and financial consulting, academic research, and executive education.

Publications - Dr. James H. Vander Weide

The Lock-Box Location Problem: a Practical Reformulation, *Journal of Bank Research*, Summer, 1974, pp. 92-96 (with S. Maier). Reprinted in *Management Science in Banking*, edited by K. J. Cohen and S. E. Gibson, Warren, Gorham and Lamont, 1978.

A Finite Horizon Dynamic Programming Approach to the Telephone Cable Layout Problem, *Conference Record*, 1976 International Conference on Communications (with S. Maier and C. Lam).

A Note on the Optimal Investment Policy of the Regulated Firm, *Atlantic Economic Journal*, Fall, 1976 (with D. Peterson).

A Unified Location Model for Cash Disbursements and Lock-Box Collections, *Journal of Bank Research*, Summer, 1976 (with S. Maier). Reprinted in *Management Science in Banking*, edited by K. J. Cohen and S. E. Gibson, Warren Gorham and Lamont, 1978. Also reprinted in *Readings on the Management of Working Capital*, edited by K. V. Smith, West Publishing Company, 1979.

Capital Budgeting in the Decentralized Firm,' *Management Science*, Vol. 23, No. 4, December 1976, pp. 433-443 (with S. Maier).

A Monte Carlo Investigation of Characteristics of Optimal Geometric Mean Portfolios, *Journal of Financial and Quantitative Analysis*, June, 1977, pp. 215-233 (with S. Maier and D. Peterson).

A Strategy which Maximizes the Geometric Mean Return on Portfolio Investments, *Management Science*, June, 1977, Vol. 23, No. 10, pp. 1117-1123 (with S. Maier and D. Peterson).

A Decision Analysis Approach to the Computer Lease-Purchase Decision, *Computers and Operations Research*, Vol. 4, No. 3, September, 1977, pp. 167-172 (with S. Maier).

A Practical Approach to Short-run Financial Planning, *Financial Management*, Winter, 1978 (with S. Maier). Reprinted in *Readings on the Management of Working Capital*, edited by K. V. Smith, West Publishing Company, 1979.

Effectiveness of Regulation in the Electric Utility Industry,' *Journal of Economics and Business*, May, 1979 (with F. Tapon).

On the Decentralized Capital Budgeting Problem Under Uncertainty, *Management Science*, September 1979 (with B. Obel).

Expectations Data and the Predictive Value of Interim Reporting: A Comment, *Journal of Accounting Research*, Spring 1980 (with L. D. Brown, J. S. Hughes, and M. S. Rozeff).

Deregulation and Oligopolistic Price-Quality Rivalry, *American Economic Review*, March 1981 (with J. Zalkind).

Incentive Considerations in the Reporting of Leveraged Leases, *Journal of Bank Research*, April 1982 (with J. S. Hughes).

Forecasting Disbursement Float, *Financial Management*, Spring 1981 (with S. Maier and D. Robinson).

Recent Developments in Management Science in Banking, *Management Science*, October 1981 (with K. Cohen and S. Maier).

General Telephone's Experience with a Short-run Financial Planning Model, *Cash Management Forum*, June 1980, Vol. 6, No. 1 (with J. Austin and S. Maier).

An Empirical Bayes Estimate of Market Risk, *Management Science*, July 1982 (with S. Maier and D. Peterson).

The Bond Scheduling Problem of the Multi-subsidiary Holding Company, *Management Science*, July 1982 (with K. Baker).

A Decision-Support System for Managing a Short-term Financial Instrument Portfolio, *Journal of Cash Management*, March 1982 (with S. Maier).

Deregulation and Locational Rents in Banking: a Comment, *Journal of Bank Research*, Summer 1983.

What Lockbox and Disbursement Models Really Do, *Journal of Finance*, May 1983 (with S. Maier).

Financial Management in the Short Run, *Handbook of Modern Finance*, edited by Dennis Logue, published by Warren, Gorham, & Lamont, Inc., New York, 1984.

Measuring Investors' Growth Expectations: the Analysts vs. History, *The Journal of Portfolio Management*, Spring 1988 (with W. Carleton).

Entry Auctions and Strategic Behavior under Cross-Market Price Constraints, *International Journal of Industrial Organization*, 20 (2002) 611-629 (with J. Anton and N. Vettas).

Principles for Lifetime Portfolio Selection: Lessons from Portfolio Theory, *Handbook of Portfolio Construction: Contemporary Applications of Markowitz Techniques*, John B. Guerard, (Ed.), Springer, forthcoming 2009.

Managing Corporate Liquidity: an Introduction to Working Capital Management, John Wiley and Sons, 1984 (with S. Maier).

DERIVATION OF THE QUARTERLY DCF MODEL

The simple DCF model assumes that a firm pays dividends only at the end of each year. Since firms in fact pay dividends quarterly and investors appreciate the time value of money, the annual version of the DCF model generally underestimates the value investors are willing to place on the firm's expected future dividend stream. In these workpapers, we review two alternative formulations of the DCF model that allow for the quarterly payment of dividends.

When dividends are assumed to be paid annually, the DCF model suggests that the current price of the firm's stock is given by the expression:

$$P_0 = \frac{D_1}{(1+k)} + \frac{D_2}{(1+k)^2} + \ldots + \frac{D_n + P_n}{(1+k)^n} \quad (1)$$

where

P_0	=	current price per share of the firm's stock,
$D_1, D_2, \ldots, D_n$	=	expected annual dividends per share on the firm's stock,
P_n	=	price per share of stock at the time investors expect to sell the stock, and
k	=	return investors expect to earn on alternative investments of the same risk, i.e., the investors' required rate of return.

Unfortunately, expression (1) is rather difficult to analyze, especially for the purpose of estimating k. Thus, most analysts make a number of simplifying assumptions. First, they assume that dividends are expected to grow at the constant rate g into the indefinite future. Second, they assume that the stock price at time n is simply the present value of all dividends expected in periods subsequent to n. Third, they assume that the investors' required rate of return, k, exceeds the expected dividend growth rate g. Under the above simplifying assumptions, a firm's stock price may be written as the following sum:

$$P_0 = \frac{D_0(1+g)}{(1+k)} + \frac{D_0(1+g)^2}{(1+k)^2} + \frac{D_0(1+g)^3}{(1+k)^3} + \ldots, \qquad (2)$$

where the three dots indicate that the sum continues indefinitely.

As we shall demonstrate shortly, this sum may be simplified to:

$$P_0 = \frac{D_0(1+g)}{(k-g)}$$

First, however, we need to review the very useful concept of a geometric progression.

Geometric Progression

Consider the sequence of numbers 3, 6, 12, 24,..., where each number after the first is obtained by multiplying the preceding number by the factor 2. Obviously, this sequence of numbers may also be expressed as the sequence 3, 3 x 2, 3×2^2, 3×2^3, etc. This sequence is an example of a geometric progression.

Definition: A geometric progression is a sequence in which each term after the first is obtained by multiplying some fixed number, called the common ratio, by the preceding term.

A general notation for geometric progressions is: a, the first term, r, the common ratio, and n, the number of terms. Using this notation, any geometric progression may be represented by the sequence:

$$a, ar, ar^2, ar^3, \ldots, ar^{n-1}.$$

In studying the DCF model, we will find it useful to have an expression for the sum of n terms of a geometric progression. Call this sum S_n. Then

$$S_n = a + ar + \ldots + ar^{n-1} . \qquad (3)$$

However, this expression can be simplified by multiplying both sides of equation (3) by r and then subtracting the new equation from the old. Thus,

$$rS_n = ar + ar^2 + ar^3 + \ldots + ar^n$$

and

$$S_n - rS_n = a - ar^n \quad ,$$

or

$$(1 - r)\, S_n = a\,(1 - r^n) \; .$$

Solving for S_n, we obtain:

$$S_n = \frac{a(1 - r^n)}{(1 - r)} \qquad (4)$$

as a simple expression for the sum of n terms of a geometric progression. Furthermore, if $|r| < 1$, then S_n is finite, and as n approaches infinity, S_n approaches $a \div (1\text{-}r)$. Thus, for a geometric progression with an infinite number of terms and $|r| < 1$, equation (4) becomes:

$$S = \frac{a}{1 - r} \qquad (5)$$

Application to DCF Model

Comparing equation (2) with equation (3), we see that the firm's stock price (under the DCF assumption) is the sum of an infinite geometric progression with the first term

$$a = \frac{D_0(1 + g)}{(1 + k)}$$

and common factor

$$r = \frac{(1+g)}{(1+k)}$$

Applying equation (5) for the sum of such a geometric progression, we obtain

$$S = a \bullet \frac{1}{(1-r)} = \frac{D_0(1+g)}{(1+k)} \bullet \frac{1}{1-\frac{1+g}{1+k}} = \frac{D_0(1+g)}{(1+k)} \bullet \frac{1+k}{k-g} = \frac{D_0(1+g)}{k-g}$$

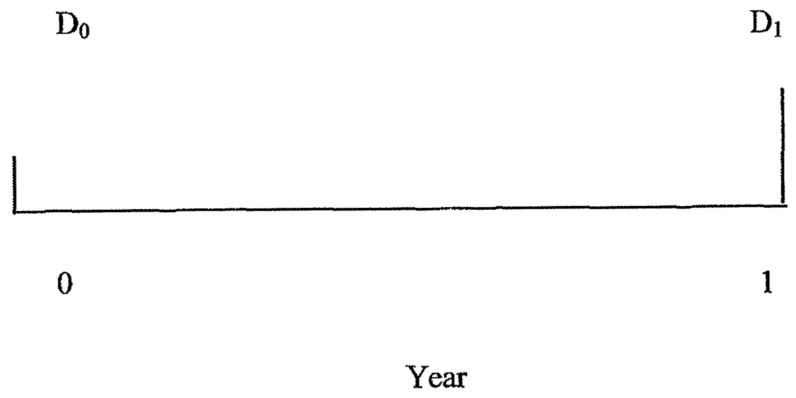
as we suggested earlier.

Quarterly DCF Model

The annual DCF model assumes that dividends grow at an annual rate of g% per year (see Figure 1).

Figure 1

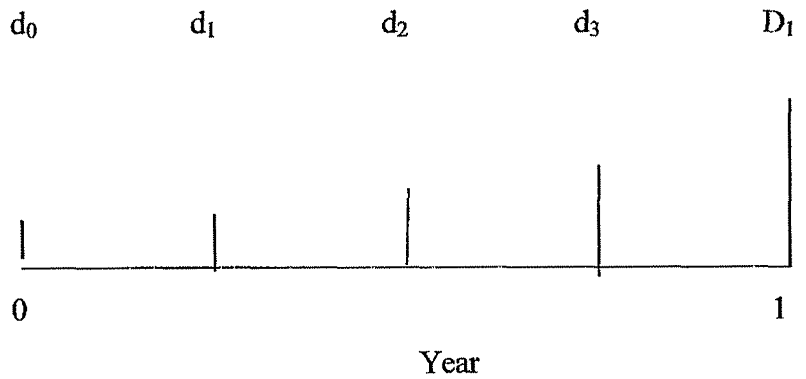
Annual DCF Model



$D_0 = 4d_0$ $\qquad$ $D_1 = D_0(1 + g)$

Figure 2

Quarterly DCF Model (Constant Growth Version)



$d_1 = d_0(1+g)^{.25}$ $\qquad$ $d_2 = d_0(1+g)^{.50}$

$d_3 = d_0(1+g)^{.75}$ $\qquad$ $d_4 = d_0(1+g)$

In the quarterly DCF model, it is natural to assume that quarterly dividend payments differ from the preceding quarterly dividend by the factor $(1 + g)^{.25}$, where g is expressed in terms of percent per year and the decimal .25 indicates that the growth has only occurred for one quarter of the year. (See Figure 2.) Using this assumption, along with the assumption of constant growth and $\boldsymbol{k > g}$, we obtain a new expression for the firm's stock price, which takes account of the quarterly payment of dividends. This expression is:

$$P_0 = \frac{d_0(1+g)^{\frac{1}{4}}}{(1+k)^{\frac{1}{4}}} + \frac{d_0(1+g)^{\frac{2}{4}}}{(1+k)^{\frac{2}{4}}} + \frac{d_0(1+g)^{\frac{3}{4}}}{(1+k)^{\frac{3}{4}}} + \ldots \qquad (6)$$

where d_0 is the last quarterly dividend payment, rather than the last annual dividend payment. (We use a lower case d to remind the reader that this is not the annual dividend.)

Although equation (6) looks formidable at first glance, it too can be greatly simplified using the formula [equation (4)] for the sum of an infinite geometric progression. As the reader can easily verify, equation (6) can be simplified to:

$$P_0 = \frac{d_0(1+g)^{\frac{1}{4}}}{(1+k)^{\frac{1}{4}} - (1+g)^{\frac{1}{4}}} \quad (7)$$

Solving equation (7) for k, we obtain a DCF formula for estimating the cost of equity under the quarterly dividend assumption:

$$k = \left[\frac{d_0(1+g)^{\frac{1}{4}}}{P_0} + (1+g)^{\frac{1}{4}} \right]^4 - 1 \qquad (8)$$

An Alternative Quarterly DCF Model

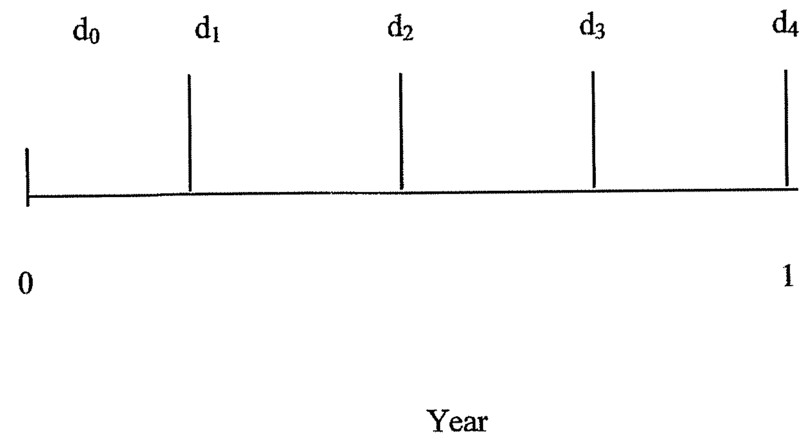
Although the constant growth quarterly DCF model [equation (8)] allows for the quarterly timing of dividend payments, it does require the assumption that the firm increases its dividend payments each quarter. Since this assumption is difficult for some analysts to accept, we now discuss a second quarterly DCF model that allows for constant quarterly dividend payments within each dividend year.

Assume then that the firm pays dividends quarterly and that each dividend payment is constant for four consecutive quarters. There are four cases to consider, with each case distinguished by varying assumptions about where we are evaluating the firm in relation to the time of its next dividend increase. (See Figure 3.)

Figure 3

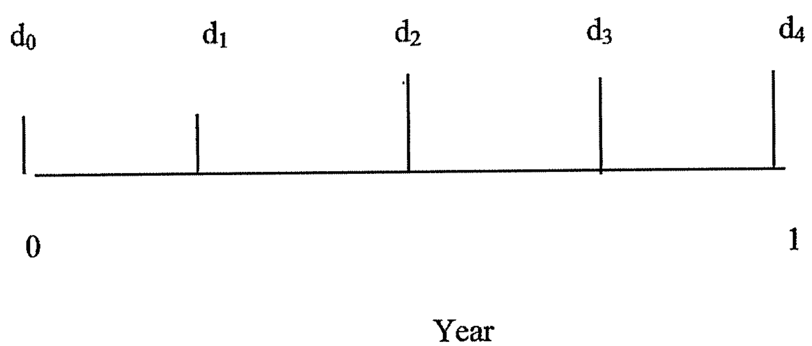
Quarterly DCF Model (Constant Dividend Version)

Case 1



$$d_1 = d_2 = d_3 = d_4 = d_0(1+g)$$

Case 2

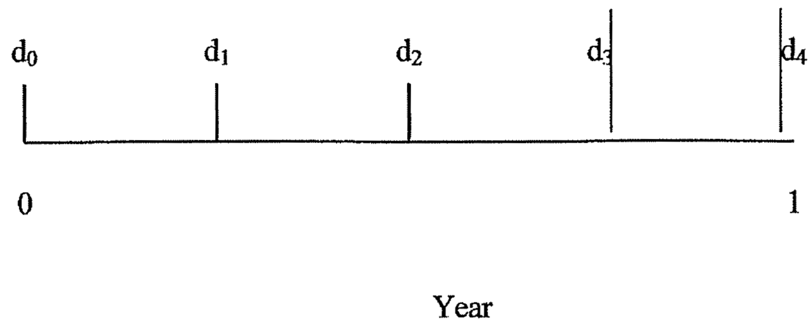


$$d_1 = d_0$$

$$d_2 = d_3 = d_4 = d_0(1+g)$$

Figure 3 (continued)

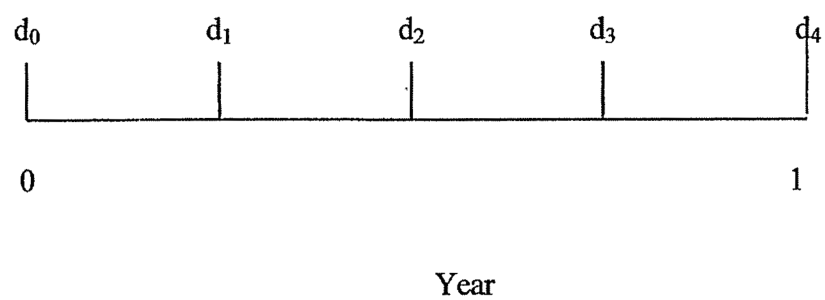
Case 3



$d_1 = d_2 = d_0$

$d_3 = d_4 = d_0(1+g)$

Case 4



$d_1 = d_2 = d_3 = d_0$

$d_4 = d_0(1+g)$

If we assume that the investor invests the quarterly dividend in an alternative investment of the same risk, then the amount accumulated by the end of the year will in all cases be given by

$$D_1^* = d_1\,(1+k)^{3/4} + d_2\,(1+k)^{1/2} + d_3\,(1+k)^{1/4} + d_4$$

where d_1, d_2, d_3 and d_4 are the four quarterly dividends. Under these new assumptions, the firm's stock price may be expressed by an annual DCF model of the form (2), with the exception that

$$D_1^* = d_1\,(1+k)^{3/4} + d_2\,(1+k)^{1/2} + d_3\,(1+k)^{1/4} + d_4 \qquad \textbf{(9)}$$

is used in place of $D_0(1+g)$. But, we already know that the annual DCF model may be reduced to

$$P_0 = \frac{D_0(1+g)}{k-g}$$

Thus, under the assumptions of the second quarterly DCF model, the firm's cost of equity is given by

$$k = \frac{D_1^*}{P_0} + g \quad \textbf{(10)}$$

with D_1^* given by (9).

Although equation (10) looks like the annual DCF model, there are at least two very important practical differences. First, since D_1^* is always greater than $D_0(1+g)$, the estimates of the cost of equity are always larger (and more accurate) in the Quarterly Model (10) than in the Annual Model. Second, since D_1^* depends on k through equation (9), the unknown "k" appears on both sides of (10), and an iterative procedure is required to solve for k.

APPENDIX 3
ADJUSTING FOR FLOTATION COSTS
IN DETERMINING A PUBLIC UTILITY'S ALLOWED
RATE OF RETURN ON EQUITY

Introduction

Regulation of public utilities is guided by the principle that utility revenues should be sufficient to allow recovery of all prudently incurred expenses, including the cost of capital. As set forth in the 1944 *Hope Natural Gas* Case [*Federal Power Comm'n v. Hope Natural Gas Co.* 320 U. S. 591 (1944) at 603], the U. S. Supreme Court states:

> From the investor or company point of view it is important that there be enough revenue not only for operating expenses but also for the capital costs of the business. These include service on the debt and dividends on the stock....By that standard the return to the equity owner should be commensurate with returns on investments in other enterprises having corresponding risks.

Since the flotation costs arising from the issuance of debt and equity securities are an integral component of capital costs, this standard requires that the company's revenues be sufficient to fully recover flotation costs.

Despite the widespread agreement that flotation costs should be recovered in the regulatory process, several issues still need to be resolved. These include:

1. How is the term "flotation costs" defined? Does it include only the out-of-pocket costs associated with issuing securities (e. g., legal fees, printing costs, selling and underwriting expenses), or does it also include the reduction in a security's price that frequently accompanies flotation (i. e., market pressure)?

2. What should be the time pattern of cost recovery? Should a company be allowed to recover flotation costs immediately, or should flotation costs be recovered over the life of the issue?

3. For the purposes of regulatory accounting, should flotation costs be included as an expense? As an addition to rate base? Or as an additional element of a firm's allowed rate of return?

4. Do existing regulatory methods for flotation cost recovery allow a firm ***full*** recovery of flotation costs?

In this paper, I review the literature pertaining to the above issues and discuss my own views regarding how this literature applies to the cost of equity for a regulated firm.

Definition of Flotation Cost

The value of a firm is related to the future stream of net cash flows (revenues minus expenses measured on a cash basis) that can be derived from its assets. In the process of acquiring assets, a firm incurs certain expenses which reduce its value. Some of these expenses or costs are directly associated with revenue production in one period (e. g., wages, cost of goods sold), others are more properly associated with revenue production in many periods (e. g., the acquisition cost of plant and equipment). In either case, the word "cost" refers to any item that reduces the value of a firm.

If this concept is applied to the act of issuing new securities to finance asset purchases, many items are properly included in issuance or flotation costs. These include: (1) compensation received by investment bankers for underwriting services, (2) legal fees, (3) accounting fees, (4) engineering fees, (5) trustee's fees, (6) listing fees, (7) printing and engraving expenses, (8) SEC registration fees, (9) Federal Revenue Stamps, (10) state taxes, (11) warrants granted to underwriters as extra compensation, (12) postage expenses, (13) employees' time, (14) market pressure, and (15) the offer discount. The finance literature generally divides these flotation cost items into three categories, namely, underwriting expenses, issuer expenses, and price effects.

Magnitude of Flotation Costs

The finance literature contains several studies of the magnitude of the flotation costs associated with new debt and equity issues. These studies differ primarily with regard to the time period studied, the sample of companies included, and the source of data. The flotation cost studies generally agree, however, that for large issues, underwriting expenses represent approximately one and one-half percent of the proceeds of debt issues and three to five percent of the proceeds of seasoned equity issues. They also agree that issuer expenses represent approximately 0.5 percent of both debt and equity issues, and that the announcement of an equity issue reduces the company's stock price by at least two to three percent of the proceeds from the stock issue. Thus, total flotation costs represent approximately two percent[12] of the proceeds from debt issues, and five and one-half to eight and one-half percent of the proceeds of equity issues.

Lee *et. al.* [14] is an excellent example of the type of flotation cost studies found in the finance literature. The Lee study is a comprehensive recent study of the underwriting and issuer costs associated with debt and equity issues for both utilities and non-utilities. The results of the Lee *et. al.* study are reproduced in Tables 1 and 2. Table 1 demonstrates that the total underwriting and issuer expenses for the 1,092 debt issues in their study averaged 2.24 percent of the proceeds of the issues, while the total underwriting and issuer costs for the 1,593 seasoned equity issues in their study averaged 7.11 percent of the proceeds of the new issue. Table 1 also demonstrates that the total underwriting and issuer costs of seasoned equity offerings, as a percent of proceeds, decline with the size of the issue. For issues above $60 million, total underwriting and issuer costs amount to from three to five percent of the amount of the proceeds.

Table 2 reports the total underwriting and issuer expenses for 135 utility debt issues and 136 seasoned utility equity issues. Total underwriting and issuer expenses for utility bond offerings averaged 1.47 percent of the amount of the proceeds and for seasoned utility equity offerings averaged 4.92 percent of the amount of the proceeds. Again, there are some economies of scale associated with larger equity offerings. Total underwriting and issuer expenses for equity offerings in excess of 40 million dollars generally range from three to four percent of the proceeds.

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12 The two percent flotation cost on debt only recognizes the cost of newly-issued debt. When interest rates decline, many companies exercise the call provisions on higher cost debt and reissue debt at lower rates. This process involves reacquisition costs that are not included in the academic studies. If reacquisition costs were included in the academic studies, debt flotation costs could increase significantly.

The results of the Lee study for large equity issues are consistent with results of earlier studies by Bhagat and Frost [4], Mikkelson and Partch [17], and Smith [24]. Bhagat and Frost found that total underwriting and issuer expenses average approximately four and one-half percent of the amount of proceeds from negotiated utility offerings during the period 1973 to 1980, and approximately three and one-half percent of the amount of the proceeds from competitive utility offerings over the same period. Mikkelson and Partch found that total underwriting and issuer expenses average five and one-half percent of the proceeds from seasoned equity offerings over the 1972 to 1982 period. Smith found that total underwriting and issuer expenses for larger equity issues generally amount to four to five percent of the proceeds of the new issue.

The finance literature also contains numerous studies of the decline in price associated with sales of large blocks of stock to the public. These articles relate to the price impact of: (1) initial public offerings; (2) the sale of large blocks of stock from one investor to another; and (3) the issuance of seasoned equity issues to the general public. All of these studies generally support the notion that the announcement of the sale of large blocks of stock produces a decline in a company's share price. The decline in share price for initial public offerings is significantly larger than the decline in share price for seasoned equity offerings; and the decline in share price for public utilities is less than the decline in share price for non-public utilities. A comprehensive study of the magnitude of the decline in share price associated specifically with the sale of new equity by public utilities is reported in Pettway [19], who found the market pressure effect for a sample of 368 public utility equity sales to be in the range of two to three percent. This decline in price is a real cost to the utility, because the proceeds to the utility depend on the stock price on the day of issue.

In addition to the price decline associated with the announcement of a new equity issue, the finance literature recognizes that there is also a price decline associated with the actual issuance of equity securities. In particular, underwriters typically sell seasoned new equity securities to investors at a price lower than the closing market price on the day preceding the issue. The Rules of Fair Practice of the National

Association of Securities Dealers require that underwriters not sell shares at a price above the offer price. Since the offer price represents a binding constraint to the underwriter, the underwriter tends to set the offer price slightly below the market price on the day of issue to compensate for the risk that the price received by the underwriter may go down, but can not increase. Smith provides evidence that the offer discount tends to be between 0.5 and 0.8 percent of the proceeds of an equity issue. I am not aware of any similar studies for debt issues.

In summary, the finance literature provides strong support for the conclusion that total underwriting and issuer expenses for public utility debt offerings represent approximately two percent of the amount of the proceeds, while total underwriting and issuer expenses for public utility equity offerings represent at least four to five percent of the amount of the proceeds. In addition, the finance literature supports the conclusion that the cost associated with the decline in stock price at the announcement date represents approximately two to three percent as a result of a large public utility equity issue.

<u>Time Pattern Of Flotation Cost Recovery</u>

Although flotation costs are incurred only at the time a firm issues new securities, there is no reason why an issuing firm ought to recognize the expense only in the current period. In fact, if assets purchased with the proceeds of a security issue produce revenues over many years, a sound argument can be made in favor of recognizing flotation expenses over a reasonably lengthy period of time. Such recognition is certainly consistent with the generally accepted accounting principle that the time pattern of expenses match the time pattern of revenues, and it is also consistent with the normal treatment of debt flotation expenses in both regulated and unregulated industries.

In the context of a regulated firm, it should be noted that there are many possible time patterns for the recovery of flotation expenses. However, if it is felt that flotation expenses are most appropriately recovered over a period of years, then it should be recognized that investors must also be compensated for the passage of time. That is to say, the value of an investor's capital will be reduced if the expenses are merely distributed over time, without any allowance for the time value of money.

Accounting For Flotation Cost In A Regulatory Setting

In a regulatory setting, a firm's revenue requirements are determined by the equation:

Revenue Requirement = Total Expenses + Allowed Rate of Return x Rate Base

Thus, there are three ways in which an issuing firm can account for and recover its flotation expenses: (1) treat flotation expenses as a current expense and recover them immediately; (2) include flotation expenses in rate base and recover them over time; and (3) adjust the allowed rate of return upward and again recover flotation expenses over time. Before considering methods currently being used to recover flotation expenses in a regulatory setting, I shall briefly consider the advantages and disadvantages of these three basic recovery methods.

Expenses. Treating flotation costs as a current expense has several advantages. Because it allows for recovery at the time the expense occurs, it is not necessary to compute amortized balances over time and to debate which interest rate should be applied to these balances. A firm's stockholders are treated fairly, and so are the firm's customers, because they pay neither more nor less than the actual flotation expense. Since flotation costs are relatively small compared to the total revenue requirement, treatment as a current expense does not cause unusual rate hikes in the year of flotation, as would the introduction of a large generating plant in a state that does not allow Construction Work in Progress in rate base.

On the other hand, there are two major disadvantages of treating flotation costs as a current expense. First, since the asset purchased with the acquired funds will likely generate revenues for many years into the future, it seems unfair that current ratepayers should bear the full cost of issuing new securities, when future ratepayers share in the benefits. Second, this method requires an estimate of the underpricing effect on each security issue. Given the difficulties involved in measuring the extent of underpricing, it may be more accurate to estimate the average underpricing allowance for many securities than to estimate the exact figure for one security.

Rate Base. In an article in *Public Utilities Fortnightly*, Bierman and Hass [5] recommend that flotation costs be treated as an intangible asset that is included in a firm's rate base along with the assets acquired with the stock proceeds. This approach has many advantages. For ratepayers, it provides a better match between benefits and expenses: the future ratepayers who benefit from the financing costs contribute the revenues to recover these costs. For investors, if the allowed rate of return is equal to the investors' required rate of return, it is also theoretically fair since they are compensated for the opportunity cost of their investment (including both the time value of money and the investment risk).

Despite the compelling advantages of this method of cost recovery, there are several disadvantages that probably explain why it has not been used in practice. First, a firm will only recover the proper amount for flotation expenses if the rate base is multiplied by the appropriate cost of capital. To the extent that a commission under or over estimates the cost of capital, a firm will under or over recover its flotation expenses. Second, it is may be both legally and psychologically difficult for commissioners to include an intangible asset in a firm's rate base. According to established legal doctrine, assets are to be included in rate base only if they are "used and useful" in the public service. It is unclear whether intangible assets such as flotation expenses meet this criterion.

Rate of Return. The prevailing practice among state regulators is to treat flotation expenses as an additional element of a firm's cost of capital or allowed rate of return. This method is similar to the second method above (treatment in rate base) in that some part of the initial flotation cost is amortized over time. However, it has a disadvantage not shared by the rate base method. If flotation cost is included in rate base, it is fairly easy to keep track of the flotation cost on each new equity issue and see how it is recovered over time. Using the rate of return method, it is not possible to track the flotation cost for specific issues because the flotation cost for a specific issue is never recorded. Thus, it is not clear to participants whether a current allowance is meant to recover (1) flotation costs actually incurred in a test period, (2) expected future flotation costs, or (3) past flotation costs. This confusion never arises in the treatment of debt flotation costs. Because the exact costs are recorded and explicitly amortized over time, participants recognize that current allowances for debt flotation costs are meant to recover some fraction of the flotation costs on all past debt issues.

Existing Regulatory Methods

Although most state commissions prefer to let a regulated firm recover flotation expenses through an adjustment to the allowed rate of return, there is considerable controversy about the magnitude of the required adjustment. The following are some of the most frequently asked questions: (1) Should an adjustment to the allowed return be made every year, or should the adjustment be made only in those years in which new equity is raised? (2) Should an adjusted rate of return be applied to the entire rate base, or should it be applied only to that portion of the rate base financed with paid-in capital (as opposed to retained earnings)? (3) What is the appropriate formula for adjusting the rate of return?

This section reviews several methods of allowing for flotation cost recovery. Since the regulatory methods of allowing for recovery of debt flotation costs is well known and widely accepted, I will begin my discussion of flotation cost recovery procedures by describing the widely accepted procedure of allowing for debt flotation cost recovery.

Debt Flotation Costs

Regulators uniformly recognize that companies incur flotation costs when they issue debt securities. They typically allow recovery of debt flotation costs by making an adjustment to both the cost of debt and the rate base (see Brigham [6]). Assume that: (1) a regulated company issues $100 million in bonds that mature in 10 years; (2) the interest rate on these bonds is seven percent; and (3) flotation costs represent four percent of the amount of the proceeds. Then the cost of debt for regulatory purposes will generally be calculated as follows:

$$\begin{aligned}\text{Cost of Debt} &= \frac{\text{Interest expense} + \text{Amortization of flotation costs}}{\text{Principal value} - \text{Unamortized flotation costs}} \\ &= \frac{\$7{,}000{,}000 + \$400{,}000}{\$100{,}000{,}000 - \$4{,}000{,}000} \\ &= 7.71\%\end{aligned}$$

Thus, current regulatory practice requires that the cost of debt be adjusted upward by approximately 71 basis points, in this example, to allow for the recovery of debt flotation costs. This example does not include losses on reacquisition of debt. The flotation cost allowance would increase if losses on reacquisition of debt were included.

The logic behind the traditional method of allowing for recovery of debt flotation costs is simple. Although the company has issued $100 million in bonds, it can only invest $96 million in rate base because flotation costs have reduced the amount of funds received by $4 million. If the company is not allowed to earn a 71 basis point higher rate of return on the $96 million invested in rate base, it will not generate sufficient cash flow to pay the seven percent interest on the $100 million in bonds it has issued. Thus, proper regulatory treatment is to increase the required rate of return on debt by 71 basis points.

Equity Flotation Costs

The finance literature discusses several methods of recovering equity flotation costs. Since each method stems from a specific model, (i. e., set of assumptions) of a firm and its cash flows, I will highlight the assumptions that distinguish one method from another.

Arzac and Marcus. Arzac and Marcus [2] study the proper flotation cost adjustment formula for a firm that makes continuous use of retained earnings and external equity financing and maintains a constant capital structure (debt/equity ratio). They assume at the outset that underwriting expenses and underpricing apply only to new equity obtained from external sources. They also assume that a firm has previously recovered all underwriting expenses, issuer expenses, and underpricing associated with previous issues of new equity.

To discuss and compare various equity flotation cost adjustment formulas, Arzac and Marcus make use of the following notation:

k	=	an investors' required return on equity
r	=	a utility's allowed return on equity base

S	=	value of equity in the absence of flotation costs
S_f	=	value of equity net of flotation costs
K_t	=	equity base at time t
E_t	=	total earnings in year t
D_t	=	total cash dividends at time t
b	=	$(E_t\text{-}D_t) \div E_t$ = retention rate, expressed as a fraction of earnings
h	=	new equity issues, expressed as a fraction of earnings
m	=	equity investment rate, expressed as a fraction of earnings, $m = b + h < 1$
f	=	flotation costs, expressed as a fraction of the value of an issue.

Because of flotation costs, Arzac and Marcus assume that a firm must issue a greater amount of external equity each year than it actually needs. In terms of the above notation, a firm issues $hE_t \div (1\text{-}f)$ to obtain hE_t in external equity funding. Thus, each year a firm loses:

Equation 3

$$L = \frac{hE_t}{1-f} - hE_t = \frac{f}{1-f} \times hE_t$$

due to flotation expenses. The present value, V, of all future flotation expenses is:

Equation 4

$$V = \sum_{t=1}^{\infty} \frac{fhE_t}{(1-f)(1+k)^t} = \frac{fh}{1-f} \times \frac{rK_0}{k-mr}$$

To avoid diluting the value of the initial stockholder's equity, a regulatory authority needs to find the value of r, a firm's allowed return on equity base, that equates the value of equity net of flotation costs to the initial equity base ($S_f = K_0$). Since the value of equity net of flotation costs equals the value of equity in the absence of flotation costs minus the present value of flotation costs, a regulatory authority needs to find that value of *r* that solves the following equation:

$$S_f = S - L.$$

This value is:

Equation 5

$$r = \frac{k}{1 - \frac{fh}{1-f}}$$

To illustrate the Arzac-Marcus approach to adjusting the allowed return on equity for the effect of flotation costs, suppose that the cost of equity in the absence of flotation costs is 12 percent. Furthermore, assume that a firm obtains external equity financing each year equal to 10 percent of its earnings and that flotation expenses equal 5 percent of the value of each issue. Then, according to Arzac and Marcus, the allowed return on equity should be:

$$r = \frac{.12}{1 - \frac{(.05).(.1)}{.95}} = .1206 = 12.06\%$$

Summary. With respect to the three questions raised at the beginning of this section, it is evident that Arzac and Marcus believe the flotation cost adjustment should be applied each year, since continuous external equity financing is a fundamental assumption of their model. They also believe that the adjusted rate of return should be applied to the entire equity-financed portion of the rate base because their model is based on the assumption that the flotation cost adjustment mechanism will be applied to the entire equity financed portion of the rate base. Finally, Arzac and Marcus recommend a flotation cost adjustment formula, Equation (3), that implicitly excludes recovery of financing costs associated with financing in previous periods and includes only an allowance for the fraction of equity financing obtained from external sources.

Patterson. The Arzac-Marcus flotation cost adjustment formula is significantly different from the conventional approach (found in many introductory textbooks) which recommends the adjustment equation:

Equation 6

$$r = \frac{D_t}{P_{t-1}(1-f)} + g$$

where P_{t-1} is the stock price in the previous period and g is the expected dividend growth rate. Patterson [18] compares the Arzac-Marcus adjustment formula to the conventional approach and reaches the conclusion that the Arzac-Marcus formula effectively expenses issuance costs as they are incurred, while the conventional approach effectively amortizes them over an assumed infinite life of the equity issue. Thus, the conventional formula is similar to the formula for the recovery of debt flotation costs: it is not meant to compensate investors for the flotation costs of future issues, but instead is meant to compensate investors for the flotation costs of previous issues. Patterson argues that the conventional approach is more appropriate for rate making purposes because the plant purchased with external equity funds will yield benefits over many future periods.

Illustration. To illustrate the Patterson approach to flotation cost recovery, assume that a newly organized utility sells an initial issue of stock for $100 per share, and that the utility plans to finance all new investments with retained earnings. Assume also that: (1) the initial dividend per share is six dollars; (2) the expected long-run dividend growth rate is six percent; (3) the flotation cost is five percent of the amount of the proceeds; and (4) the payout ratio is 51.28 percent. Then, the investor's required rate of return on equity is [k = (D/P) + g = 6 percent + 6 percent = 12 percent]; and the flotation-cost-adjusted cost of equity is [6 percent (1/.95) + 6 percent = 12.316 percent].

The effects of the Patterson adjustment formula on the utility's rate base, dividends, earnings, and stock price are shown in Table 3. We see that the Patterson formula allows earnings and dividends to grow at the expected six percent rate. We also see that the present value of expected future dividends, $100, is just sufficient to induce investors to part with their money. If the present value of expected future dividends were less than $100, investors would not have been willing to invest $100 in the firm. Furthermore, the present value of future dividends will only equal $100 if the firm is allowed to earn the 12.316 percent flotation-cost-adjusted cost of equity on its entire rate base.

Summary. Patterson's opinions on the three issues raised in this section are in stark contrast to those of Arzac and Marcus. He believes that: (1) a flotation cost adjustment should be applied in every year, regardless of whether a firm issues any new equity in each year; (2) a flotation cost adjustment should be applied to the entire equity-financed portion of the rate base, including that portion financed by retained earnings; and (3) the rate of return adjustment formula should allow a firm to recover an appropriate fraction of all previous flotation expenses.

Conclusion

Having reviewed the literature and analyzed flotation cost issues, I conclude that:

Definition of Flotation Cost: A regulated firm should be allowed to recover both the total underwriting and issuance expenses associated with issuing securities and the cost of market pressure.

Time Pattern of Flotation Cost Recovery. Shareholders are indifferent between the alternatives of immediate recovery of flotation costs and recovery over time, as long as they are fairly compensated for the opportunity cost of their money. This opportunity cost must include both the time value of money and a risk premium for equity investments of this nature.

Regulatory Recovery of Flotation Costs. The Patterson approach to recovering flotation costs is the only rate-of-return-adjustment approach that meets the *Hope* case criterion that a regulated company's revenues must be sufficient to allow the company an opportunity to recover all prudently incurred expenses, including the cost of capital. The Patterson approach is also the only rate-of-return-adjustment approach that provides an incentive for investors to invest in the regulated company.

Implementation of a Flotation Cost Adjustment. As noted earlier, prevailing regulatory practice seems to be to allow the recovery of flotation costs through an adjustment to the required rate of return. My review of the literature on this subject indicates that there are at least two recommended methods of making this adjustment: the Patterson approach and the Arzac-Marcus approach. The Patterson approach assumes that a firm's flotation expenses on new equity issues are treated in the same manner as flotation expenses on new bond issues, i. e., they are amortized over future time periods. If this assumption is true (and I believe it is), then the flotation cost adjustment should be applied to a firm's entire equity base, including retained earnings. In practical terms, the Patterson approach produces an increase in a firm's cost of equity of approximately thirty basis points. The Arzac-Marcus approach assumes that flotation costs on new equity issues are recovered entirely in the year in which the securities are sold. Under the Arzac-Marcus assumption, a firm should not be allowed any adjustments for flotation costs associated with previous flotations. Instead, a firm should be allowed only an adjustment on future security sales as they occur. Under reasonable assumptions about the rate of new equity sales, this method produces an increase in the cost of equity of approximately six basis points. Since the Arzac-Marcus approach does not allow the company to recover the entire amount of its flotation cost, I recommend that this approach be rejected and the Patterson approach be accepted.

BIBLIOGRAPHY

1. Armknecht, Raymond, Fred Grygiel and Patrick Hess, "Market Pressure: The Sales of New Common Equity and Rate of Return Regulation, "Proceedings of the Business and Economic Statistics Section of the American Statistical Association, 1974, pp. 80—91.

2. Arzac, E. R., and M. Marcus, "Flotation Cost Allowance in Rate of Return Regulation: A Note," *Journal of Finance*, December 1981, pp. 1199—1202.

3. Barclay, M. J. and R. H. Litzenberger, 1988, "Announcement Effects of New Equity Issues and the Use of Intraday Price Data," *Journal of Financial Economics* 21, 71—99.

4. Bhagat, S. and P. A. Frost, 1986, "Issuing Costs to Existing Shareholders in Competitive and Negotiated Underwritten Public Utility Equity Offerings," *Journal of Financial Economics* 15, 233—59.

5. Bierman, H., and J. E. Hass, "Equity Flotation Cost Adjustments in Utilities' Cost of Service," *Public Utilities Fortnightly*, March 1, 1983, pp.46—49 .

6. Bowyer, Jr., John W., and Jess B. Yawitz, "The Effect of New Equity Issues on Utility Stock Prices," *Pubic Utilities Fortnightly*, May 22, 1980.

7. Brigham, Eugene F., Dana Aberwald, and Louis C. Gapenski, "Common Equity Flotation Costs and Rate Making," *Public Utilities Fortnightly*, May 2, 1985, pp. 28—26.

8. Calomiris, C. W. and D. M. G Raff, 1995, "The Evolution of Market Structure, Information, and Spreads in American Investment Banking," in M. B. Bordo and R. Sylla, eds., *Anglo-American Finance: Financial Markets and Institutions in 20th Century North America and the U. K.* (Business One-Irwin Homewood, IL), 103—60.

9. Dunbar, C. G., 1995, "The Use of Warrants as Underwriter Compensation in Initial Public Offerings," *Journal of Financial Economics* 38, 59—78.

10. Evans, Robert E., "On the Existence, Measurement, and Economic Significance of Market Pressure in the Pricing of New Equity Shares," unpublished dissertation, University of Wisconsin, 1978.

11. Howe, K. M., "Flotation Cost Allowance in Rate of Return Regulation: Comment," *Journal of Finance*, March 1984, pp. 289—290.

12. Howe, K. M., "Flotation Cost Allowance for the Regulated Firm: A Comparison of Alternatives," unpublished working paper, School of Business, Iowa State University.

13. Ibbotson, R. C., "Price Performance of Common Stock New Issues," *Journal of Financial Economics*, 1975, pp. 235—272.

14. Lee, Inmoo, Scott Lochhead, Jay Ritter, and Quanshui Zhao, "The Costs of Raising Capital," *The Journal of Financial Research*, Vol XIX No 1 (Spring 1996), 59—74

15. Logue, D. E., "On the Pricing of Unseasoned Equity Offerings: 1965—1969," *Journal of Financial and Quantitative Analysis*, January 1973, pp. 91—103.

16. McDonald, J. G. and A. K. Fisher, "New Issue Stock Price Behavior," *Journal of Finance*, March 1972, pp. 97—102.

17. Mikkelson, Wayne H. and M. Megan Partch, "Valuation Effects of Security Offerings and the Issuance Process," *Journal of Financial Economics* 15 (1986), pp. 31-60.

18. Patterson, C. S., "Flotation Cost Allowance in Rate of Return Regulation: Comment," *Journal of Finance*, September 1983, pp. 1335—1338.

19. Pettway, R. H., "The Effects of New Equity Sales Upon Utility Share Prices," *Public Utilities Fortnightly*, May 10, 1984, pp. 35—39.

20. Reilly, F. K. and K. Hatfield, "Investor Experience with New Stock Issues," *Financial Analysts' Journal*, September--October 1969, pp. 73—80.

21. Richter, P. H., "The Ever Present Need for an Underpricing Allowance," *Public Utilities Fortnightly*, February 18, 1982, pp. 58—61.

22. Scholes, M., "The Market for New Securities: Substitution versus Price Pressure and the Effects of Information on Share Prices," *Journal of Business*, April 1972, pp. 179—211.

23. Securities and Exchange Commission, Report of Special Study on Securities Markets, U. S. Government Printing Office, Washington, D. C. 1963.

24. Smith, Clifford W. Jr., "Alternative Methods for Raising Capital," *Journal of Financial Economics* 5 (1977) 273-307.

Table 1

Direct Costs as a Percentage of Gross Proceeds
for Equity (IPOs and SEOs) and Straight and Convertible Bonds
Offered by Domestic Operating Companies 1990—1994[13]

Equities

		IPOs				SEOs			
Line No.	Proceeds ($ in millions)	No. of Issues	Gross Spreads	Other Direct Expenses	Total Direct Costs	No. of Issues	Gross Spreads	Other Direct Expenses	Total Direct Costs
1	2-9.99	337	9.05%	7.91%	16.96%	167	7.72%	5.56%	13.28%
2	10-19.99	389	7.24%	4.39%	11.63%	310	6.23%	2.49%	8.72%
3	20-39.99	533	7.01%	2.69%	9.70%	425	5.60%	1.33%	6.93%
4	40-59.99	215	6.96%	1.76%	8.72%	261	5.05%	0.82%	5.87%
5	60-79.99	79	6.74%	1.46%	8.20%	143	4.57%	0.61%	5.18%
6	80-99.99	51	6.47%	1.44%	7.91%	71	4.25%	0.48%	4.73%
7	100-199.99	106	6.03%	1.03%	7.06%	152	3.85%	0.37%	4.22%
8	200-499.99	47	5.67%	0.86%	6.53%	55	3.26%	0.21%	3.47%
9	500 and up	10	5.21%	0.51%	5.72%	9	3.03%	0.12%	3.15%
10	**Total/Average**	**1,767**	**7.31%**	**3.69%**	**11.00%**	**1,593**	**5.44%**	**1.67%**	**7.11%**

Bonds

		Convertible Bonds				Straight Bonds			
Line No.	Proceeds ($ in millions)	No. of Issues	Gross Spreads	Other Direct Expenses	Total Direct Costs	No. of Issues	Gross Spreads	Other Direct Expenses	Total Direct Costs
1	2-9.99	4	6.07%	2.68%	8.75%	32	2.07%	2.32%	4.39%
2	10-19.99	14	5.48%	3.18%	8.66%	78	1.36%	1.40%	2.76%
3	20-39.99	18	4.16%	1.95%	6.11%	89	1.54%	0.88%	2.42%
4	40-59.99	28	3.26%	1.04%	4.30%	90	0.72%	0.60%	1.32%
5	60-79.99	47	2.64%	0.59%	3.23%	92	1.76%	0.58%	2.34%
6	80-99.99	13	2.43%	0.61%	3.04%	112	1.55%	0.61%	2.16%
7	100-199.99	57	2.34%	0.42%	2.76%	409	1.77%	0.54%	2.31%
8	200-499.99	27	1.99%	0.19%	2.18%	170	1.79%	0.40%	2.19%
9	500 and up	3	2.00%	0.09%	2.09%	20	1.39%	0.25%	1.64%
10	**Total/Average**	**211**	**2.92%**	**0.87%**	**3.79%**	**1,092**	**1.62%**	**0.62%**	**2.24%**

[13] Inmoo Lee, Scott Lochhead, Jay Ritter, and Quanshui Zhao, "The Costs of Raising Capital," *Journal of Financial Research* Vol 19 No 1 (Spring 1996) pp. 59-74.

Notes:

Closed-end funds and unit offerings are excluded from the sample. Rights offerings for SEOs are also excluded. Bond offerings do not include securities backed by mortgages and issues by Federal agencies. Only firm commitment offerings and non-shelf-registered offerings are included.
Gross Spreads as a percentage of total proceeds, including management fee, underwriting fee, and selling concession.
Other Direct Expenses as a percentage of total proceeds, including management fee, underwriting fee, and selling concession.
Total Direct Costs as a percentage of total proceeds (total direct costs are the sum of gross spreads and other direct expenses).

Table 2
Direct Costs of Raising Capital 1990—1994
Utility versus Non-Utility Companies[14]

Equities

	Non-Utilities	IPOs			SEOs		
Line No.	Proceeds ($ in millions)	No. of Issues	Gross Spreads	Total Direct Costs	No. Of Issues	Gross Spreads	Total Direct Costs
1	2-9.99	332	9.04%	16.97%	154	7.91%	13.76%
2	10-19.99	388	7.24%	11.64%	278	6.42%	9.01%
3	20-39.99	528	7.01%	9.70%	399	5.70%	7.07%
4	40-59.99	214	6.96%	8.71%	240	5.17%	6.02%
5	60-79.99	78	6.74%	8.21%	131	4.68%	5.31%
6	80-99.99	47	6.46%	7.88%	60	4.35%	4.84%
7	100-199.99	101	6.01%	7.01%	137	3.97%	4.36%
8	200-499.99	44	5.65%	6.49%	50	3.27%	3.48%
9	500 and up	10	5.21%	5.72%	8	3.12%	3.25%
10	**Total/Average**	1,742	7.31%	11.01%	1,457	5.57%	7.32%
11	**Utilities Only**						
12	2-9.99	5	9.40%	16.54%	13	5.41%	7.68%
13	10-19.99	1	7.00%	8.77%	32	4.59%	6.21%
14	20-39.99	5	7.00%	9.86%	26	4.17%	4.96%
15	40-59.99	1	6.98%	11.55%	21	3.69%	4.12%
16	60-79.99	1	6.50%	7.55%	12	3.39%	3.72%
17	80-99.99	4	6.57%	8.24%	11	3.68%	4.11%
18	100-199.99	5	6.45%	7.96%	15	2.83%	2.98%
19	200-499.99	3	5.88%	7.00%	5	3.19%	3.48%
20	500 and up	0			1	2.25%	2.31%
21	**Total/Average**	25	7.15%	10.14%	136	4.01%	4.92%

[14] Lee *et al, op. cit.*

Table 2 (continued)
Direct Costs of Raising Capital 1990—1994
Utility versus Non-Utility Companies[15]

Bonds

	Non- Utilities	Convertible Bonds			Straight Bonds		
Line No.	Proceeds ($ in millions)	No. of Issues	Gross Spreads	Total Direct Costs	No. of Issues	Gross Spreads	Total Direct Costs
1	2-9.99	4	6.07%	8.75%	29	2.07%	4.53%
2	10-19.99	12	5.54%	8.65%	47	1.70%	3.28%
3	20-39.99	16	4.20%	6.23%	63	1.59%	2.52%
4	40-59.99	28	3.26%	4.30%	76	0.73%	1.37%
5	60-79.99	47	2.64%	3.23%	84	1.84%	2.44%
6	80-99.99	12	2.54%	3.19%	104	1.61%	2.25%
7	100-199.99	55	2.34%	2.77%	381	1.83%	2.38%
8	200-499.99	26	1.97%	2.16%	154	1.87%	2.27%
9	500 and up	3	2.00%	2.09%	19	1.28%	1.53%
10	**Total/Average**	203	2.90%	3.75%	957	1.70%	2.34%
11	**Utilities Only**						
12	2-9.99	0			3	2.00%	3.28%
13	10-19.99	2	5.13%	8.72%	31	0.86%	1.35%
14	20-39.99	2	3.88%	5.18%	26	1.40%	2.06%
15	40-59.99	0			14	0.63%	1.10%
16	60-79.99	0			8	0.87%	1.13%
17	80-99.99	1	1.13%	1.34%	8	0.71%	0.98%
18	100-199.99	2	2.50%	2.74%	28	1.06%	1.42%
19	200-499.99	1	2.50%	2.65%	16	1.00%	1.40%
20	500 and up	0			1	3.50%	na[16]
21	**Total/Average**	8	3.33%	4.66%	135	1.04%	1.47%

Notes:
Total proceeds raised in the United States, excluding proceeds from the exercise of over allotment options.
Gross spreads as a percentage of total proceeds (including management fee, underwriting fee, and selling concession).
Other direct expenses as a percentage of total proceeds (including registration fee and printing, legal, and auditing costs).

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[15] Lee *et al, op. cit.*

[16] Not available because of missing data on other direct expenses.

Table 3
Illustration of Patterson Approach to Flotation Cost Recovery

Line No.	Time Period	Rate Base	Earnings @ 12.32%	Earnings @ 12.00%	Dividends	Amortization Initial FC
1	0	95.00				
2	1	100.70	11.70	11.40	6.00	0.3000
3	2	106.74	12.40	12.08	6.36	0.3180
4	3	113.15	13.15	12.81	6.74	0.3371
5	4	119.94	13.93	13.58	7.15	0.3573
6	5	127.13	14.77	14.39	7.57	0.3787
7	6	134.76	15.66	15.26	8.03	0.4015
8	7	142.84	16.60	16.17	8.51	0.4256
9	8	151.42	17.59	17.14	9.02	0.4511
10	9	160.50	18.65	18.17	9.56	0.4782
11	10	170.13	19.77	19.26	10.14	0.5068
12	11	180.34	20.95	20.42	10.75	0.5373
13	12	191.16	22.21	21.64	11.39	0.5695
14	13	202.63	23.54	22.94	12.07	0.6037
15	14	214.79	24.96	24.32	12.80	0.6399
16	15	227.67	26.45	25.77	13.57	0.6783
17	16	241.33	28.04	27.32	14.38	0.7190
18	17	255.81	29.72	28.96	15.24	0.7621
19	18	271.16	31.51	30.70	16.16	0.8078
20	19	287.43	33.40	32.54	17.13	0.8563
21	20	304.68	35.40	34.49	18.15	0.9077
22	21	322.96	37.52	36.56	19.24	0.9621
23	22	342.34	39.77	38.76	20.40	1.0199
24	23	362.88	42.16	41.08	21.62	1.0811
25	24	384.65	44.69	43.55	22.92	1.1459
26	25	407.73	47.37	46.16	24.29	1.2147
27	26	432.19	50.21	48.93	25.75	1.2876
28	27	458.12	53.23	51.86	27.30	1.3648
29	28	485.61	56.42	54.97	28.93	1.4467
30	29	514.75	59.81	58.27	30.67	1.5335
31	30	545.63	63.40	61.77	32.51	1.6255
32	Present Value@12%		195.00	190.00	100.00	5.00

ATMOS ENERGY
APPENDIX 4
EX ANTE RISK PREMIUM APPROACH

My ex ante risk premium method is based on studies of the DCF expected return on proxy companies compared to the interest rate on Moody's A-rated utility bonds. Specifically, for each month in my study period, I calculate the risk premium using the equation,

$$RP_{PROXY} = DCF_{PROXY} - I_A$$

where:

RP_{PROXY}	=	the required risk premium on an equity investment in the proxy group of companies,
DCF_{PROXY}	=	average DCF estimated cost of equity on a portfolio of proxy companies; and
I_A	=	the yield to maturity on an investment in A-rated utility bonds.

For my ex ante risk premium analysis, I begin with my comparable group of natural gas companies shown in Schedule 1. Previous studies have shown that the ex ante risk premium tends to vary inversely with the level of interest rates, that is, the risk premium tends to increase when interest rates decline, and decrease when interest rates go up. To test whether my studies also indicate that the ex ante risk premium varies inversely with the level of interest rates, I perform a regression analysis of the relationship between the ex ante risk premium and the yield to maturity on A-rated utility bonds, using the equation,

$$RP_{PROXY} = a + (b \times I_A) + e$$

where:

RP_{PROXY} = risk premium on proxy company group;

I_A = yield to maturity on A-rated utility bonds;

e = a random residual; and

a, b = coefficients estimated by the regression procedure.

Regression analysis assumes that the statistical residuals from the regression equation are random. My examination of the residuals reveals that there is a significant probability that the residuals are serially correlated (non-zero serial correlation indicates that the residual in one time period tends to be correlated with the residual in the previous time period). Therefore, I make adjustments to my data to correct for the possibility of serial correlation in the residuals.

The common procedure for dealing with serial correlation in the residuals is to estimate the regression coefficients in two steps. First, a multiple regression analysis is used to estimate the serial correlation coefficient, *r*. Second, the estimated serial correlation coefficient is used to transform the original variables into new variables whose serial correlation is approximately zero. The regression coefficients are then re-estimated using the transformed variables as inputs in the regression equation. Based on my knowledge of the statistical relationship between the yield to maturity on A-rated utility bonds and the required risk premium, my estimate of the ex ante risk premium on an investment in my proxy natural gas company group as compared to an investment in A-rated utility bonds is given by the equation:

$$RP_{PROXY} = \underset{(8.69)}{0.0677} - \underset{(-2.706)}{.3068 \times I_A}.$$ [17]

Using the 5.97 percent average yield to maturity on A-rated utility bonds at July 2009, the regression equation produces an ex ante risk premium based on the natural gas proxy group equal to 4.94 percent (0.0677 – .3068 x 5.97 = 4.94).

To estimate the cost of equity using the ex ante risk premium method, one may add the estimated risk premium over the yield on A-rated utility bonds to the yield to maturity on

[17] The t-statistics are shown in parentheses.

A-rated utility bonds. As described above, my analyses produce an estimated risk premium over the yield on A-rated utility bonds equal to 4.94 percent. Adding an estimated risk premium of 4.94 percent to the 5.97 percent average yield to maturity on A-rated utility bonds produces a cost of equity estimate of 10.9 percent for the natural gas company proxy group using the ex ante risk premium method.

ATMOS ENERGY
APPENDIX 5
EX POST RISK PREMIUM APPROACH

SOURCE OF DATA

Stock price and yield information is obtained from Standard & Poor's Security Price publication. Standard & Poor's derives the stock dividend yield by dividing the aggregate cash dividends (based on the latest known annual rate) by the aggregate market value of the stocks in the group. The bond price information is obtained by calculating the present value of a bond due in 30 years with a $4.00 coupon and a yield to maturity of a particular year's indicated Moody's A-rated Utility bond yield. The values shown on the ex post risk premium schedules are the January values of the respective indices.

CALCULATION OF STOCK AND BOND RETURNS

Sample calculation of "Stock Return" column:

$$\text{Stock Return (2008)} = \left[\frac{\text{Stock Price (2009) - Stock Price (2008) + Dividend (2008)}}{\text{Stock Price (2008)}} \right]$$

where Dividend (2008) = Stock Price (2008) x Stock Div. Yield (2008)

Sample calculation of "Bond Return" column:

$$\text{Bond Return (2008)} = \left[\frac{\text{Bond Price (2009) - Bond Price (2008) + Interest (2008)}}{\text{Bond Price (2008)}} \right]$$

where Interest = $4.00.

REQUEST:

[Rate of Return] - Please provide copies of all articles, publications, and or other documents cited in the testimony of Dr. Vander Weide.

RESPONSE:

Please see the attached for the articles, publications and documents cited in the testimony.

- Attachment 1 - Lee, Inmoo, Scott Lochhead, Jay Ritter, and Quanshui Zhao, "The Costs of Raising Capital," The Journal of Financial Research, Vol. XIX No 1 (Spring 1996), 59 74.
- Attachment 2 - Clifford W. Smith, "Alternative Methods for Raising Capital," Journal of Financial Economics 5 (1977) 273-307.
- Attachment 3 - Richard H. Pettway, "The Effects of New Equity Sales Upon Utility Share Prices," Public Utilities Fortnightly, May 10, 1984, 35—39.
- Attachment 4 - Blue Chip Financial Forecasts, August 1, 2009.
- Attachment 5 - Fischer Black, Michael C. Jensen, and Myron Scholes, "The Capital Asset Pricing Model: Some Empirical Tests," in Studies in the Theory of Capital Markets, M. Jensen, ed. New York: Praeger, 1972.
- Attachment 6 - Eugene Fama and James MacBeth, "Risk, Return, and Equilibrium: Empirical Tests," Journal of Political Economy 81 (1973), pp. 607 36.
- Attachment 7 - Robert Litzenberger and Krishna Ramaswamy, "The Effect of Personal Taxes and Dividends on Capital Asset Prices: Theory and Empirical Evidence," Journal of Financial Economics 7 (1979), pp. 163 95.
- Attachment 8 - Rolf Banz, "The Relationship between Return and Market Value of Common Stocks," Journal of Financial Economics (March 1981), pp. 3 18.
- Attachment 9 - Eugene Fama and Kenneth French, "The Cross Section of Expected Returns," Journal of Finance (June 1992), pp. 427 465.

ATTACHMENTS:

ATTACHMENT 1 - Atmos Energy Corporation, Lee Lochhead Ritter Flotation 1996, 16 Pages.

ATTACHMENT 2 - Atmos Energy Corporation, Smith 1977 Alternative Methods Raising Capital, 35 Pages.

ATTACHMENT 3 - Atmos Energy Corporation, Pettway 1984, 5 Pages.

ATTACHMENT 4 - Atmos Energy Corporation, Blue Chip Aug 1 2009, 2 Pages.

ATTACHMENT 5 - Atmos Energy Corporation, Black Jensen 1972, 24 Pages.

ATTACHMENT 6 - Atmos Energy Corporation, Fama MacBeth 1973 Risk Return, 30 Pages.

ATTACHMENT 7 - Atmos Energy Corporation, Litzenberger 1979, 17 Pages.

ATTACHMENT 8 - Atmos Energy Corporation, Banz 1981, 16 Pages.

ATTACHMENT 9 - Atmos Energy Corporation, Fama French 1992 Expct Rtrns, 40 Pages.

Respondent: Dr. James Vander Weide

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THE COSTS OF RAISING CAPITAL

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Abstract

We report the average costs of raising external debt and equity capital for U.S. corporations from 1990 to 1994. For initial public offerings (IPOs) of equity, the direct costs average 11.0 percent of the proceeds. For seasoned equity offerings (SEOs), the direct costs average 7.1 percent. For convertible bonds, the direct costs average 3.8 percent. For straight debt issues, the direct costs average 2.2 percent, although they are strongly related to the credit rating of the issue. All classes of securities exhibit economies of scale, although they are less pronounced for straight debt issues. IPOs also incur a substantial indirect cost due to short-run underpricing. Most large equity offers include an international tranche, although debt issues do not.

I. Introduction

In this article we present the average costs of raising external capital for U.S. corporations from 1990 to 1994. Specifically, we report the average spreads on public equity offerings and debt offerings, along with the other direct costs of raising capital, as a percentage of the proceeds. We find substantial economies of scale for initial public offerings (IPOs) of equity and seasoned equity offerings (SEOs). We also find substantial economies of scale for both straight bond offerings and convertible bond offerings. Spreads on bond offerings are highly sensitive to the credit rating of the offering. This article is descriptive in nature; no theories are tested. Its purpose is to provide benchmark numbers for use by issuers of securities. We do not address why firms issue the securities they do. This much broader corporate finance question would have to address taxes, corporate control, debt capacity, long-run performance patterns, investment-financing interactions, etc.

We would like to thank Charles Calomiris and Tim Loughran for useful comments on an earlier draft.

II. Data and Terminology

Securities Data Company's (SDC) New Issues database is the primary source of information. After downloading SDC's data, we identified outliers and checked suspicious numbers in other publicly available sources. The New Issues database includes publicly placed firm commitment offerings only. In all of our tables, we exclude ADRs and unit offerings.[1] We restrict our sample to securities offered by domestic operating companies, and so exclude closed-end fund and real estate investment trust (REIT) offerings. We also exclude rights offerings and shelf registrations.[2]

We use security offerings from January 1990 to December 1994, a five-year period of relatively low inflation. Consequently, we do not make any inflation adjustments; all proceeds are the nominal proceeds. Proceeds reflect the gross proceeds raised in the U.S. and do not include money raised from the exercise of overallotment options or an international tranche, if any. In the case of equity offerings, the proceeds include the amount raised from both primary and secondary components. Primary shares are those being sold by the company, thereby increasing the number of shares outstanding. Secondary shares are those being sold by existing shareholders (managers, venture capitalists, etc.), which neither increase the number of shares outstanding nor provide capital for the company. Many IPOs include both primary and secondary components, with the fraction that is primary generally higher for younger companies. A few IPOs, sometimes involving spin-offs from parent companies, are pure secondaries. All of our SEOs involve primary shares; we exclude "registered secondaries," in which the entire issue is composed of shares being sold by existing shareholders, from our SEO sample.

For our sample of bond offerings, we exclude issues with a maturity date of one year or less. Our sample includes both zero-coupon, original-issue discount bonds, and coupon bonds. We include serial, floating-rate, and reset bonds, as

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[1]ADRs are American Depository Receipts (also called American Depository Shares) that are traded in the United States for foreign issuers. Unit offerings are bundles of securities (frequently, a share plus a warrant to buy a share at some exercise price), commonly issued in small IPOs by young, speculative companies taken public by less-prestigious investment bankers.

[2]Rights offerings give existing shareholders the right to buy the securities offered. While they are common in many countries, rights offerings have been rare in the United States during the last twenty years. See Smith (1977), Hansen and Pinkerton (1982), and Hansen (1988) for a discussion of rights offerings. Shelf registrations are offerings whereby a company meeting certain qualifications is permitted to issue securities without issuing a prospectus (taking the securities "off the shelf" and selling them). In our sample period, shelf equity offerings are practically nonexistent, although there are many bond offerings (typically smaller issues) using shelf registrations that we exclude.

well as traditional coupon bonds.[3] We exclude mortgage-backed bonds. For zero-coupon and original-issue discount bonds that are sold for less than their par value, our percentage spreads and costs are based upon the offer price, and not the face value. Our convertible bond sample includes only issues that are convertible into shares of the issuing company. Exchangeable bonds, where the bond is convertible into shares of a different company, are not in our sample. None of our convertible bonds has a maturity date of less than five years.

We refer to new equity issues by publicly traded companies as seasoned equity offerings, reserving the use of "secondary" to identify the source of shares. Among practitioners, the term "secondary offering" is frequently used to refer to an SEO. Seasoning refers to whether the security being offered is already publicly traded; IPOs are unseasoned new issues. For that matter, the term "new issues" is sometimes used to refer to any security offering, and sometimes used to refer to equity IPOs alone. Although a new bond issue is an unseasoned new issue, and therefore a debt initial public offering, we use the term IPO to refer to unseasoned equity offerings exclusively.

Gross spreads are the commissions paid to investment bankers when securities are issued. Since buyers do not pay commissions on new security issues, these spreads implicitly reflect both the buyer and seller commissions. Other direct costs include the legal, auditing, and printing costs associated with putting together a prospectus.

III. Evidence

Average Spreads and Total Direct Costs

In Table 1 we report the average investment banker commissions (gross spreads) and other direct expenses for four classes of securities: IPOs, SEOs, convertible bonds, and straight bonds. In addition to reporting the average direct costs for each class, we also classify issues by proceeds categories. By going across a row, a reader can see how the expenses vary by security type, holding proceeds constant. By going down a column, a reader can see the magnitude of the economies of scale for a given type of security. Also reported is the number of observations in each category.

In Table 1 the median IPO is $24.4 million, the median SEO is $33.8 million, the median convertible bond is $75 million, and the median straight

[3] Serial bonds have the individual bonds maturing on different dates, with the coupons varying depending upon the maturity date. Reset and floating-rate bonds have the interest rate changing periodically, with the new interest rate determined either by an auction (reset) or a formula (floaters).

TABLE 1. Direct Costs as a Percentage of Gross Proceeds for Equity (IPOs and SEOs) and Straight and Convertible Bonds Offered by Domestic Operating Companies, 1990–94.

	Equity								Bonds							
	IPOs				SEOs				Convertible Bonds				Straight Bonds			
Proceeds[a] ($ millions)	N[b]	GS[c]	E[d]	TDC[e]	N	GS	E	TDC	N	GS	E	TDC	N	GS	E	TDC
2–9.99	337	9.05	7.91	16.96	167	7.72	5.56	13.28	4	6.07	2.68	8.75	32	2.07	2.32	4.39
10–19.99	389	7.24	4.39	11.63	310	6.23	2.49	8.72	14	5.48	3.18	8.66	78	1.36	1.40	2.76
20–39.99	533	7.01	2.69	9.70	425	5.60	1.33	6.93	18	4.16	1.95	6.11	89	1.54	0.88	2.42
40–59.99	215	6.96	1.76	8.72	261	5.05	0.82	5.87	28	3.26	1.04	4.30	90	0.72	0.60	1.32
60–79.99	79	6.74	1.46	8.20	143	4.57	0.61	5.18	47	2.64	0.59	3.23	92	1.76	0.58	2.34
80–99.99	51	6.47	1.44	7.91	71	4.25	0.48	4.73	13	2.43	0.61	3.04	112	1.55	0.61	2.16
100–199.99	106	6.03	1.03	7.06	152	3.85	0.37	4.22	57	2.34	0.42	2.76	409	1.77	0.54	2.31
200–499.99	47	5.67	0.86	6.53	55	3.26	0.21	3.47	27	1.99	0.19	2.18	170	1.79	0.40	2.19
500–up	10	5.21	0.51	5.72	9	3.03	0.12	3.15	3	2.00	0.09	2.09	20	1.39	0.25	1.64
Total	1767	7.31	3.69	11.00	1593	5.44	1.67	7.11	211	2.92	0.87	3.79	1092	1.62	0.62	2.24

Notes: Closed-end funds (SIC 6726), REITs (SIC 6798), ADRs, and unit offerings are excluded from the sample. Rights offerings for SEOs are also excluded. Bond offerings do not include securities backed by mortgages and issues by Federal agencies (SIC 6011, 6019, 6111, and 999B). Only firm commitment offerings and nonshelf-registered offerings are included. Standard Industrial Classification (SIC) codes are from Securities Data Co. (SDC).

[a]Total proceeds raised in the United States, excluding proceeds from the exercise of overallotment options (SDC variable: PROCDS).
[b]Number of issues.
[c]Gross spreads as a percentage of total proceeds (including management fee, underwriting fee, and selling concession) (SDC variable: GPCTP).
[d]Other direct expenses as a percentage of total proceeds (including registration fee and printing, legal, and auditing costs) (SDC variables: EXPTH/(PROCDS)*10).
[e]Total direct costs as a percentage of total proceeds (total direct costs are the sum of gross spreads and other direct expenses).

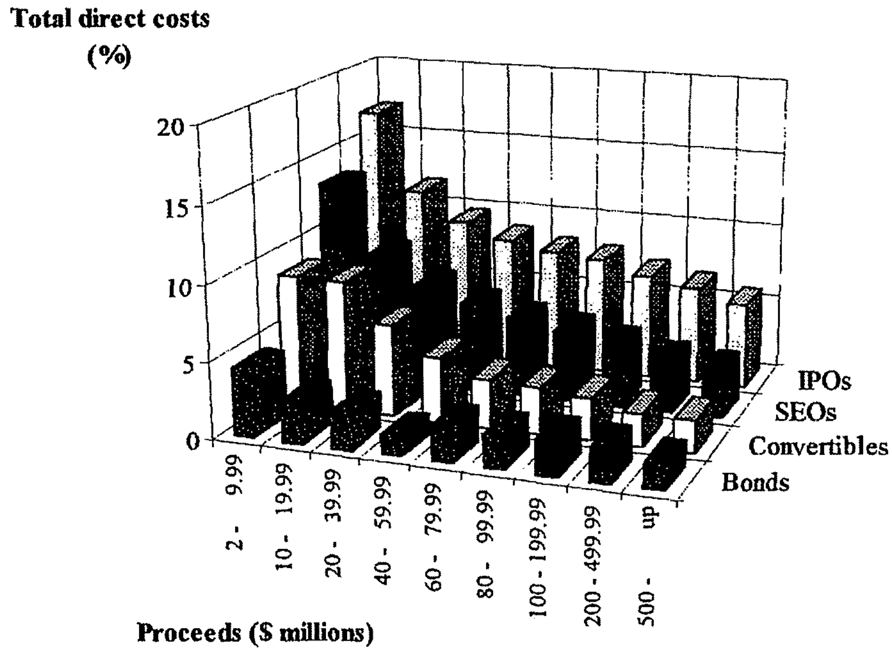

Figure I. Total Direct Costs as a Percentage of Gross Proceeds. The total direct costs for initial public offerings (IPOs), seasoned equity offerings (SEOs), convertible bonds, and straight bonds are composed of underwriter spreads and other direct expenses. Closed-end funds (SIC 6726), REITs (SIC 6798), ADRs, and unit offerings are excluded. Rights offerings for SEOs are also excluded. Bond offerings do not include securities backed by mortgages and issues by federal agencies (SIC 6011, 6019, 6111, and 999B). Only firm commitment offerings and nonshelf-registered offerings are included. The numbers plotted are reported in Table 1 for issues from 1990 to 1994.

bond is $100 million. For both IPOs and SEOs, substantial economies of scale exist in both the gross spreads and the other expenses.

For SEOs, the lack of any diseconomies, even for offerings over $500 million, is inconsistent with the findings of Hansen and Torregrosa (1992), who report diseconomies of scale for offers over $100 million. Hansen and Torregrosa use a sample of SEOs from 1978–86, in contrast to our 1990–94 sample period. Our conjecture is that while diseconomies of scale may have existed for very large issues before the mid 1980s, a structural change has probably occurred since then, possibly because of the market's greater experience with absorbing large numbers of big offerings. While they are not in our sample, the large number of multibillion dollar privatizations that have occurred around the world in the last decade have made megaofferings routine events.

In all of our tables, we report the averages based upon the number of observations for which we have data. For the gross spreads, SDC reports numbers for our entire sample. For the other direct expenses, however, many observations are missing. Consequently, the averages for the expenses are based upon a

TABLE 2. Direct Costs of Raising Capital, 1990–94: Utility versus Nonutility Companies.

	Equity						Bonds					
	IPOs			SEOs			Convertible			Straight		
Proceeds[a] ($ millions)	N[b]	GS[c]	TDC[d]	N	GS	TDC	N	GS	TDC	N	GS	TDC
Panel A. Nonutility Offerings Only												
2–9.99	332	9.04	16.97	154	7.91	13.76	4	6.07	8.75	29	2.07	4.53
10–19.99	388	7.24	11.64	278	6.42	9.01	12	5.54	8.65	47	1.70	3.28
20–39.99	528	7.01	9.70	399	5.70	7.07	16	4.20	6.23	63	1.59	2.52
40–59.99	214	6.96	8.71	240	5.17	6.02	28	3.26	4.30	76	0.73	1.37
60–79.99	78	6.74	8.21	131	4.68	5.31	47	2.64	3.23	84	1.84	2.44
80–99.99	47	6.46	7.88	60	4.35	4.84	12	2.54	3.19	104	1.61	2.25
100–199.99	101	6.01	7.01	137	3.97	4.36	55	2.34	2.77	381	1.83	2.38
200–499.99	44	5.65	6.49	50	3.27	3.48	26	1.97	2.16	154	1.87	2.27
500–up	10	5.21	5.72	8	3.12	3.25	3	2.00	2.09	19	1.28	1.53
Total	1742	7.31	11.01	1457	5.57	7.32	203	2.90	3.75	957	1.70	2.34
Panel B. Utility Offerings Only												
2–9.99	5	9.40	16.54	13	5.41	7.68	0	—	—	3	2.00	3.28
10–19.99	1	7.00	8.77	32	4.59	6.21	2	5.13	8.72	31	0.86	1.35
20–39.99	5	7.00	9.86	26	4.17	4.96	2	3.88	5.18	26	1.40	2.06
40–59.99	1	6.98	11.55	21	3.69	4.12	0	—	—	14	0.63	1.10
60–79.99	1	6.50	7.55	12	3.39	3.72	0	—	—	8	0.87	1.13
80–99.99	4	6.57	8.24	11	3.68	4.11	1	1.13	1.34	8	0.71	0.98
100–199.99	5	6.45	7.96	15	2.83	2.98	2	2.50	2.74	28	1.06	1.42
200–499.99	3	5.88	7.00	5	3.19	3.48	1	2.50	2.65	16	1.00	1.40
500–up	0	—	—	1	2.25	2.31	0	—	—	1	3.50	na[e]
Total	25	7.15	10.14	136	4.01	4.92	8	3.33	4.66	135	1.04	1.47

Notes: Closed-end funds (SIC 6726), REITs (SIC 6798), ADRs, and unit offerings are excluded from the sample. Rights offerings for SEOs are also excluded. Bond offerings do not include securities backed by mortgages and issues by Federal agencies (SIC 6011, 6019, 6111, and 999B). Only firm commitment offerings and nonshelf-registered offerings are included. Standard Industrial Classification (SIC) codes are from Securities Data Co. (SDC).

[a]Total proceeds raised in the United States, excluding proceeds from the exercise of overallotment options (SDC variable: PROCDS).
[b]Number of issues.
[c]Gross spreads as a percentage of total proceeds (including management fee, underwriting fee, and selling concession) (SDC variable: GPCTP).
[d]Other direct expenses as a percentage of total proceeds (including registration fee and printing, legal, and auditing costs) (SDC variables: EXPTH/(PROCDS)*10).
[e]Not available because of missing data on other direct expenses.

more limited number of observations.[4] For computing the average total direct costs in Table 1 (and other tables), we add the average gross spread and the average other expenses. In Figure I we show the average total direct costs for the four classes of securities, categorized by their gross proceeds.

The Appendix table reports the interquartile ranges for both the gross spreads and the total direct costs. (We report the interquartile range of the offerings for which we have complete data.) The largest variability of spreads occurs for bonds. As we document below, this can largely be explained based on differences in the credit quality of the issues.

Utility versus Nonutility Offerings

In Table 2 we report the direct costs of raising capital after categorizing offerings into utility and nonutility offerings. During the early 1990s, utilities were relatively minor issuers, representing roughly 10 percent of SEOs and straight bond offerings, and less than 5 percent of IPOs and convertibles. Spreads and direct costs are lower for utilities than for nonutilities. This pattern, previously documented by Bhagat and Frost (1986), may be partly due to the use of competitive bidding, rather than negotiated deals, for choosing an investment banker. Alternatively, it may be partly due to the relative noncomplexity of typical utility offerings.

Debt Offerings and Credit Quality

In Table 3 we report the costs of raising debt capital after categorizing issues by whether they are investment grade or noninvestment grade.[5] Following industry practice, we classify offerings as investment grade issues if they have a Standard & Poor's credit rating of BBB- or higher.[6]

Inspection of Table 3 discloses that for both convertibles and straight bonds, spreads are lower for investment-grade issues. For straight bonds, this difference is especially pronounced. Note that for issues raising less than $60

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[4]If the offerings with missing expense information have systematically higher or lower expenses than those for which SDC reports information, our procedure would result in biased estimates of average expenses. To check this, for a sample of bond offerings in 1994 that are missing expense information, we used the Securities and Exchange Commission's Edgar electronic database (http://www.sec.gov/cgi-bin/srch-edgar) to find the expense information. The expenses for these issues are representative of those for which SDC reports information, suggesting our numbers do not have important biases.

[5]Following the practice of SDC, we report as separate offerings two bond issues by the same company on the same day if they have different maturity dates, provided they are not explicitly serial bonds. For example, on September 22, 1994, Southern Pacific Transport issued two bonds, one with proceeds of $8.1 million with a coupon rate of 7.61 percent, and the other with proceeds of $8.8 million and a coupon rate of 7.77 percent. We treat these as two distinct offerings.

[6]The highest credit rating is AAA, followed by AA, A, BBB, BB, B, C, and D, in order of their perceived default probabilities. These ratings are further partitioned by pluses and minuses.

TABLE 3. Average Gross Spreads and Total Direct Costs for Domestic Debt Issues, 1990–94.

	Convertible Bonds						Straight Bonds					
	Investment Grade[a]			Noninvestment Grade[b]			Investment Grade			Noninvestment Grade		
Proceeds[c] ($ millions)	N[d]	GS[e]	TDC[f]	N	GS	TDC	N	GS	TDC	N	GS	TDC
2–9.99	0	—	—	0	—	—	14	0.58	2.19	0	—	—
10–19.99	0	—	—	1	4.00	5.67	56	0.50	1.19	2	5.13	7.41
20–39.99	1	1.75	2.75	9	3.29	4.92	64	0.86	1.48	9	3.11	4.42
40–59.99	3	1.92	2.43	19	3.37	4.58	78	0.47	0.94	9	2.48	3.35
60–79.99	4	1.31	1.76	41	2.76	3.37	49	0.61	0.98	43	3.07	3.84
80–99.99	2	1.07	1.34	10	2.83	3.48	65	0.66	0.94	47	2.78	3.75
100–199.99	20	2.03	2.33	37	2.51	3.00	181	0.57	0.81	222	2.75	3.44
200–499.99	17	1.71	1.87	10	2.46	2.70	60	0.50	0.93	105	2.56	2.96
500–up	3	2.00	2.09	0	—	—	11	0.39	0.57	9	2.60	2.90
Total	50	1.81	2.09	127	2.81	3.53	578	0.58	0.94	446	2.75	3.42

Notes: Closed-end funds (SIC 6726), REITs (SIC 6798), ADRs, and unit offerings are excluded from the sample. Bond offerings do not include securities backed by mortgages and issues by Federal agencies (SIC 6011, 6019, 6111, and 999B). Only nonshelf-registered offerings are included. Standard Industrial Classification (SIC) codes are from Securities Data Co. (SDC).

[a]Firms with a BBB- or higher Standard & Poor's credit rating.
[b]Firms with a BB+ or lower Standard & Poor's credit rating.
[c]Total proceeds raised in the United States, excluding proceeds from the exercise of overallotment options (SDC variable: PROCDS).
[d]Number of issues.
[e]Gross spreads as a percentage of total proceeds (including management fee, underwriting fee, and selling concession) (SDC variable: GPCTP).
[f]Other direct expenses as a percentage of total proceeds (including registration fee and printing, legal, and auditing costs) (SDC variables: EXPTH/(PROCDS)*10).

million, very few noninvestment-grade issues exist. This reflects that smaller issues with lower credit quality are commonly placed privately, and thus do not appear in our sample.

This correlation of credit quality and issue size also explains why in Tables 1 and 2 straight bond issues do not appear to display large economies of scale: as the issue size increases, the credit quality of public issuers decreases, masking some of the economies of scale. Still, in Table 3, where we hold credit quality constant, the economies of scale for debt issues are more modest than those for equity issues in Tables 1 and 2. The correlation between issue size and credit quality also explains why the average spread is so low for bonds with $40-$59.9 million in proceeds. The average spread of only seventy-two basis points in Table 1 reflects that for this issue size, economies of scale are largely realized, while, at the same time, very few noninvestment-grade issuers exist. For smaller offerings, the lack of economies of scale keeps the average spread high. For larger offerings, the high proportion of noninvestment-grade issues pushes

TABLE 4. Direct and Indirect Costs, in Percent, of Equity IPOs, 1990–94.

Proceeds[a] ($ millions)	Gross Spreads[b]	Other Expenses[c]	Total Direct Costs[d]	Average Initial Return[e]	Average Direct and Indirect Costs[f]
2–9.99	9.05	7.91	16.96	16.36	25.16
10–19.99	7.24	4.39	11.63	9.65	18.15
20–39.99	7.01	2.69	9.70	12.48	18.18
40–59.99	6.96	1.76	8.72	13.65	17.95
60–79.99	6.74	1.46	8.20	11.31	16.35
80–99.99	6.47	1.44	7.91	8.91	14.14
100–199.99	6.03	1.03	7.06	7.16	12.78
200–499.99	5.67	0.86	6.53	5.70	11.10
500–up	5.21	0.51	5.72	7.53	10.36
Total	7.31	3.69	11.00	12.05	18.69

Notes: There are 1,767 domestic operating company IPOs in the sample. The first four columns express costs as a percentage of the offer price, and the last column expresses costs as a percentage of the market price.

[a]Total proceeds raised in the United States, excluding proceeds from the exercise of overallotment options (SDC variable: PROCDS).
[b]Gross spreads as a percentage of total proceeds (including management fee, underwriting fee, and selling concession) (SDC variable: GPCTP).
[c]Other direct expenses as a percentage of total proceeds (including registration fee and printing, legal, and auditing costs) (SDC variables: EXPTH/(PROCDS)*10).
[d]Total direct costs as a percentage of total proceeds (the average total direct costs are the sum of average gross spreads and average other direct expenses).
[e]Initial return = 100*{[closing price one day after the offering date (SDC variable: PR1DAY)/offering price (SDC variable: P)] − 1}. If PR1DAY is missing, PR2DAY is used.
[f]Total direct and indirect costs = (d + e)/(1 + e/100), computed for each issue individually (excluding firms with other expenses or initial returns missing), and then averaged, where *d* is the percentage of total direct costs, and *e* is the percentage initial return.

the average spread up. In other words, the average spread of only seventy-two basis points for this category is not a typographical error.

Although not reported in any table, the average maturity of bond offerings is about ten years for all of the proceeds categories and investment grades.

Initial Public Offerings

In Table 4 we report not only the direct costs for IPOs, but also the indirect costs of short-run underpricing.[7] Inspection of the table reveals that, consistent with previous findings, IPOs are underpriced on average. With average direct costs of 11.0 percent and average initial returns of 12.0 percent, a typical

[7]We compute the average initial return only for those offerings for which SDC reports the market price at the end of the first day of trading or, if this is missing, at the end of the second day of trading. In computing the average direct and indirect cost, we compute this number for each individual firm for which we have the gross spread, other expenses, and the initial return, and then compute the average.

issuer with an offer price of $10.00 receives net proceeds of $8.90 on a share that trades at $11.20. Taking the difference between the market price and the amount realized of $8.90, the total direct and indirect costs amount to $2.30, which is 20.5 percent of the market value of $11.20. In Table 4 the average direct and indirect cost as a percentage of market value is 18.7 percent, since the average that is reported is the average of this percentage for each firm. (The average ratio of costs to market value is different from the ratio of the averages.) This number is less than the 21.2 percent that Ritter (1987) reports for firm commitment offerings from 1977 to 1982 for several reasons. First, our 1990–94 sample period reveals less underpricing than in 1977-1982. Second, we exclude offerings of less than $2 million, whereas he includes them. Third, spreads have experienced some downward movement the past fifteen years.[8] Still, the direct and indirect costs of going public are substantial.[9]

Note that we may be understating the extent of the economies of scale. This is because we are not including the value of any warrants granted to underwriters as part of their compensation. These warrants are common among small, speculative offerings underwritten by less-prestigious underwriters. Their inclusion would boost the average costs of the smallest offerings, but not the larger offerings. For evidence on the quantitative effect of this omission, see Barry, Muscarella, and Vetsuypens (1991) and Dunbar (1995).

While the average gross spread on IPOs is 7.31 percent, we find a large "bunching" at exactly 7.00 percent. Most issues with proceeds of $20–$60 million have a spread of exactly 7 percent, as shown in the Appendix table.

For IPOs, we include the indirect cost of underpricing in Table 4, but we do not include this as a cost for other security offerings. This is because of the lack of economically important underpricing effects for other offerings. Smith (1977) documents underpricing of 0.5 percent for SEOs. We suspect that much of this represents the practice of pricing the offering at the bid price, rather than the mean of the bid and the ask price, and the tendency to round down to the nearest eighth or integer. For example, if a stock traded at $30.125 bid and $30.375 ask, it would be common to set a $30.00 offer price. Depending upon which price had been the most recent transaction price, this would be measured as underpricing of either 0.4 percent or 1.2 percent. Barclay and Litzenberger (1988) report excess returns of 1.5 percent for SEOs during the month after issuing. Since companies typically issue after a large stock price run-up, it is not clear how much of this 1.5 percent is due to momentum effects, and how

[8]Calomiris and Raff (1995) report that for convertible bonds, the average spread in 1963–65 was 3.7 percent and in 1971–72 it was 3.2 percent. Our 1990–94 sample has an average spread of 2.9 percent.

[9]Beatty and Welch (1996) report the average direct and indirect costs for a sample of 980 IPOs from 1992 to 1994. Whereas we aggregate auditing, legal, printing, and other direct expenses, they report audit expenses and legal expenses separately. For all proceeds classes, legal expenses are slightly higher than auditor expenses.

TABLE 5. Number of Issues Containing an International Tranche for Domestic Operating Companies That Are Issuing, 1990–94.

	Equity				Bonds			
	IPOs Int'l Tranche?[a]		SEOs Int'l Tranche?		Convertible Int'l Tranche?		Straight Int'l Tranche?	
Proceeds ($ millions)	Yes	No	Yes	No	Yes	No	Yes	No
2–9.99	2	335	4	163	0	4	1	31
10–19.99	12	377	12	298	1	13	0	78
20–39.99	45	488	36	389	3	15	0	89
40–59.99	40	175	42	219	0	28	4	86
60–79.99	33	46	45	98	1	46	8	84
80–99.99	25	26	30	41	9	4	2	110
100–199.99	81	25	72	80	22	35	14	395
200–499.99	39	8	48	7	14	13	13	157
500–up	10	0	8	1	2	1	2	18
Total	287	1480	297	1296	52	159	44	1048

Notes: Closed-end funds (SIC 6726), REITs (SIC 6798), ADRs, and unit offerings are excluded from the sample. Rights offerings for SEOs are also excluded. Bond offerings do not include securities backed by mortgages and issues by Federal agencies (SIC 6011, 6019, 6111, and 999B). Only firm commitment offerings and nonshelf-registered offerings are included. Standard Industrial Classification (SIC) codes are from Securities Data Co. (SDC).

[a]If (TOTDOLAMT/PROCDS) > 1.05, the issue is treated as having an international tranche. TOTDOLAMT is the total proceeds raised globally, and PROCDS is the total proceeds raised in the United States.

much is due to issue effects. Kang and Lee (1996) document that convertible bonds are underpriced by about 1 percent on average. Straight bonds, especially those with high credit ratings, seem to be underpriced very little.

International Tranches

In Table 5 we report the frequency with which domestic operating companies include an international tranche in their offerings. Recall that we are excluding Eurobonds from our debt offerings and ADRs from our equity offerings. Inspection of the table reveals that equity offerings and convertibles that raise less than $60 million in domestic trading rarely include an international tranche. Straight debt offerings, no matter what their size, rarely include an international tranche. Now, foreign investors can always participate in a domestic offering regardless of whether it is explicitly marketed overseas. Thus, the existence/nonexistence of an international tranche largely reflects the degree to which the selling efforts are expanded to find international buyers. Domestic operating companies issuing debt with foreign buyers in mind frequently issue Eurobonds.[10]

Overallotment Options

The Rules of Fair Practice of the National Association of Security Dealers (NASD) permit firm commitment offerings to include an overallotment option, where more securities can be sold if demand is strong.[11] Since August 1983, the size of this overallotment option has been limited to 15 percent of the issue size. Investment bankers typically have thirty days to exercise this option. In practice, investment bankers typically presell at least 115 percent of the offering, and then stand ready to buy back the incremental 15 percent if demand is weak when some of the buyers immediately sell their securities (a practice known as "flipping").[12]

The NASD Rules of Fair Practice require that investment bankers sell securities at or below the stated offer price. Normally, all of the securities are sold at the offer price, but occasionally, if demand is weak, the investment banker winds up selling some of the securities below the offer price. In this arrangement the underwriter writes a put option to the issuing firm, with the value of this put included in the gross spread. The overallotment option can be viewed as a call option that the issuing firm has written, where investors hold this call.

On securities sold through the exercise of overallotment options, investment bankers collect the same gross spread as on the rest of the issue. However, since the direct expenses do not change, these fixed costs are spread over a larger issue size. Thus, the total direct cost numbers that we report would be lower if overallotment options were included in the gross proceeds. On the other hand, since overallotment options are generally exercised only if the issue is underpriced, the value of this call option is a cost to the issuing firm that we do not include in our total cost calculations.

In Table 6 we report the frequency with which overallotment options are used and the frequency with which they are exercised. Inspection of the table reveals that in recent years, essentially all IPOs have included an overallotment option. The vast majority of SEOs and convertibles include an overallotment option, but straight bond issues rarely do.

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[10]The relative yields on Eurobonds versus domestic bonds also play a role in the decision of what to issue (see Kim and Stulz (1988)).

[11]Overallotment options are sometimes called Green Shoe options. The Green Shoe Company was apparently the first company to use one.

[12]See Schultz and Zaman (1994) for evidence on the exercise of overallotment options on IPOs. With IPOs, if the underwriter expects aftermarket demand to be weak, 135 percent of the issue may be presold, with the underwriter's taking a naked short position equal to the amount exceeding 115 percent of the offering. This allows the underwriter to support, or stabilize, the price by buying back the increment in open market purchases. These shares are then treated as if they were never issued. If the underwriter expects the price to jump, typically only 115 percent of the issue size will be presold, to avoid losing money on a naked short position.

TABLE 6. Number of Issues Containing an Overallotment Option, for Domestic Operating Companies That Are Issuing, 1990–94.

	Equity								Bonds							
	IPOs Overallotment Option?				SEOs Overallement Option?				Convertible Overallotment Option?				Straight Overallotment Option?			
	Yes			No[d]	Yes			No	Yes			No	Yes			No
	Sold?				Sold?				Sold?				Sold?			
Proceeds ($ millions)	Yes[a]	No[b]	?[c]		Yes	No	?		Yes	No	?		Yes	No	?	
2–9.99	159	115	51	12	100	41	21	5	0	0	4	0	1	0	4	27
10–19.99	198	151	40	0	209	58	38	5	1	2	8	3	2	1	4	71
20–39.99	306	164	60	3	269	100	49	7	4	2	8	4	6	0	9	74
40–59.99	123	67	25	0	173	50	33	5	6	6	13	3	1	0	1	88
60–79.99	45	27	7	0	81	37	21	4	21	6	16	4	3	0	0	89
80–99.99	25	17	9	0	44	9	15	53	10	0	3	0	0	1	1	10
100–199.99	54	34	16	2	96	24	28	4	23	2	28	4	4	1	3	401
200–499.99	21	17	8	1	35	4	14	2	7	2	15	3	3	1	1	165
500–up	6	0	3	1	6	2	1	0	0	0	3	0	0	0	1	19
Total	937	592	219	19	1013	325	220	35	72	20	98	21	20	4	24	1044

Notes: Closed-end funds (SIC 6726), REITs (SIC 6798), ADRs, and unit offerings are excluded from the sample. Rights offerings for SEOs are also excluded. Bond offerings do not include securities backed by mortgages and issues by Federal agencies (SIC 6011, 6019, 6111, and 999B). Only firm commitment offerings and nonshelf-registered offerings are included. Standard Industrial Classification (SIC) codes are from Securities Data Co. (SDC).

[a]If OVERAMT > 0 and OVERC = Yes, where OVERAMT is the amount that can be raised through the overallotment option and OVERC is "Yes" if any overallotment option is exercised.
[b]If OVERAMT > 0 and OVERC = No.
[c]If OVERAMT > 0 and OVERC = Missing.
[d]If OVERAMT = "–"; this may include offerings with missing data on OVERAMT.

APPENDIX. Interquartile Range of Direct Costs as a Percentage of Gross Proceeds for Equity (IPOs and SEOs) and Straight and Convertible Bonds Offered by Domestic Operating Companies, 1990–94.

Proceeds[a] ($ millions)	Equity				Bonds			
	IPOs		SEOs		Convertible Bonds		Straight Bonds	
	GS[b]	TDC[c]	GS	TDC	GS	TDC	GS	TDC
2–9.99	8.00–10.00	14.34–19.23	6.50–10.00	10.03–16.16	5.45–6.69	7.38–10.04	0.64–3.38	3.47–6.21
10–19.99	7.00–7.14	9.94–12.44	5.74–6.94	7.42–9.63	4.25–6.00	6.65–9.70	0.35–2.90	1.55–5.68
20–39.99	7.00–7.00	8.82–10.09	5.22–6.00	6.19–7.57	3.00–5.00	4.56–6.50	0.57–3.00	1.10–4.55
40–59.99	7.00–7.00	8.23–9.00	4.73–5.48	5.26–6.31	2.88–3.50	3.63–4.65	0.15–0.71	0.91–2.88
60–79.99	6.55–7.00	7.69–8.51	4.24–5.00	4.51–5.70	2.50–3.00	2.83–3.54	0.65–3.00	0.94–3.64
80–99.99	6.21–6.85	7.26–8.44	3.87–4.75	4.22–5.38	2.25–3.00	2.56–3.66	0.63–2.76	0.94–3.70
100–199.99	5.72–6.47	6.43–7.49	3.15–4.47	3.38–4.89	2.15–2.75	2.36–3.19	0.65–2.75	1.01–3.55
200–499.99	5.29–5.86	5.92–6.78	2.79–3.58	2.92–3.79	1.25–2.50	1.40–2.69	0.65–2.63	1.43–3.16
500–up	5.00–5.37	5.33–5.95	2.75–3.00	2.82–3.17	1.00–2.50	1.11–2.60	0.29–2.75	1.05–3.18
Total	7.00–7.05	8.57–12.04	4.51–6.08	5.12–8.20	2.25–3.00	2.66–3.96	0.60–2.75	1.02–3.60

Notes: Closed-end funds (SIC 6726), REITs (SIC 6798), ADRs, and unit offerings are excluded from the sample. Rights offerings for SEOs are also excluded. Bond offerings do not include securities backed by mortgages and issues by Federal agencies (SIC 6011, 6019, 6111, and 999B). Only firm commitment offerings and nonshelf-registered offerings are included. Standard Industrial Classification (SIC) codes are from Securities Data Co. (SDC).

[a]Total proceeds raised in the United States, excluding proceeds from the exercise of overallotment options (SDC variable: PROCDS).
[b]Gross spreads as a percentage of total proceeds (including management fee, underwriting fee, and selling concession) (SDC variable: GPCTP).
[c]Total direct costs as a percentage of total proceeds (total direct costs are the sum of gross spreads and other direct expenses).

The frequency with which overallotment options are exercised varies across security type. In Table 6 we use the SDC classification where an overallotment option is considered to be exercised as long as at least part of it is exercised. In practice, most overallotment options are for 15 percent of the issue size. Most commonly, either all or none of the additional shares are sold, but sometimes only part of the overallotment option is exercised. On securities sold as part of an overallotment option, the spread is the same as on the rest of the issue.

IV. Conclusions

Firms have many choices for financing their activities: internal versus external, private versus public, and debt versus equity. This article focuses on public external financing and documents the cost of this financing from 1990 to 1994. We report the direct costs of raising capital for IPOs, SEOs, convertible bonds, and straight bonds. These are, respectively, 11.0 percent, 7.1 percent, 3.8 percent, and 2.2 percent of the proceeds. We find substantial economies of scale for all types of securities, although for straight bond offerings, these are largely exhausted for proceeds over $40 million. Spreads on bonds are sensitive to credit quality, with gross spreads more than 200 basis points higher on noninvestment-grade issues. Except for bonds, most large issues include an international tranche.

References

Barclay, M. J. and R. H. Litzenberger, 1988, Announcement effects of new equity issues and the use of intraday price data, *Journal of Financial Economics* 21, 71–99.

Barry, C., C. J. Muscarella, and M. R. Vetsuypens, 1991, Underwriter warrants, underwriter compensation, and the costs of going public, *Journal of Financial Economics* 29, 113–35.

Beatty, R P. and I. Welch, 1996, Issuer expenses and legal liability in initial public offerings, *Journal of Law and Economics*, Forthcoming.

Bhagat, S. and P. A. Frost, 1986, Issuing costs to existing shareholders in competitive and negotiated underwritten public utility equity offerings, *Journal of Financial Economics* 15, 233–59.

Calomiris, C. W. and D. M. G. Raff, 1995, The evolution of market structure, information, and spreads in American investment banking, in M. B. Bordo and R. Sylla, eds., *Anglo-American Finance: Financial Markets and Institutions in 20th Century North America and the U.K.* (Business One-Irwin, Homewood, IL), 103–60.

Dunbar, C. G., 1995, The use of warrants as underwriter compensation in initial public offerings, *Journal of Financial Economics* 38, 59–78.

Hansen, R. S., 1988, The demise of the rights issue, *Review of Financial Studies* 1, 289–309.

Hansen, R. S. and J. Pinkerton, 1982, Direct equity financing: A resolution of a paradox, *Journal of Finance* 37, 651–65.

Hansen, R. S. and P. Torregrosa, 1992, Underwriter compensation and corporate monitoring, *Journal of Finance* 47, 1537–55.

Kang, J. and Y. Lee, 1996, The pricing of convertible debt offerings, *Journal of Financial Economics*, Forthcoming.

Kim, Y. C. and R. M, Stulz, 1988, The Eurobond market and corporate financial policy: A test of the clientele hypothesis, *Journal of Financial Economics* 22, 189–205.

Ritter, J. R., 1987, The costs of going public, *Journal of Financial Economics* 19, 269–81.

Schultz, P. H. and M. A. Zaman, 1994, Aftermarket support and the pricing of initial public offerings, *Journal of Financial Economics* 35, 199–219.

Smith, C. W., 1977, Alternative methods for raising capital: Rights versus underwritten offerings, *Journal of Financial Economics* 5, 273–307.

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ALTERNATIVE METHODS FOR RAISING CAPITAL

Rights Versus Underwritten Offerings

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This paper provides an analysis of the choice of method for raising additional equity capital by listed firms Examination of expenses reported to the SEC indicates that rights offerings involve significantly lower costs, yet underwriters are employed in over 90 percent of the offerings The underwriting industry, finance textbooks, and corporate proxy statements offer several justifications for the use of underwriters However estimates of the magnitudes of these arguments indicate that they are insufficient to justify the additional costs of the use of underwriters The use of underwriters thus appears to be inconsistent with rational, wealth-maximizing behavior by the owners of the firm The paper concludes with an examination of alternate explanations of the observed choice of financing method

1. Introduction and summary

In this paper I examine an apparent paradox Based on a comparison of costs, simple finance theory suggests that listed firms should use rights offerings to raise additional equity capital, rather than employing underwriters Yet the majority of firms choose underwritten offerings, rather than rights offerings

In an underwritten offering, underwriters contract to purchase shares from the issuing firm at a price usually set within 24 hours of the offering, and then resell the shares to the public In a rights offering the shareholder receives a right from the firm giving him the option to purchase new shares for each share owned In section 2, I show that with the proper specification of the subscription price, the proceeds of a rights offering are identical to the proceeds of an underwritten offering

Not identical, however, are costs In section 3, I examine the out-of-pocket costs of underwritten and rights offerings reported to the Securities and Exchange

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Commission for issues registered under the Securities Act of 1933 between January 1971 and December 1975. Rights offerings are significantly less expensive. I also examine additional out-of-pocket expenses associated with both types of offerings. These include extras (options sold to underwriters), unreported expenses such as employee compensation, and the costs of rights offerings imposed directly on the owners of the firm. With these costs considered, I find rights offerings still are less expensive than underwritten offerings.

It has been suggested that selling efforts by underwriters raise stock prices while rights offerings lower them. In section 4 I study price behavior around the date of the offering. I find no empirical support for the hypothesis that abnormal positive returns are associated with underwritten offerings. Moreover, underwriters appear to set the offer price below the market value of the stock by at least 0.5 percent. While stock prices fall when rights are issued, the fall equals the market value of the rights received by the shareholder. Examination of the total rate of return to shareholders around the offer date indicates no abnormal returns; thus the wealth of the firm's owners is not reduced by a rights offering.

Section 5 provides an examination of other benefits presumed to accrue from the use of underwriters. Finance texts, corporate proxy statements, and the underwriting industry itself claim the existence of advantages in timing, insurance, distribution of ownership and from future consulting advice. My estimates of the magnitudes of the costs and benefits associated with these arguments are not sufficient to outweigh the lower costs of rights offerings as a means of raising capital. I can find no differential legal liability associated with the use of rights offerings which might explain the observed use of underwriters. Furthermore, there is no apparent difference in the sets of firms employing the alternative methods which could attribute the reported cost differences to selection bias.

In section 6, I offer a two-part hypothesis which is consistent with the observed frequency of employment of underwriters, with their higher costs, by the majority of listed firms. First, since managers' and directors' interests are different from those of shareholders in general, their financing decisions are not always in the best interests of the owners; benefits flow to management from the use of underwriters although not to shareholders. Second, I hypothesize that the cost to shareholders of monitoring their directors and managers is greater than the cost imposed by the choice of the more expensive financing method.

In section 7 I briefly present my conclusions.

A detailed description of the institutional arrangements for rights offerings and underwritten offerings is not easily available; I have provided one in Appendix 1. The reader unfamiliar with this institutional material will find it valuable to read this appendix before the body of the paper.

Appendix 2 presents a Black–Scholes (1973) option pricing analysis of rights issues and underwriting contracts, given here since general equilibrium analyses of these contracts have not been published.

2. Comparison of proceeds from rights and underwritten offerings

In a firm commitment underwritten offering, the underwriting syndicate purchases the new shares from the firm at an agreed upon price, and offers the shares for sale to the public at the offer price If the shares cannot be sold at the offer price, the underwriting syndicate breaks and the shares are sold for whatever price they will bring The underwriters bear the risk associated with adverse price movements, the proceeds to the firm are guaranteed Of course the difference between the offer price and the proceeds to the firm are expected to compensate the underwriter for bearing this risk

In a rights offering, each shareholder receives one right for each share owned This right is an option issued by the firm to purchase new shares The right states the relevant terms of the option, specifying the number of rights required to purchase each new share, the subscription price for each new share, and the expiration date of the option Since issuing rights is costly, it is in the firm's interest to insure the success of the offering A lower subscription price for the rights provides this insurance, a lower subscription price raises the market value of the right and reduces the probability that at the expiration date of the rights offering the stock price will be below the subscription price There is a corresponding fall in the market value of the stock, but this fall is like a stock split It does not affect the wealth of the owners of the firm [1]

If the shareholder does not exercise his rights, or does not sell his rights to someone who will exercise the rights, his wealth is reduced by the market value of the rights Thus the firm can make the probability of failure of the rights offering arbitrarily small by setting the subscription price low enough

Thus, since rights offerings and underwritten offerings can be specified so that the amount of capital raised by each is essentially equivalent, the decision as to which method to employ depends on the costs, the firm should employ that method which has lower net costs

3. Out-of-pocket expenses of rights and underwritten issues

"Expenses involved in a preemptive common stock rights offering are significantly greater than expenses involved in a direct offering of common stock

[1]The adjustment for the 'split effect' of a rights offering can be calculated as follows The ex-rights price of the shares, P_x, equals the with-rights price, P_w, minus the value of the right, R

$$P_x = P_w - R.$$

Ignoring the 'option value' of the right, the market value of a right is the difference between the ex-rights price and the subscription price, P_s, divided by the number of rights required to purchase one share, n

$$R = (P_x - P_s)/n$$

Substituting the second expression into the first and simplifying yields

$$P_x = (nP_w + P_s)/(n+1)$$

to the public due to additional printing and mailing costs, expenses associated with the handling of rights and the processing of subscriptions, higher underwriters' commissions and the longer time required for the consummation of financing."[2]

3.1. Reported out-of-pocket expenses

To examine the out-of-pocket expenses referred to in the quotation above (from Commonwealth Edison's 1976 proxy statement) I obtained a tape from the Securities and Exchange Commission covering the reported costs of all issues registered under the Securities Act of 1933 between January, 1971 and December, 1975. The tape contains data covering the following costs: (1) compensation received by investment bankers for underwriting services, (2) legal fees, (3) accounting fees, (4) engineering fees, (5) trustee's fees, (6) listing fees, (7) printing and engraving expenses, (8) Securities and Exchange Commission registration fees, (9) Federal Revenue Stamps, and (10) state taxes.

To restrict my analysis to equity issues by listed firms, I established the following criteria for inclusion: (1) the offering is of common stock and contains no other classes of securities; (2) the company's stock is listed on the New York Stock Exchange, American Stock Exchange, or a regional stock exchange prior to the offering; and (3) any associated secondary distribution is less than 10 percent of the gross proceeds of the issue. Table 1 is based on the issues meeting these criteria.

The data summarized in table 1 contradict Commonwealth Edison's Proxy Statement. My information, consistent with findings of previous SEC studies,[3] indicates that costs are *highest* for underwritten public offerings, and *lowest* for pure rights offerings. Furthermore, the difference in costs is striking. For a $15 million issue, the reported cost difference between an underwritten public offering and a pure rights offering is 4.83 percent, or $720,000; and for a $100 million issue the cost difference is 3.82 percent, or $3,820,000.[4] Yet underwriters were employed in over 93 percent of the issues examined.

3.2. Extras

Systematic understatement of the costs of underwriting presented in table 1 occurs because extras are omitted. Extras refer to the warrants which are associated with some underwritten issues and are used as partial payment to the underwriter. The warrants are options which are usually convertible into the

[2]Commonwealth Edison Proxy Statement, 1976.

[3]See SEC (1940, 1941, 1944, 1949, 1951, 1957, 1970, 1974).

[4]One empirical regularity in the data presented in table 1 should be noted. To a first approximation, the differences in costs among financing methods are explained by the differences in underwriter compensation. Compare 'Other Expenses' for Underwriting and Rights with Standby Underwriting with 'Total Costs' for Rights.

Table I

Costs of flotation as a percentage of proceeds for 578 common stock issues registered under the Securities Act of 1933 during 1971–1975. The issues are subdivided by size of issue and method of financing: underwriting, rights with standby underwriting, and pure rights offering[a]

Size of issue ($ millions)	Underwriting: Number	Underwriting: Compensation as a percent of proceeds	Underwriting: Other expenses as a percent of proceeds	Underwriting: Total cost as a percent of proceeds	Rights with standby underwriting: Number	Rights with standby underwriting: Compensation as a percent of proceeds	Rights with standby underwriting: Other expenses as a percent of proceeds	Rights with standby underwriting: Total cost as a percent of proceeds	Rights: Number	Rights: Total cost as a percent of proceeds
Under 0.50	0	–	–	–	0	–	–	–	3	8.99
0.50 to 0.99	6	6.96	6.78	13.74	2	3.43	4.80	8.24	2	4.59
1.00 to 1.99	18	10.40	4.89	15.29	5	6.36	4.15	10.51	5	4.90
2.00 to 4.99	61	6.59	2.87	9.47	9	5.20	2.85	8.06	7	2.85
5.00 to 9.99	66	5.50	1.53	7.03	4	3.92	2.18	6.10	6	1.39
10.00 to 19.99	91	4.84	0.71	5.55	10	4.14	1.21	5.35	3	0.72
20.00 to 49.99	156	4.30	0.37	4.67	12	3.84	0.90	4.74	1	0.52
50.00 to 99.99	70	3.97	0.21	4.18	9	3.96	0.74	4.70	2	0.21
100.00 to 500.00	16	3.81	0.14	3.95	5	3.50	0.50	4.00	9	0.13
Total/Average	484	5.02	1.15	6.17	56	4.32	1.73	6.05	38	2.45

[a]Issues are included only if the company's stock was listed on the NYSE, AMEX or regional exchanges prior to the offering, any associated secondary distribution represents less than ten percent of the total proceeds of the issue, and the offering contains no other types of securities. The costs reported are (1) compensation received by investment bankers for underwriting services rendered, (2) legal fees, (3) accounting fees, (4) engineering fees, (5) trustees' fees, (6) listing fees, (7) printing and engraving expenses, (8) Securities and Exchange Commission registration fees, (9) Federal Revenue Stamps, and (10) state taxes.

stock of the firm at prices ranging from well below to considerably above the offering price. When the underwriters acquire these warrants at a price below their market value, this represents a form of compensation to the underwriter, and it is not included in table 1.

Although extras have historically been most often associated with new issues, their use in the compensation of underwriters of seasoned firms is not unusual. For the years 1971–1972, the SEC (1974) reported that of the 1,599 issues which were underwritten, 530, or 33.1 percent, included extras. However, since extras were included primarily with the smaller offerings, the total dollar volume of issues with extra compensation was only 7 percent of the gross proceeds from all underwritten offerings.

The average exercise price of the warrants granted as a percentage of the offering price was 11.72 percent. A lower bound on the value of the option is the difference between the subscription price of the offering and the exercise price of the extras, here that is 88.28 percent of the subscription price.[5] Since these warrants are typically purchased by the managing investment banker at a minimal price, usually one to ten cents, the options appear to be significantly underpriced. The SEC also found that the average ratio of shares granted the underwriters through extras to the number of shares offered in the underwriting was 7.99 percent. To assess the impact on the figures reported in table 1, assume that the value of the warrant is 80 percent of the offering price, that the underwriter pays 5 percent of the offering price for the extras, and that the ratio of warrants received as extras to shares offered through the underwriting is 0.07, then the compensation represented by the extras would be 4.95 percent of the total proceeds. These numbers suggest that for the issues employing extras, the figures in table 1 understate the underwriters' compensation on the order of 50 to 100 percent.

3.3. *Unreported out-of-pocket expenses*

Such items as the opportunity cost of the time of the firm's employees and postage expenses[6] are not included in the summary of costs reported in table 1. However, unreported employee expenses are unlikely to explain the deviations reported in table 1. For a $15 million issue, the $720,000 difference would not be explained if 20 employees with an average salary of $30 thousand worked

[5] This is a conservative estimate of the value. Merton (1973) has demonstrated that the lower bound on the value of an option is the difference between the stock price and the discounted exercise price.

[6] Although postage expenses are not reported to the SEC, estimates were obtained from summaries of expenses reported to the New York State Public Utilities Commission for a sample of firms. For the sample, the maximum postage expense as a percentage of total proceeds was one-tenth of one percent. Even if this were understated by a factor of ten, it would be of insufficient magnitude to explain even the smallest reported difference in costs. Moreover, the marginal postage expense could be reduced to zero by mailing the rights with other required mailings, such as dividend checks or quarterly reports.

full time on a rights offering for a year For a $300 million issue the difference in reported costs of underwriting versus a rights issue exceeds $11 million, it would require over 350 man-years to explain this difference

It should be noted that expenses allocated to raising capital do *not* reduce the tax liability of the firm [7] These expenses are deducted from the capital account without affecting the income statement Thus, the use of internal resources can lower the tax liability of the firm if it is more expensive for the Internal Revenue Service to monitor the allocation of internal resources between capital raising activities and other activities In the above examples, if the firm's marginal tax rate is 50 percent, and if they were able to deduct all their wages for tax purposes, the required number of man-years to explain the reported cost differential would be doubled

There are strong reasons to believe that table 1 also omits significant unreported costs of the issuing firm's employees' time for underwritten offerings There are important parameters (e g , the offering price and the fee structure) which must be negotiated between the underwriter and the representatives of the firm, these parameters have wealth implications for the owners of the firm as well as the underwriter Such negotiation can be lengthy and usually directly involves top management These unreported costs of underwriting must be significantly greater than the costs of setting a subscription price for a rights issue, since the subscription price has no wealth implications for the owners of the firm as long as it is low enough to ensure that the rights will be exercised

Moreover, with an underwritten issue the firm has the same tax incentives to substitute internal for external resources if it is more expensive for the IRS to monitor the allocation of costs of internally acquired resources to capital raising activities than of those which are externally acquired Thus, it is not clear that rights offerings employ fewer unreported internal resources than do underwritten offerings

3 4 Costs imposed directly on shareholders

If a shareholder chooses to sell his rights, he incurs transactions costs and tax liabilities These costs, although not borne by the firm, are relevant because they affect the wealth of the owners [8]

[7]If the firm sells bonds rather than stock, the costs of selling the issue can be amortized over the life of the issue In no case, however, may these costs be expensed either for tax or reporting purposes

[8]There is a limited benefit from issuing rights to the owners of the firm under Regulation T, the Federal Reserve regulation restricting margin credit For an owner who wishes to borrow to acquire additional stock, Reg T provides for the establishment of a 'Special Subscription Account' which lowers the effective margin requirement by permitting a customer to purchase on an installment basis a margin security acquired through the exercise of subscription rights expiring within 90 days Under this provision, 75 percent of the market value of the acquired stock can be borrowed initially Quarterly installments are required over a 12 month period to bring the position up to proper margin

To determine the impact of the selling costs, let us assume generally extreme values for the relevant parameters For small dollar transactions (less than $1,000), the brokerage fee can be as much as 10 percent And for rights, the bid-ask spread can be as high as 10 percent, this represents another selling cost If half the bid-ask spread is taken as an implicit selling cost the total cost can be as much as 15 percent of the value of the rights To make the figures comparable to those in table 1, calculate transactions costs as a fraction of the proceeds of the offering to the firm The 15 percent must be multiplied by the ratio of the value of the rights to the total proceeds For the offerings in the sample, this ratio was approximately 10 percent If all individuals sold their rights, transactions costs would be 1 50 percent of the proceeds, a figure less than the difference in transactions costs for any reported issue size [9] But rights offerings are generally 50 percent subscribed by existing shareholders who do not bear these transactions costs [10] Therefore this cost appears to be less than one percent

Selling rights also has tax consequences for the shareholder For tax purposes, the cost basis of the stock must be allocated between the stock and the rights when the rights are received based on the market values of the rights and stock at that time [11] The acquisition date of the rights for tax purposes is the date on which the stock issuing the rights is acquired If the stock has risen in value since it was acquired, a relevant cost of employing a rights offering is the difference between the shareholder tax liability incurred now and the present value of the taxes which would have been paid had the rights issue not occurred [12]

To determine the impact of this cost again postulate generally extreme values for the relevant parameters Assume (1) that the marginal tax rate for the average shareholder is 50 percent (note this would be an unattainably high rate if the capital gain were long term), (2) that in the absence of the rights offering the taxes could have been postponed forever (3) that the allocated cash basis for the rights is 50 percent of the current rights price (4) that the ratio of the value of the rights to the proceeds of the issue is 10 percent, and (5) that only 20 percent of the current stockholders subscribe to the rights offering In this

[9] Note that since the expenses associated with raising equity capital are not tax deductible, these figures are comparable without further adjustment

[10] Estimates vary but ballpark figures on how investors react [to rights offerings] are as follows 50% exercise their rights 40% sell out for cash, and 10% do nothing ['Vanishing Rights' (May 2, 1977) *Barron's* p 25]

[11] If the fair market value of the rights is less than fifteen percent of the fair market value of the stock, the shareholder can choose to set the basis of the rights at zero leaving unaffected the basis of the stock The shareholder might choose this alternative if the cost of the bookkeeping exceeded the present value of the tax saving or if he anticipated being in a higher tax bracket when his remaining holdings were sold

[12] See Bailey (1969) for a discussion of the effective rate of capital gains tax, discounted to reflect the liability deferral

case, the cost would be 2 percent of the capital raised by the firm This is less than any reported cost differential in table 1 [15]

One other argument involving shareholder-borne costs has been offered by Weston and Brigham (1975) They argue that in a rights offering some stockholders may neither exercise nor sell, and by allowing their rights to expire unexercised they incur a loss [16] However, if an oversubscription privilege is employed with the offering, current owners in the aggregate receive full market value for the shares sold Admittedly, the oversubscription privilege affects the distribution of wealth among the owners, but it does not impose costs on owners as a whole

4. Security price behavior associated with rights and underwritten offering

4 1 Rights offerings lower the stock price

"A rights offering, under market conditions then existing, could well have a long-term depressing effect on the market price of the stock " [17]

Given the investment policy of the firm, a rights offering *will* lower the price of the stock in both the short run and in the long run as AT&T s Proxy Statement suggests But this is irrelevant to the choice of financing methods because the drop in price is *not* a reduction in the wealth of the owners and thus cannot be considered a cost of a rights issue

The fall in the stock price when rights are issued can be illustrated by the following argument Rights give the shareholders the option to purchase new shares at less than market prices Other things equal, the total market value of the firm after a rights offering, V, will then be the previous value, V' plus the subscription payments, S

$$V = V' + S \tag{1}$$

The per share price before the offering is V'/n, where n is the number of old shares If m new shares are sold, the per share price after the offering, $(V'+S)/(n+m)$ must be less than the price per share before the offering [18]

[15]If taxes were important, firms would avoid rights offerings when share prices had risen However the evidence presented in table 2 shows that, on average, firms have had abnormal positive price changes during the 12 months before an offering

[16]Stockbrokers holding securities for safekeeping do not allow the warrants to expire unexercised If no instructions are received, the broker will sell the rights immediately before expiration

[17]American Telephone and Telegraph Co, Notice of 1976 Annual Meeting and Proxy Statement

[18]Also note that arbitrage profits must not be available When a stock trades ex rights, a right is issued for each share outstanding At the ex rights date, the expected change in the stock price must equal the expected value of the right, or profit opportunities would exist if the sum of the ex rights value of the stock plus the value of the right at the ex rights date were

The fall in the stock price on the ex rights day is similar to the expected fall in the stock price at the ex dividend date. The two cases differ only in what is distributed – in the latter instance cash, in the former rights. Thus, the fall in the stock price simply reflects the fact that the shareholders have been given a valuable asset, the right.

The argument that the fall in the stock price is a relevant cost of a rights offering also appears in two related forms: (1) if an underwriter is used, the firm can raise a greater amount of capital with the same number of shares; (2) a rights offering lowers the earnings per share of the firm.[19] Both statements are true but if the fall in the stock price equals the market value of the rights, then the impact of the additional shares issued through the rights offering is the same as that of a stock split and the wealth of the owners of the firm is unaffected.

To examine whether, after correcting for the expected normal fall in the stock price, there were also abnormal price changes,[20] I studied the 853 rights offerings on the CRSP master file between 1926 and 1975. Following Fama, Fisher, Jensen and Roll (1967), I estimated the regression,

$$R_{jt} = \alpha_j + \beta_j R_{mt} + \varepsilon_{jt}, \tag{2}$$

where R_{jt} is the return to security j in month t, adjusted for capital structure changes (including rights offerings) and R_{mt} is the return to the market portfolio in month t. I estimated (2) for each of the 853 offerings, using data from the CRSP monthly return file, excluding the 25 months around the date of the offering. Setting $t = 0$ for the month of the rights offering, I used the estimated α_j and β_j to calculate the ε_{jt} for each security for the 25 months around the offering. I then calculated the average residual over all firms for each month in the interval -12 to $+12$. The average residuals were then cumulated from month -12 to the event month. The results are presented in table 2 and figure 1.

In the months subsequent to 'event month minus two' the average residuals

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systematically different from the value of the stock immediately before the ex rights date, then profits could be made by taking an appropriate position in the stock upon the announcement of the rights issue.

[19]'Thus, if the amendment [to remove the preemptive right from the corporate charter] is adopted, the company will be able to obtain the amount of capital needed through the issuance of fewer shares. Over a period of time this will result in slightly less dilution, higher equity value per share and better earnings per share.' [Commonwealth Edison Proxy Statement, 1976.]

[20]E.g., Commonwealth Edison suggests, 'Selling pressures often unduly depress both stock and rights values during the two or three week offering period which is a practical necessity when stock is sold with preemptive rights. Because the majority of stockholders do not exercise their rights but offer them for sale, the market value of the rights is driven far too low. Outsiders are then able to benefit by selling large amounts of stock during the offering period while buying rights for almost nothing and then exercising their rights to purchase stock at a discount to cover their sales. As a result, rights offerings tend to cost the company more than the rights themselves are worth to the stockholders who get them.'

are all insignificantly different from zero[21] and there is no significant sign pattern in the time series of average residuals. The cumulative average residuals in table 2 are also at approximately the same level three months before the

Table 2

Summary of average residual and cumulative average residual analysis of 853 rights offerings between 1926 and 1975 for the 25 event months [−12 to +12] surrounding the offer date.

Event month	Average residual	Cumulative average
−12	0.00721	0.00721
−11	0.01004	0.01725
−10	0.00255	0.01980
− 9	0.00629	0.02609
− 8	0.00388	0.02997
− 7	0.01062[a]	0.04059
− 6	0.00750	0.04809
− 5	0.00622	0.05431
− 4	0.01334[a]	0.06765
− 3	0.00662	0.07427
− 2	0.01624[a]	0.09051
− 1	−0.00649	0.08401
0	−0.00739	0.07663
+ 1	0.00779	0.08441
+ 2	0.00412	0.08853
+ 3	0.00405	0.09258
+ 4	−0.00110	0.09149
+ 5	−0.00047	0.09102
+ 6	0.00053	0.09155
+ 7	−0.00338	0.08817
+ 8	−0.00387	0.08430
+ 9	0.00256	0.08686
+10	−0.00264	0.08422
+11	−0.00013	0.08408
+12	−0.00476	0.07933

[a]Greater than 2σ. (Computation of the standard deviation is described in footnote 21.)

offering, on the date of the offering and 12 months after the offering. The significant positive residuals prior to the offer date are to be expected because of selection bias; firms which raise capital tend to have been doing well.

[21]As an estimate of the dispersion of an average residual, the approximation

$$\sigma^2 = (\sigma^2_M/r^2)(1-r^2)/N$$

was employed where σ^2_M is the variance of the market return, r^2 is the squared correlation coefficient between the return to an asset and the market return, and N is the number of securities in the sample. If σ_M is 0.089 [from Black Jensen Scholes (1972)], $r^2 = 0.25$, and $N = 853$ then $\sigma^2 = 0.000028$ and $\sigma = 0.00528$.

The results presented in table 2 are consistent with previous studies of this question Nelson (1965) examined all the rights offerings by firms listed on the New York Stock Exchange between January 1, 1946 and December 31, 1957. He found after the price series is adjusted for the 'split effect' in the rights offerings and general market movements are removed, prices six months after a rights offering are not significantly different from prices six months before the offering [22] Scholes (1972) found that the price of shares generally rose in value before the issue, fell 0 3 percent during the month of the issue, but experienced no abnormal gains or losses after the issue

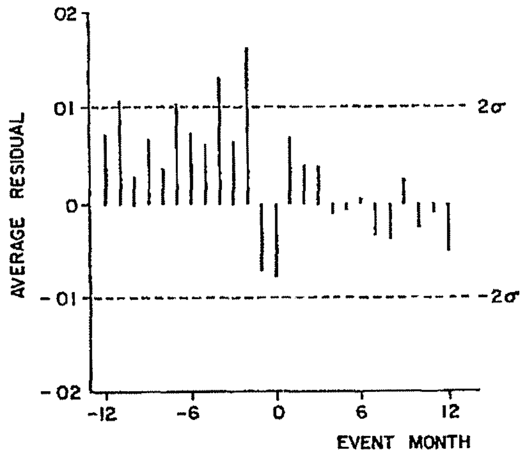

Fig 1 Plot of average residuals for 853 rights offerings between 1926 and 1975 for the 25 event months [−12 to +12] surrounding the offer date

4 2 Underwriters increase the stock price

Some argue that underwriters cause an increase in the stock price (1) by increasing 'public confidence' through external certification of the legal, accounting, and engineering analyses and (2) by the selling efforts of the underwriting syndicate.[23]

To examine the behavior of stock prices around the offer date of underwritten offerings and rights offerings, I obtained the returns for those securities which were included both in the sample of 578 firms covered in table 1 and on the CRSP daily return file There were 344 underwritten offerings and 52 rights offerings in this sample I set the offer date equal to day zero for all offerings and formed a portfolio of underwritten offerings and a portfolio of rights offerings I weighted securities in the portfolio of underwritten offerings so that

[22] The 'split effect' adjustment used by Nelson is derived in footnote 1

[23] See e g Brigham (1977, pp 473–474)

the two portfolios had equal betas. Then I calculated the difference in the portfolio returns for the 130 days before and 130 days after the offerings. The difference in average returns between two portfolios with equal risk will measure abnormal returns from either underwritten offerings or rights offerings. Table 3 presents the results for the period 20 days before the offering to 20 days after the offering; and figure 2 graphically presents the results for the period 40 days before to 40 days after the offering.

The average difference in returns to the two portfolios over the 260 days around the offer date is +0.00006, with a sample standard deviation of 0.00265. Therefore rights offerings have marginally higher returns during the 40 days around the offer date, but there is no obvious abnormal price behavior around the offer date for either underwritten offerings or rights offerings.

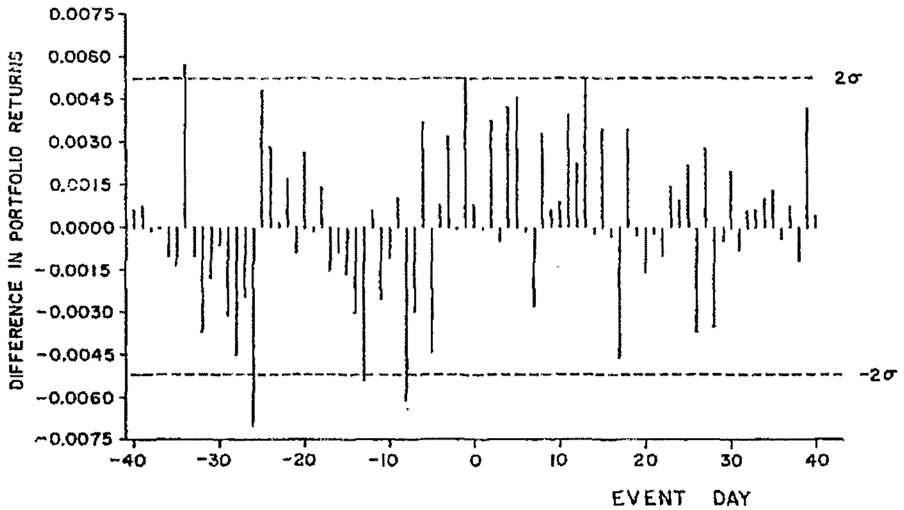

Fig. 2. Differences in daily returns between a portfolio of 52 rights offerings and a portfolio of 344 underwritten offerings for the 81 event days [−40 to +40] surrounding the offer date. (Portfolio weights are adjusted so that the two portfolios have the same beta.)

That underwriters are unable to generate abnormal positive price behavior should not be surprising. The firm always has the option of disclosing more information than is required by the Securities and Exchange Commission. The firm will expend resources on certification by external legal, accounting, and engineering firms until the net increase in the value of the firm is zero. Since the firm can contract for external certification of any disclosure, the benefit of whatever 'expert' valuation by the investment banker associated with an underwriting is limited to the difference in costs between certification through the underwriting process and independent certification.

But if underwriters are employed they influence the firm's decision about the

Table 3

Differences in daily returns between a portfolio of 52 rights offerings and a portfolio of 344 underwritten offerings between January 1971 and December 1975 for the 41 event days [−20 to +20] surrounding the offer date (Portfolio weights are adjusted so that the two portfolios have the same beta)

Event day	Rights average return	Underwritten average return	Difference (rights-und)	Cumulative difference
−20	−0 000361	−0 003007	0 002646	0 002646
−19	−0 001642	−0 001523	−0 000120	0 002526
−18	0 000072	−0 001361	0 001433	0 003959
−17	−0 001325	0 000175	−0 001500	0 002458
−16	−0 001134	−0 000231	−0 000902	0 001556
−15	−0 002865	−0 001229	−0 001636	−0 000080
−14	−0 002245	0 000732	−0 002977	−0 003057
−13	−0 004471	0 000949	−0 005420	−0 008477
−12	0 001722	0 001110	0 000611	−0 007866
−11	−0 002834	−0 000264	−0 002570	−0 010436
−10	−0 001226	−0 000125	−0 001102	−0 011538
− 9	0 001961	0 000960	0 001000	−0 010537
− 8	−0 004966	0 001151	−0 006117	−0 016654
− 7	0 001031	0 001327	−0 000296	−0 016950
− 6	0 002433	−0 001257	0 003690	−0 013260
−	−0 002373	0 002069	−0 004442	−0 017702
4	0 002180	0 001384	0 000797	−0 016905
− 3	0 001978	−0 001284	0 003262	−0 013642
− 2	−0 000570	−0 000557	−0 000013	−0 013656
− 1	0 004425	−0 000803	0 005228	−0 008428
0	0 001413	0 000583	0 000829	−0 007598
1	−0 000000	0 000054	−0 000054	−0 007653
2	0 003127	−0 000605	0 003732	−0 003921
3	−0 001182	−0 000700	−0 000482	−0 004403
4	0 003059	−0 001195	0 004254	−0 000149
5	0 005288	0 000710	0 004577	0 004428
6	0 000311	0 000477	−0 000166	0 004262
7	−0 002551	0 000206	−0 002757	0 001505
8	0 004396	0 001072	0 003324	0 004829
9	0 000851	0 000221	0 000630	0 005458
10	0 001601	0 000720	0 000881	0 006339
11	0 004703	0 000768	0 003934	0 010273
12	0 002369	0 000099	0 002271	0 012544
13	0 004764	−0 000502	0 005267	0 017811
14	−0 000734	−0 000495	−0 000239	0 017572
15	0 002944	−0 000527	0 003471	0 021043
16	−0 001089	−0 000790	−0 000299	0 020744
17	−0 001809	0 003065	−0 004874	0 015870
18	0 001228	−0 002196	0 003424	0 019294
19	0 000169	0 000458	−0 000289	0 019004
20	−0 000823	0 000711	−0 001534	0 017471

level of disclosure The underwriters will request that level of disclosure for which the marginal private costs and benefits to the underwriter are equal Given the legal liability of underwriters under the 1933 Act, the incentives of the firm and underwriter can differ Any divergence from the level of disclosure which maximizes the market value of the firm imposes a cost on the shareholders, and underwriters do ask for 'comfort letters' from accountants, frequently requiring expensive auditing procedures not produced without underwriters Thus, I conclude that the disclosure incentives of the underwriters lead to an over-investment in information production However, the costs of this over-investment should be reflected in the figures in table I

4 3 Do underwriters underprice the securities?

In Ibbotson's (1975) study of unseasoned new issues he found that the offer price on average is set 11 4 percent below the market value of the shares If seasoned new issues are also underpriced, the difference between market value and offer price would represent another cost of employing underwriters

There are reasons to believe that underwriters underprice the seasoned new issues For a firm commitment underwriting agreement the Rules of Fair Practice of the National Association of Securities Dealers[24] require that once the offer price is set, the underwriter cannot sell the shares at a higher price. If the offer price is set above the market value of the shares excess supply results If the offer price presents a binding constraint to the underwriter, the limit order placed with the specialist by the managing underwriter results in the purchase of additional shares at the offer price If continued this purchasing would cause the underwriting syndicate to break Since very few underwriting syndicates break,[25] the implication must be either that the offer price is generally set below the market value of the shares, or that the offer price constraint can be circumvented

There are two ways in which the offer price could be circumvented First, for hot issues (i e, underpriced issues for which there is significant excess demand) the underwriters allocate the shares to preferred customers One way to achieve preferred customer status is to purchase issues for which there is an excess supply Second, underwriters employ 'swaps' In a swap, the underwriter buys another security from a customer while selling the underwritten security at the offer price Through this tie-in sale, the underwriter can shift the profit or loss These two tying arrangements allow the underwriter to minimize the impact of the regulation

[24]Although the rules of fair practice were established by the NASD, and not Congress or the SEC, there is little difference in the impact These rules are a response to the SEC's self regulatory position If the SEC found them unsatisfactory the SEC could establish superseding regulation

[25]See *History of Corporate Finance for the Decade* (1972)

To see if seasoned new issues are underpriced I calculated the return from the closing price the day prior to the offer date to the offer price, and the return from the offer price to the close on the offer date. For the 328 firms with the requisite data, the average return from the close to the offer price is -0.0054 and the average return from the offer price to the close on the offer date is $+0.0082$. For the 260 days around the offer date the average daily return is 0.0005 with a sample standard deviation in the time series of average returns of 0.0013. Therefore, both figures, although much smaller than the 11.4 percent found by Ibbotson, are significantly different from the average daily return.[26] Thus the underpricing imposes an additional cost on the owners of the firm of between 0.5 and 0.8 percent of the proceeds of the issue, a cost which is not reflected in table 1.

5. Miscellaneous arguments favoring underwritten offerings

5.1. Insurance

It is frequently argued that employing an underwriter provides an 'insurance policy', reducing uncertainty of the offering's success.[27] In effect, the firm

[26]One difference between Ibbotson's unseasoned issues and the seasoned issues examined here is that the unseasoned shares trade on the OTC market. One hypothesis which has been suggested to explain the differences in the results is that the underpricing is a method of compensating the underwriter for maintaining a secondary market in the security. Although the argument can explain why underwriter's compensation (including underpricing costs) for unseasoned issues is higher than for seasoned issues, it does not explain the differential underpricing.

[27]Another type of 'insurance' might be relevant. If material errors are found in the registration statement of a public issue, parties who allege damage can bring suit. The suit typically names as co-defendants the firm, the board of directors of the firm, the firm's accountants, and the firm's underwriter. If the underwriter assumes a large share of the liability for the error, sheltering the firm from suit, then the underwriter will receive a normal compensation for bearing that risk.

Direct evidence on the hypothesis that underwriters reduce the firm's liability in case of a suit is expensive to obtain, economic studies of securities fraud suits have not been published. However indirect evidence suggests that this factor cannot be of a sufficiently large magnitude to make this an important factor in the choice of underwritten issues over rights issues. First, damage must be demonstrated – i.e., in addition to finding a material misstatement in the registration statement, the share price must have fallen after the offering. Second, the underwriters explicitly seek to limit their liability as much as is legally feasible. '[Issuer-Underwriter Indemnification] agreements are universally used in today's underwriting. ... These agreements, although varying in specific language, provide essentially for indemnification of the 'passively' guilty party by the party whose omissions or misstatements were the source of the liability.' (See 'The Expanding Liability of Security Underwriters', *Duke Law Journal*, Dec. 1969, pp. 1191–1246.) Thus underwriters contracts seek to minimize their exposure in this area. Third, if the courts imposed a significant share of the responsibility for material errors on the underwriter, it would be expected that accounting firms would recognize this by offering lower rates for securities work to firms employing underwriters. This does not seem to be the case. At least when this issue was raised with several partners of eight big accounting firms, this effect was denied. The judicial procedure tends to make the liability of each of the groups of defendants in this type of suit virtually independent.

purchases an option to sell the shares to the underwriter at the offer price (See Appendix 2) Note four things about this option First, in an underwritten issue, the offer price is not set generally until within 24 hours of the offering when the final agreement is signed, and hence the net proceeds are not determined until that time Second, as shown in section 4 3, the offer price on average is set below the market value of the stock Thus, the firm purchases a one-day option to sell shares at a discount of $\frac{1}{2}$ percent below their market value Third, subject to certain conditions specified in the letter of intent, the underwriter has the option of backing out of the tentative agreement until the date the final agreement is signed Thus, the 'insurance policy' is of limited value because its effective duration is short Fourth, as argued above, the subscription price for a rights offering can be set low enough so that the probability of failure of the rights offering becomes arbitrarily close to zero So an alternate source of 'self-insurance' is available through the rights offering For these reasons, the possible value of the 'insurance policy' associated with underwritten issues must be small

5 2 Timing

Commonwealth Edison claims that the proceeds of an underwritten issue are available to the firm sooner than in a rights issue [28] But timing benefits provided by underwriters must be small First, the settlement date for an underwritten issue is generally seven days after the offer date, while the settlement date for a rights offering is generally seven days after the expiration of the offering Since the offering generally lasts about 18 days, any reasonable estimate of the cost in terms of the lost interest which would be imposed on the firm by waiting that short period of time would have to be small Second, since it is not expected that the rights will be exercised prior to their expiration,[29] the owners of the firm have the use of the funds during the period of the offering Thus, the time period which entails an opportunity cost of the funds is reduced to a seven- to ten-day period both for rights and underwritten offerings Third, if the services provided by the underwriter and transfer agents are competitively supplied, the fees charged will reflect the opportunity cost of the funds at their disposal This would imply that the timing cost is impounded in the figures in table 1 And fourth, unless there is an unforeseen urgency associated with obtaining the funds, the firm can simply initiate the rights procedure at an earlier date

Moreover, under certain circumstances, the registration procedure with the SEC is simpler when a rights issue is employed It is my belief that with a rights offering, the SEC is more likely to presume a regular dialogue between the firm and its owners and thus impose less restrictive disclosure requirements There-

[28]Commonwealth Edison Proxy Statement, 1976

[29]See Merton (1973) or Smith (1976)

fore, the time until the registration becomes effective can be expected to be shorter with a rights offering than with an underwritten offering. This shorter registration time reduces the total time from the point where the decision is made to raise additional capital to the receipt of the proceeds.

5 3 *Distribution of ownership*

Weston and Brigham (1975) argue that underwriters provide a wider distribution of the securities sold, 'lessening any possible control problem' Since change in control may result in a change in management, this is likely to be a relevant issue for the current management. Yet it is not clear that possible control problems should be a concern of the owners I know of no reason to believe that one group of owners is any better (i e , will price the firm any higher) than another group

Furthermore, it is not obvious that underwriters will achieve a wider distribution of ownership than will a rights offering For most rights offerings of listed firms, the consensus among investment bankers is that the subscription rate of the current owners of the firm ranges from 20 to 50 percent It is difficult to estimate what percentage of an underwritten issue is purchased by the current owners of the firm, but there is no reason to believe it is zero Further, underwritten issues seem to attract more institutional interest, resulting in large block purchases and therefore more concentration of ownership

These factors preclude any general conclusions about the effect of financing method on ownership distribution With this uncertainty it is not clear that management, even if concerned with control issues, should prefer the use of an underwriter

5 4 *Consulting advice*

Van Horne (1974) suggests that 'advice from investment bankers may be of a continuing nature, with the company consulting a certain investment banker or group of bankers regularly' It is more expensive for the firm to compensate the investment banker for future consulting services by including in the underwriting fee a payment for the present value of the expected advice Costs incurred in raising capital are not tax deductible, they directly reduce the capital account and do not enter the income statement Thus, compared to separate billing for services rendered, paying for future consulting through a higher underwriting fee doubles its cost for a firm with a marginal tax rate of 50 percent

5 5 *Expected legal costs*

If there were a law, regulation, or merely an unresolved judicial principle which might impose additional liability on a firm using rights offerings, then the expected legal costs of using rights could explain the observed use of underwriters. But I can find no differential legal liability associated with the use of rights offerings.

5.6. Selection bias

If the firms which employ rights offerings were systematically different from the firms which employ underwritten offerings, then the observed cost differences could be attributable to selection bias. It could be that if the firms which employed underwriters had used rights, their expenses would have been greater.

There is a significant difference in the betas of the firms in the two groups. I calculated the betas for those firms in the sample which were listed on the New York Stock Exchange and included on the daily CRSP tape. The average beta for the 344 underwritten offerings is 0.731 with a standard deviation of 0.560, and the average beta for the 52 rights offerings is 0.493 with a standard deviation of 0.330. But I can find no other systematic difference between the two populations.

Examination of the data shows similar distributions of firms across industries; 80.8 percent of the firms employing rights and 73.2 percent of the firms employing underwritten offerings were utilities (electric, gas, or telephone companies). I attempted to predict the choice of underwritten versus rights offering based on the following variables: (1) the percentage of the firm which is sold through the offering, (2) the market value of the firm, and (3) the variance of the returns on the stock. The r^2 for the regression is 0.016. None of the t statistics for the variables appears to be significant.

Although differences exist between the two sets of firms, the nature and magnitude of the differences seem insufficient to account for the observed cost differences.

6. A monitoring cost hypothesis

6.1. Why not monitor the choice of financing method?

My examination of alternative financing methods suggests that rights offerings are significantly less expensive than underwritten offerings. Yet underwriters are employed in over 90 percent of the offerings studied. One hypothesis consistent with the evidence is: (1) managers and members of the board of directors receive benefits from the use of underwriters which do not accrue to the other owners of the firm, and (2) the expenses which would be imposed on the owners of the firm by monitoring the managers and directors in the choice of financing method are greater than the costs without monitoring.

Managers or members of the board of directors may recommend that offerings be underwritten because their welfare increases as a by-product of the use of underwriters in several ways [30] First, firms frequently include an investment banker as a member of the board of directors It is in his interest to lobby for the use of underwriters, particularly the use of his investment banking firm as managing underwriter Second, there is the possibility of 'bribery' This may be simply consumption for the managers and directors through 'wining and dining' by the underwriters But there is a more important possibility In an underwritten issue, if the offer price is set below the market value of the shares, the issue will be oversubscribed To handle this excess demand, underwriters ration the shares In the rationing process the underwriters presumably favor their preferred customers, and preferred customer status could be given to key management people or members of the board of directors of firms employing the underwriter This form of payment would be virtually impossible to detect, since the shares the officer of Company A would favorably acquire are those of Company B and would therefore call for no disclosure [31]

Further possible benefits to managers include the reduction of possible control problems, if underwritten offerings produce a wider distribution of ownership than rights offerings Finally, managers whose compensation is a function of reported profits will prefer an underwriter's fee which includes a payment for future consulting advice, the manager's compensation will be higher because payment through underwriting does not affect reported profits while separate billing for consulting does

Jensen and Meckling (1976) show that the costs which the managers and directors can impose on the other owners of the firm are limited by the costs of monitoring their activities Thus the cost to shareholders of monitoring the method of raising capital must be greater than the costs imposed by the financing method chosen Given the dispersion of ownership in modern corporations, the benefit to any single shareholder from voting his shares is small Thus the costs that he would rationally incur in voting are small,[32] and the resources the shareholder would rationally devote to deciding whether a 'yes' or 'no' vote is more in his interest are few Moreover, voting procedures in most corporations ensure that management has a disproportionate voice in the outcome Management is often assigned votes by proxy, and in many firms management has the

[30] Certain management compensation plans, such as stock option plans, make managers' compensation a function of the price of the firm's shares If the compensation plan were not adjusted to reflect the effect of the rights offering on the share price, management could be expected to provide a strong lobby in favor of employing underwriters In fact, however, employee stock option plans have general clauses calling for adjustment of the terms of the plan to reflect relevant capital structure changes Furthermore, most plans include specific reference to rights issues Thus, agency costs resulting from compensation plans do not seem to offer an explanation of the observed behavior

[31] This argument is similar to that of Manne (1966), especially Chapter V

[32] See Downs (1957) Basically, if a person owns 100 shares in a firm, his vote only matters if the vote is tied or his 'side' would have lost by 100 votes or less The probability is low that out of 50 million votes, the issue will split that way Thus the expected benefit (benefit times probability) of voting is very small

power to vote unreturned proxies They are also permitted to vote proxies on specific questions when the stockholder does not specify a choice These factors raise the cost of monitoring management

6 2 *The preemptive right as a monitoring tool*

There appears to be a low cost method of monitoring the use of underwriters the preemptive right The preemptive right is a provision which can be included in a firm s charter requiring the firm to offer any new common stock first to its existing shareholders But the inclusion of the preemptive right does not solve the problem firms can still employ underwriters through a standby under-

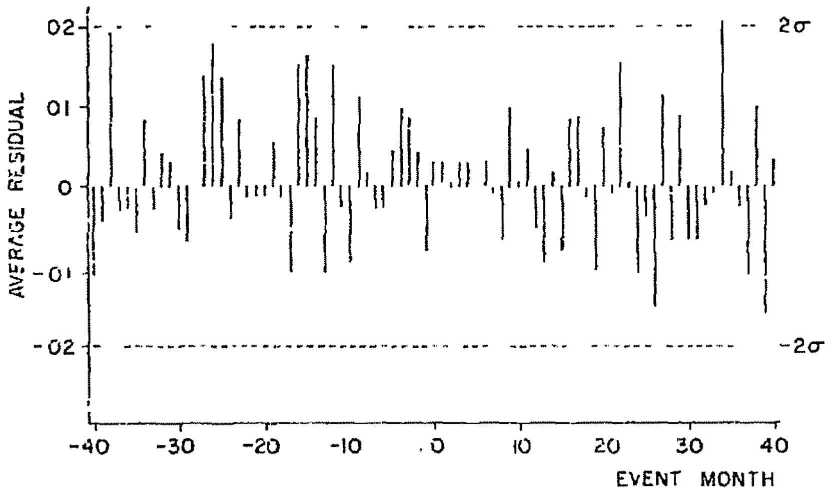

Fig 3 Plot of average residuals from 89 firms which removed the preemptive right from their corporate charter for the 81 event months [−40 to +40] surrounding the month of removal

writing agreement Since the figures in table I suggest a negligible difference in costs between a firm commitment underwritten offering and a rights offering with a standby underwriting agreement what becomes important is not a requirement to use rights, but a prohibition against using underwriters

To test the hypothesis that the impact of removing the preemptive right from the corporate charter is negligible, I collected a sample of 89 firms listed on the New York Stock Exchange which have removed the preemptive right The results of this study are presented in table 4 and figure 3 The average residual in the month of removal is 0 277 percent, and the mean average residual for the six prior months is 0 309 percent There is no apparent impact

I believe the results in table 4 provide a plausible explanation for why the intellectual level of the argument involving the preemptive right is so low on both sides of the question For example, the above quotes from Commonwealth

Table 4

Summary of residual analysis of 89 firms which removed the preemptive right from their corporate charter for the 81 event months [−40 to +40] surrounding the month of removal

Event month	Average residual	Cumulative average residual	Event month	Average residual	Cumulative average residual
−40	−0 00995	−0 00995	1	0 00363	0 11718
−39	−0 00382	−0 01376	2	0 00028	0 11745
−38	0 01999	0 00623	3	0 00293	0 12038
−37	−0 00258	0 00365	4	0 00276	0 12315
−36	−0 00160	0 00205	5	0 00101	0 12415
−35	−0 00414	−0 00209	6	0 00336	0 12751
−34	0 00842	0 00633	7	−0 00017	0 12734
−33	−0 00238	0 00395	8	−0 00537	0 12196
−32	0 00483	0 00878	9	0 00963	0 13159
−31	0 00375	0 01254	10	0 00002	0 13162
−30	−0 00419	0 00834	11	0 00406	0 13568
−29	−0 00632	0 00202	12	−0 00446	0 13122
−28	0 00082	0 00284	13	−0 00855	0 12766
−27	0 01337	0 01621	14	0 00210	0 12476
−26	0 01839	0 03460	15	−0 00696	0 11780
−25	0 01440	0 04900	16	0 00903	0 12683
−24	−0 00397	0 04503	17	0 00752	0 13435
−23	0 00800	0 05303	18	−0 00096	0 13339
−22	−0 00102	0 05201	19	−0 00942	0 12397
−21	−0 00007	0 05195	20	0 00701	0 13097
−20	−0 00072	0 05123	21	−0 00021	0 13077
−19	0 00602	0 05725	22	0 01591	0 14668
−18	−0 00067	0 05658	23	0 00090	0 14758
−17	−0 01032	0 04626	24	−0 01043	0 13715
−16	0 01575	0 06201	25	−0 00281	0 13434
−15	0 01608	0 07809	26	−0 01389	0 12046
−14	0 00828	0 08637	27	0 01069	0 13115
−13	−0 00943	0 07694	28	−0 00566	0 12548
−12	0 01496	0 09190	29	0 00901	0 13449
−11	−0 00183	0 09007	30	−0 00592	0 12857
−10	−0 00833	0 08174	31	−0 00624	0 12233
−9	0 01103	0 09277	32	−0 00240	0 11993
−8	0 00138	0 09415	33	−0 00071	0 11922
−7	−0 00185	0 09230	34	0 02059	0 13981
−6	−0 00170	0 09060	35	0 00183	0 14165
−5	0 00508	0 09568	36	−0 00263	0 13901
−4	0 00998	0 10566	37	−0 01103	0 12799
−3	0 00816	0 11382	38	0 00971	0 13770
−2	0 00477	0 11859	39	−0 01524	0 12246
−1	−0 00782	0 11078	40	0 00300	0 12546
0	0 00277	0 11355			

Edison's Proxy Statement are demonstrably false, and the quote from AT&T's Proxy Statement is irrelevant The primary lobbying effort in favor of the preemptive right is from Lewis D Gilbert, John J Gilbert and Wilma Soss who regularly introduce proposals to reincorporate the preemptive right into the corporate charter of corporations which have removed it However, their reason for the use of rights is so that shareholders can maintain their proportionate interest in the firm For large firms this 'benefit' has negligible value [33]

6 3 Other considerations

It should be emphasized that the monitoring cost hypothesis is consistent with both observed institutional arrangements and rational, wealth-maximizing behavior by the stockholders Rational behavior implies that actions will be taken if the benefits exceed the costs I have pointed out certain costs associated with the voting mechanism within corporations inclusion of an investment banker on the board of directors, and certain management compensation plans These practices, while costly, would still be in the stockholders' best interests if there are offsetting benefits

Furthermore, the monitoring cost hypothesis does not imply that there are rents which accrue to the underwriting industry There are two available 'technologies' with which additional equity capital can be raised If the underwriting industry is competitive, the underwriting fees reported in table 1 would reflect a normal return to the resources required in employing that technology

However, the monitoring cost hypothesis does present some problems I do not observe the costs of monitoring management Hence the hypothesis is not directly tested Furthermore, while the incentives set up through the voting mechanism suggest that it is plausible that monitoring costs are large enough to explain the observed use of underwriters, competition in the market for management should reduce the required monitoring expenditures If the use of rights offerings is in the best interests of stockholders, then it will pay potential managers to incur bonding costs to guarantee not to use underwriters

7. Conclusions

In my examination of the choice of method for raising additional equity capital by listed firms I demonstrate that properly constructed rights offerings provide proceeds which are equivalent to those of an underwritten offering Furthermore, estimates of expenses from reports filed with the Securities and

[33] For a firm with 50 million shares outstanding, a ten percent increase in the number of outstanding shares would change the percentage ownership for someone with 100 shares only in the sixth decimal place With so many inexpensive alternate ways for a stockholder to maintain his proportionate interest in the firm the proportionate interest argument lacks importance

Exchange Commission indicate that rights offerings involve lower out-of-pocket costs than underwritten offerings Yet underwriters are employed in over 90 percent of the issues Examination of the arguments to justify the use of underwriters advanced by the underwriting industry, finance textbooks, corporate officers, and securities lawyers suggest that none of the arguments are capable of explaining the observed choice of financing method in terms of rational, wealth-maximizing behavior by the stockholders of the firm

The one hypothesis I find which is consistent with the available evidence relates to the costs of monitoring management Although direct expenses imposed on shareholders are higher per dollar raised through the use of underwriters, I hypothesize that management derives benefits from their use From the shareholders' standpoint, the firm's use of underwriters is optimal because the cost of monitoring management exceeds the savings in out-of-pocket expenses from using rights If this hypothesis is correct, then the present value of the stream of differences in costs reported in this paper provides a lower bound on the costs of getting shareholders together to monitor and control management on the method of raising capital Thus, the present value of the differences in costs establishes a lower bound on the expected costs of control mechanisms such as proxy fights, tender offers, and takeover bids

The monitoring cost hypothesis does present some problems I do not observe directly the costs of monitoring management While it is possible that the monitoring costs are large enough to explain the observed choice of underwriters, consideration of competition in the market for management reduces the plausibility of this hypothesis But if the monitoring cost hypothesis is rejected, then the observed choice of financing method cannot be explained in terms of rational, wealth-maximizing behavior by the owners of the firm, unless it can be shown that I have either ignored or misestimated a relevant cost of using rights or benefit from using underwriters

Appendix 1: A description of the institutional arrangements for rights and underwritten offerings

A description of the procedures followed in the various types of offerings specified in sufficient detail to answer the questions addressed in this study is not available This appendix provides that information Some of this material comes from written sources [34] However, much of the material comes from conversations with underwriters, corporate financial officers, and SEC officials.

Underwritten offerings

The firm typically selects an underwriter in one of two ways – either by competitive bidding or by negotiated underwriting In competitive bidding, the firm

[34] See Weston and Brigham (1975), SEC (1974), and Pessin (1976)

files appropriate papers with the SEC, then specifies the terms of the issue and has potential underwriters submit sealed bids Government regulation requires the use of this procedure by electric utility holding companies the primary users of competitive bidding In a negotiated underwriting bid, the important variables in the underwriting contract are determined by direct negotiation between firm and underwriter

Negotiated underwriting begins with a series of pre-underwriting conferences, when decisions as to the amount of capital, type of security, and other terms of the offering are discussed Several general forms of the underwriting agreement can be employed [35] The first is a 'firm commitment' underwriting agreement, under which the underwriter agrees to purchase the whole issue from the firm at a particular price for resale to the public Almost all large underwriters employ this form In the second form, a 'best efforts' underwriting, the underwriter acts only as a marketing agent for the firm The underwriter does not agree to purchase the issue at a predetermined price, but sells the security for whatever price it will bring The underwriters take a predetermined spread and the firm takes the residual A variant of this agreement employs a fixed price but no guarantee on the quantity to be sold The third possibility is an 'all-or-nothing' commitment which requires the underwriter to sell the entire issue at a given price, usually within thirty days, otherwise the underwriting agreement is voided

If the corporation and underwriter agree to proceed,[36] the underwriter will begin his underwriting investigation, in which he assesses the prospects for the offering This investigation includes an audit of the firm's financial records by a public accounting firm, which aids in preparing the registration statements required by the Securities and Exchange Commission A legal opinion of the offering will be obtained from lawyers who typically participate in writing the registration statement Reports may also be obtained from the underwriter's engineering staff when applicable

Before a company can raise capital through a public offering of new stock it must comply with the Federal Law that governs such a sale – the Securities Act of 1933, and the Securities Exchange Act of 1934 The Securities and Exchange Commission, established to administer both laws, requires full disclosure of all pertinent facts about the company before it makes a public offering of new stock The firm must file a lengthy registration statement with the SEC setting forth data about its financial condition For underwritten issues,

[35]The underwriter may make a 'standby commitment' during a rights offering under which he will purchase and distribute to the public any amount of the rights issue not purchased by the present security holders This form will be discussed further below

[36]Agreements are usually subject to conditions, most allow the underwriters to void their obligation in the event of specified adverse developments For example, a negative finding in the lawyer's or auditor's reports may allow voiding the contract

the firm usually files the form S-1 or S-7 registration statement Form S-7 is less expensive, but requires certain conditions to qualify [37]

The SEC has 20 days to examine the registration statement for material omissions or misrepresentations If any error is found, a deficiency letter is sent to the corporation and the offering is delayed until the deficiency is corrected If no deficiency letter is sent, a registration statement automatically becomes effective 20 days after filing, except when the SEC notifies the firm that the commission's workload is such that it requires more time to review the registration statement [38] The firm will typically amend the registration statement to include the offer price and the offer date after the SEC has examined the rest of the statement This procedure allows the firm and underwriter to postpone the effective date of the registration statement until they agree the offering should proceed

In addition to the registration requirements under the Securities Act of 1933, firms must qualify their securities under the state securities laws, the so-called 'Blue Sky Laws', in those states where the securities are to be sold Some states are satisfied with SEC approval, others require a registration statement be filed with state securities commissioners

The underwriter usually does not handle the purchase and distribution of the issue alone, except for the smallest of security issues The investment banker usually forms a syndicate of other investment bankers and security dealers to assist the underwriting [39] During the waiting period between the filing and the offer date, no written sales literature other than the so-called 'red herring'

[37]For example, the majority of the board of directors have been members for the last three years, there have been no defaults on preferred stock or bond payments for the past 10 years, net income after taxes was at least $500,000 for the past five years, and earnings exceeded any dividend payments made over the past five years

[38]In 1960 and 1961, delays of four to six months occurred for this reason

[39]Prior to the passage of the Securities Act in 1933 most new issues were purchased by an originating house The originating house would resell the issue at a small increase in price to a so-called banking group, generally a few large houses The banking group would then sell the issue to an underwriting group, which in turn sold it to a selling syndicate – each sale occurred at a fractional increase in price The selling syndicate members, however, were liable for their proportional interest of any securities remaining unsold Late in the 1920s it became frequent practice to make the final group a so-called selling group, the members of which had no liability except for securities which they had purchased from the underwriting syndicate

The Securities Act, as amended shortly after its passage, contained a provision limiting an underwriter's liability for misstatements and omissions in the registration statement to an amount not 'in excess of the total price at which securities underwritten by him and distributed to the public were offered to the public' This Act changed the method of wholesaling securities, the use of the joint syndicate in handling registered securities disappeared Because of the provisions of the Act, it was to the advantage of the manager of the offering to have his fellow participants purchase direct from the company, since then the manager's liability under the Act became limited to the amount which the firm itself underwrote Liability for transfer taxes that would have been payable on the sale by the manager to the underwriters was thus avoided At the present time, underwriters of securities registered under the Act contract to buy directly from the issuer even though the manager of the offering signs the agreement with the issuer on behalf of each of the underwriting firms prospectus[40] and 'tombstone' advertisements[41] are permitted by the SEC However, oral selling efforts are permitted, and underwriters can and do note interest from their clients to buy at various prices These do not represent legal commitments, but are used to help the underwriter decide on the offer price for the issue Underwriters typically attempt to obtain indications of interest for approximately 10 percent more shares than will be available through the offering [42]

Before the effective date of the registration, the corporation's officers meet with the members of the underwriting group Given the personal liability provisions of the 1933 Act, this meeting is often identified as a due diligence meeting An investment banker who is dissatisfied with any of the terms or conditions discussed at this session can still withdraw from the group with no legal or financial liability Discussed at this meeting are (1) the information in the firm's registration statement, (2) the material in the prospectus, (3) the specific provisions of the formal underwriting agreement As a rule, all the provisions of the formal underwriting agreement are set except the final sales price

The 'Rules of Fair Practice' of the National Association of Security Dealers require that new issues must be offered at a fixed price and that a maximum offering price be announced two weeks in advance of the offering However, the actual offering price need not be established until immediately before the offering date In fact, the binding underwriting agreement which specifies the offer price is not normally signed until within 24 hours of the effective date of the registration

Once the underwriter files the final offering price with the SEC, the underwriters are precluded from selling the shares above this price The SEC permits the managing underwriter to place a standing order with the specialist to buy the stock at the public offer price If the underwriter buys more than 10 percent of the shares to be issued through this order, the syndicate usually breaks, permitting the stock to be sold below the offer price The syndicate can also be broken if the managing underwriter feels that the issue cannot be sold at the offer price [43] On the other hand, if all the indications of interest become orders

[40]The red herring prospectus derives its name from the required disclaimer on the front printed in red

A registration statement relating to these securities has been filed with the Securities and Exchange Commission but has not yet become effective Information contained herein is subject to completion or amendment These securities may not be sold nor may offers to buy be accepted prior to the time the registration statement becomes effective This prospectus shall not constitute an offer to sell or the solicitation of an offer to buy nor shall there be any sale of these securities in any state in which such offer, solicitation or sale would be unlawful prior to registration or qualification under the securities laws of any such state

[41]The very limited notice of the offering permitted is often presented in a form resembling the inscription on a tombstone – hence the name

[42]This procedure is like 'over-booking' on airplane flights

[43]Syndicates break infrequently, my impression is that this occurs less than five percent of the time See *History of Corporate Finance For the Decade* (1972)

for shares, the issue is oversold In that case the managing underwriter typically sells additional shares short and covers these short sales in the aftermarket

The final settlement with the underwriter usually takes place seven to ten days after the registration statement becomes effective At that time, the firm receives the proceeds of the sale, net of the underwriting compensation

Rights offering

Offering of stock to existing shareholders on a pro rata basis is called a rights offering Each stockholder owning shares of common stock at the issue date receives an instrument (formally called a warrant) giving the owner the option to buy new shares [44] One warrant or right is issued for each share of stock held [45] This instrument states the relevant terms of the option (1) the number of rights required to purchase one new share, (2) the exercise price (or subscription price) for the rights offering, (3) the expiration date of the rights offering

Before the offering, the firm must file a registration statement for these securities For rights offerings, the firm typically files either a form S-1 or S-16 registration S-16 is simpler, but has usage requirements similar to those of form S-7

After the SEC approves the registration statement, the firm establishes a holder of record date The stock exchange establishes the date five business days earlier as the ex rights date [46] All individuals who hold the stock on the ex rights date will appear in the company's records on the holder of record date and will receive the rights However, the rights can be traded on a 'when issued' basis Usually trading begins after the formal announcement of the rights offering To ensure that there is adequate time for the stockholders to exercise or sell their rights, the New York Stock Exchange requires that the minimum period during which rights may be exercised is 14 days Rights trade on the exchange where the stock is listed

Issuing rights is costly in terms of management s time, postage and other expenses, so it is in the best interest of the firm to ensure the success of the offering Therefore, the firm has an incentive to set the subscription price of the rights low enough to ensure that the rights will be exercised But some of

[44] In the 1880s it was customary to require a stockholder to appear in person in the office of the corporation to subscribe to the issue After the 1880s, it became customary to send out a printed slip of paper so the stockholders could sign and subscribe for the stock without actually having to appear Later, it became the practice to make these slips of paper transferable, so that they could be sold Around 1910 the engraved form of warrant was first issued

[45] The Uniform Practice Code of the National Association of Security Dealers, Inc , provides that subscription rights issued to security holders shall be traded in the market on the basis of one right accruing on each share of outstanding stock, except when otherwise designated by the National Uniform Practice Committee Thus, the price quotation will be based on a single right even though several rights may be necessary to purchase one new share

[46] This procedure is comparable to that used in setting the ex dividend date

the warrants of most offerings do expire unexercised These unexercised rights can be offered through an over-subscription privilege to subscribing shareholders on a pro rata basis Shares not distributed through the rights offering or through the over-subscription privilege can be sold by the firm either to investment bankers or directly to the public

Rights offerings with a standby underwriting agreement

A formal commitment with an underwriter to take the shares not distributed through a rights offering is called a standby underwriting agreement Several types of fee schedules are generally employed in standby underwriting agreements A single fee may be negotiated, the firm paying the underwriter to exercise any unexercised rights at the subscription price A two fee agreement employs both a standby fee', based on the total number of shares to be distributed through the offering and a take-up fee, based on the number of warrants handled The take-up' fee may be a flat fee or a proportioned fee [47] These agreements generally include a profit sharing arrangement on unsubscribed shares (e g, if the underwriter sells the shares for more than the subscription price, this difference in prices is split between the underwriter and the firm according to an agreed formula)

Underwriters are prohibited from trading in the rights until 24 hours after the rights offering is made [48] After that time, they can sell shares of the stock short and purchase and exercise rights to cover their short position in the stock, thus hedging the risk that they bear

Appendix 2: A contingent claims analysis of rights and underwriting contracts

The derivation of general equilibrium pricing implications of rights and underwriting contracts has not been presented Black and Scholes (1973) suggest the approach I employ to value rights, but they do not carry out the analysis or present the solution Ederington (1975) provides a model of under-

[47] A proportioned fee involves more than one price for the shares handled by the underwriter For example there may be one price for the first 15% of the issue, a higher price for from 15% to 30% of the issue, and a still higher price for any of the issue over 30% which is unexercised through the rights offering and must be purchased by the underwriter

[48] Through the late 1940s underwriters were prohibited from trading in the rights during the offering This arrangement increased the underwriter s risk because the 14-day time period allowed large adverse price movements in the stock The NYSE instituted a study in 1947 after the failure of three rights offerings They found than on 43 rights offerings which had been successful the total underwriting profit was approximately $2 4 million, while on the three unsuccessful offerings, their losses were in excess of $3 million Underwriters were reportedly refusing to sign standby agreements unless the offering period were as short as five days Since this violated NYSE rules no NYSE listed firms used rights issues with standby underwriting agreements In response to this impasse, the NYSE now allows underwriters to trade in the rights 24 hours after the rights offering is made

writer behavior, but his model assumes underwriters maximize expected profits, and thus does not represent a general equilibrium solution in a market where the agents are risk averse The option pricing framework employed here will yield a solution which is consistent with general equilibrium, no matter what the risk preferences of the agents in the market.

I employ the contingent claims pricing techniques to derive a specification of the equilibrium value of these contracts For valuing both contracts I assume

(1) There are homogeneous expectations about the dynamics of firm asset values and of security prices The distribution of firm values at the end of any finite time interval is log normal The variance rate, σ^2, is constant
(2) Capital markets are perfect There are no transactions costs or taxes and all traders have free and costless access to all available information Borrowing and perfect short sales of assets are allowed Traders are price takers in the capital markets
(3) There is a known constant instantaneously riskless rate of interest, r, which is the same for borrowers and lenders
(4) Trading takes place continuously, price changes are continuous and assets are infinitely divisible
(5) The firm pays no dividends

Rights offerings

To derive the equilibrium value of the rights offering I make the following assumptions about the specification of the rights offering

The total proceeds to the firm if the rights are exercised is X (the exercise price per share times the total number of shares sold through the rights issue) The rights expire after T time periods If the rights are exercised, the shares sold through the offering will be a fraction, γ, of the total number of shares outstanding ($\gamma \equiv Q_R/(Q_S+Q_R)$, where Q_R is the number of shares sold through the rights offering and Q_S is the existing number of shares) Any assets acquired with the proceeds of the rights offering are acquired at competitive prices [49]

Given the above assumption, Merton (1974) has demonstrated that any contingent claim, whose value can be written solely as a function of asset value and time must satisfy the partial differential equation

$$\frac{\partial f}{\partial t}=\frac{1}{2}\frac{\partial^2 f}{\partial V^2}\sigma^2 V^2+rV\frac{\partial f}{\partial V}-rf, \tag{A1}$$

[49]This last assumption is necessary to avoid the problem of the dependence of the dynamic behavior of the stock price on the probability of the rights being exercised

where $f(V, t)$ is the function representing the value of the contingent claim [e.g., $R = R(V, t)$]. To solve this equation, normally two boundary conditions are required, one in the time dimension and one in the firm value dimension.

To derive the appropriate boundary condition in the time dimension, note that when the time to expiration is zero, R^*, the value of the rights at the expiration date will be either zero (in which case the rights will not be exercised) or, if the rights are valuable and are exercised, their value is their claim on the total assets of the firm, $\gamma(V^*+X)$ (where V^* is the value of the firm's assets and X is the proceeds from the exercise of the rights) minus the payment the right-holders must make, X:

$$R^* = \mathrm{Max}[0, \gamma(V^*+X)-X], \tag{A2}$$

where:

V^* is the value of the firm's assets at the expiration date of the issue.

X is the proceeds to the firm of the exercise of the rights.

γ is the fraction of new shares issued through the rights offering to the total shares of the firm (both old and new).

The most natural boundary condition in the firm value dimension is that when the value of the firm is zero, the value of the rights issue, R, is zero. However, the first assumption, that the distribution of firm values is log normal, insures that V can never be zero; therefore, this boundary condition will never be binding.

This equation can be solved by noting that no assumptions about risk preferences have been made, thus the solution must be the same for any preference structure which permits equilibrium. Therefore choose that structure which is mathematically simplest.[50] Assume that the market is composed of risk-neutral investors. In that case, the equilibrium rate of return on all assets will be equal. Specifically, the expected rate of return on the firm, and the rights will equal the riskless rate. Then the current rights price must be the discounted terminal price:

$$R = \mathrm{e}^{-rT} \int_{((1-\gamma)/\gamma)X}^{\infty} [\gamma V^* - (1-\gamma)X] L'(V^*) \mathrm{d}V^*, \tag{A3}$$

where $L'(V^*)$ is the log normal density function.

Eq. (A3) can be solved to yield:[51]

[50]See Cox and Ross (1976) or Smith (1976). For a mathematical derivation of this solution technique, see Friedman (1975), especially page 148.

[51]See Smith (1976, p. 16) for a theorem which can be employed to immediately solve (A3) to yield (A4).

$$R = \gamma V N\left\{\frac{\ln(\gamma V/(1-\gamma)X)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}\right\}$$
$$-e^{-rT}(1-\gamma)XN\left\{\frac{\ln(\gamma V/(1-\gamma)X)+(r-\sigma^2/2)T}{\sigma\sqrt{T}}\right\}$$
$$= R(V, T, X, \gamma, \sigma^2, r) \tag{A4}$$

where $\partial R/\partial V$, $\partial R/\partial T$, $\partial R/\partial \gamma$, $\partial R/\partial \sigma^2$, $\partial R/\partial r > 0$ and $\partial R/\partial X < 0$

The indicated partial effects have intuitive interpretations Increasing the value of the firm, decreasing the exercise price (holding the proportion of the firm's shares offered through the rights offering constant), or increasing the proportion of the firm's shares offered through the rights offering (holding the total proceeds of the issue constant) increase the expected payoff to the rights and thus increases the current market value of the rights offering An increase in the time to expiration of the riskless rate lowers the present value of the exercise payment, and thus increases the value of the rights Finally, an increase in the variance rate gives a higher probability of a large increase in the value of the firm and increases the value of the rights

Underwriting agreements

To analyze the appropriate compensation to the underwriter for the risk he bears in the distribution of the securities make the following assumptions about the underwriting contract

Underwriters submit a bid, B, today which specifies that on the offer date, T time periods from now, the underwriter will pay B dollars and receive shares of stock representing fraction γ of the total shares of the firm He can sell the securities at the offer price and receive a total payment of Ω, or (if the share price is below the offer price) at the market price, $\gamma(V^*+B)$ If his bid is accepted, he will be notified immediately

Again, (A1) can be employed where $f(V, t)$ is the function representing the value of the underwriting contract (i e, $U - U(V, t)$) The boundary condition for this problem is

$$U^* = \mathrm{Min}[\gamma(V^*+B)-B, \Omega-B] \tag{A5}$$

This assumes that at the offer date the underwriter will pay the firm B dollars The shares which the underwriter receives represent a claim to a fraction γ of the total assets of the firm, V^*+B If the offer price is greater than the value of the shares, $\gamma(V^*+B)$, then the underwriter will be unable to sell the shares at the offer price, hence he will receive $\gamma(V^*+B)$ If, at the offer date the offer price is less than the value of the shares, the underwriter receives the offer price Therefore, the boundary condition is that at the offer date the underwriting contract is worth the minimum of the market value of the shares minus the bid, B, or the proceeds of the sale at the offer price minus the bid

Again, the above solution technique can be employed to solve (A1) subject to (A5). In a risk-neutral world, the expected value of the underwriting contract can be expressed as [52]

$$U = \int_0^{(\Omega/\gamma)-B} [\gamma(V^*+B)-B]L'(V^*)\mathrm{d}V^* + \int_{(\Omega/\gamma)-B}^{\infty} [\Omega-B]L'(V^*)\mathrm{d}V^*. \tag{A6}$$

Note that this can be rewritten as

$$U = \int_0^{\infty} [\gamma(V^*+B)-B]L'(V^*)\mathrm{d}V^* - \int_{(\Omega/\gamma)-B}^{\infty} \gamma\left[V^*-\left(\frac{\Omega}{\gamma}-B\right)\right]L'(V^*)\mathrm{d}V^* \tag{A7}$$

Eq (A7) can be solved for the risk-neutral case to yield

$$U = \mathrm{e}^{rT}\gamma V-(1-\gamma)B-\mathrm{e}^{rT}\gamma VN\left\{\frac{\ln(\gamma V/(\Omega-\gamma B))+(r+\sigma^2/2)T}{\sigma\sqrt{T}}\right\} + (\Omega-B\gamma)N\left\{\frac{\ln(\gamma V/(\Omega-\gamma B)+(r-\sigma^2/2)T}{\sigma\sqrt{T}}\right\} \tag{A8}$$

Examination of (A8) reveals that the underwriting contract is equivalent to a portfolio consisting of a long position in the firm, a cash payment, and writing a call on γ of the firm with an exercise price equal to $(\Omega-\gamma B)$

$$\begin{aligned} U &= \mathrm{e}^{rT}\gamma V-(1-\gamma)B-\mathrm{e}^{rT}C(\gamma V, T, \Omega-\gamma B) \\ &= \mathrm{e}^{rT}\gamma V-(1-\gamma)B-\mathrm{e}^{rT}\gamma C\left(V, T, \frac{\Omega}{\gamma}-B\right), \end{aligned} \tag{A9}$$

where C( ) is the Black-Scholes call option function

If the process of preparing and submitting a bid is costless, then in a competitive equilibrium, the value of the underwriting contract must be zero [53]

[52]Since the contract calls for the payment only at t^*, to find the current value of the underwriting contract does not require discounting

[53]If this were not the case, arbitrage profits could be earned by acquiring an underwriting contract and establishing the above hedge

Therefore the bid which would represent a normal compensation for the risk he bears is implicitly defined by the equation [54]

$$B - e^{rT}\frac{\gamma}{1-\gamma}\left[V - C\left(V, T, \frac{\Omega}{\gamma} - B\right)\right] = 0 \tag{A10}$$

The firm generally receives less than the market value of the stock[55] given the specification of the underwriting contract, if the equilibrium stock price at the offer date is above the offer price then the initial purchaser of the issue receives 'rents', he obtains the shares for less than the market value of the shares Therefore, if the offer price in the underwriting agreement represents a binding constraint to the underwriter, then in a perfect market underwriting must be a more expensive method of raising additional capital than is a rights issue Therefore, under these conditions, underwriting would not be employed

The above analysis implicitly assumes that the terms of the underwriting contract represent a binding constraint to the underwriter, i e , if the security price is above the offer price, then the offer price presents a constraint to the underwriter and a pure profit opportunity to the potential investor However, in a market without transactions costs, this could not be the case If the security price is above the offer price there will be excess demand for the issue To the extent that the underwriter can, through the rationing process, extract those profits, they will accrue to the underwriter rather than to the initial purchaser In this situation competition among underwriters would ensure that the profits were in fact garnered by the firm In that case the offer price presents no effective constraint and the competitive bid becomes simply

$$B = e^{rT}\left(\frac{\gamma}{1-\gamma}\right)V \tag{A11}$$

Therefore, if through tie-in sales or other means the offer price in an underwriting agreement can be circumvented, then underwriting is no more expensive a method of raising additional capital than a rights offering

[54] This equation implicitly defines the bid because B appears twice in the equation The explicit solution for equilibrium bid can be found by standard numerical analysis techniques

[55] A sufficient condition for the bid to be less than the market value of the shares is that $(1-\gamma)$ be less than $e^{\gamma T}$ Since T is generally a matter of days, this condition should be met

References

Bailey, M J , 1969, Capital gains and income taxation, in A C Harberger and M J Bailey, eds , The taxation of income from capital (Brookings, Washington, D C)

Black, F , M C Jensen and M S Scholes, 1972, The capital asset pricing model Some empirical tests, in M C Jensen, ed , Studies in the theory of capital markets (Praeger, New York)

Black, F and M Scholes, 1973 The pricing of options and corporate liabilities, Journal of Political Economy 81, 637–654
Brigham, E F, 1977, Financial management (Dryden, Hinesdale, Illinois)
Cox, J C and S A Ross, 1975, The pricing of options for jump processes, Rodney L White Center for Financial Research, Working Paper 2-75 (University of Pennsylvania, Philadelphia Pennsylvania)
Downs, A C, 1957 An economic theory of democracy (Harper, New York)
Ederington, Louis H 1975, Uncertainty, competition, and costs in corporate bond underwriting, Journal of Financial Economics 2 71–94
Fama E F L Fisher, M C Jensen, and R Roll, 1969, The adjustment of stock prices to new information International Economic Review 10 1–21
Friedman A 1975, Stochastic differential equations and applications Volume 1 (Academic Press, New York)
Gilbert, Lewis D and John J Gilbert, various dates Annual report of stockholder activities at annual meetings, New York
Hess Alan C, 1977 The role of investment banking in an efficient capital market, unpublished manuscript Securities and Exchange Commission, Washington, D C
History of Corporate Finance for the Decade, 1972 (Investment Dealers Digest, New York)
Ibbotson, Roger C, 1975, Price performance of common stock new issues, Journal of Financial Economics 2, 235-272
Jensen M C and W H Meckling 1976, Theory of the firm Managerial behavior, agency costs and ownership structure Journal of Financial Economics 3, 305–360
Manne, Henry G, 1966, Insider trading and the stock market (The Free Press, New York)
Merton, Robert C 1973 Theory of rational option pricing, Bell Journal of Economics and Management Science 4, 141–183
Merton, Robert C 1974 On the pricing of corporate debt The risk structure of interest rates, Journal of Finance 29
National Association of Security Dealers, NASD manual
Nelson, J R, 1965, Price effects in rights offerings Journal of Finance 20, 647–650
Pessin, A H 1973, The work of the securities industry (New York Institute of Finance, New York)
Schneider, Carl W and Joseph M Manko, 1970, Going public -- Practice, procedure and consequences, Villanova Law Review 15, 283-312
Scholes, Myron S, 1972, The market for securities Substitution versus price pressure and the effects of information on share price, Journal of Business 45, 179–211
Smith, Clifford W, Jr, 1976 Option pricing A review, Journal of Financial Economics 3, 3–51
U S Securities and Exchange Commission, 1940, Cost of flotation for small issues, 1925–1929 and 1935-1938
U S Securities and Exchange Commission, 1941, Cost of flotation for registered securities 1930–1939
U S Securities and Exchange Commission, 1944, Cost of flotation of equity securities of small companies (Statistical Series Release no 744)
U S Securities and Exchange Commission, 1949, Cost of flotation, 1945–1947
U S Securities and Exchange Commission, 1951, Cost of flotation, 1945–1949
U S Securities and Exchange Commission, 1957, Cost of flotation of corporate securities, 1951–1955
U S Securities and Exchange Commission, 1970, Cost of flotation of registered equity issues, 1963–1965
U S Securities and Exchange Commission, 1974, Cost of flotation of registered issues, 1971–1972
Van Horne, J D, 1974, Financial management and policy (Englewood Cliffs, New Jersey)
Weston, J Fred and Eugene F Brigham, 1975, Managerial finance (Hinesdale, Illinois)
Zwick, Jack and Nathaniel R Norton, III, 1970, Investment banking and underwriting, in F G Zarb and G T Kerekes, eds, The stock market handbook, 54–71

The Effects of New Equity Sales Upon Utility Share Prices

By RICHARD H. PETTWAY*

Public knowledge of a forthcoming sale of new equity by a utility company often precipitates a decline in the market price of that equity and continues to impact share prices after the sale has taken place. Such price changes are part of the real cost of selling the new issue. The market pressure costs of new equity capital have been the subject of much speculation in utility rate cases, but have received little detailed study. The author of this article has made such a study and here presents a quantitative analysis of price-return movements encountered by utility stocks in the market, after first defining market pressure as it applies particularly to the regulated utility environment. He concludes that investors clearly view a new sale of equity shares with disfavor and regulators, as well as company managements, should be concerned with the resultant decline in utility stock prices.

When a public utility decides to sell a new issue of equity capital and publicly discloses this information, share prices are thought to decline. Often these selling firms ask for an adjustment to their costs of equity capital for the effects of this market pressure upon share prices. The subsequent argument and debate about the magnitude of an adjustment for market pressure at rate hearings is well known.

The electric utility industry has been one of the largest issuers of new equity shares during the past twenty-five years. Therefore, it is surprising that there has not been much more research to determine the magnitude of market pressure of these numerous new equity sales in this industry. The objective of this article is to report on the results of an analysis of 368 equity sales by 73 different electric utilities from January 1, 1973, through December 31, 1980. The analysis will measure two effects of new common equity sales upon share prices: market pressure and sales effect. Specifically, this article will determine the magnitude of market pressure defined as the effect of the sale upon share prices which reduces the funds received by the issuing company at the sale date, and will determine the size of the sales effect defined as the total effect of the sale upon share prices from before the announcement until after the sale.

There have been studies into the size of market pressure defined as a temporary price decline in share values when a large block of shares is said to be "overhanging" the market. However, most of this research concentrates upon the price effects of new issues of industrial companies sold in the primary markets or of large blocks of existing stock sold in the secondary market [1, 2, 4, 5, 6, 9].** This literature defines market pressure as the amount of recovery in market prices *after* the issue has been sold. A review of this literature indicates either no market pressure existing in large block trades of outstanding shares, or only a small amount of pressure associated with primary market sales of new issues.

Under utility regulation, the concern is with a different definition of market pressure. Market pressure in the public utility industry is generally defined as the decline in prices *while* the issue is still overhanging, *before* it is sold. The main question is how much did the utility's stock decline in the secondary market associated with the sales announcement to the date of sale. This decline is a real cost of selling the new issue as the firm will receive only the reduced price at the sales date. An

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**Numbers in brackets refer to the list of references at the end of the article.

article by Bowyer and Yawitz (BY) [3] measured the decline in share prices between the announcement date and the sales date of 278 new equity issues of public utilities from 1973 through 1976. But that research had some obvious problems which are corrected by this study.

The first problem with BY is their definition of the announcement date (AD). They defined this critical AD as the initial Securities and Exchange Commission filing date of the issue prospectus. This may not be the true AD as often public utilities make prior announcements of their new issues to state public service commissions, to investors in the *Irving Trust Calendar*, to underwriters, or to financial analysts much earlier than the SEC filing date. This study redefines the critical announcement date through a detailed questionnaire survey of electric utility companies. Further, an analysis of price changes prior to the established announcement date for each issue will be made to determine the actual impact of new equity sales upon share prices. It is very important to measure the complete decline in market prices associated with the information about the forthcoming sale of new equity shares.

Another problem with the BY study concerns its authors' use of the Dow-Jones utility index to measure differential declines in share prices and returns. The use of this index is flawed for at least four reasons. First, the number of companies included is small, 15 firms, and only 11 are electric companies; whereas four are gas transmission and distribution companies. The inclusion of the gas companies raises serious questions concerning the similarities of risks between electric utilities tested and the companies which make up their comparison index. Second, their index does not capture the dividend portion of the return and thus only measures the changes in prices without adjusting for dividends paid. In the electric power industry, the dividend yields tend to be a high portion of the total return and the omission of dividends could impart a bias to the index. Third, if there is evidence of market pressure in new sales of equity shares by utilities as BY found, then it is certain that this market pressure is contained also in share prices of Dow-Jones utility index firms when they sold new equity shares. The effect of using an index which contains market pressure to measure the size of market pressure of a particular firm which sold new equity naturally will understate the true amount of market pressure which is present. Fourth, if utilities are impacted differently from unregulated firms, there may be an additional "industrial effect" which will not be observed by looking only at other utilities rather than a broadly based comparison index of share prices and returns.

Finally, there are some technical problems with the way that BY measured the decline in stock returns or market pressure. These problems concern the use of average residual returns versus a more correct measure (geometric residual returns) and the way BY handled underwriting costs.

Data

A questionnaire survey was conducted of the 93 New York Stock Exchange-listed, investor-owned electric utilities from which 73 usable company replies were obtained for a response rate of over 78 per cent. Each company provided all identifiable costs and critical dates for each new equity capital sale made by the firm from January 1, 1973, through December 31, 1980. The survey results contain data on 368 actual equity sales over the eight-year survey period. The data represent more than five new equity sales per company on average over the study period. The size of these equity sales ranged from $4.7 million to $198 million with a mode sale value in a range between $30 and $49.9 million per issue. The frequency of the issues over the eight years of the survey shows that 1975 was the most popular year followed by 1976 and 1980. Yet, the individual year variation was not dramatic as the range over the eight years was from a low of 37 issues in 1974 to a high of 64 issues in 1975. Eighty-two per cent of the sales were through negotiated underwriting, 16 per cent through competitive bidding, and 2 per cent through rights offerings. See [7] for a thorough review of the data and details on the flotation costs of these issues.

Data on realized share returns including dividends for each company were obtained on a daily basis for a period which began sixty-five trading days before the announcement date and ended thirty trading days after the sale date (SD). Thus, company returns were obtained from a fixed period prior to the AD through a fixed period after the SD for each issue. It is best to think of these data sets as 368 separate arrays of returns. Because the interim time period between the AD and the subsequent SD varied for each issue, the number of return observations in each array is different. Each collected array of returns is unique to the particular announcement and issue dates and is not impacted by other equity sales of the same company.

Methodology

In order to control for risk, to adjust for movements in general prices and returns, and to reduce estimating bias, a two-stage regression process was used to measure the effects of new equity sales upon share returns and prices. First, during the estimating period, the market regression model (1) was applied to a firm's daily equity returns over a uniform estimating period which began sixty-five trading days prior to the AD and ended fifteen days before the AD for each issue. The market regression model asserts that:

$$\tilde{R}_{i,t} = \hat{a}_i + \hat{B}_i \tilde{R}_{m,t} + \tilde{e}_{i,t} \qquad (1)$$

where $\tilde{R}_{i,t}$ is the daily return including dividends of the issuing company for equity issue i — i.e., one to 368 — at time t; where daily returns of the issuing company concerning issue i are defined as $(P_{i,t} + D_{i,t} - P_{i,t-1}) / (P_{i,t-1})$; P is the price and D is the dividend per share; $\tilde{R}_{m,t}$ is the daily return at time t on a market portfolio for comparison; $\hat{a}_i$ and $\hat{B}_i$ are the estimated parameters of the market model; and $\tilde{e}_{i,t}$ is the error term of the model.

In order to make comparisons, an electric utility portfolio index of returns was created over the period January 1, 1973, through December 31, 1980, containing an equal investment in each of 73 electric companies which sold equity during the period. It is a daily returns index including dividends and provides the average return for each day on a portfolio consisting of an equal dollar investment in each of the 73 electric utilities.

Thus, the first stage uses an estimating period of fifty trading days, approximately two and one-half months, to determine the parameters of the market regression model. The second stage then applies these estimated parameters to the returns series during the subsequent test period after the estimating period in each array in order to calculate the expected returns for each company on each issue i using:

$$\hat{R}_{i,t} = \hat{a}_i + \hat{B}_i \hat{R}_{m,t} \qquad (2)$$

where $\hat{R}_{i,t}$ is the expected return for the issuing company associated with issue i at time t. Then residual returns during the test period are obtained by comparing the actual versus the predicted returns using:

$$\tilde{R}_{i,t} - \hat{R}_{i,t} = \hat{u}_{i,t} \qquad (3)$$

where $\hat{u}_{i,t}$ is the daily residual return of the issuing company for issue i at time t.

In order to display these residual returns properly, a decision must be made of how to combine the individual company residuals centered on a common date during the test period. The method of combining residuals used by Bowyer and Yawitz is called cumulative average residual or CAR. This method would find the average residual return of all issues on a specific day relative to the common AD or SD and would accumulate these averages over the period in an additive way. A different way of combining residual returns, average geometric residual return (AGRR), was chosen for this study. It is a theoretically better measure of residual returns over time than CAR. AGRR does not use the average residual returns on a specific date but takes the individual issue residual ($\hat{u}_{i,t}$) from (3) and converts it into a price relative for each t and then forms a geometric return series by multiplying successive price relatives from fourteen days prior to AD to the end of the residual data for each company using formula (4). Thus, a geometric return series which precisely measures the change in investment worth for each individual issue is created. At any point in time relative to the common dates, AD and SD, the AGRR was determined as the numeric average of the geometric returns up to that point in time of all issues using formula (5).

$$GRR_{i,T} = \prod_{t=1}^{T} (1 + u_{i,t}) \qquad (4)$$

$$AGRR_T = \sum_{i=1}^{N} GRR_{i,T}/N \qquad (5)$$

where i is the issue number, t is time, T is the specific point in time (T=1, 2, 3, . . . total number of observations in the test period which was from fourteen days before the AD until thirty trading days after the SD), and N is the number of issues. For further details concerning the specifics of the methodology employed see [8].

In observing the pattern of these residuals over the test period, it is important to be able to use common definitions to describe their movements. "Market pressure" is defined as the decline of share prices and average geometric residual returns from fourteen days before the AD until the SD. "Sales effect" is defined as the change in share prices and AGRRs from fourteen days before the AD until thirty trading days after the SD. This sales effect would be the net change over the entire test period from before the announcement until well after the sale.

Price-Return Movements

Because the number of days between the AD and the SD are not identical for each issue, arrays of residual returns had to be centered on two separate common dates. The first common date is the AD and then data are centered on the common SD. To begin measuring any price effects of these new equity sales, the study first observed movements in residual returns when the data are centered on the common AD.

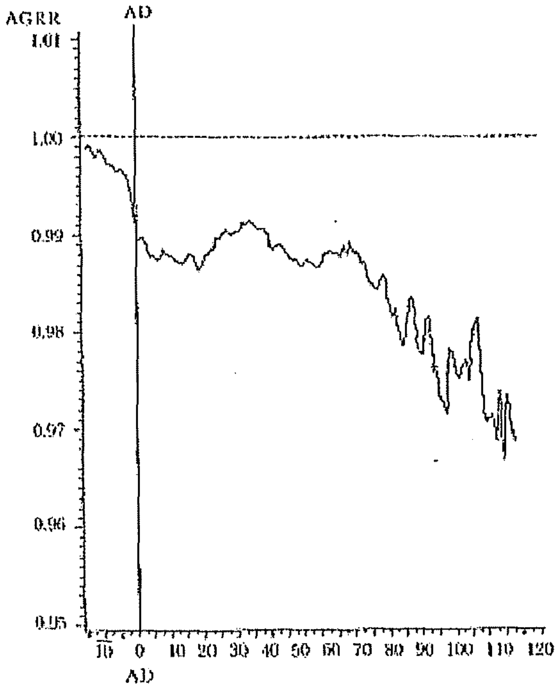
Common Announcement Date

Figure 1 illustrates the AGRRs derived from the use of the electric utility market index of returns for comparison.[†] The derived residuals are accumulated for 128 days starting fourteen days before the announcement date. All issues are centered on the AD. The trend of the AGRRs are clearly downward and below one during the entire span of 128 days. The downward trend is most noticeable immediately before and around the AD and is then followed by a period of relative stability. During this initial decline, share prices had fallen between one per cent and 1.4 per cent. The downward trend resumes again beginning about sixty-seven days after the AD. The latter downward trend may be associated with the SD, but since these data are centered on the AD, the SD did not occur at a common point in time in the data. Further, because SD is not a common point in the data, the amount of market pressure cannot be measured from the data in this format.

Panel 1 of the accompanying table contains statistical summaries of changes in AGRRs over the entire period shown in Figure 1. It is clear from the data that the change over the 128-day period centered on the AD was a negative 3.019 per cent, indicating a sales effect of this

[†]If there were no effects of new equity sales upon electric utilities which sold new shares, then the AGRRs shown on Figure 1 would be very close to one over time. A detrimental effect and a relative decline in share prices would be represented as a decline in AGRRs below one. A favorable effect would be represented as an increase in AGRRs. Also notice that the x-axis displays time with negative numbers as days before the AD and positive numbers as days after the AD. The AD, or centering date, is designated as zero.

FIGURE 1
AGRR CENTERED ON ANNOUNCEMENT DATE
(UTILITY INDEX)



magnitude. Thus, comparing the returns over the same time period of an electric utility which sold new equity shares with returns of a portfolio of electric companies which also sold equity during the eight-year study period, there appears to have been a substantial and significant decline or sales effect of −3 per cent. There appear to be two periods of rapid declines, one just before and around the AD and another which appears to begin about sixty-seven days after the AD. Measuring the initial decline during a period from fourteen days before the AD to fourteen days after the AD, the specific decline was −1.2 per cent. This first major decline which begins before the AD suggests that the market was either anticipating the new equity sale or obtaining information about the new equity sale just prior to the public announcement.

Because of the decline in these residuals, it is clear that the market considered the potential new equity sale as detrimental to the future prospects of the current equity holders of the selling firm. Since the decline begins before the AD, this article measures more precisely the *total* decline in share prices than did the work of Bowyer and Yawitz.

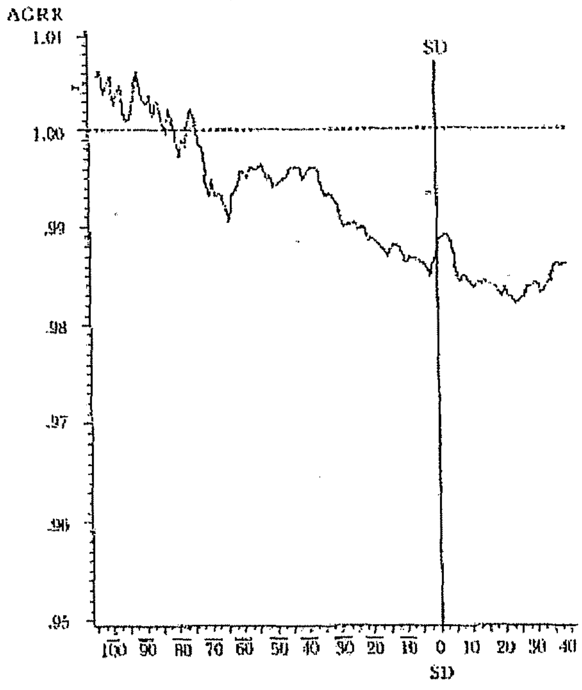
Common Sales Date

Figure 2 shows the AGRRs using the electric utility returns index for comparison with all issues centered on the SD. This plot is clearly one whose trend is also downward across the entire time period, although it appears not to begin its major decline until eighty-five to ninety days prior to the SD.

In Panel 2 of the table are found the summary statistics describing the magnitudes of the AGRRs shown on Figure 2. The changes or sales effect during the period from fourteen days before the AD to after the SD over 147 days was −2.041 per cent.

Panel 3 of the table contains the magnitudes of AGRRs shown on Figure 2 but stopping at the SD. This decline in relative share prices and returns, called market pressure, is caused by the equity sale and is the discount required to sell the new issue. These costs of new equity issues were 1.893 per cent on average. Thus, market prices of shares of electric utilities which sold new equity declined by about 1.9 per cent from before the AD until the SD over 104 days. This is the decline in price that the firm did not receive when it sold new equity shares at the SD and is the market pressure of the new equity issue.

FIGURE 2
AGRR CENTERED ON SALE DATE
(UTILITY INDEX)



EFFECTS OF NEW EQUITY SALES OF UTILITIES UPON SHARE PRICES
CHANGES IN THE AVERAGE GEOMETRIC RESIDUAL RETURNS

368 New Equity Issues of 73 Electric Utilities from January 1, 1973, through December 31, 1980

Using the Utility Index

Measurements	*Panel 1* Centered on AD (Sales Effect)	*Panel 2* Centered on SD (Sales Effect)	*Panel 3* Centered and Ending on SD (Market Pressure)
Change over the Period	−3.019%	−2.041%	−1.893%
Length of Period (Days)	128	147	104
Change from −14 AD to +14 AD	−1.170%		
Length of Period (Days)	29		

Summary and Conclusions

When electric utilities sold new equity shares between January 1, 1973, and December 31, 1980, the share prices of these companies were depressed downward because of the sale. This downward movement or market pressure measured from before the announcement date to the sales date of the new issue was −1.9 per cent when compared with returns of other electric utilities which sold new equity regularly. Further, a sales effect ranging from −3 per cent to −2 per cent was found over the period from before the announcement date until after the sales date depending upon whether the data were centered on the AD or on the SD.

These averages are conservative and the minimum estimated average declines as they were derived from using a return index of comparison (electric utility) which itself contains the effects of market pressure. Further, the use of another index of return for comparison which was composed of regulated and unregulated firms would substantially raise these average costs. (In fact, if the comparison were to be made against the return of all equities listed on the New York and American stock exchanges over the same time period, the average estimate for market pressure would rise to −3 per cent and the average estimates for sales effect would rise to −4.4 per cent centered on the AD to −3.6 per cent centered on the SD. See [8] for details.)

The sizeable sales effect over the entire period from before the announcement date to after the sales date using the portfolio of electric companies for comparison provides direct evidence that share prices of electric utilities which sell new equity continue to decline after the sale has taken place. This condition may be explained as the impact of other factors than market pressure alone upon share prices. Perhaps some of these factors are due to the investors' perceptions of increased dilution problems caused by regulatory lag and regulatory risk associated with these public utilities not being allowed a rate of return on new equity equal to the investors' required rate of return over the eight-year survey period.

Even though the exact causes are not known precisely, it is definitely clear that investors view the new sale of equity shares with disfavor and that the new equity sale results in a substantial decline in equity prices. Public utility regulators should be concerned with these impacts of new equity sales upon share prices and returns and attempt to make proper adjustments in the allowed rate of return to offset or eliminate these effects in the future.

References

1. "Market Pressure: The Sales of New Common Equity and Rate of Return Regulation," by Raymond Armknecht, Fred Grygiel, and Patrick Hess, "Proceedings of the Business and Economic Statistics Section of the American Statistical Association," 1974, pp. 80-91.

2. "The Effect of the Size of Public Offerings of Common Stocks upon Preoffering Stock Prices," by Lee Bodenhamer, unpublished dissertation, Harvard University, Graduate School of Business, May, 1968.

3. "The Effect of New Equity Issues on Utility Stock Prices," by John W. Bowyer, Jr., and Jess B. Yawitz, 105 PUBLIC UTILITIES FORTNIGHTLY 25, May 22, 1980.

4. "On the Existence, Measurement, and Economic Significance of Market Pressure in the Pricing of New Equity Shares," by Robert E. Evans, unpublished dissertation, University of Wisconsin-Madison, 1978.

5. "Price Impacts of Block Trading on the New York Stock Exchange," by Alan Kraus and Hans Stoll, *Journal of Finance*, June, 1972, pp. 569-588.

6. "New Issue Stock Price Behavior," by J. G. McDonald and A. K. Fisher, *Journal of Finance*, March, 1972, pp. 97-102.

7. "A Note on the Flotation Costs of New Equity Capital Issues of Electric Companies," by Richard H. Pettway, 109 PUBLIC UTILITIES FORTNIGHTLY 68, March 18, 1982.

8. "Impacts of New Equity Sales upon Electric Utility Share Prices," by Richard H. Pettway and Robert C. Radcliffe, working paper series, Public Utility Research Center, Graduate School of Business, University of Florida, Gainesville, Florida, May, 1983.

9. "The Market for Securities: Substitution versus Price Pressure and the Effects of Information on Share Prices," by Myron S. Scholes, *Journal of Business*, April, 1972, pp. 179-211.

Utilities Raise Their Capital Appropriations

The nation's investor-owned utilities appropriated $7.2 billion (seasonally adjusted) for new plant and equipment in the final quarter of 1983, up 25 per cent over the unusually low figure recorded in the third quarter, the Conference Board reported in April. Both the gas and electric utilities shared in this fourth-quarter gain. (Capital appropriations are authorizations to spend money in the future for new plant and equipment. Appropriations are the first step in the capital investment process, preceding the ordering of equipment, the letting of construction contracts, and finally the actual expenditures. Appropriations are considered to be a leading indicator for capital spending.)

Electric utility appropriations rose to $5.8 billion in the fourth quarter, their first quarterly increase since the third quarter of 1982. Cancellations of previously approved projects were widespread, however, amounting to $2.7 billion in the final quarter of 1983.

Gas utility appropriations climbed to $1.4 billion in the fourth quarter, a 68 per cent jump over the third quarter. It was the highest quarterly total recorded last year. For the full year, however, the gas utilities appropriated only $4.4 billion, down by a third from 1982, and canceled a record $1.3 billion worth of earlier-approved projects.

Actual capital spending by the investor-owned utilities fell to $8.3 billion in the fourth quarter, an 8 per cent dip from the third quarter. The electric utilities accounted for all of the fourth-quarter decline. For 1983 as a whole, the electric utilities spent a record $32.2 billion on new plant and equipment, up 3 per cent over 1982. Gas utility expenditures amounted to $3.5 billion in 1983, down 30 per cent from 1982.

CASE NO. 2009-00354
ATTACHMENT 4
TO AG DR SET NO. 1
QUESTION NO. 1-73

Aspen Publishers

Blue Chip Financial Forecasts®

Top Analysts' Forecasts Of U.S. And Foreign Interest Rates, Currency Values And The Factors That Influence Them

Vol. 28, No. 8, August 1, 2009

Wolters Kluwer
Law & Business

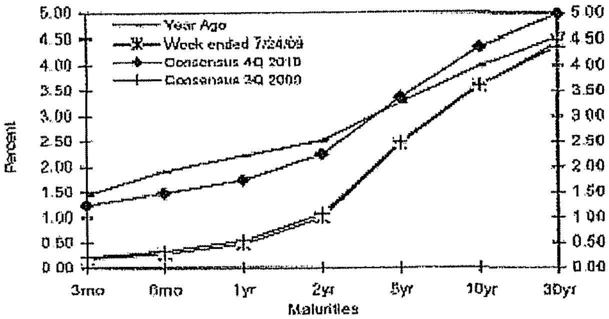
Consensus Forecasts Of U.S. Interest Rates And Key Assumptions[1]

	History								Consensus Forecasts-Quarterly Avg.					
	Average For Week End				Average For Month			Latest Q						
Interest Rates	July 24	July 17	July 10	July 3	June	May	Apr.	2Q 2009	3Q 2009	4Q 2009	1Q 2010	2Q 2010	3Q 2010	4Q 2010
Federal Funds Rate	0.15	0.14	0.17	0.19	0.21	0.18	0.15	0.18	0.2	0.2	0.2	0.4	0.8	1.1
Prime Rate	3.25	3.25	3.25	3.25	3.25	3.25	3.25	3.25	3.2	3.2	3.3	3.5	3.8	4.2
LIBOR, 3-mo.	0.50	0.51	0.52	0.58	0.62	0.82	1.12	0.85	0.7	0.7	0.7	0.9	1.2	1.6
Commercial Paper, 1-mo.	0.18	0.18	0.20	0.18	0.18	0.22	0.22	0.21	0.3	0.3	0.4	0.6	0.9	1.4
Treasury bill, 3-mo.	0.19	0.18	0.19	0.18	0.18	0.18	0.16	0.19	0.2	0.3	0.4	0.5	0.9	1.2
Treasury bill, 6-mo.	0.28	0.28	0.27	0.34	0.31	0.30	0.35	0.36	0.3	0.4	0.5	0.8	1.1	1.5
Treasury bill, 1 yr.	0.47	0.48	0.46	0.53	0.51	0.50	0.55	0.57	0.5	0.6	0.8	1.1	1.4	1.7
Treasury note, 2 yr.	0.99	0.99	0.94	1.06	1.18	0.93	0.93	1.01	1.1	1.2	1.4	1.6	1.9	2.2
Treasury note, 5 yr.	2.45	2.43	2.31	2.50	2.71	2.13	1.86	2.13	2.5	2.6	2.7	2.9	3.1	3.4
Treasury note, 10 yr.	3.59	3.55	3.42	3.53	3.72	3.29	2.93	3.16	3.6	3.7	3.8	4.0	4.2	4.4
Treasury note, 30 yr.	4.46	4.42	4.27	4.32	4.52	4.23	3.76	3.97	4.4	4.4	4.6	4.7	4.8	5.0
Corporate Aaa bond	5.44	5.44	5.34	5.40	5.61	5.54	5.39	5.50	5.5	5.6	5.6	5.7	5.8	6.0
Corporate Baa bond	7.15	7.19	7.10	7.18	7.50	8.06	8.39	8.10	7.3	7.4	7.4	7.5	7.5	7.6
State & Local bonds	4.69	4.68	4.71	4.81	4.81	4.56	4.76	4.85	4.8	4.8	4.9	4.9	5.0	5.1
Home mortgage rate	5.20	5.14	5.20	5.32	5.42	4.86	4.81	5.08	5.3	5.3	5.4	5.6	5.8	5.9
	History								Consensus Forecasts-Quarterly Avg.					
Key Assumptions	3Q 2007	4Q 2007	1Q 2008	2Q 2008	3Q 2008	4Q 2008	1Q 2009	*2Q* 2009*	3Q 2009	4Q 2009	1Q 2010	2Q 2010	3Q 2010	4Q 2010
Major Currency Index	77.0	73.3	72.0	70.9	73.5	81.3	82.7	79.4	77.6	77.3	77.4	77.1	77.5	77.5
Real GDP	4.8	-0.2	0.9	2.8	-0.5	-6.3	-5.5	*-1.3*	0.9	1.9	2.3	2.7	2.7	2.9
GDP Price Index	1.5	2.8	2.6	1.1	3.9	0.5	2.8	*0.9*	1.4	1.3	1.5	1.5	1.7	1.7
Consumer Price Index	2.4	5.8	4.5	4.5	6.2	-8.3	-2.4	1.3	2.4	1.6	1.7	1.7	2.0	2.1

Individual panel members' forecasts are on pages 4 through 9. Historical data for interest rates except LIBOR is from Federal Reserve Release (FRSR) H.15. LIBOR quotes available from *The Wall Street Journal*. Definitions reported here are same as those in FRSR H.15. Treasury yields are reported on a constant maturity basis. Historical data for the U.S. Federal Reserve Board's Major Currency Index is from FRSR H.10 and G.5. Historical data for Real GDP and GDP Chained Price Index are from the Bureau of Economic Analysis (BEA). Consumer Price Index (CPI) history is from the Department of Labor's Bureau of Labor Statistics (BLS). *Figures for 2Q 2009 Real GDP and GDP Chained Price Index are consensus forecasts based on a special question asked of the panelists this month (see page 14)*

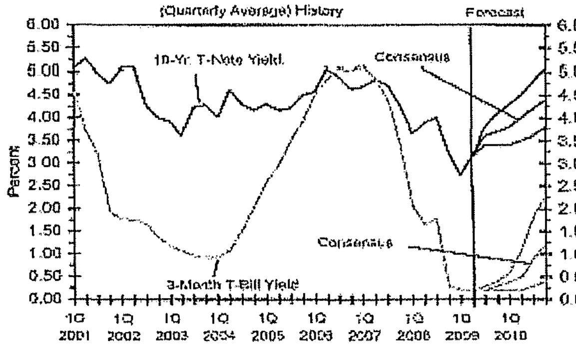
U.S. Treasury Yield Curve

Week ended July 24, 2009 and Year Ago vs. 3Q 2009 and 4Q 2010 Consensus Forecasts



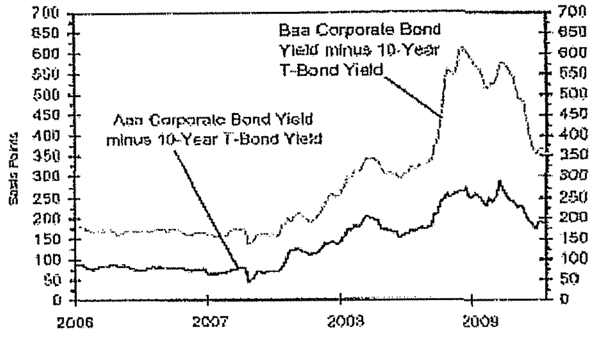
U.S. 3-Mo. T-Bills & 10-Yr. T-Note Yield

(Quarterly Average) History Forecast



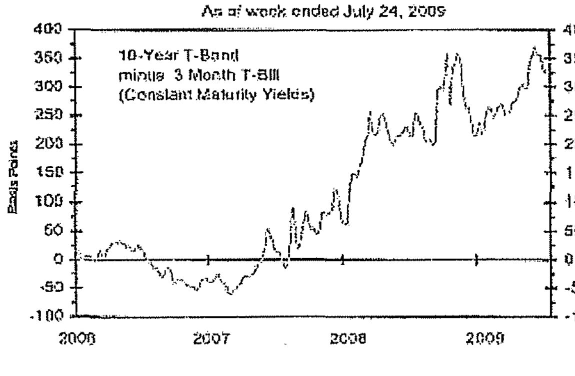
Corporate Bond Spreads

As of week ended July 24, 2009



U.S. Treasury Yield Curve

As of week ended July 24, 2009



*The Capital Asset Pricing Model: Some Empirical Tests**

CASE NO. 2009-00354
ATTACHMENT 5
TO AG DR SET NO. 1
QUESTION NO. 1-73

FISCHER BLACK,† MICHAEL C. JENSEN,‡

AND

MYRON SCHOLES§

I. Introduction and Summary

Considerable attention has recently been given to general equilibrium models of the pricing of capital assets. Of these, perhaps the best known is the mean-variance formulation originally developed by Sharpe [1964] and Treynor [1961], and extended and clarified by Lintner [1965a, b], Mossin [1966], Fama [1968a, b], and Long [1972]. In addition Treynor [1965], Sharpe [1966], and Jensen [1968, 1969] have developed portfolio evaluation models which are either based on this asset pricing model or bear a close relation to it. In the development of the asset pricing model it is assumed that (1) all investors are single period risk-averse utility of terminal wealth maximizers and can choose among portfolios solely on the basis of mean and variance, (2) there are no taxes or

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transactions costs, (3) all investors have homogeneous views regarding the parameters of the joint probability distribution of all security returns, and (4) all investors can borrow and lend at a given riskless rate of interest. The main result of the model is a statement of the relation between the expected risk premiums on individual assets and their "systematic risk." The relationship is

$$E(\tilde{R}_j) = E(\tilde{R}_M)\beta_j \tag{1}$$

where the tildes denote random variables and

$$E(\tilde{R}_j) = \frac{E(\tilde{P}_t) - P_{t-1} + E(\tilde{D}_t)}{P_{t-1}} - r_{Ft} = \text{expected excess returns}$$

on the jth asset

$\tilde{D}_t$ = dividends paid on the jth security at time t

r_{Ft} = the riskless rate of interest

$E(\tilde{R}_M)$ = expected excess returns on a "market portfolio" consisting of an investment in every asset outstanding in proportion to its value

$$\beta_j = \frac{\text{cov}(\tilde{R}_j, \tilde{R}_M)}{\sigma^2(\tilde{R}_M)} = \text{the "systematic" risk of the } j\text{th asset.}$$

Relation 1 says that the expected excess return on any asset is directly proportional to its β. If we define α_j as

$$\alpha_j = E(\tilde{R}_j) - E(\tilde{R}_M)\beta_j$$

then (1) implies that the α on every asset is zero.

If empirically true, the relation given by (1) has wide-ranging implications for problems in capital budgeting, cost benefit analysis, portfolio selection, and for other economic problems requiring knowledge of the relation between risk and return. Evidence presented by Jensen [1968, 1969] on the relationship between the expected return and systematic risk of a large sample of mutual funds suggests that (1) might provide an adequate description of the relation between risk and return for securities. On the other hand, evidence presented by Douglas [1969], Lintner [1965], and most recently Miller and Scholes [1972] seems to indicate the model does not provide a complete description of the structure of security returns. In particular, the work done by Miller and Scholes suggests that the α's on individual assets depend in a systematic way on their β's: that high-beta assets tend to have negative α's, and that low-beta stocks tend to have positive α's.

Our main purpose is to present some additional tests of this asset pricing model which avoid some of the problems of earlier studies and which, we believe, provide additional insights into the nature of the structure of security returns. All previous direct tests of the model have been conducted using cross-sectional methods; primarily regression of $\bar{R}_j$, the mean excess return over a time interval for a set of securities on estimates of the systematic risk, $\hat{\beta}_j$, of each of the securities. The equation

$$\bar{R}_j = \gamma_0 + \gamma_1\hat{\beta}_j + \tilde{u}_j$$

was estimated, and contrary to the theory, γ_0 seemed to be significantly different from zero and γ_1 significantly different from $\bar{R}_M$, the slope predicted by the model. We shall show in Section III that, because of the structure of the process which appears to be generating the data, these cross-sectional tests of significance can be misleading and therefore do not provide direct tests of the validity of (1). In Section II we provide a more powerful time series test of the validity of the model, which is free of the difficulties associated with the cross-sectional tests. These results indicate that the usual form of the asset pricing model as given by (1) does not provide an accurate description of the structure of security returns. The tests indicate that the expected excess returns on high-beta assets are lower than (1) suggests and that the expected excess returns on low-beta assets are higher than (1) suggests. In other words, that high-beta stocks have negative α's and low-beta stocks have positive α's.

The data indicate that the expected return on a security can be represented by a two-factor model such as

$$E(\tilde{r}_j) = E(\tilde{r}_Z)(1-\beta_j) + E(\tilde{r}_M)\beta_j \tag{2}$$

where the r's indicate total returns and $E(\tilde{r}_Z)$ is the expected return on a second factor, which we shall call the "beta factor," since its coefficient is a function of the asset's β. After we had observed this phenomenon, Black [1970] was able to show that relaxing the assumption of the existence of riskless borrowing and lending opportunities provides an asset pricing model which implies that, in equilibrium, the expected return on an asset will be given by (2). His results furnish an explicit definition of the beta factor, $\tilde{r}_Z$, as the return on a portfolio that has a zero covariance with the return on the market portfolio $\tilde{r}_M$. Although this model is entirely consistent with our empirical results (and provides a convenient interpretation of them), there are perhaps other plausible hypotheses consistent with the data (we shall briefly discuss several in Section V). We hasten to add that we have not attempted here to supply any direct tests of these alternative hypotheses.

The evidence presented in Section II indicates the expected excess return on an asset is not strictly proportional to its β, and we believe that this evidence, coupled with that given in Section IV, is sufficiently strong to warrant rejection of the traditional form of the model given by (1). We then show in Section III how the cross-sectional tests are subject to measurement error bias, provide a solution to this problem through grouping procedures, and show how cross-sectional methods are relevant to testing the expanded two-factor form of the model. Here we find that the evidence indicates the existance of a linear relation between risk and return and is therefore consistent with a form of the two-factor model which specifies the realized returns on each asset to be a linear function of the returns on the two factors $\tilde{r}_Z$ and $\tilde{r}_M$,

$$\tilde{r}_j = \tilde{r}_Z(1-\beta_j) + \tilde{r}_M\beta_j + \tilde{w}_j \qquad (2)$$

The fact that the α's of high-beta securities are negative and that the α's of low-beta securities are positive implies that the mean of the beta factor is greater than r_F. The traditional form of the capital asset pricing model as expressed by (1), could hold exactly, even if asset returns were generated by (2'), if the mean of the beta factor were equal to the risk-free rate. We show in Section IV that the mean of the beta factor has had a positive trend over the period 1931–65 and was on the order of 1.0 to 1.3% per month in the two sample intervals we examined in the period 1948–65. This seems to have been significantly different from the average risk-free rate and indeed is roughly the same size as the average market return of 1.3 and 1.2% per month over the two sample intervals in this period. This evidence seems to be sufficiently strong enough to warrant rejection of the traditional form of the model given by (1). In addition, the standard deviation of the beta factor over these two sample intervals was 2.0 and 2.2% per month, as compared with the standard deviation of the market factor of 3.6 and 3.8% per month. Thus the beta factor seems to be an important determinant of security returns.

II. Time Series Tests of the Model

A. Specification of the Model. Although the model of (1) which we wish to test is stated in terms of expected returns, it is possible to use realized returns to test the theory. Let us represent the returns on any security by the "market model" originally proposed by Markowitz [1959] and extended by Sharpe [1963] and Fama [1968a]

$$\tilde{R}_j = E(\tilde{R}_j) + \beta_j\tilde{R}'_M + \tilde{e}_j \qquad (3)$$

where $\tilde{R}'_M = \tilde{R}_M - E(\tilde{R}_M) =$ the "unexpected" excess market return, and $\tilde{R}'_M$ and $\tilde{e}_j$ are normally distributed random variables that satisfy:

$$E(\tilde{R}'_M) = 0 \qquad (4a)$$

$$E(\tilde{e}_j) = 0 \qquad (4b)$$

$$E(\tilde{e}_j\tilde{R}'_M) = 0 \qquad (4c)$$

The specifications of the market model, extensively tested by Fama et al. [1969] and Blume [1968], are well satisfied by the data for a large number of securities on the New York Stock Exchange. The only assumption violated to any extent is the normality assumption[1]—the estimated residuals seem to conform to the infinite variance members of the stable class of distributions rather than the normal. There are those who would explain these discrepancies from normality by certain nonstationarities in the distributions (cf. Press [1967]), which still yield finite variances. However, Wise [1963] has shown that the least-squares estimate of β_j in (3) is unbiased (although not efficient) even if the variance does not exist, and simulations by Blattberg and Sargent [1968] and Fama and Babiak [1968] also indicate that the least-squares procedures are not totally inappropriate in the presence of infinite variance stable distributions. For simplicity, therefore, we shall ignore the nonnormality issues and continue to assume normally distributed random variables where relevant.[2] However, because of these problems caution should be exercised in making literal interpretations of any significance tests.

Substituting from (1) for $E(\tilde{R}_j)$ in (3) we obtain

$$\tilde{R}_j = \tilde{R}_M\beta_j + \tilde{e}_j \qquad (5)$$

where $\tilde{R}_M$ is the ex post excess return on the market portfolio over the holding period of interest. If assets are priced in the market such that (1) holds over each short time interval (say a month), then we can test the traditional form of the model by adding an intercept α_j to (5) and subscripting each of the variables by t to obtain

$$\tilde{R}_{jt} = \alpha_j + \beta_j \tilde{R}_{Mt} + \tilde{e}_{jt} \qquad (6)$$

which, given the assumptions of the market model, is a regression equation. If the asset pricing and the market models given by (1), (3), and (4) are valid, then the intercept α_j in (6) will be zero. Thus a direct test of the model can be obtained by estimating (6) for a security over some time period and testing to see if α_j is significantly different from zero.[3,4]

B. An Aggregation Problem. The test just proposed is simple but inefficient, since it makes use of information on only a single security whereas data is available on a large number of securities. We would like to design a test that allows us to aggregate the data on a large number of securities in an efficient manner. If the estimates of the α_j's were independent with normally distributed residuals, we could proceed along the lines outlined by Jensen [1968] and compare the frequency distributions of the "t" values for the intercepts with the theoretical distribution. However, the fact that the e_{jt} are not cross-sectionally independent, (that is, $E(\tilde{e}_{jt}\tilde{e}_{it}) \neq 0$ for $i \neq j$, cf. King [1966]); makes this procedure much more difficult.

One procedure for solving this problem which makes appropriate allowance for the effects of the nonindependence of the residuals on the standard error of estimate of the average coefficient, $\bar{\alpha}$, is to run the tests on grouped data. That is, we form portfolios (or groups) of the individual securities and estimate (6) defining $\tilde{R}_{Kt}$ to be the average return on all securities in the Kth portfolio for time t. Given this definition of R_{Kt}, $\hat{\beta}_K$ will be the average risk of the securities in the portfolio and $\hat{\alpha}_K$ will be the average intercept. Moreover, since the residual variance from this regression will incorporate the effects of any cross-sectional interdependencies in the $\tilde{e}_{jt}$ among the securities in each portfolio, the standard error of the intercept $\hat{\alpha}_K$ will appropriately incorporate the nonindependence of $\tilde{e}_{jt}$.

In addition, we wish to group our securities such that we obtain the maximum possible dispersion of the risk coefficients, β_K. If we were to construct our portfolios by using the ranked values of the $\hat{\beta}_j$, we would introduce a selection bias into the procedure. This would occur because those securities

entering the first or high-beta portfolio would tend to have positive measurement errors in their $\hat{\beta}_j$, and this would introduce positive bias in $\hat{\beta}_K$, the estimated portfolio risk coefficient. This positive bias in $\hat{\beta}_K$ will, of course, introduce a negative bias in our estimate of the intercept, $\hat{\alpha}_K$, for that portfolio. On the other hand, the opposite would occur for the lowest beta portfolio; its $\hat{\beta}_K$ would be negatively biased, and therefore our estimate of the intercept for this low-risk portfolio would be positively biased. Thus even if the traditional model were true, this selection bias would tend to cause the low-risk portfolios to exhibit positive intercepts and high-risk portfolios to exhibit negative intercepts. To avoid this bias, we need to use an instrumental variable that is highly correlated with $\hat{\beta}_j$, but that can be observed independently of $\hat{\beta}_j$. The instrumental variable we have chosen is simply an independent estimate of the β of the security obtained from past data. Thus when we estimate the group risk parameter on sample data not used in the ranking procedures, the measurement errors in these estimates will be independent of the errors in the coefficients used in the ranking and we therefore obtain unbiased estimates of $\hat{\beta}_K$ and $\hat{\alpha}_K$.

C. The Data. The data used in the tests to be described were taken from the University of Chicago Center for Research in Security Prices Monthly Price Relative File, which contains monthly price, dividend, and adjusted price and dividend information for all securities listed on the New York Stock Exchange in the period January, 1926–March, 1966. The monthly returns on the market portfolio R_{Mt} were defined as the returns that would have been earned on a portfolio consisting of an equal investment in every security listed on the NYSE at the beginning of each month. The risk-free rate was defined as the 30-day rate on U.S. Treasury Bills for the period 1948–66. For the period 1926–47 the dealer commercial paper rate[5] was used because Treasury Bill rates were not available.

D. The Grouping Procedure

1. *The ranking procedure.* Ideally we would like to assign the individual securities to the various groups on the basis of the ranked β_j (the true coefficients), but of course these are unobservable. In addition we cannot assign them on the basis of the $\hat{\beta}_j$, since this would introduce the selection bias prob-

lems discussed previously. Therefore, we must use a ranking procedure that is independent of the measurement errors in the $\hat{\beta}_j$. One way to do this is to use part of the data—in our case five years of previous monthly data—to obtain estimates $\hat{\beta}_{j0}$, of the risk measures for each security. The ranked values of the $\hat{\beta}_{j0}$ are used to assign membership to the groups. We then use data from a subsequent time period to estimate the group risk coefficients $\hat{\beta}_K$, which then contain measurement errors for the individual securities, which are independent of the errors in $\hat{\beta}_{j0}$ and hence independent of the original ranking and independent among the securities in each group.

2. *The stationarity assumptions.* The group assignment procedure just described will be satisfactory as long as the coefficients β_j are stationary through time. Evidence presented by Blume [1968] indicates this assumption is not totally inappropriate, but we have used a somewhat more complicated procedure for grouping the firms which allows for any nonstationarity in the coefficients through time.

We began by estimating the coefficient β_j, (call this estimate $\hat{\beta}_{j0}$) in (6) for the five-year period January, 1926–December, 1930 for all securities listed on the NYSE at the beginning of January 1931 for which at least 24 monthly returns were available. These securities were then ranked from high to low on the basis of the estimates $\hat{\beta}_{j0}$, and were assigned to ten portfolios[6]—the 10% with the largest $\hat{\beta}_{j0}$ to the first portfolio, and so on. The return in each of the next 12 months for each of the ten portfolios was calculated. Then the entire process was repeated for all securities listed as of January, 1932 (for which at least 24 months of previous monthly returns were available) using the immediately preceding five years of data (if available) to estimate new coefficients to be used for ranking and assignment to the ten portfolios. The monthly portfolio returns were again calculated for the next year. This process was then repeated for January, 1933, January, 1934, and so on, through January, 1965.

In this way we obtained 35 years of monthly returns on ten portfolios from the 1,952 securities in the data file. Since at each stage we used all listed securities for which at least 24 months of data were available in the immediately preceding five-year period, the total number of securities used in the analysis varied through time ranging from 582 to 1,094, and thus the number of securities contained in each portfolio changed from year to year.[7] The total number of securities

from which the portfolios were formed at the beginning of each year is given in Table 1. Each of the portfolios may be thought of as a mutual fund portfolio, which has an identity of its own, even though the stocks it contains change over time.

TABLE 1
Total Number of Securities Entering All Portfolios, by Year

Year	Number of Securities	Year	Number of Securities
1931	582	1949	893
1932	673	1950	928
1933	688	1951	943
1934	653	1952	966
1935	676	1953	994
1936	674	1954	1000
1937	666	1955	1006
1938	690	1956	994
1939	718	1957	994
1940	743	1958	1000
1941	741	1959	995
1942	757	1960	1021
1943	772	1961	1014
1944	778	1962	1024
1945	773	1963	1056
1946	791	1964	1081
1947	812	1965	1094
1948	842		

E. The Empirical Results

1. *The entire period.* Given the 35 years of monthly returns on each of the ten portfolios calculated as explained previously, we then calculated the least-squares estimates of the parameters α_K and β_K in (6) for each of the ten portfolios ($K = 1, \ldots, 10$) using all 35 years of monthly data (420 observations). The results are summarized in Table 2. Portfolio number 1 contains the highest-risk securities and portfolio number 10 contains the lowest-risk securities. The estimated risk coefficients range from 1.561 for portfolio 1 to 0.499 for portfolio 10. The critical intercepts, the $\hat{\alpha}_K$, are given in the second line of Table 2 and the Student "t" values are given directly below them. The correlation between the portfolio returns and the market returns, $r(\tilde{R}_K, \tilde{R}_M)$, and the autocorrelation of the residuals, $r(\tilde{e}_t, \tilde{e}_{t-1})$, are also given in Table 2. The autocorrelation appears to be quite small and the correlation between the portfolio and market returns are, as expected, quite

TABLE 2
Summary of Statistics for Time Series Tests, Entire Period (January, 1931–December, 1965)
(Sample Size for Each Regression = 420)

Item*	Portfolio Number 1	2	3	4	5	6	7	8	9	10	$\bar{R}_M$
$\hat{\beta}$	1.5614	1.3838	1.2483	1.1625	1.0572	0.9229	0.8531	0.7534	0.6291	0.4992	1.0000
$\hat{\alpha}\cdot 10^2$	−0.0829	−0.1938	−0.0649	−0.0167	−0.0543	0.0593	0.0462	0.0812	0.1968	0.2012	
$t(\hat{\alpha})$	−0.4274	−1.9935	−0.7597	−0.2468	−0.3869	0.7878	0.7050	1.1837	2.3126	1.8684	
$r(\bar{R}, \bar{R}_M)$	0.9625	0.9875	0.9882	0.9914	0.9915	0.9833	0.9851	0.9703	0.9560	0.8981	
$r(\tilde{e}_t, \tilde{e}_{t-1})$	0.0549	−0.0638	0.0366	0.0073	−0.0708	−0.1248	0.1294	0.1041	0.0444	0.0992	
$\sigma(\tilde{e})$	0.0393	0.0197	0.0173	0.0137	0.0124	0.0152	0.0133	0.0139	0.0172	0.0218	
$\bar{R}$	0.0213	0.0177	0.0171	0.0163	0.0145	0.0137	0.0126	0.0115	0.0109	0.0091	0.0142
σ	0.1445	0.1248	0.1126	0.1045	0.0950	0.0936	0.0772	0.0685	0.0586	0.0495	0.0891

* $\bar{R}$ = average monthly excess returns, σ = standard deviation of the monthly excess returns, r = correlation coefficient.

high. The standard deviation of the residuals $\sigma(\tilde{e}_K)$, the average monthly excess return $\bar{R}_K$, and the standard deviation of the monthly excess return, σ, are also given for each of the portfolios.

Note first that the intercepts $\hat{\alpha}$ are consistently negative for the high-risk portfolios ($\hat{\beta} > 1$) and consistently positive for the low-risk portfolios ($\hat{\beta} < 1$). Thus the high-risk securities earned less on average over this 35-year period than the amount predicted by the traditional form of the asset pricing model. At the same time, the low-risk securities earned more than the amount predicted by the model.

The significance tests given by the "t" values in Table 2 are somewhat inconclusive, since only 3 of the 10 coefficients have "t" values greater than 1.85 and, as we pointed out earlier, we should use some caution in interpreting these "t" values since the normality assumptions can be questioned. We shall see, however, that due to the existence of some nonstationarity in the relations and to the lack of more complete aggregation, these results vastly understate the significance of the departures from the traditional model.

2. *The subperiods*. In order to test the stationarity of the empirical relations, we divided the 35-year interval into four equal subperiods each containing 105 months. Table 3 presents a summary of the regression statistics of (6) calculated using the data for each of these periods for each of the ten portfolios. Note that the data for $\hat{\beta}$ in Table 3 indicate that, except for portfolios 1 and 10, the risk coefficients $\hat{\beta}_K$ were fairly stationary.

Note, however, in the sections for α and $t(\hat{\alpha})$ that the critical intercepts $\hat{\alpha}_K$, were most definitely nonstationary throughout this period. The positive α's for the high-risk portfolios in the first subperiod (January, 1931–September, 1939) indicate that these securities earned more than the amount predicted by the model, and the negative α's for the low-risk portfolios indicate they earned less than what the model predicted. In the three succeeding subperiods (October, 1939–June, 1948; July, 1948–March, 1957, and April, 1957–December, 1965) this pattern was reversed and the departures from the model seemed to become progressively larger; so much larger that six of the ten coefficients in the last subperiod seem significant. (Note that all six coefficients are those with β's most different from unity—a point we shall return to. Thus it seems unlikely that these changes were the result of chance; they most probably reflect changes in the α_K's).

TABLE 3
Summary of Coefficients for the Subperiods

Item*	Sub-period†	Portfolio Number 1	2	3	4	5	6	7	8	9	10	M_{it}
$\hat{\beta}$	1	1.5416	1.3993	1.2620	1.1813	1.0750	0.9197	0.8569	0.7510	0.6222	0.4843	1.0000
	2	1.7157	1.3106	1.1938	1.0861	0.9697	0.9254	0.8114	0.7675	0.6647	0.5626	1.0000
	3	1.5427	1.3598	1.1822	1.1216	1.0474	0.9851	0.9180	0.7714	0.6547	0.4868	1.0000
	4	1.4423	1.2764	1.1818	1.0655	0.9957	0.9248	0.8601	0.7800	0.6614	0.6226	1.0000
$\hat{\alpha}\cdot 10^2$	1	0.7366	0.1902	0.3978	0.1314	−0.0650	−0.0501	−0.2190	−0.3786	−0.2128	−0.0710	
	2	−0.2197	−0.1300	−0.1224	0.0653	−0.0805	0.0014	0.1306	0.0760	0.2685	0.1478	
	3	−0.4614	−0.3994	−0.1159	0.0052	0.0002	−0.0070	0.1266	0.2428	0.3032	0.2035	
	4	−0.4475	−0.2536	−0.2329	−0.0654	0.0840	0.1356	0.1218	0.3257	0.3338	0.3685	
$t(\hat{\alpha})$	1	1.3881	0.6121	1.4037	0.6484	−0.3687	−0.1882	−1.0341	−1.7601	−0.7882	−0.1978	
	2	−0.4256	−0.7605	−0.8719	0.5019	−0.6288	0.8988	1.1377	0.6178	1.7853	0.8377	
	3	−2.0030	−3.6760	−1.5160	0.0742	0.0029	−0.1010	1.8261	3.3768	3.3939	1.9879	
	4	−2.8761	−2.4603	−2.7886	−0.7722	1.1016	1.7937	1.6769	3.8772	3.0651	3.2439	
$\bar{R}$	1	0.0412	0.0326	0.0317	0.0272	0.0239	0.0197	0.0166	0.0127	0.0115	0.0099	0.0220
	2	0.0233	0.0183	0.0165	0.0168	0.0136	0.0147	0.0134	0.0122	0.0126	0.0098	0.0149
	3	0.0126	0.0112	0.0120	0.0126	0.0117	0.0109	0.0115	0.0110	0.0103	0.0075	0.0112
	4	0.0082	0.0082	0.0081	0.0087	0.0096	0.0095	0.0088	0.0101	0.0092	0.0092	0.0088
σ	1	0.2504	0.2243	0.2023	0.1886	0.1715	0.1484	0.1377	0.1211	0.1024	0.0850	0.1587
	2	0.1187	0.0841	0.0758	0.0690	0.0618	0.0586	0.0519	0.0494	0.0441	0.0392	0.0624
	3	0.0581	0.0505	0.0436	0.0413	0.0385	0.0364	0.0340	0.0289	0.0253	0.0203	0.0363
	4	0.0577	0.0503	0.0463	0.0420	0.0391	0.0365	0.0340	0.0312	0.0277	0.0265	0.0366

* $\bar{R}$ = average monthly excess returns, σ = standard deviation of monthly excess returns.
† Subperiod 1 = January, 1931–September, 1939; 2 = October, 1939–June 1948; 3 = July, 1948–March, 1957; 4 = April, 1957–December, 1965.

Note that the correlation coefficients between $\tilde{R}_{Nt}$ and $\tilde{R}_{Mt}$ given in Table 2 for each of the portfolios are all greater than 0.95 except for portfolio number 10. The lowest of the 40 coefficients in the subperiods (not shown) was 0.87, and all but two were greater than 0.90. A a result, the standard deviation of the residuals from each regression is quite small and hence so is the standard error of estimate of α, and this provides the main advantage of grouping in these tests.

III. Cross-sectional Tests of the Model

A. Tests of the Two-Factor Model. Although the time series tests discussed in Section II provide a test of the traditional form of the asset pricing model, they cannot be used to test the two-factor model directly. The cross-sectional tests, however, do furnish an opportunity to test the linearity of the relation between returns and risk implied by (2) or (2′) without making any explicit specification of the intercept. Recall that the traditional form of the model implies $\gamma_0 = 0$ and $\gamma_1 = R_M$. The two factor model merely requires the linearity of (2) to hold for any specific cross section and allows the intercept to be nonzero. At this level of specification we shall not specify the size or even the sign of γ_0. We shall be able to make some statements on this point after a closer examination of the theory. However, we shall first examine the empirical evidence to motivate that discussion.

B. Measurement Errors and Bias in Cross-sectional Tests. We consider here the problems caused in cross-sectional tests of the model by measurement errors in the estimation of the security risk measures.[8] Let β_j represent the true (and unobservable) systematic risk of firm j and $\hat{\beta}_j = \beta_j + \tilde{\epsilon}_j$ be the measured value of the systematic risk of firm j where we assume that $\tilde{\epsilon}_j$, the measurement error, is normally distributed and for all j satisfies

$$E(\tilde{\epsilon}_j) = 0 \tag{7a}$$

$$E(\tilde{\epsilon}_j \beta_j) = 0 \tag{7b}$$

$$E(\tilde{\epsilon}_j \tilde{\epsilon}_i) = \begin{cases} 0 & i \neq j \\ \sigma^2(\tilde{\epsilon}) & i = j \end{cases} \tag{7c}$$

The traditional form of the asset pricing model and the assumptions of the market model imply that the mean excess return on a security

$$\bar{R}_j = \frac{\sum_{t=1}^{T} \tilde{R}_{jt}}{T} \tag{8}$$

observed over T periods can be written as

$$\bar{R}_j = E(\bar{R}_j|\bar{R}_M) + \bar{e}_j = \bar{R}_M\beta_j + \bar{e}_j \tag{9}$$

where $\bar{R}_M = \Sigma_{t=1}^{T} \tilde{R}_{Mt}/T$, $\bar{e}_j = \Sigma_{t=1}^{T} e_{jt}/T$. Now an obvious test of the traditional form of the asset pricing model is to fit

$$\bar{R}_j = \gamma_0 + \gamma_1\hat{\beta}_j + \bar{e}_j^* \tag{10}$$

to a cross section of firms (where $\hat{\beta}_j$ is the estimated risk coefficient for each firm and $\bar{e}_j^* = \bar{e}_j - \gamma_1\tilde{\epsilon}_j$) and test to see if, as implied by the theory

$$\gamma_0 = 0 \quad \text{and} \quad \gamma_1 = \bar{R}_M$$

There are two major difficulties with this procedure; the first involves bias due to the measurement errors in $\hat{\beta}_j$, and the second involves the apparent inadequacy of (9) as a specification of the process generating the data. The two-factor asset pricing model given by (2') implies that γ_0 and γ_1 are random coefficients—that is, in addition to the theoretical values above, they involve a variable that is random through time. If the two-factor model is the true model, the usual significance tests on γ_0 and γ_1 are misleading, since the data from a given cross section cannot provide any evidence on the standard deviation of $\tilde{r}_Z$ and hence results in a serious underestimate of the sampling error of $\hat{\gamma}_0$ and $\hat{\gamma}_1$. Ignoring this second difficulty for the moment, we shall first consider the measurement error problems and the cross-sectional empirical evidence. The random coefficients issue and appropriate significance tests in the context of the two-factor model are discussed in more detail in Section IV.

As long as the $\hat{\beta}_j$ contain the measurement errors $\tilde{\epsilon}_j$, the least-squares estimates $\hat{\gamma}_0$ and $\hat{\gamma}_1$ in (10) will be subject to the well-known errors in variables bias and will be inconsistent, (cf. Johnston [1963, Chap. VI]). That is, assuming that $\bar{e}_j$ and $\tilde{\epsilon}_j$ are independent and are independent of the β_j in the cross-sectional sample,

$$\text{plim}\, \hat{\gamma}_1 = \frac{\gamma_1}{1 + \sigma^2(\tilde{\epsilon})/S^2(\beta_j)} \tag{11}$$

where $S^2(\beta_j)$ is the cross-sectional sample variance of the true risk parameters β_j. Even for large samples, then, as long as the variance of the errors in the risk measure $\sigma^2(\tilde{\epsilon})$ is positive, the estimated coefficient $\hat{\gamma}_1$ will be biased toward zero and $\hat{\gamma}_0$ will therefore be biased away from zero. Hence tests of the significance of the differences $\hat{\gamma}_0 - 0$ and $\hat{\gamma}_1 - \bar{R}_M$ will be misleading.

C. The Grouping Solution to the Measurement Error Problem. We show in the Appendix that by appropriate grouping of the data to be used in estimating (10) one can substantially reduce the bias introduced through the existence of measurement errors in the $\hat{\beta}_j$. In essence the procedure amounts to systematically ordering the firms into groups (in fact by the same procedure that formed the ten portfolios used in the time series tests in Section II) and then calculating the risk measures $\hat{\beta}$ for each portfolio using the time series of portfolio returns. This procedure can greatly reduce the sampling error in the estimated risk measures; indeed, for large samples and independent errors, the sampling error is virtually eliminated. We then estimate the cross-sectional parameters of (10) using the portfolio mean returns over the relevant holding period and the risk coefficients obtained from estimation of (6) from the time series of portfolio returns. If appropriate grouping procedures are employed, this procedure will yield consistent estimates of the parameters γ_0 and γ_1 and thus will yield virtually unbiased estimates for samples in which the number of securities entering each group is large. Thus, by applying the cross-sectional test to our ten portfolios rather than to the underlying individual securities, we can virtually eliminate the measurement error problem.[9]

D. The Cross-sectional Empirical Results. Given the 35 years of monthly returns on each of the ten portfolios calculated as explained in Section II, we then estimated $\hat{\beta}_K$ and $\bar{R}_K$ $(K = 1, 2, \ldots, 10)$ for each portfolio, using all 35 years of monthly data. These estimates (see Table 2) were then used in estimating the cross-sectional relation given by (10) for various holding periods.

Figure 1 is a plot of $\bar{R}_K$ versus $\hat{\beta}_K$ for the 35-year holding period January, 1931–December, 1965. The symbol × denotes the average monthly excess return and risk of each of the ten portfolios. The symbol ⊡ denotes the average excess

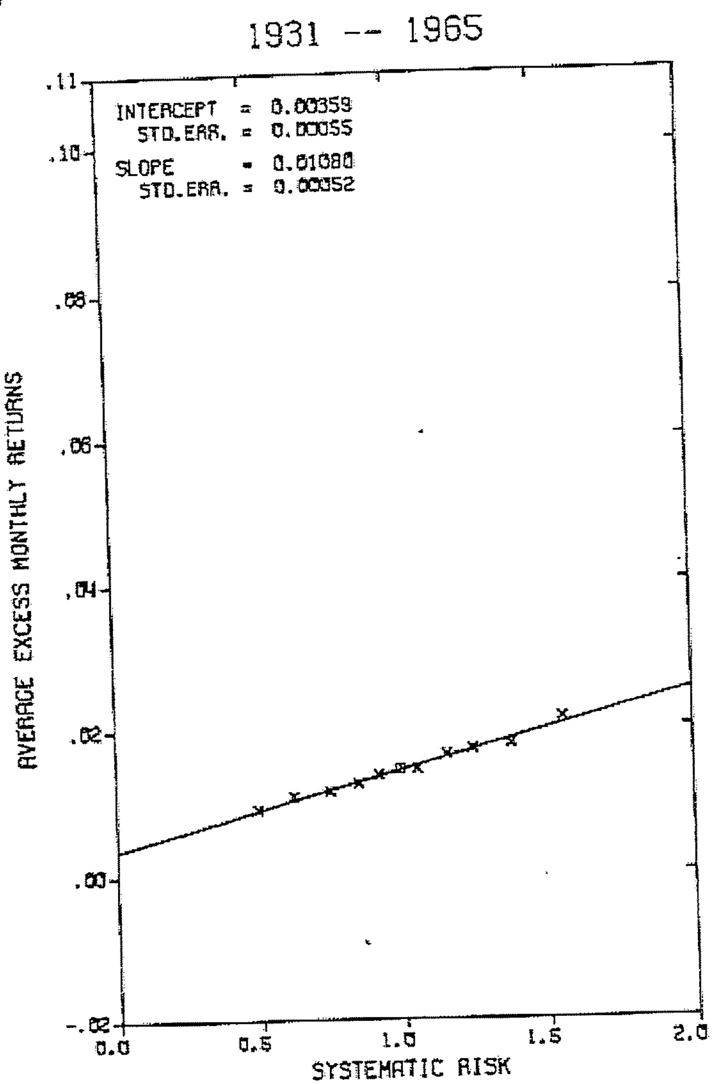

FIGURE 1 Average excess monthly returns versus systematic risk for the 35-year period 1931–65 for each of ten portfolios (denoted by ×) and the market portfolio (denoted by ⊡).

return and risk of the market portfolio (which by the definition of β is equal to unity). The line represents the least-squares estimate of the relation between $\bar{R}_K$ and $\hat{\beta}_K$. The "intercept" and "slope" (with their respective standard errors given in parentheses) in the upper portion of the figure are the coefficients γ_0 and γ_1 of (10).

The traditional form of the asset pricing model implies that the intercept γ_0 in (10) should be equal to zero and the slope γ_1 should be equal to $\bar{R}_M$, the mean excess return on the market portfolio. Over this 35-year period, the average monthly excess return on the market portfolio $\bar{R}_M$, was 0.0142, and the theoretical values of the intercept and slope in Figure 1 are

$$\gamma_0 = 0 \qquad \text{and} \qquad \gamma_1 = 0.0142$$

The "t" values

$$t(\hat{\gamma}_0) = \frac{\hat{\gamma}_0}{s(\hat{\gamma}_0)} = \frac{0.00359}{0.00055} = 6.52$$

$$t(\hat{\gamma}_1) = \frac{\gamma_1 - \hat{\gamma}_1}{s(\hat{\gamma}_1)} = \frac{0.0142 - 0.0108}{0.00052} = 6.53$$

seem to indicate the observed relation is significantly different from the theoretical one. However, as we shall see, because (9) is a misspecification of the process generating the data, these tests vastly overstate the significance of the results.

We also divided the 35-year interval into four equal subperiods, and Figures 2 through 5 present the plots of the $\bar{R}_K$ versus the $\hat{\beta}_K$ for each of these intervals. In order to obtain better estimates of the risk coefficients for each of the subperiods, we used the coefficients previously estimated over the entire 35-year period.[19] The graphs indicate that the relation between return and risk is linear but that the slope is related in a nonstationary way to the theoretical slope for each period. Note that the traditional model implies that the theoretical relationship (not drawn) always passes through the two points given by the origin (0, 0) and the average market excess returns represented by ⊡ in each figure. In the first subperiod (see Fig. 2) the empirical slope is steeper than the theoretical slope and then becomes successively flatter in each of the following three periods. In the last subperiod (see Fig. 5) the slope $\hat{\gamma}_1$ even has the "wrong" sign.

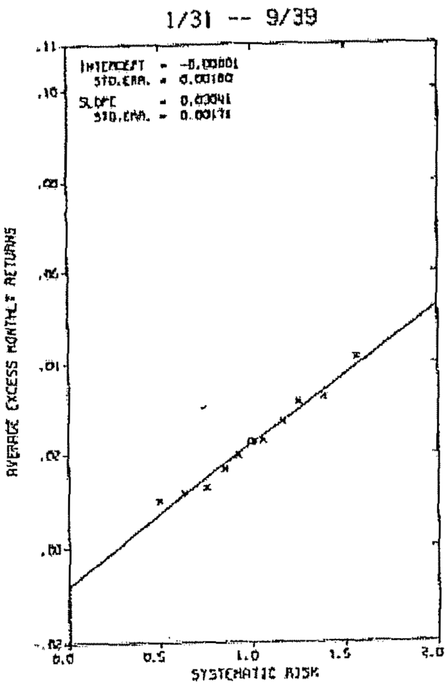

FIGURE 2 Average excess monthly returns versus systematic risk for the 105-month period January, 1931—September, 1939. Symbols as in Figure 1.

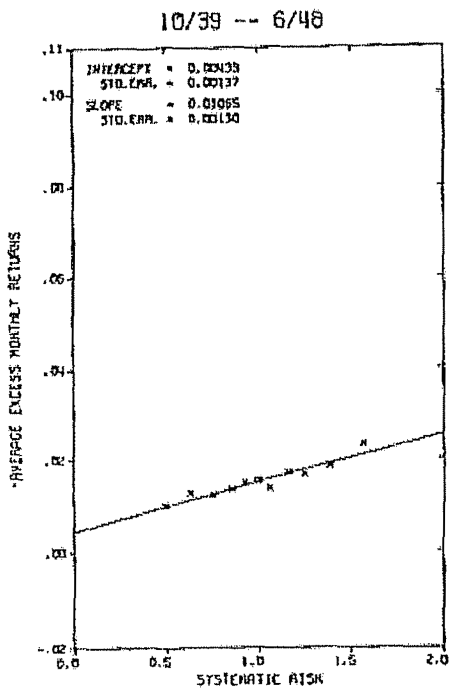

FIGURE 3 Average excess monthly returns versus systematic risk for the 105-month period October, 1939—June, 1948. Symbols as in Figure 1.

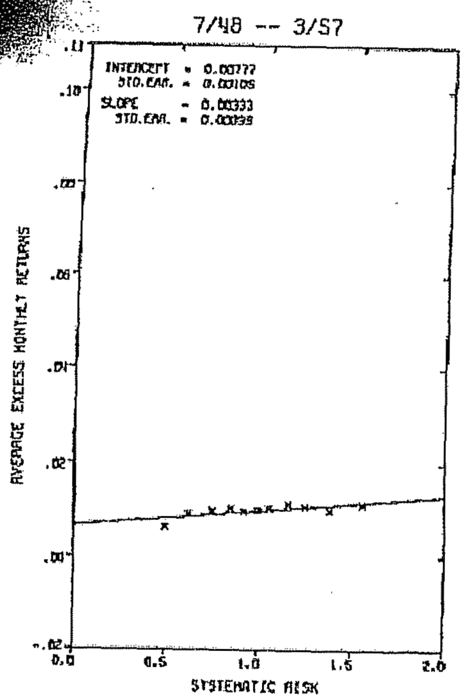

FIGURE 4 Average excess monthly returns versus systematic risk for the 105-month period July, 1948—March, 1957. Symbols as in Figure 1.

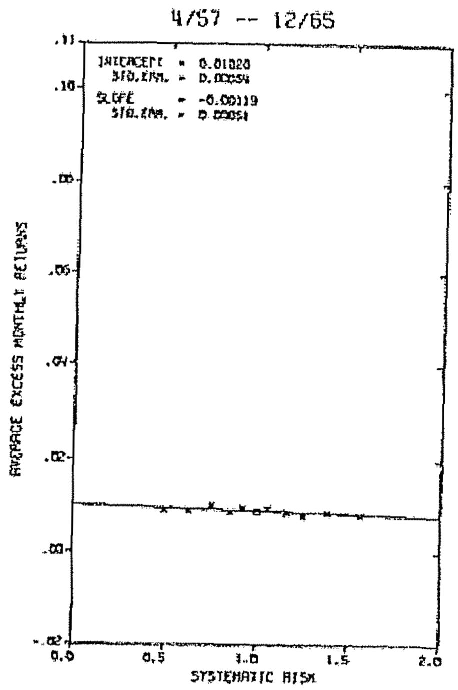

FIGURE 5 Average excess monthly returns versus systematic risk for the 105-month period April, 1957—December, 1965. Symbols as in Figure 1.

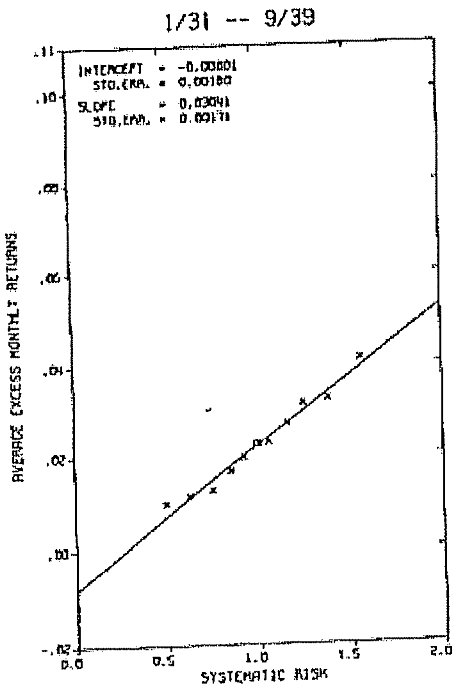

FIGURE 2 Average excess monthly returns versus systematic risk for the 105-month period January, 1931—September, 1939. Symbols as in Figure 1.

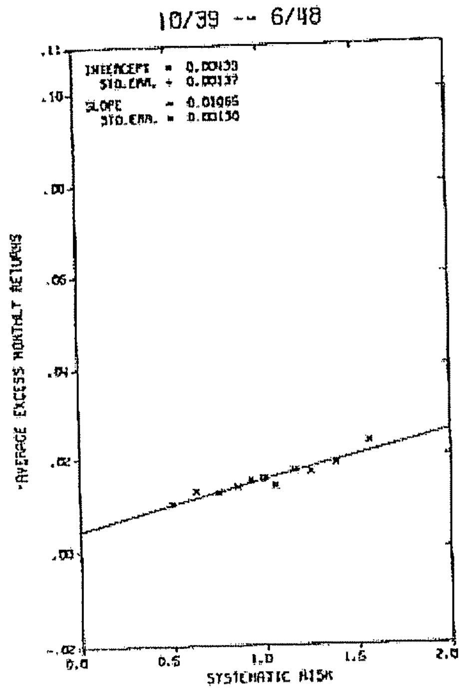

FIGURE 3 Average excess monthly returns versus systematic risk for the 105-month period October, 1939—June, 1948. Symbols as in Figure 1.

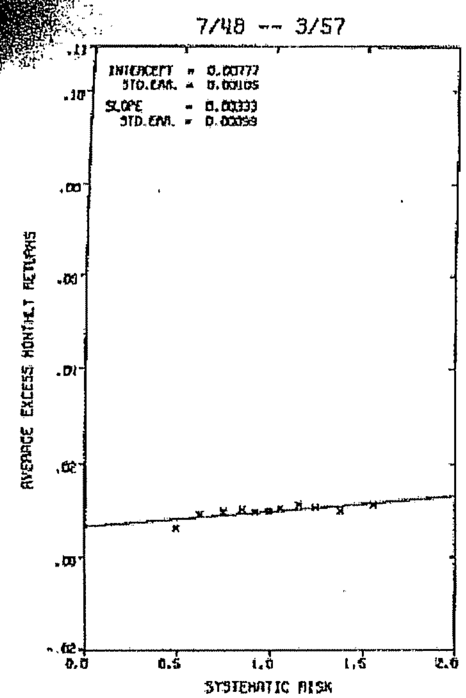

FIGURE 4 Average excess monthly returns versus systematic risk for the 105-month period July, 1948—March, 1957. Symbols as in Figure 1.

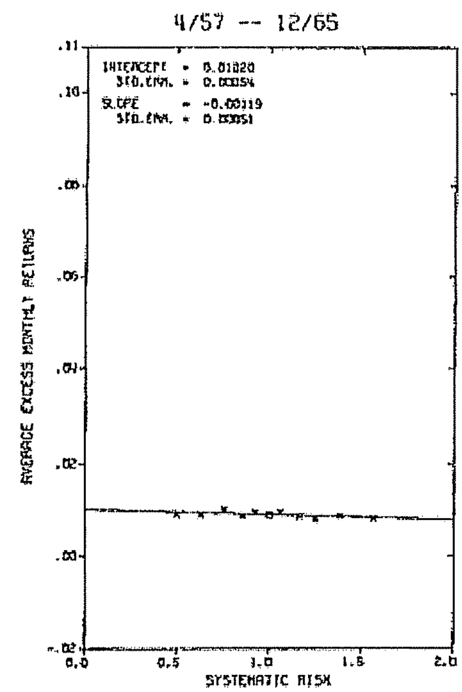

FIGURE 5 Average excess monthly returns versus systematic risk for the 105-month period April, 1957—December, 1965. Symbols as in Figure 1.

TABLE 4
Summary of Cross-sectional Regression Coefficients and Their *t* Values

	Time Period				
	Total Period	Subperiods			
	1/31–12/65	1/31–9/39	10/39–6/48	7/48–3/57	4/57–12/65
$\hat{\gamma}_0$	0.00359	−0.00801	0.00439	0.00777	0.01020
$\hat{\gamma}_1$	0.0108	0.0304	0.0107	0.0033	−0.0012
$\gamma_1 = \bar{R}_M$	0.0142	0.0220	0.0149	0.0112	0.0088
$t(\hat{\gamma}_0)$	6.52	−4.45	3.20	7.40	18.89
$t(\gamma_1 - \hat{\gamma}_1)$	6.53	−4.91	3.23	7.98	19.61

The coefficients $\hat{\gamma}_0$, $\hat{\gamma}_1$, γ_1 and the "*t*" values of $\hat{\gamma}_0$ and $\gamma_1 - \hat{\gamma}_1$ are summarized in Table 4 for the entire period and for each of the four subperiods. The smallest "*t*" value given there is 3.20, and all seem to be "significantly" different from their theoretical values. However, as we have already maintained, these "*t*" values are somewhat misleading because the estimated coefficients fluctuate far more in the subperiods than the estimated sampling errors indicate. This evidence suggests that the model given by (9) is misspecified. We shall now attempt to deal with this specification problem and to furnish an alternative formulation of the model.

IV. A Two-Factor Model

A. *Form of the Model.* As mentioned in the introduction, Black [1970] has shown under assumptions identical to that of the asset pricing model that, if riskless borrowing opportunities do not exist, the expected return on any asset *j* will be given by

$$E(\tilde{r}_j) = E(\tilde{r}_Z)(1-\beta_j) + E(\tilde{r}_M)\beta_j \tag{12}$$

where $\tilde{r}_Z$ represents the return on a "zero beta" portfolio—a portfolio whose covariance with the returns on the market portfolio $\tilde{r}_M$ is zero.[11]

Close examination of the empirical evidence from both the cross-sectional and the time series tests indicates that the return results are consistent with a model that expresses the return on a security as a linear function of the market factor r_M, (with a coefficient of β_j) and a second factor r_Z, (with a coefficient of

$1-\beta_j$). The function is

$$\tilde{r}_{jt} = \tilde{r}_{Zt}(1-\beta_j) + \tilde{r}_{Mt}\beta_j + \tilde{\omega}_{jt} \tag{13}$$

Because the coefficient of the second factor is a function of the security's β, we call this factor the beta factor. For a given holding period *T*, the average value of $\tilde{r}_{Zt}$ will determine the relation between $\hat{\alpha}$ and $\hat{\beta}$ for different securities or portfolios. If the data are being generated by the process given by (13) and if we estimate the single variable time series regression given by (6), then the intercept $\hat{\alpha}$ in that regression will be

$$\hat{\alpha} = (\bar{r}_Z - \bar{r}_F)(1-\hat{\beta}_j) = \bar{R}_Z(1-\hat{\beta}_j) \tag{14}$$

where $\bar{r}_Z = \Sigma_{t=1}^{T} \tilde{r}_{Zt}/T$ is the mean return on the beta factor over the period, $\bar{r}_F$ is the mean risk-free rate over the period, and $\bar{R}_Z$ is the difference between the two. Thus if $\bar{R}_Z$ is positive, high-beta securities will tend to have negative $\hat{\alpha}$'s, and low-beta securities will tend to have positive $\hat{\alpha}$'s. If $\bar{R}_Z$ is negative, high-beta securities will tend to have positive $\hat{\alpha}$'s, and low-beta securities will tend to have negative $\hat{\alpha}$'s.

In addition, if we estimate the cross-sectional regression given by (10), the expanded two-factor model implies that the true values of the parameters γ_0 and γ_1 will not be equal to zero and $\bar{R}_M$ but instead will be given by

$$\gamma_0 = \bar{R}_Z \quad \text{and} \quad \gamma_1 = \bar{R}_M - \bar{R}_Z$$

Hence if $\bar{R}_Z$ is positive, γ_0 will be positive and γ_1 will be less than $\bar{R}_M$. If $\bar{R}_Z$ is negative, γ_0 will be negative and γ_1 will be greater than $\bar{R}_M$.

Thus we can interpret Table 3 and Figures 2 through 5 as indicating that $\bar{R}_Z$ was negative in the first subperiod and became positive and successively larger in each of the following subperiods.

Examining (12), we see that the traditional form of the capital asset pricing model, as expressed in (1), is consistent with the present two-factor model if

$$E(\tilde{R}_Z) = 0 \tag{15}$$

and (questions of statistical efficiency aside) any test for whether α_K for a portfolio is zero is equivalent to a test for whether $E(\tilde{R}_Z)$ is zero. The results in Table 3 suggest that $E(\tilde{R}_Z)$ is not stationary through time. For example, $\hat{\alpha}_K$ for the lowest risk portfolio (number 10) is negative in the first subperiod and positive in the last subperiod, with a "*t*" value of 8. Thus it is unlikely that the true values of α_K were the same in

TABLE 4

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$\hat{\gamma}_0$	0.00359	−0.00801	0.00439	0.00777	0.01020
$\hat{\gamma}_1$	0.0108	0.0304	0.0107	0.0033	−0.0012
$\gamma_1 = \bar{R}_M$	0.0142	0.0220	0.0149	0.0112	0.0085
$t(\hat{\gamma}_0)$	6.52	−4.45	3.20	7.40	18.89
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where $\tilde{r}_Z$ represents the return on a "zero beta" portfolio—a portfolio whose covariance with the returns on the market portfolio $\tilde{r}_M$ is zero.[11]

Close examination of the empirical evidence from both the cross-sectional and the time series tests indicates that the results are consistent with a model that expresses the return on a security as a linear function of the market factor r_M, (with a coefficient of β_j) and a second factor r_Z, (with a coefficient of $1-\beta_j$). The function is

$$\tilde{r}_{jt} = \tilde{r}_{Zt}(1-\beta_j) + \tilde{r}_{Mt}\beta_j + \tilde{\omega}_{jt} \qquad (13)$$

Because the coefficient of the second factor is a function of the security's β, we call this factor the beta factor. For a given holding period T, the average value of $\tilde{r}_{Zt}$ will determine the relation between $\hat{\alpha}$ and $\hat{\beta}$ for different securities or portfolios. If the data are being generated by the process given by (13) and if we estimate the single variable time series regression given by (6), then the intercept $\hat{\alpha}$ in that regression will be

$$\hat{\alpha} = (\bar{r}_Z - \bar{r}_F)(1-\hat{\beta}_j) = \bar{R}_Z(1-\hat{\beta}_j) \qquad (14)$$

where $\bar{r}_Z = \Sigma_{t=1}^{T} \tilde{r}_{Zt}/T$ is the mean return on the beta factor over the period, $\bar{r}_F$ is the mean risk-free rate over the period, and $\bar{R}_Z$ is the difference between the two. Thus if $\bar{R}_Z$ is positive, high-beta securities will tend to have negative $\hat{\alpha}$'s, and low-beta securities will tend to have positive $\hat{\alpha}$'s. If $\bar{R}_Z$ is negative, high-beta securities will tend to have positive $\hat{\alpha}$'s, and low-beta securities will tend to have negative $\hat{\alpha}$'s.

In addition, if we estimate the cross-sectional regression given by (10), the expanded two-factor model implies that the true values of the parameters γ_0 and γ_1 will not be equal to zero and $\bar{R}_M$ but instead will be given by

$$\gamma_0 = \bar{R}_Z \qquad \text{and} \qquad \gamma_1 = \bar{R}_M - \bar{R}_Z$$

Hence if $\bar{R}_Z$ is positive, γ_0 will be positive and γ_1 will be less than $\bar{R}_M$. If $\bar{R}_Z$ is negative, γ_0 will be negative and γ_1 will be greater than $\bar{R}_M$.

Thus we can interpret Table 3 and Figures 2 through 5 as indicating that $\bar{R}_Z$ was negative in the first subperiod and became positive and successively larger in each of the following subperiods.

Examining (12), we see that the traditional form of the capital asset pricing model, as expressed in (1), is consistent with the present two-factor model if

$$E(\tilde{R}_Z) = 0 \qquad (15)$$

and (questions of statistical efficiency aside) any test for whether α_K for a portfolio is zero is equivalent to a test for whether $E(\tilde{R}_Z)$ is zero. The results in Table 3 suggest that $E(\tilde{R}_Z)$ is not stationary through time. For example, $\hat{\alpha}_K$ for the lowest risk portfolio (number 10) is negative in the first subperiod and positive in the last subperiod, with a "t" value of 8. Thus it is unlikely that the true values of α_K were the same in the two subperiods (each of which contains 105 observations) and thus unlikely that the true values of $E(\tilde{R}_Z)$ were the same in the two subperiods, and we shall derive formal tests of this proposition below.

The existence of a factor $\tilde{R}_Z$ with a weight proportional to $1-\beta_j$ in most securities is also suggested by the unreasonably high "t" values[12] obtained in the cross-sectional regressions, as given in Table 4. Since γ_0 and γ_1 involve $\tilde{R}_Z$, which is a random variable from cross section to cross section, and since no single cross-sectional run can provide any information whatsoever on the variability of $\tilde{R}_Z$, this element is totally ignored in the usual calculation of the standard errors of γ_0 and γ_1. It is not surprising, therefore, that each individual cross-sectional result seems so highly significant but so totally different from any other cross-sectional relationship. Of course the presence of infinite-variance stable distributions will also contribute to this type of phenomenon.

In addition, in an attempt to determine whether the linearity observed in Figures 1 through 5 was in some way due to the averaging involved in the long periods presented there, we replicated those plots for our ten portfolios for 17 separate two-year periods from 1932 to 1965. These results, which also exhibit a remarkable linearity, are presented in Figures 6a and 6b. Since the evidence seems to indicate that the all-risky asset model describes the data better than the traditional model, and since the definition of our "riskless" interest rate was somewhat arbitrary in any case, these plots were derived from calculations on the raw return data with no reference whatsoever to the "risk-free" rate defined earlier (including the recalculation of the ten portfolios and the estimation of the β_j). Figures 7 through 11 contain a replication of Figures 1 through 5 calculated on the same basis. These results indicate that the basic findings summarized previously cannot be be attributed to misspecification of the riskless rate.

In summary, then, the empirical results suggest that the returns on different securities can be written as a linear function of two factors as given in (13), that the expected excess return on the beta factor $\tilde{R}_Z$ has in general been positive, and that the expected return on the beta factor has been higher in more recent subperiods than in earlier subperiods.

B. Explicit Estimation of the Beta Factor and a Crucial Test of the Model. Since the traditional form of the asset

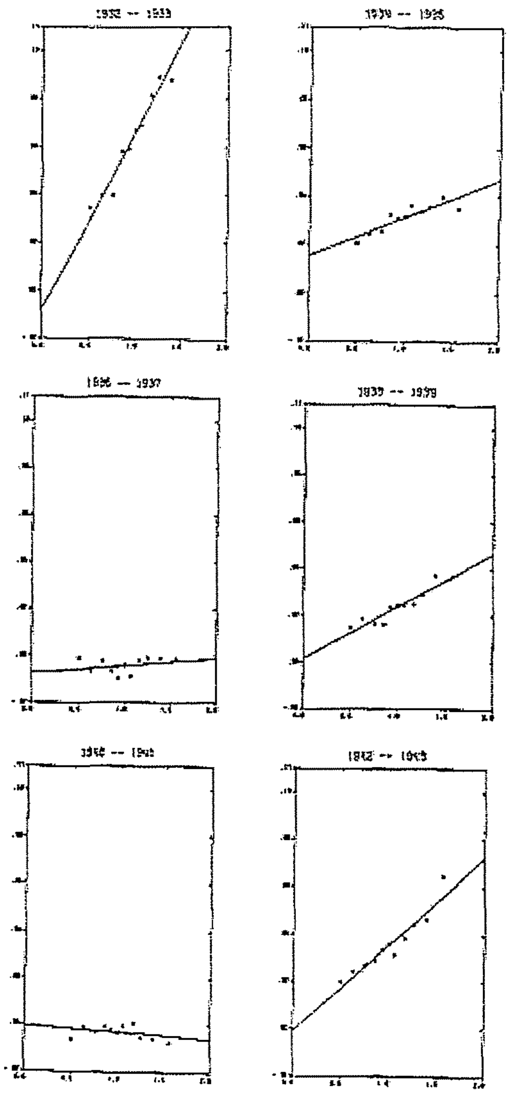
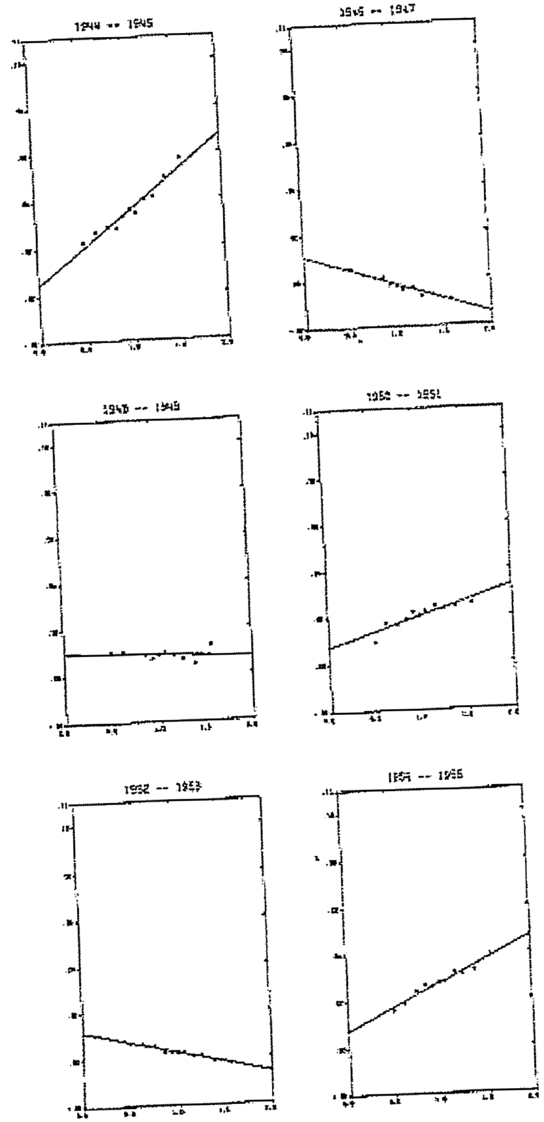


FIGURE 6 Average monthly returns versus systematic risk for 17 non-overlapping two-year periods from 1932 to 1965.

FIGURE 6 (continued)



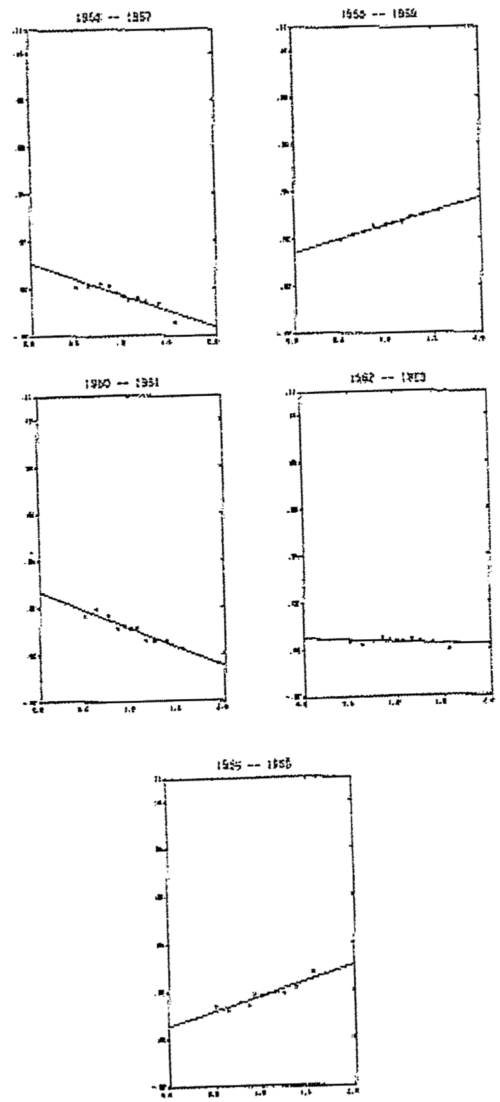


FIGURE 6 (continued)

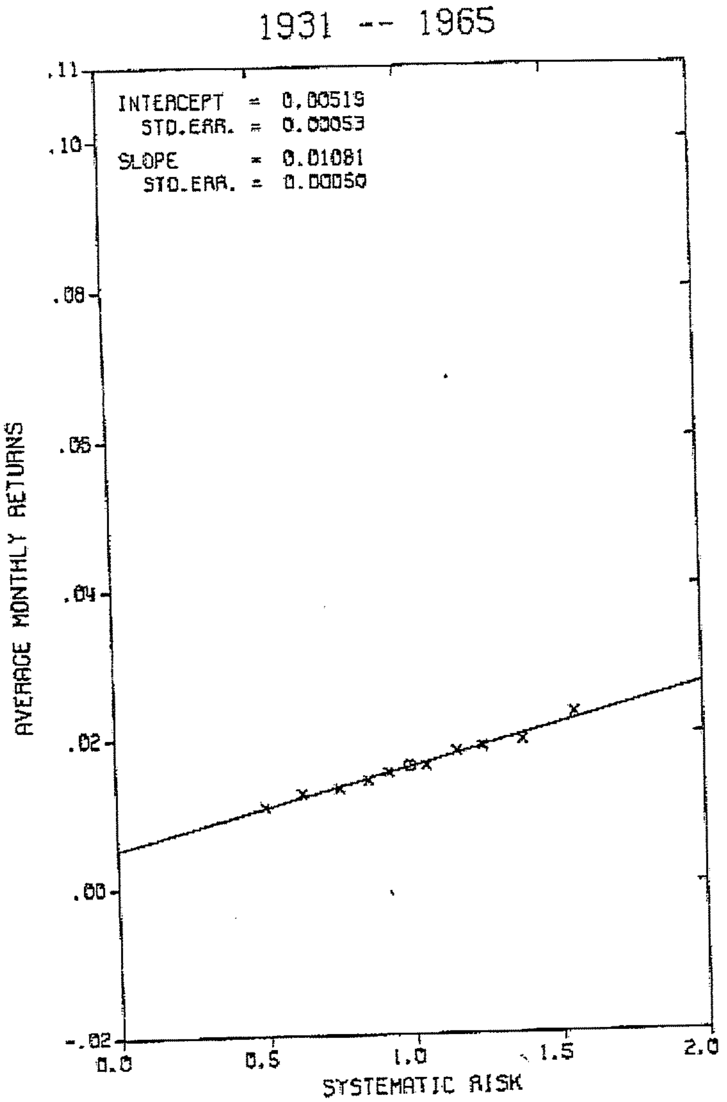


FIGURE 7 Average monthly returns versus systematic risk for the 35-year period 1931–65 for the ten portfolios and the market portfolio.

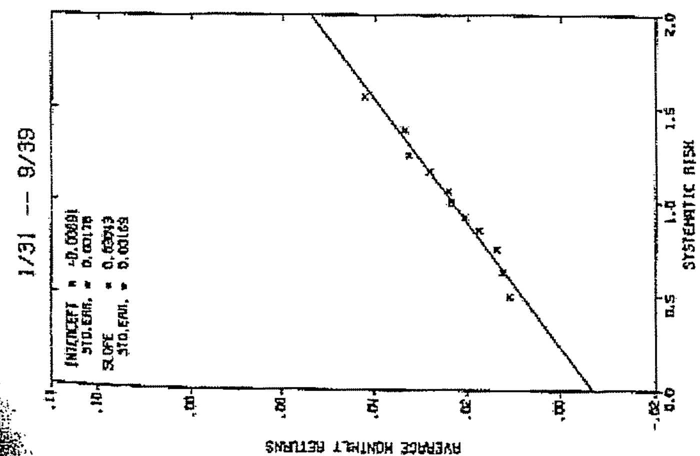


FIGURE 8 Average monthly returns versus systematic risk for the 105-month period January, 1931 – September, 1939.

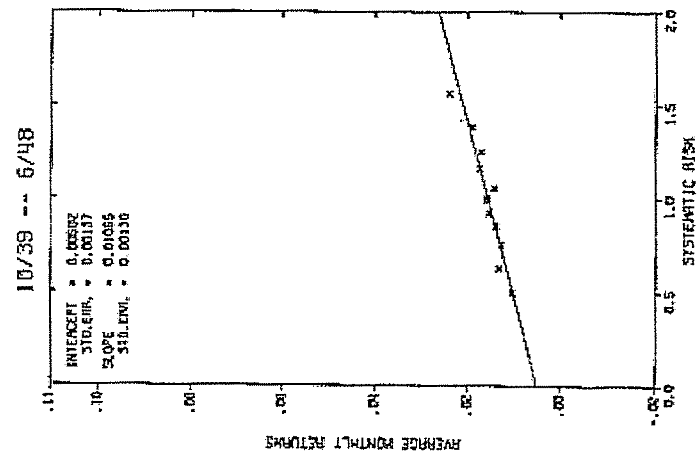


FIGURE 9 Average monthly returns versus systematic risk for the 105-month period October, 1939 – June, 1948.

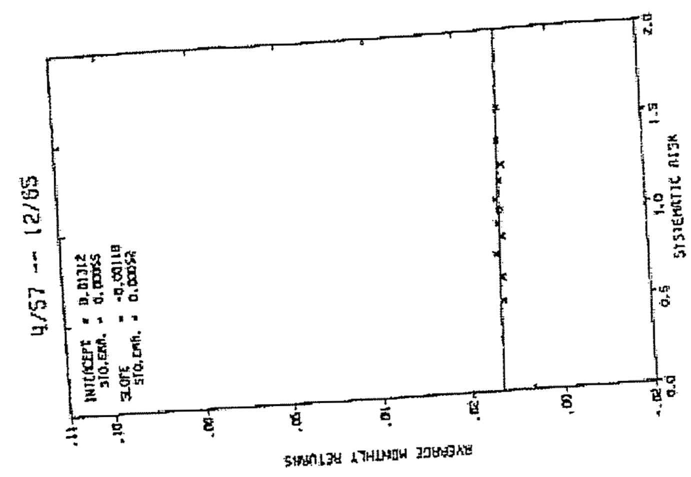


FIGURE 11 Average monthly returns versus systematic risk for the 105-month period April, 1957 – December, 1965.

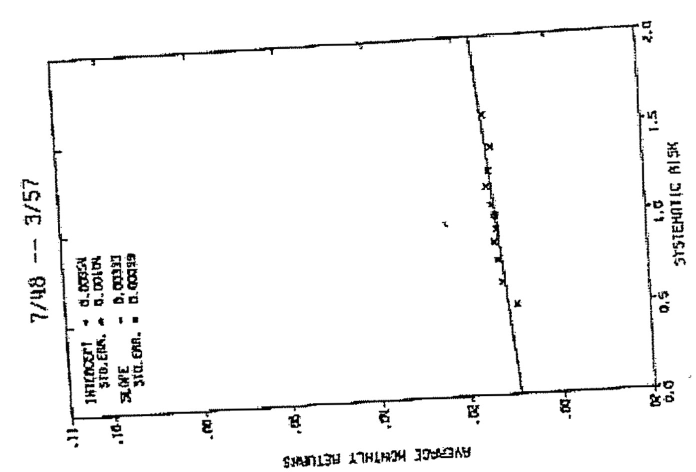


FIGURE 10 Average monthly returns versus systematic risk for the 105-month period July, 1948 – March, 1957.

pricing model is consistent with the existence of the beta factor as long as the excess returns on the beta factor have a zero mean,[13] our purpose here is to provide a procedure for explicit estimation of the time series of the factor. Given such a time series, we can then make explicit estimates of the significance of its mean excess return rather than depending mainly on an examination of the $\hat\alpha_j$ for high- and low-beta securities. Solving (13) for $\tilde r_{Zt}$ plus the error term, we have an estimate $\hat r_{Zjt}$, of $\tilde r_{Zt}$

$$\hat r_{Zjt} = \frac{1}{(1-\beta_j)}[\tilde r_j - \beta_j \tilde r_{Mt}] = \tilde r_{Zt} + \tilde u_{jt} \qquad (16)$$

where $\tilde u_{jt} = \tilde e_{jt}/(1-\beta_j)$. We subscript $\hat r_{Zjt}$ by j to denote that this is an estimate of $\tilde r_{Zt}$ obtained from the jth asset or portfolio. Now, since we can obtain as many separate estimates of $\tilde r_{Zt}$ as we have securities or portfolios, we can formulate a combined estimate

$$r^{o}_{Zt} = \sum_j h_j \hat r_{Zjt} \qquad (17)$$

which is a linear combination of the $\hat r_{Zjt}$, to provide a much more efficient estimate of $\tilde r_{Zt}$. The problem is to find that linear combination of the $\hat r_{Zjt}$ which minimizes the error variance in the estimate of $\tilde r_{Zt}$. That is, we want to

$$\min_{h_j} E(r^{o}_{Zt} - \tilde r_{Zt})^2 = \min_{h_j} E\Big(\sum_j h_j \hat r_{Zjt} - \tilde r_{Zt}\Big)^2$$

subject to $\sum_j h_j = 1$, since we want an unbiased estimate. From the Lagrangian we obtain the first-order conditions

$$h_j \sigma^2(\tilde u_j) - \lambda = 0 \qquad j = 1, 2, \ldots, N \qquad (18)$$

where λ is the Lagrangian multiplier and N is the total number of securities or nonoverlapping portfolios. These conditions imply that

$$\frac{h_j}{h_i} = \frac{\sigma^2(\tilde u_i)}{\sigma^2(\tilde u_j)} \qquad \text{for all } i \text{ and } j \qquad (19)$$

which implies that the optimal weights h_j are proportional to $1/\sigma^2(\tilde u_j)$. That is,

$$h_j = \frac{K}{\sigma^2(\tilde u_j)} \qquad j = 1, 2, \ldots, N \qquad (20)$$

where $K = 1/\Sigma_j [1/\sigma^2(\tilde u_j)]$ is a normalizing constant. But from the definition of $\tilde{u}_j$ we know that $\sigma^2(\tilde{u}_j) = \sigma^2(\tilde{w}_j)/(1-\beta_j)^2$, so

$$h_j = \frac{K(1-\beta_j)^2}{\sigma^2(\tilde{w}_j)} \tag{21}$$

Equation (21) makes sense, for we are then weighting the estimates in proportion to $(1-\beta_j)^2$ and inversely proportional to $\sigma^2(\tilde{w}_j)$. However, since we cannot observe $\sigma^2(\tilde{w}_j)$ directly,[14] we are forced, for lack of explicit estimates, to assume that the $\sigma^2(\tilde{w}_j)$ are all identical and to use as our weights

$$h_j = K'(1-\beta_j)^2 \tag{22}$$

where $K' = 1/\Sigma_j(1-\beta_j)^2$.

Equations (17) and (22) thus provide an unbiased and (approximately) efficient procedure for estimating $\tilde{r}_{Zt}$ utilizing all available information. However, there is a problem of bias involved in actually applying this procedure to the security data. The coefficient β_j is of course unobservable, and in general if we use our estimates $\hat{\beta}_j$ in the weighting procedure we will introduce bias into our estimate of $\tilde{r}_{Zt}$. To understand this, recall that $\hat{\beta}_j = \beta_j + \epsilon_j$, substitute this into (13) with the necessary additions and subtractions, and solve for the estimate

$$\hat{r}_{Zjt} = \frac{\tilde{r}_{jt} - \hat{\beta}_j \tilde{r}_{Mt}}{(1-\hat{\beta}_j)} = \frac{\tilde{r}_{Zt}(1-\beta_j) + \tilde{w}_j - \tilde{\epsilon}_j \tilde{r}_{Mt}}{(1-\hat{\beta}_j)}$$

Substituting this into (17), using (22), rearranging terms, and taking the probability limit, we have

$$\operatorname*{plim}_{N\to\infty} r^{\circ}_{Zt} = \frac{C_t[S^2(\beta) + (1-\bar{\beta})^2] + \sigma^2(\bar{\epsilon}) r_{Mt}}{[S^2(\beta) + (1-\bar{\beta})^2] + \sigma^2(\bar{\epsilon})} \tag{23}$$

where $S^2(\beta)$ is the cross-sectional variance of the β_j and $\bar{\beta}$ is the mean. However, the average standard deviation of the measurement error $\sigma(\tilde{\epsilon}_j)$ for our portfolios is only 0.0101 (implying an average variance on the order of 0.0001), and since $S^2(\beta)$ for our ten portfolios is 0.1144 and $\bar{\beta} = 1.007$, this bias will be negligible and we shall ignore it.

To begin, let us apply the foregoing procedures to the excess return data to obtain an estimate of $\tilde{R}_{Zt} = \tilde{r}_{Zt} - r_{Ft}$, the excess return on the beta factor. Substituting R_{jt} for r_{jt} and R_{Mt} for r_{Mt} in (16), the $\hat{R}_{Zjt}$ were estimated for each of our ten

portfolios. These were then averaged to obtain the estimate

$$R^{\circ}_{Zt} = \sum_j h_j \hat{R}_{Zjt} = K' \sum_j (1-\beta_j)^2 \left[\frac{\tilde{R}_{jt} - \hat{\beta}_j R_{Mt}}{1-\hat{\beta}_j}\right]$$

for each month t. The average of the R°_{Zt} for the entire period and for each of the four subperiods are given in Table 5, along with their t values. Table 5 also presents the serial correlation

TABLE 5

Estimated Mean Values and Serial Correlation of the Excess Returns on the Beta Factor over the Entire Periods and the Four Subperiods*

Period	$\bar{R}^{\circ}_Z$	$\sigma(R^{\circ}_Z)$	$t(\bar{R}^{\circ}_Z)$	$r(R^{\circ}_{Zt}, R^{\circ}_{Z,t-1})$	$t(r)$
1/31–12/65	0.00338	0.0426	1.62	0.113	2.33
1/31–9/39	−0.00849	0.0641	−1.35	0.194	1.48
10/39–6/48	0.00420	0.0455	0.946	0.208	2.19
7/48–3/57	0.00782	0.0199	4.03	−0.181	−1.87
4/57–12/65	0.00997	0.0228	4.49	0.414	4.60

*The values of $t(\bar{R}^{\circ}_Z)$ were calculated under the assumption of normal distributions.

coefficients $r(R^{\circ}_{Zt}, R^{\circ}_{Z,t-1})$.[15] Note that the mean value $\bar{R}^{\circ}_Z$ of the beta factor over the whole period has a "t" value of only 1.64. However, as hypothesized earlier, it was negative in the first subperiod and positive and successively larger in each of the following subperiods. Moreover, in the last two subperiods its "t" values were 4.03 and 4.49, respectively. These results seem to us to be strong evidence favoring rejection of the traditional form of the asset pricing model which says that $\bar{R}^{\circ}_Z$ should be insignificantly different from zero.

In order to be sure that the significance levels reported in Table 5 are not spurious and due only to the misapplication of normal distribution theory to a situation in which the variables may actually be distributed according to the infinite variance members of the stable class of distributions. We have performed the significance tests using the stable distribution theory outlined by Fama and Roll [1968]. Table 6 presents the standardized variates (i.e., the "t" values) for $\bar{R}^{\circ}_Z$ for each of the sample periods given in Table 5 along with the "t" values at the 5% level of significance (two-tail) under

TABLE 6

Normalized Variate [i.e., t Value $t(\bar{R}_Z^o, \alpha) = \bar{R}_Z^o / \sigma(\bar{R}_Z^o, \alpha)$] of the Excess Return on the Beta Factor Under the Assumption of Infinite Variance Symmetric Stable Distributions

	α					
Period	1.5	1.6	1.7	1.8	1.9	2.0
1/31–12/65	1.33	1.71	2.14	2.61	3.11*	3.65*
1/31–9/39	−1.11	−1.44	−1.71	−2.00	−2.29	−2.58
10/39–6/48	0.82	1.00	1.18	1.38	1.58	1.79
7/48–3/57	2.60	3.16	3.75*	4.37*	5.00*	5.66*
4/57–12/65	3.05	3.70	4.40*	5.11*	5.86*	6.63*
t Value at the 5% level of significance (two-tail)†	4.49	3.90	3.48	3.16	2.93	2.77

Note: α = characteristic exponent, $\sigma(\bar{R}_Z^o, \alpha)$ = dispersion parameter of the distribution.

†Cf. Fama and Roll [1968].

alternative assumptions regarding the value of α, the characteristic exponent of the distribution. The smaller is α, the higher are the extreme tails of the probability distribution; $\alpha = 2$ corresponds to the normal distribution and $\alpha = 1$ to the Cauchy distribution. Evidence presented by Fama [1965] seems to indicate that α is probably in the range 1.7 to 1.9 for common stocks. We have not attempted to obtain explicit estimates of α for our data, since currently known estimation procedures are quite imprecise and require extremely large samples (up to 2,000 observations). Therefore we have simply presented the "t" values calculated according to the procedures suggested by Fama and Roll [1968] for six values of α ranging from 1.5 to 2.0. The coefficients in Table 6 that are significant at the 5% level are noted with an asterisk. Clearly, if α is greater than 1.7, the results confirm the impression gained from the normal tests given in Table 5.

Note that the estimates in Tables 5 and 6 were obtained from the excess return data; therefore, although the figures are of interest for testing the traditional form of the model, they do not give the appropriate level of the mean value of $\bar{r}_Z$. The estimates $\bar{r}_Z^o$ and $\bar{r}_M$ obtained from the total return data used in Figures 6 through 11 appear in Table 7, along with $\sigma(\bar{r}_Z^o)$ and $\sigma(\bar{r}_M)$ and the estimated values of γ_0 and γ_1 for the cross-sectional regressions [given by (10)] for each of the var-

TABLE 7

Mean and Standard Deviation of Returns on the Zero Beta and Market Portfolios and the Cross-sectional Regression Coefficients [from (10)] for Various Sample Periods

Time Period	$\bar{r}_Z^o$	$\bar{r}_M$	$\bar{r}_M - \bar{r}_Z^o$	$\sigma(r_Z^o)$	$\sigma(r_M)$	$\hat{\gamma}_0$[a]	$\hat{\gamma}_1$[a]
1931–1965	0.004980	0.015800	0.010820	0.042584	0.069054	0.005190	0.010807
1/31–9/39	−0.007393	0.023067	0.030459	0.063927	0.158707	−0.006913	0.030429
10/39–6/48	0.004833	0.015487	0.010665	0.045520	0.062414	0.005021	0.010652
7/48–3/57	0.009591	0.012915	0.003324	0.019895	0.036204	0.009537	0.003327
4/57–12/65	0.012889	0.011723	−0.001167	0.022631	0.038470	0.013115	−0.001181
1931	−0.047243	−0.037573	0.009660	0.040827	0.152024	−0.045492	0.009557
1932–1933	−0.009180	0.065574	0.074754	0.059741	0.245281	−0.008266	0.074696
1934–1935	0.015549	0.031250	0.015701	0.048551	0.097739	0.015542	0.015702
1936–1937	−0.007749	−0.004538	0.005211	0.032589	0.084786	−0.007336	0.003194
1938–1939	0.001919	0.024436	0.022517	0.100490	0.147129	0.001514	0.022543
1940–1941	−0.001308	−0.003902	−0.002596	0.043481	0.072454	−0.000646	−0.002638
1942–1943	−0.009898	0.035782	0.036780	0.066552	0.066451	−0.001069	0.036784
1944–1945	0.004511	0.036117	0.031507	0.032522	0.043560	0.004451	0.031517
1946–1947	0.010153	−0.002357	−0.013010	0.033074	0.056139	0.010946	−0.013061
1948–1949	0.009721	0.008529	−0.001192	0.019590	0.051471	0.009709	−0.001191
1950–1951	0.007163	0.020253	0.013090	0.028656	0.039764	0.007215	0.013097
1952–1953	0.012258	0.003054	−0.009204	0.014559	0.026896	0.012050	−0.009191
1954–1955	0.007432	0.027266	0.019834	0.019232	0.030604	0.007392	0.019836
1956–1957	0.010463	−0.003097	−0.013559	0.017638	0.032340	0.010555	−0.013565
1958–1959	0.014582	0.025060	0.011478	0.019982	0.028261	0.014205	0.011502
1960–1961	0.026825	0.010867	−0.015958	0.023178	0.036505	0.026753	−0.015953
1962–1963	0.004300	0.002728	−0.001571	0.026231	0.052144	0.005054	−0.001620
1964–1965	0.005032	0.017771	0.012738	0.014433	0.026761	0.005519	0.012707

[a]Cf. eq. (10).

ious sample periods portrayed in Figures 6 through 11. (Recall that the two-factor model implies $\gamma_0 = \tilde{r}_Z$ and $\gamma_1 = \tilde{r}_M - \tilde{r}_Z$.) One additional item of interest in judging the importance of the beta factor in the determination of security returns is its standard deviation relative to that of the market returns. As Table 7 reveals, $\sigma(\tilde{r}_Z^\circ)$ is roughly 50% as large as $\sigma(\tilde{r}_M)$. Comparison of $\tilde{r}_Z^\circ$ and $\tilde{r}_M$ in Table 7 for the four 105-month subperiods indicates that the mean returns on the beta factor were approximately equal to the average market returns in the last two periods covering the interval July, 1948–December, 1965. Apparently, then, the relative magnitudes of $\tilde{r}_Z^\circ$ and $\tilde{r}_M$ indicate that the beta factor is economically as well as statistically significant.

V. Conclusion

The traditional form of the capital asset pricing model states that the expected excess return on a security is equal to its level of systematic risk, β, times the expected excess return on the market portfolio. That is, in capital market equilibrium, prices of assets adjust such that

$$E(\tilde{R}_j) = \gamma_1 \beta_j \tag{24}$$

where $\gamma_1 = E(\tilde{R}_M)$, the expected excess return on the market portfolio.

An alternative hypothesis of the pricing of capital assets arises from the relaxation of one of the assumptions of the tranditional form of the capital asset pricing model. Relaxation of the assumption that riskless borrowing and lending opportunities are available leads to the formulation of the two-factor model. In equilibrium, the expected returns $E(\tilde{r}_j)$ on an asset will be given by

$$E(\tilde{r}_j) = E(\tilde{r}_Z) + [E(\tilde{r}_M) - E(\tilde{r}_Z)]\beta_j \tag{25}$$

where $E(\tilde{r}_Z)$ is the expected return on a portfolio that has a zero covariance (and thus $\beta_Z = 0$) with the return on the market portfolio $\tilde{r}_M$. In the context of this model, the return on 30-day Treasury Bills (which we have used as a proxy for a "riskless" rate) simply represents the return on a particular asset in the system. Thus, subtracting r_F from both sides of (25), we can rewrite (25) in terms of "excess" returns as

$$E(\tilde{R}_j) = \gamma_0 + \gamma_1 \beta_j \tag{26}$$

where $\gamma_0 = E(\tilde{R}_Z)$ and $\gamma_1 = E(\tilde{R}_M) - E(\tilde{R}_Z)$.

The traditional form of the asset pricing model implies that $\gamma_0 = 0$ and $\gamma_1 = E(\tilde{R}_M)$ and the two-factor model implies that $\gamma_0 = E(\tilde{R}_Z)$, which is not necessarily zero and that $\gamma_1 = E(\tilde{R}_M) - E(\tilde{R}_Z)$. In addition, several other models arise from relaxing some of the assumptions of the traditional asset pricing model which imply $\gamma_0 \neq 0$ and $\gamma_1 \neq E(R_M)$. These models involve explicit consideration of the problems of measuring R_M, the existence of nonmarketable assets, and the existence of differential taxes on capital gains and dividends, and we shall briefly outline them. Our main emphasis has been to test the strict traditional form of the asset pricing model; that is, is $\gamma_0 \neq 0$? We have made no attempt to provide direct tests of these other alternative hypotheses.

To test the traditional model, we used all securities listed on the New York Stock Exchange at any time in the interval between 1926 and 1966. The problem we faced was to obtain efficient estimates of the mean of the beta factor and its variance. It would be possible to test the alternative hypotheses by selecting one security at random and estimating its beta from the time series and ascertaining whether its mean return was significantly different from that predicted by the traditional form of the capital asset pricing model. However, this would be a very inefficient test procedure.

To gain efficiency, we grouped the securities into ten portfolios in such a way that the portfolios had a large spread in their β's. However, we knew that grouping the securities on the basis of their estimated β's would not give unbiased estimates of the portfolio "Beta," since the β's used to select the portfolios would contain measurement error. Such a procedure would introduce a selection bias into the tests. To eliminate this bias we used an instrumental variable, the previous period's estimated beta, to select a security's portfolio grouping for the next year. Using these procedures, we constructed ten portfolios whose estimated β's were unbiased estimates of the portfolio "Beta." We found that much of the sampling variability of the β's estimated for individual securities was eliminated by using the portfolio groupings. The β's of the portfolios constructed in this manner ranged from 0.49 to 1.5, and the estimates of the portfolio β's for the subperiods exhibited considerable stationarity.

The time series regressions of the portfolio excess returns on the market portfolio excess returns indicated that high-beta securities had significantly negative intercepts and low-beta securities had significantly positive intercepts, contrary to the predictions of the traditional form of the model. There was also considerable evidence that this effect became stronger through time, being strongest in the 1947–65 period. The cross-sectional plots of the mean excess returns on the portfolios against the estimated β's indicated that the relation between mean excess return and β was linear. However, the intercept and slope of the cross-sectional relation varied in different subperiods and were not consistent with the traditional form of the capital asset pricing model. In the two prewar 105-month subperiods examined, the slope was steeper in the first period than that predicted by the traditional form of the model, and it was flatter in the second period. In each of the two 105-month postwar periods it was considerably flatter than predicted. From the evidence of both the time series and cross-sectional runs, we were led to reject the hypothesis that γ_0 in (26) was equal to zero; we therefore concluded that the traditional form of the asset pricing model is not consistent with the data.

We also attempted to make explicit estimates of the time series of returns on the beta factor in order to obtain a more efficient estimate of its mean and variance and thereby enable ourselves to directly test whether or not the mean excess return on the beta factor was zero. We derived a minimum-variance, unbiased linear estimator of the returns on the β factor using our portfolio return data. We showed that, given the independence of the residuals the optimum estimator requires knowledge of the unobservable residual variances of each of the portfolios but that this problem could be avoided if they were equal. Under this assumption of equal residual variances, we estimated the time series of returns on the beta factor. However, if these assumptions (i.e., the independence of the residuals and equality of their variances) are not valid—and there is reason to believe they are not—more complicated procedures are necessary to obtain minimum-variance estimates. Such estimators, which use the complete covariance structure of the portfolio returns are available (although not derived here). However, we feel that a straightforward application of these procedures to the return data would result in the introduction of serious *ex post* bias in the estimates. Thus we have left a complete investigation of these problems, as well as more detailed tests of the two-factor model, to a future paper. In order to fully utilize the properties of the two-factor model in a number of applied problems (such as portfolio evaluation, see Jensen [1971] and various issues in valuation

theory), it will be necessary to have minimum-variance unbiased estimates of the time series of returns on the beta factor, and we hope to provide such estimates in the not-too-distant future.

The evidence obtained from the time series of returns on the beta factor indicated that the beta factor had a nonzero mean and that the mean was nonstationary over time. It seems to us that we have established the presence and significance of the beta factor in explaining security returns but, as mentioned earlier, we have not provided any direct tests aimed at explaining the existence of the beta factor. We have, however, suggested an economic rationale for why capital market equilibrium is consistent with the finding of this second factor. Black [1970] has shown that if riskless borrowing opportunities are not available, the equilibrium expected returns on an asset will be a linear function of two factors, one the β factor, the other the market factor.

In addition, Black and Jensen [1970] have demonstrated that if assets are omitted from the estimated market return, a model similar in some ways to the two-factor model would result. (Roll's analysis [1969] is relevant to this issue as well.) That is, it yields a model similar in structure to (26) and implies that $\gamma_0 \neq 0$. However, it is clear from Figures 6a and 6b and Table 7 that the beta factor (the intercept in the figures and γ_0 in Table 7) is *highly* variable and any alternative hypothesis must be consistent with this phenomenon. In other words, it is not sufficient for an alternative model to simply imply a nonzero but constant intercept in (26).

Others have provided alternative models that are similar in structure to the Black–Jensen results. For example, Mayers [1972] has developed an equilibrium model incorporating the existence of nonmarketable assets and has shown that the basic linear relation of the traditional model is unaltered, but the constant term γ_0 will be nonzero and γ_1 will not equal $E(\tilde{R}_M)$. The implications of his model for the structure of asset returns are virtually identical to those of the omitted assets model. Brennan [1970] has derived the equilibrium structure of security returns when the effects of a differential tax on dividends and capital gains are considered. He also concludes that the basic linearity of the traditional model is unchanged, but a nonzero constant term must be included and γ_1 will not equal $E(\tilde{R}_M)$. Black and Scholes [1970], however, have tested for the existence of dividend effects and have found that the differential tax on dividends and capital gains does not affect the structure of security returns and hence cannot explain the results reported here.

There are undoubtedly other economic hypotheses that are consistent with the findings of the existence of a second factor and consistent also with capital market equilibrium. Each hypothesis must be tested directly to determine whether it can account for the presence of the β factor. The Black-Scholes investigation of dividend effects is an example of such a test.

Appendix: The Grouping Solution to the Measurement Error Problem

Consider first the estimate $\hat{\beta}_j$ of the risk parameter in more detail. We will want to test (10) over some holding period, but we must first obtain the estimates of the risk parameter $\hat{\beta}_j$, from the time series equation given by (6). For simplicity, we shall assume that the $\tilde{e}_{jt}$ are independently distributed and have constant variance for all j and t. The least-squares estimate of β_j in (6), $\hat{\beta}_j$, is thus unbiased but subject to a sampling error $\tilde{\epsilon}_j$ as in (7), and the variance of the sampling error of the estimate $\hat{\beta}_j$ is

$$\text{var}(\hat{\beta}_j|\beta_j) = \sigma^2(\tilde{\epsilon}_j) = \frac{\sigma^2(\tilde{e}_j)}{\phi} = \frac{\sigma^2(\tilde{e})}{\phi} \tag{A.1}$$

since $\sigma^2(\tilde{e}_j)$ was assumed equal for all j, and where

$$\phi = \sum_{t=1}^{T} (R_{Mt} - R_M)^2 \tag{A.2}$$

is the sample sum of squared deviations of the independent variable over the T observations used in the time series estimating equation. Hence using (11) we see that

$$\text{plim}\ \hat{\gamma} = \frac{\gamma_1}{1 + \sigma^2(\tilde{e})/\phi S^2(\beta_j)} \tag{A.3}$$

Let us assume that we can order the firms on the basis of β_j or on the basis of some instrumental variable highly correlated with β_j but independent of $\tilde{\epsilon}_j$. Given the N ordered firms, we group them into M equal-size contiguous subgroups, represented by $K = 1, 2, \ldots, M$ and calculate the average return

for each group for each month t according to

$$\tilde{R}_{Kt} = \frac{1}{L}\sum_{j=1}^{L} \tilde{R}_{Kjt} \qquad K = 1, 2, \ldots, M \tag{A.4}$$

$$L = \frac{N}{M} \quad \text{(assumed to be integer)} \tag{A.5}$$

where $\tilde{R}_{Kjt}$ is the return for month t for security j in group K. We then estimate the systematic risk of the group by applying least squares to

$$\tilde{R}_{Kt} = \alpha_K + \beta_K \tilde{R}_{Mt} + \tilde{e}_{Kt} \qquad \begin{cases} K = 1, 2, \ldots, M \\ t = 1, 2, \ldots, T \end{cases} \tag{A.6}$$

where

$$\tilde{e}_{Kt} = \frac{1}{L}\sum_{j=1}^{L} \tilde{e}_{Kjt} \tag{A.7}$$

and

$$\sigma^2(\tilde{e}_{Kt}) = \frac{\sigma^2(\tilde{e})}{L} \tag{A.8}$$

Equation (A.8) holds, since, by assumption, the $\tilde{e}_{Kjt}$ are independently distributed with equal variance. The least-squares estimate of β_K in (A.6) is $\hat{\beta}_K = \beta_K + \tilde{\epsilon}_K$ and its variance is

$$\text{var}(\hat{\beta}_K|\beta_K) = \sigma^2(\tilde{\epsilon}_K) = \frac{\sigma^2(\tilde{e})}{\phi L} \tag{A.9}$$

Now if we estimate the cross-sectional relation (10) using our M observations on $\bar{R}_K = \sum_{t=1}^{T} R_{Kt}/T$ and $\hat{\beta}_K$ for some holding period, we have

$$\bar{R}_K = \gamma_0 + \gamma_1\hat{\beta}_K + \bar{e}_K^* \tag{A.10}$$

where

$$\bar{e}_K^* = \sum_{t=1}^{T} \frac{\tilde{e}_{Kt}^*}{T} = \bar{e}_K - \gamma_1\tilde{\epsilon}_K \tag{A.11}$$

Now the large sample estimate of γ_1 in (A.10)

$$\text{plim}\ \hat{\gamma}_1 = \frac{\gamma_1}{1 + \dfrac{\text{plim}\ \sigma^2(\tilde{\epsilon}_K)}{\text{plim}\ S^2(\beta_K)}} = \frac{\gamma_1}{1 + \dfrac{\text{plim}\ \frac{1}{L}\sigma^2(\tilde{e})}{\phi S^2(\beta_K)}} = \gamma_1 \tag{A.12}$$

since plim $\sigma^2(\tilde{e})/L = 0$ as long as $L \to \infty$ as $N \to \infty$, and this is true as long as we hold the number of groups constant. Thus these grouping procedures will result in unbiased estimates of the parameters of (10) for large samples. Note that $S^2(\beta_K)$, the cross-sectional sample variance of the true group risk coefficients, is constant with increasing L so long as securities are assigned to groups on the basis of the ranked $\hat{\beta}_j$. Note also, however, that if we randomly assigned securities to the M groups we would have $\text{plim } S^2(\hat{\beta}_K) = \text{plim } S^2(\beta_j)/L$ and (A.12) would thus be identical to (A.3). Therefore, random grouping would be of no help in eliminating the bias. As can be seen, the grouping procedures we have already described in the time series tests accomplish these results. While we expect these procedures to substantially reduce the bias[16] they cannot completely eliminate it in our case because the $\hat{e}_j$ and therefore the $\bar{e}_j$ are not independent across firms. However, as discussed in Section III, we expect the remaining bias to be trivially small.

Notes

1. Note that (4c) can be valid even though R_M is a weighted average of the R_j and therefore R'_M contains e_j. This may be clarified as follows: taking the weighted sum of (3) using the weights, X_j, of each security in the market portfolio we know by the definition of R_M that $\Sigma_j X_j R_j = R_M$, $\Sigma_j X_j \beta_j = 1$, and $\Sigma_j X_j e_j = 0$. Thus by the last equality we know $X_j e_j = -\Sigma_{i \neq j} X_i e_i$, and by substitution $E(e_j X_j e_j) = E[e_j(-\Sigma_{i \neq j} X_i e_i)] = X_j \sigma^2(e_j)$, and this implies condition (4c) since $E(e_j R'_M) = X_j \sigma^2(e_j) + E[e_j \Sigma_{i \neq j} X_i e_i] = 0$.
2. We could develop the model and tests under the assumption of infinite variance stable distributions, but this would unnecessarily complicate some of the analysis. We shall take explicit account of these distributional problems in some of the crucial tests of significance in Section IV.
3. Recall that the R_j and R_M are defined as excess returns. The model can be formulated with r_{Ft} omitted from (6) and therefore assumed constant (then $\alpha_j = r_F(1 - \beta_j)$) or included as a variable (as we have done), which strictly requires them to be known for all t. But experiments with estimates obtained with the inclusion of r_{Ft} as a variable in (6) yield results virtually identical to those obtained with the assumption of constant r_F [and hence the exclusion of r_{Ft} as a variable in (6)], so we shall ignore this problem here. See also Roll [1969] and Miller and Scholes [1972] for a thorough discussion of the bias introduced through misspecification of the riskless rate. Miller and Scholes conclude as we do that these problems are not serious.
4. Unbiased measurement errors in $\hat{\beta}_j$ cause severe difficulties with the cross-sectional tests of the model, and it is important to note that the time series form of the tests given by (6) are free of this source of bias. Unbiased measurement errors in $\hat{\beta}_j$, which is estimated simultaneously with α_j in the time series formulation, cause errors in the estimate of α_j but no systematic bias. Measurement errors in R_M may cause difficulties in both the cross-sectional and time series forms of the tests, but we shall ignore this issue here. For an analysis of the problems associated with measurement errors in R_M, see Black and Jensen [1970], Miller and Scholes [1972], and Roll [1969].

5. Treasury Bill rates were obtained from the Salomon Brothers & Hutzler quote sheets at the end of the previous month for the following month. Dealer commercial paper rates were obtained from Banking and Monetary Statistics, Board of Governors of the Federal Reserve System, Washington, D.C.
6. The choice of the number of portfolios is somewhat arbitrary. As we shall see below, we wanted enough portfolios to provide a continuum of observations across the risk spectrum to enable us to estimate the suspected relation between α_K and β_K.
7. Note that in order for the risk parameters of the groups β_K, to be stationary through time, our procedures require that firms leave and enter the sample symmetrically across the entire risk spectrum.
8. See also Miller and Scholes [1972], who provide a careful analysis (using procedures that are complementary to but much different from those suggested here) of many of these problems with cross-sectional tests and their implications for the interpretation of previous empirical work.
9. Intuitively one can see that the measurement error problem is virtually eliminated by these procedures because the errors in $\hat{\beta}_K$ become extremely small. Since the correlations $r(\tilde{R}_K, \tilde{R}_M)$ are so high in Table 2, the standard errors of estimate of the coefficients β_K are all less than 0.022, and nine of them are less than 0.012. The average standard error of estimate for the ten $\hat{\beta}_K$ coefficients given in Table 2 for the entire period was 0.0101 and the cross-sectional variance of the $\hat{\beta}_K$, $S^2(\hat{\beta}_K)$ was 0.1144. Hence, assuming $S^2(\hat{\beta}_K) \approx S^2(\beta_K)$, squaring 0.0101, and using (11), we see that our estimate of γ_1 will be greater than 99.9% of its true value.
10. The analysis was also performed where the coefficients were reestimated for each subperiod, and the results were very similar because the $\hat{\beta}_K$ were quite stable over time. We report these results since this estimation procedure seemed to result in a slightly larger spread of the $\hat{\beta}_K$ and since the increased sample sizes tends to further reduce the bias caused by the variance of the measurement error in $\hat{\beta}_K$.
11. In fact, there is an infinite number of such zero β portfolios. Of all such portfolios, however, r_z is the return on the one with minimum variance. (We are indebted to John Long for the proof of this point.)
12. We say unreasonably high because the coefficients change from period to period by amounts ranging up to almost seven times their estimated standard errors.
13. Although the traditional form of the model is consistent with the existence of the β factor if its excess return had a zero mean, clearly it would not provide as complete an explanation of the structure of asset returns as a model that explicitly incorporated such a factor. In particular, under these circumstances the traditional form would provide an adequate description of security returns over fairly lengthy periods of time, say three years or more, but it would probably not furnish an adequate description of security returns over much shorter intervals.
14. We only observe the residual variance from the single variable regression, and, as we can see from (13), this will be equal to $(1 - \beta_j)^2\sigma^2(\tilde{r}_z) + \sigma^2(\tilde{u}_j)$. However, there are more general procedures for estimating $\tilde{r}_{zt}$ in

the situation of nonidentical $\sigma^2(\tilde{u}_j)$ and $\text{cov}(\tilde{u}_j, \tilde{u}_i) = 0$ for $j \neq i$. But we leave an investigation of the properties of these estimates and some additional tests of the two-factor model for a future paper. If the assumption of identical $\sigma^2(\tilde{u}_i)$ made here is inappropriate, we still obtain an unbiased estimate of the $\hat{R}_z$. However, the estimated variance of $\hat{R}_z$, which is of some interest, will be greater than the true variance.

15. The serial correlation for the entire period appears significant. Indeed, the serial correlation in the last period, 0.414, seems very large and even highly significant, with a t value of 4.6. However, the coefficients in the earlier periods seem to border on significance but show an inordinately large amount of variability, thus indicating substantial nonstationarity.
16. As mentioned earlier, the choice of the number of groups is somewhat arbitrary and, for any given sample size, involves a tradeoff between the bias and the degree of sampling error in the estimates of the parameters in (10). In an unpublished study of the properties of the grouping procedures by simulation techniques, Jensen and Mendu Rao have found that, when $\sigma^2(\tilde{\epsilon}_j) = S^2(\beta_j)$, the use of ten groups with a total sample size of $N = 400$, yields estimates of the coefficient γ_1 in (10) which, on the average, are biased downward by less than 0.9% of their true value and have a standard error of estimate about 50% higher than that obtained with ungrouped data. The ungrouped sample estimates were, of course, 50% of their true values on the average [as implied by (11) for these assumed variances].

References

BLACK, FISCHER. "Capital Market Equilibrium With No Riskless Borrowing or Lending" (forthcoming in the *Journal of Business*).

——, and JENSEN, MICHAEL C. "Incomplete Measurement of Market Returns and Its Implications for Tests of the Asset Pricing Model" (unpublished manuscript, November, 1970).

——, and SCHOLES, MYRON. "Dividend Yields and Common Stock Returns: A New Methodology" (Cambridge: Sloan School of Management, Massachusetts Institute of Technology, Working Paper #488-70, September, 1970).

BLATTBERG, ROBERT, and SARGENT, THOMAS. "Regression with Paretian Disturbances: Some Sampling Results," *Econometrics*, V, 39 (May, 1971) 501–10.

BLUME, MARSHALL. "The Assessment of Portfolio Performance" (Ph.D. dissertation, University of Chicago, 1968).

DOUGLAS, GEORGE W. "Risk in the Equity Markets: An Empirical Appraisal of Market Efficiency," *Yale Economic Essays*, IX (Spring, 1969), 3–45.

FAMA, EUGENE F. "Risk, Return, and Equilibrium: Some Clarifying Comments," *Journal of Finance* (March, 1968), 29–40.

——. "Risk, Return, and Equilibrium," *Journal of Political Economy*, LXXIX (January–February, 1971).

——, and BABIAK, HARVEY. "Dividend Policy: An Empirical Analysis," *Journal of the American Statistical Association*, LXIII (December, 1968), 1132–61.

——, FISHER, LAWRENCE; JENSEN, MICHAEL C.; and ROOL, RICHARD. "The Adjustment of Stock Prices to New Information," *International Economic Review*, X (February, 1969), 1–26.

——, and ROLL, RICHARD. "Some Properties of Symmetric Stable Distributions," *Journal of the American Statistical Association*, LXIII (September, 1968), 817–36.

JENSEN, MICHAEL C. "The Performance of Mutual Funds in the Period 1945–64," *Journal of Finance*, XXIII (May, 1968), 389–416.

——. "Risk, the Pricing of Capital Assets, and the Evaluation of Investment Portfolios," *Journal of Business*, XLII (April, 1969), 167–247.

——. "Optimal Utilization of Market Forecasts and The Evaluation of Investment Performance" (Working Paper No. 7109, University of Rochester School of Management, September, 1971).

KING, BENJAMIN F. "Market and Industry Factors in Stock Price Behavior," *Journal of Business*, XXXIX (January 1966, Part II), 134–90.

LINTNER, JOHN. "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *Review of Economics and Statistics*, XLVII (February, 1965a), 13–37.

——. "Security Prices, Risk, and Maximal Gains from Diversification," *Journal of Finance*, XX (December, 1965b), 587–616.

LONG, JOHN B., JR. "Consumption-Investment Decisions and Equilibrium in the Securities Market," this volume, 1972.

MARKOWITZ, HARRY M. Portfolio Selection: Efficient Diversification of Investments, Cowles Foundation Monograph No. 16 (New York: John Wiley & Sons, 1959).

MAYERS, DAVID. "Nonmarketable Assets and Capital Market Equilibrium Under Uncertainty," *this volume*, 1971.

MILLER, MERTON H., and SCHOLES, MYRON. "Rates of Return in Relation to Risk: A Re-examination of Some Recent Findings," this volume, 1972.

MOSSIN, JAN. "Equilibrium in a Capital Asset Market," *Econometrica*, XXXIV (October, 1966), 768–83.

PRESS, S. JAMES. "A Compound Events Model for Security Prices," *Journal of Business*, XL (July, 1967), 317–37.

ROLL, RICHARD. "Bias in Fitting the Sharpe Model to Time Series Data," *Journal of Financial and Quantitative Analysis*, IV (September, 1969), 271–89.

SHARPE, WILLIAM F. "A Simplified Model for Portfolio Analysis," *Management Science* (January, 1963), 277–93.

——. "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," *Journal of Finance*, XIX (September, 1964), 425–42.

——. "Risk Aversion in the Stock Market," *Journal of Finance*, XX (September, 1965), 416–22.

——. "Mutual Fund Performance," *Journal of Business*, XXXIX, Part 2 (January, 1966), 119–38.

TREYNOR, JACK L. "Toward a Theory of Market Value of Risky Assets" (unpublished manuscript, 1961).

——. "How to Rate Management of Investment Funds," *Harvard Business Review*, XLIII (January–February, 1965), 63–75.

WALD, ABRAHAM. "The Fitting of Straight Lines if Both Variables are Subject to Error," *Annals of Mathematical Statistics*, II (1940), 284–300.

WISE, JOHN. "Linear Estimators for Linear Regression Systems Having Infinite Variances" (paper presented at the Berkeley–Stanford Mathematical Economics Seminar, October, 1963).

Risk, Return, and Equilibrium: Empirical Tests

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This paper tests the relationship between average return and risk for New York Stock Exchange common stocks. The theoretical basis of the tests is the "two-parameter" portfolio model and models of market equilibrium derived from the two-parameter portfolio model. We cannot reject the hypothesis of these models that the pricing of common stocks reflects the attempts of risk-averse investors to hold portfolios that are "efficient" in terms of expected value and dispersion of return. Moreover, the observed "fair game" properties of the coefficients and residuals of the risk-return regressions are consistent with an "efficient capital market"—that is, a market where prices of securities fully reflect available information.

I. Theoretical Background

In the two-parameter portfolio model of Tobin (1958), Markowitz (1959), and Fama (1965*b*), the capital market is assumed to be perfect in the sense that investors are price takers and there are neither transactions costs nor information costs. Distributions of one-period percentage returns on all assets and portfolios are assumed to be normal or to conform to some other two-parameter member of the symmetric stable class. Investors are assumed to be risk averse and to behave as if they choose among portfolios on the basis of maximum expected utility. A perfect capital market, investor risk aversion, and two-parameter return distributions imply the important "efficient set theorem": The optimal portfolio for any investor must be efficient in the sense that no other portfolio with the same or higher expected return has lower dispersion of return.[1]

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[1] Although the choice of dispersion parameter is arbitrary, the standard deviation

In the portfolio model the investor looks at individual assets only in terms of their contributions to the expected value and dispersion, or risk, of his portfolio return. With normal return distributions the risk of portfolio p is measured by the standard deviation, $\sigma(\tilde{R}_p)$, of its return, $\tilde{R}_p$,[2] and the risk of an asset for an investor who holds p is the contribution of the asset to $\sigma(\tilde{R}_p)$. If x_{ip} is the proportion of portfolio funds invested in asset i, $\sigma_{ij} = \mathrm{cov}(\tilde{R}_i, \tilde{R}_j)$ is the covariance between the returns on assets i and j, and N is the number of assets, then

$$\sigma(\tilde{R}_p) = \sum_{i=1}^{N} x_{ip} \left[\frac{\sum_{j=1}^{N} x_{jp}\sigma_{ij}}{\sigma(\tilde{R}_p)} \right] = \sum_{i=1}^{N} x_{ip} \frac{\mathrm{cov}(\tilde{R}_i, \tilde{R}_p)}{\sigma(\tilde{R}_p)}.$$

Thus, the contribution of asset i to $\sigma(\tilde{R}_p)$—that is, the risk of asset i in the portfolio p—is proportional to

$$\sum_{j=1}^{N} x_{jp}\sigma_{ij}/\sigma(\tilde{R}_p) = \mathrm{cov}(\tilde{R}_i, \tilde{R}_p)/\sigma(\tilde{R}_p).$$

Note that since the weights x_{jp} vary from portfolio to portfolio, the risk of an asset is different for different portfolios.

For an individual investor the relationship between the risk of an asset and its expected return is implied by the fact that the investor's optimal portfolio is efficient. Thus, if he chooses the portfolio m, the fact that m is efficient means that the weights x_{im}, $i = 1, 2, \ldots, N$, maximize expected portfolio return

$$E(\tilde{R}_m) = \sum_{i=1}^{N} x_{im} E(\tilde{R}_i),$$

subject to the constraints

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is common when return distributions are assumed to be normal, whereas an interfractile range is usually suggested when returns are generated from some other symmetric stable distribution.

It is well known that the mean-standard deviation version of the two-parameter portfolio model can be derived from the assumption that investors have quadratic utility functions. But the problems with this approach are also well known. In any case, the empirical evidence of Fama (1965*a*), Blume (1970), Roll (1970), K. Miller (1971), and Officer (1971) provides support for the "distribution" approach to the model. For a discussion of the issues and a detailed treatment of the two-parameter model, see Fama and Miller (1972, chaps. 6–8).

We also concentrate on the special case of the two-parameter model obtained with the assumption of normally distributed returns. As shown in Fama (1971) or Fama and Miller (1972, chap. 7), the important testable implications of the general symmetric stable model are the same as those of the normal model.

[2] Tildes (~) are used to denote random variables. And the one-period percentage return is most often referred to just as the return.

$$\sigma(\tilde{R}_p) = \sigma(\tilde{R}_m) \quad \text{and} \quad \sum_{i=1}^{N} x_{im} = 1.$$

Lagrangian methods can then be used to show that the weights x_{jm} must be chosen in such a way that for any asset i in m

$$E(\tilde{R}_i) - E(\tilde{R}_m) = S_m \left[\frac{\sum_{j=1}^{N} x_{jm}\sigma_{ij}}{\sigma(\tilde{R}_m)} - \sigma(\tilde{R}_m) \right], \tag{1}$$

where S_m is the rate of change of $E(\tilde{R}_p)$ with respect to a change in $\sigma(\tilde{R}_p)$ at the point on the efficient set corresponding to portfolio m. If there are nonnegativity constraints on the weights (that is, if short selling is prohibited), then (1) only holds for assets i such that $x_{im} > 0$.

Although equation (1) is just a condition on the weights x_{jm} that is required for portfolio efficiency, it can be interpreted as the relationship between the risk of asset i in portfolio m and the expected return on the asset. The equation says that the difference between the expected return on the asset and the expected return on the portfolio is proportional to the difference between the risk of the asset and the risk of the portfolio. The proportionality factor is S_m, the slope of the efficient set at the point corresponding to the portfolio m. And the risk of the asset is its contribution to total portfolio risk, $\sigma(\tilde{R}_m)$.

II. Testable Implications

Suppose now that we posit a market of risk-averse investors who make portfolio decisions period by period according to the two-parameter model.[3] We are concerned with determining what this implies for observable properties of security and portfolio returns. We consider two categories of implications. First, there are conditions on expected returns that are implied by the fact that in a two-parameter world investors hold efficient portfolios. Second, there are conditions on the behavior of returns through time that are implied by the assumption of the two-parameter model that the capital market is perfect or frictionless in the sense that there are neither transactions costs nor information costs.

A. Expected Returns

The implications of the two-parameter model for expected returns derive from the efficiency condition or expected return-risk relationship of equation (1). First, it is convenient to rewrite (1) as

[3] A multiperiod version of the two-parameter model is in Fama (1970*a*) or Fama and Miller (1972, chap. 8).

$$E(\tilde{R}_i) = [E(\tilde{R}_m) - S_m\,\sigma(\tilde{R}_m)] + S_m\,\sigma(\tilde{R}_m)\beta_i, \tag{2}$$

where

$$\beta_i \equiv \frac{\mathrm{cov}(\tilde{R}_i, \tilde{R}_m)}{\sigma^2(\tilde{R}_m)} = \frac{\sum_{j=1}^{N} x_{jm}\sigma_{ij}}{\sigma^2(\tilde{R}_m)} = \frac{\mathrm{cov}(\tilde{R}_i, \tilde{R}_m)/\sigma(\tilde{R}_m)}{\sigma(\tilde{R}_m)}. \tag{3}$$

The parameter β_i can be interpreted as the risk of asset i in the portfolio m, measured relative to $\sigma(\tilde{R}_m)$, the total risk of m. The intercept in (2),

$$E(\tilde{R}_0) \equiv E(\tilde{R}_m) - S_m\,\sigma(\tilde{R}_m), \tag{4}$$

is the expected return on a security whose return is uncorrelated with $\tilde{R}_m$—that is, a zero-β security. Since $\beta = 0$ implies that a security contributes nothing to $\sigma(\tilde{R}_m)$, it is appropriate to say that it is riskless in this portfolio. It is well to note from (3), however, that since $x_{im}\,\sigma_{ii} = x_{im}\,\sigma^2(\tilde{R}_i)$ is just one of the N terms in β_i, $\beta_i = 0$ does not imply that security i has zero variance of return.

From (4), it follows that

$$S_m = \frac{E(\tilde{R}_m) - E(\tilde{R}_0)}{\sigma(\tilde{R}_m)}, \tag{5}$$

so that (2) can be rewritten

$$E(\tilde{R}_i) = E(\tilde{R}_0) + [E(\tilde{R}_m) - E(\tilde{R}_0)]\beta_i. \tag{6}$$

In words, the expected return on security i is $E(\tilde{R}_0)$, the expected return on a security that is riskless in the portfolio m, plus a risk premium that is β_i times the difference between $E(\tilde{R}_m)$ and $E(\tilde{R}_0)$.

Equation (6) has three testable implications: (C1) The relationship between the expected return on a security and its risk in any efficient portfolio m is linear. (C2) β_i is a complete measure of the risk of security i in the efficient portfolio m; no other measure of the risk of i appears in (6). (C3) In a market of risk-averse investors, higher risk should be associated with higher expected return; that is, $E(\tilde{R}_m) - E(\tilde{R}_0) > 0$.

The importance of condition C3 is obvious. The importance of C1 and C2 should become clear as the discussion proceeds. At this point suffice it to say that if C1 and C2 do not hold, market returns do not reflect the attempts of investors to hold efficient portfolios: Some assets are systematically underpriced or overpriced relative to what is implied by the expected return-risk or efficiency equation (6).

B. Market Equilibrium and the Efficiency of the Market Portfolio

To test conditions C1–C3 we must identify some efficient portfolio m. This in turn requires specification of the characteristic of market equilibrium when investors make portfolio decisions according to the two-parameter model.

Assume again that the capital market is perfect. In addition, suppose that from the information available without cost all investors derive the same and correct assessment of the distribution of the future value of any asset or portfolio—an assumption usually called "homogeneous expectations." Finally, assume that short selling of all assets is allowed. Then Black (1972) has shown that in a market equilibrium, the so-called market portfolio, defined by the weights

$$x_{im} \equiv \frac{\text{total market value of all units of asset } i}{\text{total market value of all assets}},$$

is always efficient.

Since it contains all assets in positive amounts, the market portfolio is a convenient reference point for testing the expected return-risk conditions C1–C3 of the two-parameter model. And the homogeneous-expectations assumption implies a correspondence between ex ante assessments of return distributions and distributions of ex post returns that is also required for meaningful tests of these three hypotheses.

C. *A Stochastic Model for Returns*

Equation (6) is in terms of expected returns. But its implications must be tested with data on period-by-period security and portfolio returns. We wish to choose a model of period-by-period returns that allows us to use observed average returns to test the expected-return conditions C1–C3, but one that is nevertheless as general as possible. We suggest the following stochastic generalization of (6):

$$\tilde{R}_{it} = \tilde{\gamma}_{0t} + \tilde{\gamma}_{1t}\beta_i + \tilde{\gamma}_{2t}\beta_i^2 + \tilde{\gamma}_{3t}s_i + \tilde{\eta}_{it}. \qquad (7)$$

The subscript t refers to period t, so that $\tilde{R}_{it}$ is the one-period percentage return on security i from $t-1$ to t. Equation (7) allows $\tilde{\gamma}_{0t}$ and $\tilde{\gamma}_{1t}$ to vary stochastically from period to period. The hypothesis of condition C3 is that the expected value of the risk premium $\tilde{\gamma}_{1t}$, which is the slope $[E(\tilde{R}_{mt}) - E(\tilde{R}_{0t})]$ in (6), is positive—that is, $E(\tilde{\gamma}_{1t}) = E(\tilde{R}_{mt}) - E(\tilde{R}_{0t}) > 0$.

The variable β_i^2 is included in (7) to test linearity. The hypothesis of condition C1 is $E(\tilde{\gamma}_{2t}) = 0$, although $\tilde{\gamma}_{2t}$ is also allowed to vary stochastically from period to period. Similar statements apply to the term involving s_i in (7), which is meant to be some measure of the risk of security i that is not deterministically related to β_i. The hypothesis of condition C2 is $E(\tilde{\gamma}_{3t}) = 0$, but $\tilde{\gamma}_{3t}$ can vary stochastically through time.

The disturbance $\tilde{\eta}_{it}$ is assumed to have zero mean and to be independent of all other variables in (7). If all portfolio return distributions are to be normal (or symmetric stable), then the variables $\tilde{\eta}_{it}$, $\tilde{\gamma}_{0t}$, $\tilde{\gamma}_{1t}$, $\tilde{\gamma}_{2t}$ and $\tilde{\gamma}_{3t}$ must have a multivariate normal (or symmetric stable) distribution.

D. Capital Market Efficiency: The Behavior of Returns through Time

C1–C3 are conditions on expected returns and risk that are implied by the two-parameter model. But the model, and especially the underlying assumption of a perfect market, implies a capital market that is efficient in the sense that prices at every point in time fully reflect available information. This use of the word efficient is, of course, not to be confused with portfolio efficiency. The terminology, if a bit unfortunate, is at least standard.

Market efficiency in combination with condition C1 requires that scrutiny of the time series of the stochastic nonlinearity coefficient $\tilde{\gamma}_{2t}$ does not lead to nonzero estimates of expected future values of $\tilde{\gamma}_{2t}$. Formally, $\tilde{\gamma}_{2t}$ must be a fair game. In practical terms, although nonlinearities are observed ex post, because $\tilde{\gamma}_{2t}$ is a fair game, it is always appropriate for the investor to act ex ante under the presumption that the two-parameter model, as summarized by (6), is valid. That is, in his portfolio decisions he always assumes that there is a linear relationship between the risk of a security and its expected return. Likewise, market efficiency in the two-parameter model requires that the non-β risk coefficient $\tilde{\gamma}_{3t}$ and the time series of return disturbances $\tilde{\eta}_{it}$ are fair games. And the fair-game hypothesis also applies to the time series of $\tilde{\gamma}_{1t} - [E(\tilde{R}_{mt}) - E(\tilde{R}_{0t})]$, the difference between the risk premium for period t and its expected value.

In the terminology of Fama (1970*b*), these are "weak-form" propositions about capital market efficiency for a market where expected returns are generated by the two-parameter model. The propositions are weak since they are only concerned with whether prices fully reflect any information in the time series of past returns. "Strong-form" tests would be concerned with the speed-of-adjustment of prices to all available information.

E. Market Equilibrium with Riskless Borrowing and Lending

We have as yet presented no hypothesis about $\tilde{\gamma}_{0t}$ in (7). In the general two-parameter model, given $E(\tilde{\gamma}_{2t}) = E(\tilde{\gamma}_{3t}) = E(\tilde{\eta}_{it}) = 0$, then, from (6), $E(\tilde{\gamma}_{0t})$ is just $E(\tilde{R}_{0t})$, the expected return on any zero-β security. And market efficiency requires that $\tilde{\gamma}_{0t} - E(\tilde{R}_{0t})$ be a fair game.

But if we add to the model as presented thus far the assumption that there is unrestricted riskless borrowing and lending at the known rate R_{ft}, then one has the market setting of the original two-parameter "capital asset pricing model" of Sharpe (1964) and Lintner (1965). In this world, since $\beta_f = 0$, $E(\tilde{\gamma}_{0t}) = R_{ft}$. And market efficiency requires that $\tilde{\gamma}_{0t} - R_{ft}$ be a fair game.

It is well to emphasize that to refute the proposition that $E(\tilde{\gamma}_{0t}) = R_{ft}$ is only to refute a specific two-parameter model of market equilibrium. Our view is that tests of conditions C1–C3 are more fundamental. We regard C1–C3 as the general expected return implications of the two-parameter model in the sense that they are the implications of the fact that in the two-parameter portfolio model investors hold efficient portfolios, and they are consistent with any two-parameter model of market equilibrium in which the market portfolio is efficient.

F. The Hypotheses

To summarize, given the stochastic generalization of (2) and (6) that is provided by (7), the testable implications of the two-parameter model for expected returns are:

C1 (linearity)—$E(\tilde{\gamma}_{2t}) = 0$.

C2 (no systematic effects of non-β risk)—$E(\tilde{\gamma}_{3t}) = 0$.

C3 (positive expected return-risk tradeoff)—$E(\tilde{\gamma}_{1t}) = E(\tilde{R}_{mt}) - E(\tilde{R}_{0t}) > 0$.

Sharpe-Lintner (S-L) Hypothesis—$E(\tilde{\gamma}_{0t}) = R_{ft}$.

Finally, capital market efficiency in a two-parameter world requires

ME (market efficiency)—the stochastic coefficients $\tilde{\gamma}_{2t}$, $\tilde{\gamma}_{3t}$, $\tilde{\gamma}_{1t} - [E(\tilde{R}_{mt}) - E(\tilde{R}_{0t})]$, $\tilde{\gamma}_{0t} - E(\tilde{R}_{0t})$, and the disturbances $\tilde{\eta}_{it}$ are fair games.[4]

III. Previous Work[5]

The earliest tests of the two-parameter model were done by Douglas (1969), whose results seem to refute condition C2. In annual and quarterly return data, there seem to be measures of risk, in addition to β, that contribute systematically to observed average returns. These results, if valid, are inconsistent with the hypothesis that investors attempt to hold efficient portfolios. Assuming that the market portfolio is efficient, premiums are paid for risks that do not contribute to the risk of an efficient portfolio.

Miller and Scholes (1972) take issue both with Douglas's statistical techniques and with his use of annual and quarterly data. Using different methods and simulations, they show that Douglas's negative results could be expected even if condition C2 holds. Condition C2 is tested below with extensive monthly data, and this avoids almost all of the problems discussed by Miller and Scholes.

[4] If $\tilde{\gamma}_{2t}$ and $\tilde{\gamma}_{3t}$ are fair games, then $E(\tilde{\gamma}_{2t}) = E(\tilde{\gamma}_{3t}) = 0$. Thus, C1 and C2 are implied by ME. Keeping the expected return conditions separate, however, better emphasizes the economic basis of the various hypotheses.

[5] A comprehensive survey of empirical and theoretical work on the two-parameter model is in Jensen (1972).

Much of the available empirical work on the two-parameter model is concerned with testing the S-L hypothesis that $E(\tilde{\gamma}_{0t}) = R_{ft}$. The tests of Friend and Blume (1970) and those of Black, Jensen, and Scholes (1972) indicate that, at least in the period since 1940, on average $\tilde{\gamma}_{0t}$ is systematically greater than R_{ft}. The results below support this conclusion.

In the empirical literature to date, the importance of the linearity condition C1 has been largely overlooked. Assuming that the market portfolio m is efficient, if $E(\tilde{\gamma}_{2t})$ in (7) is positive, the prices of high-β securities are on average too low—their expected returns are too high—relative to those of low-β securities, while the reverse holds if $E(\tilde{\gamma}_{2t})$ is negative. In short, if the process of price formation in the capital market reflects the attempts of investors to hold efficient portfolios, then the linear relationship of (6) between expected return and risk must hold.

Finally, the previous empirical work on the two-parameter model has not been concerned with tests of market efficiency.

IV. Methodology

The data for this study are monthly percentage returns (including dividends and capital gains, with the appropriate adjustments for capital changes such as splits and stock dividends) for all common stocks traded on the New York Stock Exchange during the period January 1926 through June 1968. The data are from the Center for Research in Security Prices of the University of Chicago.

A. General Approach

Testing the two-parameter model immediately presents an unavoidable "errors-in-the-variables" problem: The efficiency condition or expected return-risk equation (6) is in terms of true values of the relative risk measure β_i, but in empirical tests estimates, $\hat{\beta}_i$, must be used. In this paper

$$\hat{\beta}_i \equiv \frac{\widehat{\text{cov}}(\tilde{R}_i, \tilde{R}_m)}{\hat{\sigma}^2(\tilde{R}_m)},$$

where $\widehat{\text{cov}}(\tilde{R}_i, \tilde{R}_m)$ and $\hat{\sigma}^2(\tilde{R}_m)$ are estimates of $\text{cov}(\tilde{R}_i, \tilde{R}_m)$ and $\sigma^2(\tilde{R}_m)$ obtained from monthly returns, and where the proxy chosen for $\tilde{R}_{mt}$ is "Fisher's Arithmetic Index," an equally weighted average of the returns on all stocks listed on the New York Stock Exchange in month t. The properties of this index are analyzed in Fisher (1966).

Blume (1970) shows that for any portfolio p, defined by the weights x_{ip}, $i = 1, 2, \ldots, N$,

$$\hat{\beta}_p \equiv \frac{\widehat{\text{cov}}(\tilde{R}_p, \tilde{R}_m)}{\hat{\sigma}^2(\tilde{R}_m)} = \sum_{i=1}^{N} x_{ip} \frac{\widehat{\text{cov}}(\tilde{R}_i, \tilde{R}_m)}{\hat{\sigma}^2(\tilde{R}_m)} = \sum_{i=1}^{N} x_{ip}\, \hat{\beta}_i.$$

If the errors in the $\hat{\beta}_i$ are substantially less than perfectly positively correlated, the $\hat{\beta}$'s of portfolios can be much more precise estimates of true β's than the $\hat{\beta}$'s for individual securities.

To reduce the loss of information in the risk-return tests caused by using portfolios rather than individual securities, a wide range of values of portfolio $\hat{\beta}_p$'s is obtained by forming portfolios on the basis of ranked values of $\hat{\beta}_i$ for individual securities. But such a procedure, naïvely executed could result in a serious regression phenomenon. In a cross section of $\hat{\beta}_i$, high observed $\hat{\beta}_i$ tend to be above the corresponding true β_i and low observed $\hat{\beta}_i$ tend to be below the true β_i. Forming portfolios on the basis of ranked $\hat{\beta}_i$ thus causes bunching of positive and negative sampling errors within portfolios. The result is that a large portfolio $\hat{\beta}_p$ would tend to overstate the true β_p, while a low $\hat{\beta}_p$ would tend to be an underestimate.

The regression phenomenon can be avoided to a large extent by forming portfolios from ranked $\hat{\beta}_i$ computed from data for one time period but then using a subsequent period to obtain the $\hat{\beta}_p$ for these portfolios that are used to test the two-parameter model. With fresh data, within a portfolio errors in the individual security $\hat{\beta}_i$ are to a large extent random across securities, so that in a portfolio $\hat{\beta}_p$ the effects of the regression phenomenon are, it is hoped, minimized.[6]

B. Details

The specifics of the approach are as follows. Let N be the total number of securities to be allocated to portfolios and let $\text{int}(N/20)$ be the largest integer equal to or less than $N/20$. Using the first 4 years (1926–29) of monthly return data, 20 portfolios are formed on the basis of ranked $\hat{\beta}_i$ for individual securities. The middle 18 portfolios each has $\text{int}(N/20)$ securities. If N is even, the first and last portfolios each has $\text{int}(N/20) + \frac{1}{2}[N - 20\,\text{int}(N/20)]$ securities. The last (highest $\hat{\beta}$) portfolio gets an additional security if N is odd.

The following 5 years (1930–34) of data are then used to recompute the $\hat{\beta}_i$, and these are averaged across securities within portfolios to obtain 20 initial portfolio $\hat{\beta}_{pt}$ for the risk-return tests. The subscript t is added to indicate that each month t of the following four years (1935–38) these $\hat{\beta}_{pt}$ are recomputed as simple averages of individual security $\hat{\beta}_i$, thus adjusting the portfolio $\hat{\beta}_{pt}$ month by month to allow for delisting of securities. The component $\hat{\beta}_i$ for securities are themselves updated yearly—that

[6] The errors-in-the-variables problem and the technique of using portfolios to solve it were first pointed out by Blume (1970). The portfolio approach is also used by Friend and Blume (1970) and Black, Jensen, and Scholes (1972). The regression phenomenon that arises in risk-return tests was first recognized by Blume (1970) and then by Black, Jensen, and Scholes (1972), who offer a solution to the problem that is similar in spirit to ours.

is, they are recomputed from monthly returns for 1930 through 1935, 1936, or 1937.

As a measure of the non-β risk of security i we use $s(\hat{\epsilon}_i)$, the standard deviation of the least-squares residuals $\hat{\epsilon}_{it}$ from the so-called market model

$$\tilde{R}_{it} = a_i + \beta^i \tilde{R}_{mt} + \tilde{\epsilon}_{it}. \quad (8)$$

The standard deviation $s(\hat{\epsilon}_i)$ is a measure of non-β risk in the following sense. One view of risk, antithetic to that of portfolio theory, says that the risk of a security is measured by the total dispersion of its return distribution. Given a market dominated by risk averters, this model would predict that a security's expected return is related to its total return dispersion rather than just to the contribution of the security to the dispersion in the return on an efficient portfolio.[7] If $B_i \equiv \text{cov}(\tilde{R}_i, \tilde{R}_m)/\sigma^2(\tilde{R}_m)$, then in (8) $\text{cov}(\tilde{\epsilon}_i, \tilde{R}_m) = 0$, and

$$\sigma^2(\tilde{R}_i) = \beta_i^2\sigma^2(\tilde{R}_m) + \sigma^2(\tilde{\epsilon}_i) + 2\beta_i \text{cov}(\tilde{R}_m, \tilde{\epsilon}_i). \quad (9)$$

Thus, from (9), one can say that $s(\hat{\epsilon}_i)$ is an estimate of that part of the dispersion of the distribution of the return on security i that is not directly related to β_i.

The month-by-month returns on the 20 portfolios, with equal weighting of individual securities each month, are also computed for the 4-year period 1935–38. For each month t of this period, the following cross-sectional regression—the empirical analog of equation (7)—is run:

$$R_{pt} = \hat{\gamma}_{0t} + \hat{\gamma}_{1t}\hat{\beta}_{p,t-1} + \hat{\gamma}_{2t}\hat{\beta}^2_{p,t-1} + \hat{\gamma}_{3t}\bar{s}_{p,t-1}(\hat{\epsilon}_i) + \hat{\eta}_{pt}, \quad (10)$$
$$p = 1, 2, \ldots, 20.$$

The independent variable $\hat{\beta}_{p,t-1}$ is the average of the $\hat{\beta}_i$ for securities in portfolio p discussed above; $\hat{\beta}^2_{p,t-1}$ is the average of the squared values of these $\hat{\beta}_i$ (and is thus somewhat mislabeled); and $\bar{s}_{p,t-1}(\hat{\epsilon}_i)$ is likewise the average of $s(\hat{\epsilon}_i)$ for securities in portfolio p. The $s(\hat{\epsilon}_i)$ are computed from data for the same period as the component $\hat{\beta}_i$ of $\hat{\beta}_{p,t-1}$, and like these $\hat{\beta}_i$, they are updated annually.

The regression equation (10) is (7) averaged across the securities in a portfolio, with estimates $\hat{\beta}_{p,t-1}$, $\hat{\beta}^2_{p,t-1}$, and $\bar{s}_{p,t-1}(\hat{\epsilon}_i)$ used as explanatory variables, and with least-squares estimates of the stochastic coefficients $\hat{\gamma}_{0t}$, $\hat{\gamma}_{1t}$, $\hat{\gamma}_{2t}$, and $\hat{\gamma}_{3t}$. The results from (10)—the time series of month-by-month values of the regression coefficients $\hat{\gamma}_{0t}$, $\hat{\gamma}_{1t}$, $\hat{\gamma}_{2t}$, and $\hat{\gamma}_{3t}$ for the 4-year period 1935–38—are the inputs for our tests of the two-parameter model for this period. To get results for other periods, the steps described

[7] For those accustomed to the portfolio viewpoint, this alternative model may seem so naïve that it should be classified as a straw man. But it is the model of risk and return implied by the "liquidity preference" and "market segmentation" theories of the term structure of interest rates and by the Keynesian "normal backwardation" theory of commodity futures markets. For a discussion of the issues with respect to these markets, see Roll (1970) and K. Miller (1971).

above are repeated. That is, 7 years of data are used to form portfolios; the next 5 years are used to compute initial values of the independent variables in (10); and then the risk-return regressions of (10) are fit month by month for the following 4-year period.

The nine different portfolio formation periods (all except the first 7 years in length), initial 5-year estimation periods, and testing periods (all but the last 4 years in length) are shown in table 1. The choice of 4-year testing periods is a balance of computation costs against the desire to reform portfolios frequently. The choice of 7-year portfolio formation periods and 5–8-year periods for estimating the independent variables $\hat{\beta}_{p,t-1}$ and $\bar{s}_{p,t-1}(\hat{\epsilon}_i)$ in the risk-return regressions reflects a desire to balance the statistical power obtained with a large sample from a stationary process against potential problems caused by any nonconstancy of the β_i. The choices here are in line with the results of Gonedes (1973). His results also led us to require that to be included in a portfolio a security available in the first month of a testing period must also have data for all 5 years of the preceding estimation period and for at least 4 years of the portfolio formation period. The total number of securities available in the first month of each testing period and the number of securities meeting the data requirement are shown in table 1.

C. *Some Observations on the Approach*

Table 2 shows the values of the 20 portfolios $\hat{\beta}_{p,t-1}$ and their standard errors $s(\hat{\beta}_{p,t-1})$ for four of the nine 5-year estimation periods. Also shown are: $r(R_p, R_m)^2$, the coefficient of determination between R_{pt} and R_{mt}; $s(R_p)$, the sample standard deviation of R_p; and $s(\hat{\epsilon}_p)$, the standard deviation of the portfolio residuals from the market model of (8), not to be confused with $\bar{s}_{p,t-1}(\hat{\epsilon}_i)$, the average for individual securities, which is also shown. The $\hat{\beta}_{p,t-1}$ and $\bar{s}_{p,t-1}(\hat{\epsilon}_i)$ are the independent variables in the risk return regressions of (10) for the first month of the 4-year testing periods following the four estimation periods shown.

Under the assumptions that for a given security the disturbances $\tilde{\epsilon}_{jt}$ in (8) are serially independent, independent of $\tilde{R}_{mt}$, and identically distributed through time, the standard error of $\hat{\beta}_i$ is

$$\sigma(\hat{\beta}_i) = \frac{\sigma(\tilde{\epsilon}_i)}{\sqrt{n}\,\sigma(\tilde{R}_m)},$$

where n is the number of months used to compute $\hat{\beta}_i$. Likewise,

$$\sigma(\tilde{\beta}_{p,t-1}) = \frac{\sigma(\tilde{\epsilon}_p)}{\sqrt{n}\,\sigma(\tilde{R}_m)}.$$

Thus, the fact that in table 2, $s(\hat{\epsilon}_p)$ is generally on the order of one-third to one-seventh $\bar{s}_{p,t-1}(\hat{\epsilon}_i)$ implies that $s(\hat{\beta}_{p,t-1})$ is one-third to one-seventh

TABLE 1

PORTFOLIO FORMATION, ESTIMATION, AND TESTING PERIODS

	PERIODS				
	1	2	3	4	5
Portfolio formation period ...	1926–29	1927–33	1931–37	1935–41	1939–45
Initial estimation period	1930–34	1934–38	1938–42	1942–46	1946–50
Testing period	1935–38	1939–42	1943–46	1947–50	1951–54
No. of securities available	710	779	804	908	1,011
No. of securities meeting data requirement	435	576	607	704	751

$s(\hat{\beta}_i)$. Estimates of β for portfolios are indeed more precise than those for individual securities.

Nevertheless, it is interesting to note that if the disturbances $\tilde{\epsilon}_{jt}$ in (8) were independent from security to security, the relative increase in the precision of the $\hat{\beta}$ obtained by using portfolios rather than individual securities would be about the same for all portfolios. We argue in the Appendix, however, that the results from (10) imply that the $\tilde{\epsilon}_{it}$ in (8) are interdependent, and the interdependence is strongest among high-β securities and among low-β securities. This is evident in table 2: The ratios $s(\hat{\epsilon}_p)/\bar{s}_{p,t-1}(\hat{\epsilon}_i)$ are always highest at the extremes of the $\hat{\beta}_{p,t-1}$ range and lowest for $\hat{\beta}_{p,t-1}$ close to 1.0. But it is important to emphasize that since these ratios are generally less than .33, interdependence among the $\tilde{\epsilon}_{it}$ of different securities does not destroy the value of using portfolios to reduce the dispersion of the errors in estimated β's.

Finally, all the tests of the two-parameter model are predictive in the sense that the explanatory variables $\hat{\beta}_{p,t-1}$ and $\bar{s}_{p,t-1}(\hat{\epsilon}_i)$ in (10) are computed from data for a period prior to the month of the returns, the R_{pt}, on which the regression is run. Although we are interested in testing the two-parameter model as a positive theory—that is, examining the extent to which it is helpful in describing actual return data—the model was initially developed by Markowitz (1959) as a normative theory—that is, as a model to help people make better decisions. As a normative theory the model only has content if there is some relationship between future returns and estimates of risk that can be made on the basis of current information.

Now that the predictive nature of the tests has been emphasized, to simplify the notation, the explanatory variables in (10) are henceforth referred to as $\hat{\beta}_p$, $\hat{\beta}_p^2$, and $\bar{s}_p(\hat{\epsilon}_i)$.

V. Results

The major tests of the implications of the two-parameter model are in table 3. Results are presented for 10 periods: the overall period 1935–

TABLE 1 (*Continued*)

	PERIODS			
	6	7	8	9
Portfolio formation period ...	1943–49	1947–53	1951–57	1955–61
Initial estimation period	1950–54	1954–58	1958–62	1962–66
Testing period	1955–58	1959–62	1963–66	1967–68
No. of securities available	1,053	1,065	1,162	1,261
No. of securities meeting data requirement	802	856	858	845

6/68; three long subperiods, 1935–45, 1946–55, and 1956–6/68; and six subperiods which, except for the first and last, cover 5 years each. This choice of subperiods reflects the desire to keep separate the pre– and post–World War II periods. Results are presented for four different versions of the risk-return regression equation (10): Panel D is based on (10) itself, but in panels A–C, one or more of the variables in (10) is suppressed. For each period and model, the table shows: $\bar{\hat{\gamma}}_j$, the average of the month-by-month regression coefficient estimates, $\hat{\gamma}_{jt}$; $s(\hat{\gamma}_j)$, the standard deviation of the monthly estimates; and $\bar{r}^2$ and $s(r^2)$, the mean and standard deviation of the month-by-month coefficients of determination, r_t^2, which are adjusted for degrees of freedom. The table also shows the first-order serial correlations of the various monthly $\hat{\gamma}_{jt}$ computed either about the sample mean of $\hat{\gamma}_{jt}$ [in which case the serial correlations are labeled $\rho_M(\hat{\gamma}_i)$] or about an assumed mean of zero [in which case they are labeled $\rho_0(\hat{\gamma}_j)$]. Finally, t-statistics for testing the hypothesis that $\bar{\hat{\gamma}}_j = 0$ are presented. These t-statistics are

$$t(\bar{\hat{\gamma}}_j) = \frac{\bar{\hat{\gamma}}_j}{s(\hat{\gamma}_j)/\sqrt{n}},$$

where n is the number of months in the period, which is also the number of estimates $\hat{\gamma}_{jt}$ used to compute $\bar{\hat{\gamma}}_j$ and $s(\hat{\gamma}_j)$.

In interpreting these t-statistics one should keep in mind the evidence of Fama (1965*a*) and Blume (1970) which suggests that distributions of common stock returns are "thick-tailed" relative to the normal distribution and probably conform better to nonnormal symmetric stable distributions than to the normal. From Fama and Babiak (1968), this evidence means that when one interprets large t-statistics under the assumption that the underlying variables are normal, the probability or significance levels obtained are likely to be overestimates. But it is important to note that, with the exception of condition C3 (positive expected return-risk tradeoff), upward-biased probability levels lead to biases toward rejection of the hypotheses of the two-parameter model. Thus, if these hypotheses cannot

TABLE 2

SAMPLE STATISTICS FOR FOUR SELECTED ESTIMATION PERIODS

Statistic	1	2	3	4	5	6	7	8	9	10
	Portfolios for Estimation Period 1934–38									
$\hat{\beta}_{p,t-1}$	.322	.508	.651	.674	.695	.792	.921	.942	.970	1.005
$s(\hat{\beta}_{p,t-1})$	.027	.027	.025	.023	.028	.026	.032	.029	.034	.027
$r(R_p, R_m)^2$	.709	.861	.921	.936	.912	.941	.932	.946	.933	.958
$s(R_p)$	.040	.058	.072	.074	.077	.087	.101	.103	.106	.109
$s(\hat{\epsilon}_p)$	.022	.022	.020	.019	.023	.021	.026	.024	.028	.022
$\bar{s}_{p,t-1}(\hat{\epsilon}_i)$	.085	.075	.083	.078	.090	.095	.109	.106	.111	.097
$s(\hat{\epsilon}_p)/\bar{s}_{p,t-1}(\hat{\epsilon}_i)$	.259	.293	.241	.244	.256	.221	.238	.226	.252	.227
	Portfolios for Estimation Period 1942–46									
$\hat{\beta}_{p,t-1}$	.467	.537	.593	.628	.707	.721	.770	.792	.805	.894
$s(\hat{\beta}_{p,t-1})$	.045	.041	.044	.037	.027	.032	.035	.035	.028	.040
$r(R_p, R_m)^2$	.645	.745	.753	.829	.919	.898	.889	.898	.934	.896
$s(R_p)$	.035	.037	.041	.041	.044	.046	.049	.050	.050	.057
$s(\hat{\epsilon}_p)$	.021	.019	.020	.017	.013	.015	.016	.016	.013	.018
$\bar{s}_{p,t-1}(\hat{\epsilon}_i)$	.055	.055	.063	.058	.058	.063	.064	.064	.062	.069
$s(\hat{\epsilon}_p)/\bar{s}_{p,t-1}(\hat{\epsilon}_i)$	.382	.345	.317	.293	.224	.238	.250	.250	.210	.261
	Portfolios for Estimation Period 1950–54									
$\hat{\beta}_{p,t-1}$	.418	.590	.694	.751	.777	.784	.929	.950	.996	1.014
$s(\hat{\beta}_{p,t-1})$	.042	.047	.045	.037	.038	.035	.050	.038	.035	.029
$r(R_p, R_m)^2$	.629	.723	.798	.872	.878	.895	.856	.913	.933	.954
$s(R_p)$	.019	.025	.028	.029	.030	.030	.036	.036	.037	.038
$s(\hat{\epsilon}_p)$	.012	.013	.013	.010	.010	.010	.014	.011	.010	.008
$\bar{s}_{p,t-1}(\hat{\epsilon}_i)$	.040	.044	.046	.048	.051	.051	.052	.053	.054	.057
$s(\hat{\epsilon}_p)/\bar{s}_{p,t-1}(\hat{\epsilon}_i)$	.300	.295	.283	.208	.196	.196	.269	.208	.185	.140
	Portfolios for Estimation Period 1958–62									
$\hat{\beta}_{p,t-1}$	.626	.635	.719	.801	.817	.860	.920	.950	.975	.995
$s(\hat{\beta}_{p,t-1})$	.043	.048	.039	.046	.047	.033	.037	.038	.032	.037
$r(R_p, R_m)^2$	.783	.745	.851	.835	.838	.920	.913	.915	.939	.925
$s(R_p)$	.030	.031	.033	.037	.038	.038	.041	.042	.043	.044
$s(\hat{\epsilon}_p)$	.014	.016	.013	.015	.015	.011	.012	.012	.011	.012
$\bar{s}_{p,t-1}(\hat{\epsilon}_i)$	.049	.052	.056	.059	.064	.061	.070	.069	.068	.064
$s(\hat{\epsilon}_p)/\bar{s}_{p,t-1}(\hat{\epsilon}_i)$	.286	.308	.232	.254	.234	.180	.171	.174	.162	.188

be rejected when t-statistics are interpreted under the assumption of normality, the hypotheses are on even firmer ground when one takes into account the thick tails of empirical return distributions.

Further justification for using t-statistics to test hypotheses on monthly common stock returns is in the work of Officer (1971). Under the assumption that distributions of monthly returns are symmetric stable, he estimates that in the post–World War II period the characteristic exponent

TABLE 2 (*Continued*)

Statistic	11	12	13	14	15	16	17	18	19	20
	Portfolios for Estimation Period 1934–38									
$\hat{\beta}_{p,t-1}$	1.046	1.122	1.181	1.192	1.196	1.295	1.335	1.396	1.445	1.458
$s(\hat{\beta}_{p,t-1})$	.028	.031	.035	.028	.029	.032	.032	.053	.039	.053
$r(R_p, R_m)^2$	.959	.956	.951	.969	.966	.966	.967	.922	.958	.927
$s(R_p)$	.113	.122	.128	.128	.129	.140	.144	.154	.156	.160
$s(\hat{\epsilon}_p)$	.023	.026	.029	.023	.024	.026	.026	.043	.032	.043
$\bar{s}_{p,t-1}(\hat{\epsilon}_i)$	.094	.124	.120	.122	.132	.125	.129	.158	.145	.170
$s(\hat{\epsilon}_p)/\bar{s}_{p,t-1}(\hat{\epsilon}_i)$	.245	.210	.242	.188	.182	.208	.202	.272	.221	.253
	Portfolios for Estimation Period 1942–46									
$\hat{\beta}_{p,t-1}$	.949	.952	1.010	1.038	1.254	1.312	1.316	1.473	1.631	1.661
$s(\hat{\beta}_{p,t-1})$	.031	.036	.040	.030	.034	.039	.041	.084	.083	.077
$r(R_p, R_m)^2$	.942	.923	.917	.954	.958	.951	.945	.839	.867	.887
$s(R_p)$	.059	.060	.063	.064	.077	.081	.081	.097	.105	.106
$s(\hat{\epsilon}_p)$	.014	.016	.018	.014	.016	.018	.019	.039	.038	.036
$\bar{s}_{p,t-1}(\hat{\epsilon}_i)$	.073	.074	.085	.077	.096	.083	.086	.134	.117	.122
$s(\hat{\epsilon}_p)/\bar{s}_{p,t-1}(\hat{\epsilon}_i)$	.192	.216	.212	.182	.167	.217	.221	.291	.325	.295
	Portfolios for Estimation Period 1950–54									
$\hat{\beta}_{p,t-1}$	1.117	1.123	1.131	1.134	1.186	1.235	1.295	1.324	1.478	1.527
$s(\hat{\beta}_{p,t-1})$	.039	.027	.044	.033	.037	.049	.045	.046	.058	.086
$r(R_p, R_m)^2$	.934	.968	.919	.952	.944	.915	.933	.934	.917	.841
$s(R_p)$	.042	.041	.043	.042	.044	.047	.049	.050	.056	.060
$s(\hat{\epsilon}_p)$	.011	.007	.012	.009	.010	.014	.013	.013	.016	.024
$\bar{s}_{p,t-1}(\hat{\epsilon}_i)$	.066	.057	.066	.060	.064	.064	.065	.068	.076	.088
$s(\hat{\epsilon}_p)/\bar{s}_{p,t-1}(\hat{\epsilon}_i)$	.167	.123	.182	.150	.156	.219	.200	.192	.210	.273
	Portfolios for Estimation Period 1958–62									
$\hat{\beta}_{p,t-1}$	1.013	1.019	1.037	1.048	1.069	1.081	1.092	1.098	1.269	1.388
$s(\hat{\beta}_{p,t-1})$	.038	.031	.036	.033	.036	.038	.045	.045	.048	.065
$r(R_p, R_m)^2$	.922	.948	.934	.945	.936	.931	.907	.910	.922	.886
$s(R_p)$	.045	.045	.046	.046	.047	.048	.049	.049	.056	.063
$s(\hat{\epsilon}_p)$	.013	.010	.012	.011	.012	.013	.015	.015	.016	.021
$\bar{s}_{p,t-1}(\hat{\epsilon}_i)$	.069	.066	.067	.062	.070	.072	.076	.068	.070	.078
$s(\hat{\epsilon}_p)/\bar{s}_{p,t-1}(\hat{\epsilon}_i)$	.188	.152	.179	.177	.171	.180	.197	.220	.228	.269

for these distributions is about 1.8 (as compared with a value of 2.0 for a normal distribution). From Fama and Roll (1968), for values of the characteristic exponent so close to 2.0 stable nonnormal distributions differ noticeably from the normal only in their extreme tails—that is, beyond the .05 and .95 fractiles. Thus, as long as one is not concerned with precise estimates of probability levels (always a somewhat meaningless activity), interpreting t-statistics in the usual way does not lead to serious errors.

TABLE 3

SUMMARY RESULTS FOR THE REGRESSION

$$R_p = \hat{\gamma}_{0t} + \hat{\gamma}_{1t}\hat{\beta}_p + \hat{\gamma}_{2t}\hat{\beta}^2_p + \hat{\gamma}_{3t}\bar{s}_p(\hat{\epsilon}_i) + \hat{\eta}_{pt}$$

PERIOD	STATISTIC																			
	$\bar{\hat{\gamma}}_0$	$\bar{\hat{\gamma}}_1$	$\bar{\hat{\gamma}}_2$	$\bar{\hat{\gamma}}_3$	$\overline{\hat{\gamma}_0 - R_f}$	$s(\hat{\gamma}_0)$	$s(\hat{\gamma}_1)$	$s(\hat{\gamma}_2)$	$s(\hat{\gamma}_3)$	$\rho_0(\hat{\gamma}_0 - R_f)$	$\rho_M(\hat{\gamma}_1)$	$\rho_0(\hat{\gamma}_2)$	$\rho_0(\hat{\gamma}_3)$	$t(\bar{\hat{\gamma}}_0)$	$t(\bar{\hat{\gamma}}_1)$	$t(\bar{\hat{\gamma}}_2)$	$t(\bar{\hat{\gamma}}_3)$	$t(\overline{\hat{\gamma}_0 - R_f})$	$\bar{r}^2$	$s(r^2)$
Panel A:																				
1935–6/68 ..	.0061	.0085	...	...	.0048	.038	.066	...	...	.15	.02	...	...	3.24	2.57	...	...	2.55	.29	.30
1935–45	.0039	.0163	...	...	.0037	.052	.098	...	...	.10	−.03	...	...	.86	1.92	...	...	.82	.29	.29
1946–55	.0087	.0027	...	...	.0078	.026	.041	...	...	.18	.07	...	...	3.71	.70	...	...	3.31	.31	.32
1956–6/68 ..	.0060	.0062	...	...	.0034	.030	.044	...	...	.27	.15	...	...	2.45	1.73	...	...	1.39	.28	.29
1935–40	.0024	.0109	...	...	.0023	.064	.116	...	...	.07	−.09	...	...	.32	.79	...	...	.31	.23	.30
1941–45	.0056	.0229	...	...	.0054	.034	.069	...	...	.23	.15	...	...	1.27	2.55	...	...	1.22	.37	.28
1946–50	.0050	.0029	...	...	.0044	.031	.047	...	...	.20	.04	...	...	1.27	.48	...	...	1.10	.39	.33
1951–55	.0123	.0024	...	...	.0111	.019	.035	...	...	.20	.08	...	...	5.06	.53	...	...	4.56	.24	.29
1956–60	.0148	−.0059	...	...	.0128	.020	.034	...	...	.37	.18	...	...	5.68	−1.37	...	...	4.89	.22	.31
1961–6/68 ..	.0001	.0143	...	...	−.0029	.034	.048	...	...	.22	.09	...	...	.03	2.81	...	...	−.80	.32	.27
Panel B:																				
1935–6/68 ..	.0049	.0105	−.0008	...	.0036	.052	.118	.056	...	.03	−.11	−.11	...	1.92	1.79	−.29	...	1.42	.32	.31
1935–45	.0074	.0079	.0040	...	.0073	.061	.139	.074	...	−.10	−.31	−.21	...	1.39	.65	.61	...	1.36	.32	.30
1946–55	−.0002	.0217	−.0087	...	−.0012	.036	.095	.034	...	.04	.00	.00	...	−.07	2.51	−2.83	...	−.38	.36	.32
1956–6/68 ..	.0069	.0040	.0013	...	.0043	.054	.116	.053	...	.17	.07	.03	...	1.56	.42	.29	...	.97	.30	.30
1935–40	.0013	.0141	−.0017	...	.0012	.069	.160	.075	...	−.13	−.36	−.35	...	.16	.75	−.19	...	.14	.24	.30
1941–45	.0148	.0004	.0108	...	.0146	.050	.111	.073	...	−.04	−.19	−.04	...	2.28	.03	1.15	...	2.24	.39	.29
1946–50	−.0008	.0152	−.0051	...	−.0015	.037	.104	.032	...	.14	.04	.00	...	−.18	1.14	−1.24	...	−.32	.44	.32
1951–55	.0004	.0281	−.0122	...	−.0008	.030	.085	.035	...	−.17	−.14	−.01	...	.10	2.55	−2.72	...	−.20	.28	.29
1956–60	.0128	−.0015	−.0020	...	.0108	.030	.072	.029	...	.35	.11	.26	...	3.38	−.16	−.54	...	2.84	.25	.31
1961–6/68 ..	.0029	.0077	.0034	...	−.0000	.066	.138	.064	...	.14	.06	−.01	...	.42	.53	.51	...	−.01	.34	.29

TABLE 3 (*Continued*)

Period	$\overline{\hat{\gamma}}_0$	$\overline{\hat{\gamma}}_1$	$\overline{\hat{\gamma}}_2$	$\overline{\hat{\gamma}}_3$	$\overline{\hat{\gamma}_0 - R_f}$	$s(\hat{\gamma}_0)$	$s(\hat{\gamma}_1)$	$s(\hat{\gamma}_2)$	$s(\hat{\gamma}_3)$	$\rho_0(\hat{\gamma}_0 - R_f)$	$\rho_M(\hat{\gamma}_1)$	$\rho_0(\hat{\gamma}_2)$	$\rho_0(\hat{\gamma}_3)$	$t(\overline{\hat{\gamma}}_0)$	$t(\overline{\hat{\gamma}}_1)$	$t(\overline{\hat{\gamma}}_2)$	$t(\overline{\hat{\gamma}}_3)$	$t(\overline{\hat{\gamma}_0 - R_f})$	$\bar{r}^2$	$s(r^2)$
									Statistic											
Panel C:																				
1935–6/68 ..	.0054	.0072	...	.0198	.0041	.052	.065	...	.868	.04	—.12	...	—.04	2.10	2.20	...	.46	1.59	.32	.31
1935–45	.0017	.0104	...	.0841	.0015	.073	.083	...	.921	—.00	—.26	...	—.08	.26	1.41	...	1.05	.24	.32	.31
1946–55	.0110	.0075	...	—.1052	.0100	.032	.056	...	.609	.08	.02	...	—.20	3.78	1.47	...	—1.89	3.46	.34	.32
1956–6/68 ..	.0042	.0041	...	.0633	.0016	.040	.052	...	.984	.12	.08	...	.03	1.28	.96	...	.79	.50	.30	.29
1935–40	.0036	.0119	...	—.0170	.0035	.082	.105	...	.744	—.03	—.26	...	—.18	.37	.97	...	—.19	.36	.25	.30
1941–45	—.0006	.0085	...	.2053	—.0009	.061	.052	...	1.091	.07	—.29	...	—.02	—.08	1.25	...	1.46	—.11	.41	.30
1946–50	.0069	.0081	...	—.0920	.0062	.034	.066	...	.504	.14	.06	...	—.02	1.56	.95	...	—1.41	1.40	.42	.33
1951–55	.0150	.0069	...	—.1185	.0138	.029	.043	...	.702	.06	—.18	...	—.32	4.05	1.24	...	—1.31	3.72	.27	.29
1956–60	.0127	—.0081	...	.0728	.0107	.037	.045	...	1.164	.15	.15	...	.21	2.68	—1.40	...	.48	2.26	.26	.30
1961–6/68 ..	—.0014	.0122	...	.0570	—.0044	.042	.055	...	.850	.10	.00	...	—.19	—.32	2.12	...	.64	—.98	.33	.27
Panel D:																				
1935–6/68 ..	.0020	.0114	—.0026	.0516	.0008	.075	.123	.060	.929	—.09	—.09	—.12	—.10	.55	1.85	—.86	1.11	.20	.34	.31
1935–45	.0011	.0118	—.0009	.0817	.0010	.103	.146	.079	1.003	—.20	—.23	—.24	—.15	.13	.94	—.14	.94	.11	.34	.31
1946–55	.0017	.0209	—.0076	—.0378	.0008	.042	.096	.038	.619	—.10	—.00	—.01	—.20	.44	2.39	—2.16	—.67	.20	.36	.32
1956–6/68 ..	.0031	.0034	—.0000	.0966	.0005	.065	.122	.055	1.061	.12	.03	.01	—.05	.59	.34	—.00	1.11	.10	.32	.29
1935–40	.0009	.0156	—.0029	.0025	.0008	.112	.171	.085	.826	—.16	—.23	—.26	—.12	.07	.78	—.29	.03	.06	.26	.30
1941–45	.0015	.0073	.0014	.1767	.0012	.092	.109	.072	1.181	—.28	—.21	—.22	—.18	.12	.52	.15	1.16	.10	.43	.31
1946–50	.0011	.0141	—.0040	—.0313	.0004	.047	.106	.042	.590	—.10	.03	—.01	—.12	.18	1.03	—.73	—.41	.07	.44	.33
1951–55	.0023	.0277	—.0112	—.0443	.0011	.037	.085	.034	.651	—.11	—.13	—.01	—.28	.48	2.53	—2.54	—.53	.23	.29	.30
1956–60	.0103	—.0047	—.0020	.0979	.0083	.049	.078	.032	1.286	—.16	.19	—.01	.02	1.63	—.47	—.49	.59	1.31	.28	.30
1961–6/68 ..	—.0017	.0088	.0013	.0957	—.0046	.073	.144	.066	.887	.20	.00	.01	—.15	—.21	.58	.19	1.02	—.60	.35	.29

Inferences based on approximate normality are on even safer ground if one assumes, again in line with the results of Officer (1971), that although they are well approximated by stable nonnormal distributions with $\alpha \cong 1.8$, distributions of monthly returns in fact have finite variances and converge—but very slowly—toward the normal as one takes sums or averages of individual returns. Then the distributions of the means of month-by-month regression coefficients from the risk-return model are likely to be close to normal since each mean is based on coefficients for many months.

A. Tests of the Major Hypotheses of the Two-Parameter Model

Consider first condition C2 of the two-parameter model, which says that no measure of risk, in addition to β, systematically affects expected returns. This hypothesis is not rejected by the results in panels C and D of table 3. The values of $t(\bar{\hat{\gamma}}_3)$ are small, and the signs of the $t(\bar{\hat{\gamma}}_3)$ are randomly positive and negative.

Likewise, the results in panels B and D of table 3 do not reject condition C1 of the two-parameter model, which says that the relationship between expected return and β is linear. In panel B, the value of $t(\bar{\hat{\gamma}}_2)$ for the overall period 1935–6/68 is only $-.29$. In the 5-year subperiods, $t(\bar{\hat{\gamma}}_2)$ for 1951–55 is approximately -2.7, but for subperiods that do not cover 1951–55, the values of $t(\bar{\hat{\gamma}}_2)$ are much closer to zero.

So far, then, the two-parameter model seems to be standing up well to the data. All is for naught, however, if the critical condition C3 is rejected. That is, we are not happy with the model unless there is on average a positive tradeoff between risk and return. This seems to be the case. For the overall period 1935–6/68, $t(\bar{\hat{\gamma}}_1)$ is large for all models. Except for the period 1956–60, the values of $t(\bar{\hat{\gamma}}_1)$ are also systematically positive in the subperiods, but not so systematically large.

The small t-statistics for subperiods reflect the substantial month-to-month variability of the parameters of the risk-return regressions. For example, in the one-variable regressions summarized in panel A, for the period 1935–40, $\bar{\hat{\gamma}}_1 = .0109$. In other words, for this period the average incremental return per unit of β was almost 1.1 percent per month, so that on average, bearing risk had substantial rewards. Nevertheless, because of the variability of $\hat{\gamma}_{1t}$—in this period $s(\hat{\gamma}_1)$ is 11.6 percent per month (!)—$t(\bar{\hat{\gamma}}_1)$ is only .79. It takes the statistical power of the large sample for the overall period before values of $\bar{\hat{\gamma}}_1$ that are large in practical terms also yield large t-values.

But at least with the sample of the overall period $t(\bar{\hat{\gamma}}_1)$ achieves values supportive of the conclusion that on average there is a statistically observable positive relationship between return and risk. This is not the case with respect to $t(\bar{\hat{\gamma}}_2)$ and $t(\bar{\hat{\gamma}}_3)$. Even, or indeed especially, for the overall period, these t-statistics are close to zero.

The behavior through time of $\hat{\gamma}_{1t}$, $\hat{\gamma}_{2t}$, and $\hat{\gamma}_{3t}$ is also consistent with hypothesis ME that the capital market is efficient. The serial correlations $\rho_M(\hat{\gamma}_1)$, $\rho_0(\hat{\gamma}_2)$, and $\rho_0(\hat{\gamma}_3)$, are always low in terms of explanatory power and generally low in terms of statistical significance. The proportion of the variance of $\tilde{\gamma}_{jt}$ explained by first-order serial correlation is estimated by $\rho(\hat{\gamma}_j)^2$ which in all cases is small. As for statistical significance, under the hypothesis that the true serial correlation is zero, the standard deviation of the sample coefficient can be approximated by $\sigma(\hat{\rho}) = 1/\sqrt{n}$. For the overall period, $\sigma(\hat{\rho})$ is approximately .05, while for the 10- and 5-year subperiods $\sigma(\hat{\rho})$ is approximately .09 and .13, respectively. Thus, the values of $\rho_M(\hat{\gamma}_1)$, $\rho_0(\hat{\gamma}_2)$, and $\rho_0(\hat{\gamma}_3)$ in table 3 are generally statistically close to zero. The exceptions involve primarily periods that include the 1935–40 subperiod, and the results for these periods are not independent.[8]

To conserve space, the serial correlations of the portfolio residuals, $\hat{\eta}_{pt}$, are not shown. In these serial correlations, negative values predominate. But like the serial correlations of the $\hat{\gamma}$'s, those of the $\hat{\eta}$'s are close to zero. Higher-order serial correlations of the $\hat{\gamma}$'s and $\hat{\eta}$'s have been computed, and these also are never systematically large.

In short, one cannot reject the hypothesis that the pricing of securities is in line with the implications of the two-parameter model for expected returns. And given a two-parameter pricing model, the behavior of returns through time is consistent with an efficient capital market.

B. *The Behavior of the Market*

Some perspective on the behavior of the market during different periods and on the interpretation of the coefficients $\hat{\gamma}_{0t}$ and $\hat{\gamma}_{1t}$ in the risk-return regressions can be obtained from table 4. For the various periods of table 3, table 4 shows the sample means (and with some exceptions), the standard

[8] The serial correlations of $\hat{\gamma}_2$ and $\hat{\gamma}_3$ about means that are assumed to be zero provide a test of the fair game property of an efficient market, given that expected returns are generated by the two-parameter model—that is, given $E(\tilde{\gamma}_{2t}) = E(\tilde{\gamma}_{3t}) = 0$. Likewise, $\rho_0(\hat{\gamma}_{0t} - R_{ft})$ provides a test of market efficiency with respect to the behavior of $\hat{\gamma}_{0t}$ through time, given the validity of the Sharpe-Lintner hypothesis (about which we have as yet said nothing). But, at least for $\hat{\gamma}_{2t}$ and $\hat{\gamma}_{3t}$, computing the serial correlations about sample means produces essentially the same results.

To test the market efficiency hypothesis on $\tilde{\gamma}_{1t} - [E(\tilde{R}_{mt}) - E(\tilde{R}_{0t})]$, the sample mean of the $\hat{\gamma}_{1t}$ is used to estimate $E(\tilde{R}_{mt}) - E(\tilde{R}_{0t})$, thus implicitly assuming that the expected risk premium is constant. That this is a reasonable approximation [in the sense that the $\rho_M(\hat{\gamma}_1)$ are small], probably reflects the fact that variation in $E(\tilde{R}_{mt}) - E(\tilde{R}_{0t})$ is trivial relative to the month-by-month variation in $\hat{\gamma}_{1t}$.

Finally, it is well to note that in terms of the implications of the serial correlations for making good portfolio decisions—and thus for judging whether market efficiency is a workable representation of reality—the fact that the serial correlations are low in terms of explanatory power is more important than whether or not they are low in terms of statistical significance.

TABLE 4

THE BEHAVIOR OF THE MARKET

	Statistic*								
Period	$\overline{R_m}$	$\overline{R_m - R_f}$	$\overline{\hat{\gamma}_1}$	$\overline{\hat{\gamma}_0}$	$\overline{R_f}$	$\frac{\overline{R_m - R_f}}{s(R_m)}$	$\frac{\overline{\hat{\gamma}_1}}{s(R_m)}$	$s(R_m)$	$s(R_m)$
1935–6/68	.0143	.0130	.0085	.0061	.0013	.2136	.1388	.061	.066
1935–45	.0197	.0195	.0163	.0039	.0002	.2207	.1844	.089	.098
1946–55	.0112	.0103	.0027	.0087	.0009	.2378	.0614	.043	.041
1956–6/68	.0121	.0095	.0062	.0060	.0026	.2387	.1560	.040	.044
1935–40	.0132	.0132	.0109	.0024	.0001	.1221	.1009	.108	.116
1941–45	.0274	.0272	.0229	.0056	.0002	.4715	.3963	.058	.069
1946–50	.0077	.0070	.0029	.0050	.0007	.1351	.0564	.052	.047
1951–55	.0148	.0136	.0024	.0123	.0012	.4174	.0735	.033	.035
1956–60	.0090	.0070	−.0059	.0148	.0020	.2080	−.1755	.034	.034
1961–6/68	.0141	.0111	.0143	.0001	.0030	.2567	.3294	.043	.048

* Since $s(R_f)$ is so small relative to $s(R_m)$, $s(R_m - R_f)$, which is not shown, is essentially the same as $s(R_m)$. The standard deviations of $(R_m - R_f)/s(R_m)$ and $\hat{\gamma}_1/s(R_m)$, also not shown, can be obtained directly from $s(R_m - R_f)$, $s(\hat{\gamma}_1)$ and $s(R_m)$. Finally, the t-statistics for $(\overline{R_m - R_f})/s(R_m)$ and $\overline{\hat{\gamma}}_1/s(R_m)$ are identical with those for $\overline{R_m - R_f}$ and $\overline{\hat{\gamma}}_1$.

deviations, t-statistics for sample means, and first-order serial correlations for the month-by-month values of the following variables and coefficients: the market return R_{mt}; the riskless rate of interest R_{ft}, taken to be the yield on 1-month Treasury bills; $R_{mt} - R_{ft}$; $(R_{mt} - R_{ft})/s(R_m)$; $\hat{\gamma}_{0t}$ and $\hat{\gamma}_{1t}$, repeated from panel A of table 3; and $\hat{\gamma}_{1t}/s(R_m)$. The t-statistics on sample means are computed in the same way as those in table 3.

If the two-parameter model is valid, then in equation (7), $E(\tilde{\gamma}_{0t}) = E(\tilde{R}_{0t})$, where $E(\tilde{R}_{0t})$ is the expected return on any zero-β security or portfolio. Likewise, the expected risk premium per unit of β is $E(\tilde{R}_{mt}) - E(\tilde{R}_{0t}) = E(\tilde{\gamma}_{1t})$. In fact, for the one-variable regressions of panel A, table 3, that is,

$$R_{pt} = \hat{\gamma}_{0t} + \hat{\gamma}_{1t}\,\hat{\beta}_p + \hat{\eta}_{pt}, \tag{11}$$

we have, period by period,

$$\hat{\gamma}_{1t} = R_{mt} - \hat{\gamma}_{0t}. \tag{12}$$

This condition is obtained by averaging (11) over p and making use of the least-squares constraint

$$\sum_p \hat{\eta}_{pt} = 0.^{9}$$

Moreover, the least-squares estimate $\hat{\gamma}_{0t}$ can always be interpreted as the return for month t on a zero-$\hat{\beta}$ portfolio, where the weights given to each

[9] There is some degree of approximation in (12). The averages over p of R_{pt} and $\hat{\beta}_p$ are R_{mt} and 1.0, respectively, only if every security in the market is in some portfolio. With our methodology (see table 1) this is never true. But the degree of approximation turns out to be small: The average of the R_{pt} is always close to R_{mt} and the average $\hat{\beta}_p$ is always close to 1.0.

TABLE 4 (*Continued*)

				STATISTIC*						
$s(\hat{\gamma}_0)$	$s(R_f)$	$t(\bar{R}_m)$	$t(\overline{R_m - R_f})$	$t(\bar{\hat{\gamma}}_1)$	$t(\bar{\hat{\gamma}}_0)$	$\rho_M(R_m)$	$\rho_M(R_m - R_f)$	$\rho_M(\hat{\gamma}_1)$	$\rho_M(\hat{\gamma}_0)$	$\rho_M(R_f)$
.038	.0012	4.71	4.28	2.57	3.24	—.01	—.01	.02	.14	.98
.052	.0001	2.56	2.54	1.92	.86	—.07	—.07	—.03	.10	.88
.026	.0004	2.84	2.60	.70	3.71	.09	.09	.07	.10	.94
.030	.0009	3.72	2.92	1.73	2.45	.14	.14	.15	.25	.92
.064	.0001	1.04	1.04	.79	.32	—.13	—.13	—.09	.07	.72
.034	.0001	3.68	3.65	2.55	1.27	.14	.14	.15	.21	.83
.031	.0003	1.15	1.05	.48	1.27	.09	.09	.04	.18	.97
.019	.0004	3.51	3.22	.53	5.06	—.02	—.01	.08	—.07	.89
.020	.0007	2.07	1.60	—1.37	5.68	.12	.13	.18	.13	.80
.034	.0008	3.08	2.44	2.81	.03	.13	.13	.09	.21	.93

of the 20 portfolios to form this zero-$\hat{\beta}$ portfolio are the least-squares weights that are applied to the R_{pt} in computing $\hat{\gamma}_{0t}$.[10]

In the Sharpe-Lintner two-parameter model of market equilibrium $E(\tilde{\gamma}_{0t}) = E(\tilde{R}_{0t}) = R_{ft}$ and $E(\tilde{\gamma}_{1t}) = E(\tilde{R}_{mt}) - E(\tilde{R}_{0t}) = E(\tilde{R}_{mt}) - R_{ft}$. In the period 1935–40 and in the most recent period 1961–6/68, $\hat{\gamma}_{1t}$ is close to $\overline{R_m - R_f}$ and the t-statistics for the two averages are similar. In other periods, and especially in the period 1951–60, $\bar{\hat{\gamma}}_1$ is substantially less than $\overline{R_m - R_f}$. This is a consequence of the fact that for these periods $\bar{\hat{\gamma}}_0$ is noticeably greater than $\bar{R}_f$. In economic terms, the tradeoff of average return for risk between common stocks and short-term bonds has been more consistently large through time than the tradeoff of average return for risk among common stocks. Testing whether the differences between $\overline{R_m - R_f}$ and $\bar{\hat{\gamma}}_1$ are statistically large, however, is equivalent to testing the S-L hypothesis $E(\tilde{\gamma}_{0t}) = R_{ft}$, which we prefer to take up after examining further the stochastic process generating monthly returns.

Finally, although the differences between values of $\overline{R_m - R_f}$ for different periods or between values of $\bar{\hat{\gamma}}_1$ are never statistically large, there is a hint in table 4 that average-risk premiums declined from the pre– to the post–World War II periods. These are average risk premiums per unit of $\hat{\beta}$, however, which are not of prime interest to the investor. In making his portfolio decision, the investor is more concerned with the tradeoff of expected portfolio return for dispersion of return—that is, the slope of the efficient set of portfolios. In the Sharpe-Lintner model this slope is

10 That $\hat{\gamma}_{0t}$ is the return on a zero-$\hat{\beta}$ portfolio can be shown to follow from the unbiasedness of the least-squares coefficients in the cross-sectional risk-return regressions. If one makes the Gauss-Markov assumptions that the underlying disturbances $\tilde{\eta}_{pt}$ of (11) have zero means, are uncorrelated across p, and have the same variance for all p, then it follows almost directly from the Gauss-Markov Theorem that the least-squares estimate $\hat{\gamma}_{0t}$ is also the return for month t on the minimum variance zero-$\hat{\beta}$ portfolio that can be constructed from the 20 portfolio $\hat{\beta}_p$.

always $[E(\tilde{R}_{mt}) - R_{ft}]/\sigma(\tilde{R}_{mt})$, and in the more general model of Black (1972), it is $[E(\tilde{R}_{mt}) - E(\tilde{R}_{0t})]/\sigma(\tilde{R}_{mt})$ at the point on the efficient set corresponding to the market portfolio m. In table 4, especially for the three long subperiods, dividing $\overline{R_m - R_f}$ and $\bar{\hat{\gamma}}_1$, by $s(R_m)$ seems to yield estimated risk premiums that are more constant through time. This results from the fact that any declines in $\bar{\hat{\gamma}}_1$ or $\overline{R_m - R_f}$ are matched by a quite noticeable downward shift in $s(R_m)$ from the early to the later periods (cf. Blume [1970] or Officer [1971]).

C. *Errors and True Variation in the Coefficients* $\hat{\gamma}_{jt}$

Each cross-sectional regression coefficient $\hat{\gamma}_{jt}$ in (10) has two components: the true $\tilde{\gamma}_{jt}$ and the estimation error, $\tilde{\phi}_{jt} = \hat{\gamma}_{jt} - \tilde{\gamma}_{jt}$. A natural question is: To what extent is the variation in $\hat{\gamma}_{jt}$ through time due to variation in $\tilde{\gamma}_{jt}$ and to what extent is it due to $\tilde{\phi}_{jt}$? In addition to providing important information about the precision of the coefficient estimates used to test the two-parameter model, the answer to this question can be used to test hypotheses about the stochastic process generating returns. For example, although we cannot reject the hypothesis that $E(\tilde{\gamma}_{2t}) = 0$, does including the term involving $\hat{\beta}_p{}^2$ in (10) help in explaining the month-by-month behavior of returns? That is, can we reject the hypothesis that for all t, $\tilde{\gamma}_{2t} = 0$? Likewise, can we reject the hypothesis that month-by-month $\tilde{\gamma}_{3t} = 0$? And is the variation through time in $\hat{\gamma}_{0t}$ due entirely to $\tilde{\phi}_{0t}$ and to variation in R_{ft}?

The answers to these questions are in table 5. For the models and time periods of table 3, table 5 shows for each $\hat{\gamma}_j$: $s^2(\hat{\gamma}_j)$, the sample variance of the month-by-month $\hat{\gamma}_{jt}$; $\overline{s^2(\tilde{\phi}_j)}$, the average of the month-by-month values of $s^2(\tilde{\phi}_{jt})$, where $s(\tilde{\phi}_{jt})$ is the standard error of $\hat{\gamma}_{jt}$ from the cross-sectional risk-return regression of (10) for month t; $s^2(\tilde{\gamma}_j) \equiv s^2(\hat{\gamma}_j) - \overline{s^2(\tilde{\phi}_j)}$; and the F-statistic $F = s^2(\hat{\gamma}_j)/\overline{s^2(\tilde{\phi}_j)}$, which is relevant for testing the hypothesis, $s^2(\hat{\gamma}_j) = \overline{s^2(\tilde{\phi}_j)}$. The numerator of F has $n - 1$ df, where n is the number of months in the sample period; and the denominator has $n(20 - K)$ df, where K is the number of coefficients $\hat{\gamma}_j$ in the model.[11]

[11] The standard error of $\hat{\gamma}_{jt}$, $s(\tilde{\phi}_{jt})$, is proportional to the standard error of the risk-return residuals, $\hat{\eta}_{pt}$, for month t, which has $20 - K$ df. And n values of $s^2(\tilde{\phi}_{jt})$ are averaged to get $\overline{s^2(\tilde{\phi}_j)}$, so that the latter has $n(20 - K)$ df. Note that if the underlying return disturbances $\tilde{\eta}_{pt}$ of (10) are independent across p and have identical normal distributions for all p, then $\hat{\gamma}_{jt}$ is the sample mean of a normal distribution and $s^2(\tilde{\phi}_{jt})$ is proportional to the sample variance of the same normal distribution. If the process is also assumed to be stationary through time, it then follows that $s^2(\hat{\gamma}_{jt})$ and $\overline{s^2(\tilde{\phi}_{jt})}$ are independent, as required by the F-test. Finally, in the F-statistics of table 5, the values of n are 60 or larger, so that, since K is from 2 to 4, $n(20 - K) \geqslant 960$. From Mood and Graybill (1963), some upper percentage points of the F-distribution are:

One clear-cut result in table 5 is that there is a substantial decline in the reliability of the coefficients $\hat{\gamma}_{0t}$ and $\hat{\gamma}_{1t}$—that is, a substantial increase in $s^2(\overline{\hat{\phi}_0})$ and $s^2(\overline{\hat{\phi}_1})$—when $\hat{\beta}_p^2$ and/or $\bar{s}_p(\hat{\epsilon}_i)$ are included in the risk-return regressions. The variable $\hat{\beta}_p^2$ is obviously collinear with $\hat{\beta}_p$, and, as can be seen from table 2, $\bar{s}_p(\hat{\epsilon}_i)$ likewise increases with $\hat{\beta}_p$. From panels B and C of table 5, the collinearity with $\hat{\beta}_p$ is stronger for $\hat{\beta}_p^2$ than for $\bar{s}_p(\hat{\epsilon}_i)$.

In spite of the loss in precision that arises from multicollinearity, however, the F-statistics for $\hat{\gamma}_2$ (the coefficient of $\hat{\beta}_p^2$) and $\hat{\gamma}_3$ [the coefficient of $\bar{s}_p(\hat{\epsilon}_i)$] are generally large for the models of panels B and C of table 5, and for the model of panel D which includes both variables. From the F-statistics in panel D, it seems that, except for the period 1935–45, the variation through time of $\tilde{\gamma}_{2t}$ is statistically more noticeable than that of $\tilde{\gamma}_{3t}$, but there are periods (1941–45, 1956–60) when the values of F for both $\tilde{\gamma}_{2t}$ and $\tilde{\gamma}_{3t}$ are large.

The F-statistics for $\hat{\gamma}_{1t} = \tilde{\gamma}_{1t} + \tilde{\phi}_{1t}$ also indicate that $\tilde{\gamma}_{1t}$ has substantial variation through time. This is not surprising, however, since $\hat{\gamma}_{1t}$ is always directly related to $\tilde{R}_{mt}$. For example, from equation (12), for the one-variable model of panel A, $\hat{\gamma}_{1t} = \tilde{R}_{mt} - \hat{\gamma}_{0t}$.

Finally, the F-statistics for $\hat{\gamma}_{0t} = \tilde{\gamma}_{0t} + \tilde{\phi}_{0t}$ are also in general large. And the month-by-month variation in $\tilde{\gamma}_{0t}$ cannot be accounted for by variation in R_{ft}. The variance of R_{ft} is so small relative to $s^2(\hat{\gamma}_{0t})$, $s^2(\tilde{\gamma}_{0t})$, and $s^2(\overline{\tilde{\phi}_{0t}})$ that doing the F-tests in terms of $\hat{\gamma}_{0t} - R_{ft}$ produces results almost identical with those for $\hat{\gamma}_{0t}$.

Rejection of the hypothesis that $\tilde{\gamma}_{0t} - R_{ft} = 0$ does not imply rejection of the S-L hypothesis—to be tested next—that $E(\tilde{\gamma}_{0t}) = R_{ft}$. Likewise, to find that month-by-month $\tilde{\gamma}_{2t} \neq 0$ and $\tilde{\gamma}_{3t} \neq 0$ does not imply rejection of hypotheses C1 and C2 of the two-parameter model. These hypotheses, which we are unable to reject on the basis of the results in table 3, say that $E(\tilde{\gamma}_{2t}) = 0$ and $E(\tilde{\gamma}_{3t}) = 0$.

What we have found in table 5 is that there are variables in addition to $\hat{\beta}_p$ that systematically affect period-by-period returns. Some of these omitted variables are apparently related to $\hat{\beta}_p^2$ and $\bar{s}_p(\hat{\epsilon}_i)$. But the latter are almost surely proxies, since there is no economic rationale for their presence in our stochastic risk-return model.

n	$F_{.90}$	$F_{.95}$	$F_{.975}$	$F_{.99}$	$F_{.995}$
60 (120)	1.35	1.47	1.58	1.73	1.83
60 (∞)	1.29	1.39	1.48	1.60	1.69
120 (120)	1.26	1.35	1.43	1.53	1.61
120 (∞)	1.19	1.25	1.31	1.38	1.43

TABLE 5

COMPONENTS OF THE VARIANCES OF THE $\hat{\gamma}_{jt}$

PERIOD	$s^2(\tilde{\gamma}_0)$	$s^2(\hat{\gamma}_0)$	$\overline{s^2(\tilde{\phi}_0)}$	F	$s^2(\tilde{\gamma}_1)$	$s^2(\hat{\gamma}_1)$	$\overline{s^2(\tilde{\phi}_1)}$	F
Panel A:								
1935–6/68 ...	.00105	.00142	.00037	3.84	.00401	.00436	.00035	12.46
1935–45	.00182	.00273	.00091	3.00	.00863	.00950	.00087	10.92
1946–55	.00057	.00066	.00009	7.33	.00163	.00171	.00008	21.38
1956–6/68 ...	.00077	.00090	.00013	6.92	.00181	.00193	.00012	16.08
1935–40	.00265	.00404	.00139	2.91	.01212	.01347	.00135	9.98
1941–45	.00086	.00118	.00032	3.69	.00452	.00481	.00029	16.59
1946–50	.00086	.00094	.00008	11.75	.00216	.00224	.00008	28.00
1951–55	.00027	.00036	.00009	4.00	.00113	.00121	.00008	15.12
1956–60	.00032	.00041	.00009	4.56	.00104	.00112	.00008	21.50
1961–6/68 ...	.00100	.00114	.00014	8.14	.00217	.00231	.00014	16.50
Panel B:								
1935–6/68 ...	.00092	.00267	.00175	1.52	.00564	.01403	.00839	1.67
1935–45	.00057	.00377	.00320	1.18	.00372	.01941	.01569	1.24
1946–55	.00053	.00112	.00059	1.90	.00651	.00897	.00245	3.66
1956–6/68 ...	.00155	.00294	.00139	2.12	.00667	.01338	.00671	1.99
1935–40	.00018	.00476	.00458	1.04	.00374	.02555	.02181	1.17
1941–45	.00101	.00254	.00153	1.66	.00389	.01225	.00836	1.46
1946–50	.00084	.00136	.00052	2.62	.00862	.01071	.00209	5.12
1951–55	.00024	.00090	.00066	1.36	.00447	.00729	.00282	2.58
1956–60	.00037	.00087	.00050	1.74	.00289	.00517	.00228	2.27
1961–6/68 ...	.00232	.00431	.00199	2.16	.00928	.01894	.00966	1.96
Panel C:								
1935–6/68 ...	.00192	.00266	.00075	3.55	.00285	.00428	.00142	3.01
1935–45	.00394	.00533	.00139	3.83	.00433	.00717	.00283	2.52
1946–55	.00083	.00101	.00018	5.61	.00261	.00310	.00050	6.20
1956–6/68 ...	.00100	.00164	.00063	2.60	.00178	.00270	.00092	2.93
1935–40	.00473	.00669	.00196	3.41	.00732	.01094	.00362	3.02
1941–45	.00307	.00377	.00070	5.38	.00085	.00274	.00189	1.45
1946–50	.00103	.00117	.00014	8.36	.00386	.00439	.00053	8.28
1951–55	.00061	.00083	.00022	3.77	.00140	.00188	.00047	4.00
1956–60	.00079	.00134	.00055	2.44	.00106	.00204	.00098	2.08
1961–6/68 ...	.00109	.00177	.00068	2.60	.00212	.00300	.00088	3.41
Panel D:								
1935–6/68 ...	.00150	.00566	.00406	1.39	.00608	.01521	.00913	1.66
1935–45	.00233	.01065	.00832	1.28	.00402	.02118	.01716	1.23
1946–55	.00013	.00176	.00163	1.08	.00647	.00916	.00269	3.41
1956–6/68 ...	.00194	.00420	.00226	1.86	.00763	.01485	.00722	2.06
1935–40	.00157	.01263	.01106	1.14	.00457	.02910	.02453	1.19
1941–45	.00340	.00843	.00503	1.68	.00365	.01196	.00832	1.44
1946–50	.00023	.00220	.00197	1.12	.00858	.01119	.00261	4.29
1951–55	.00006	.00136	.00130	1.05	.00442	.00719	.00277	2.60
1956–60	.00092	.00239	.00147	1.62	.00328	.00602	.00274	2.20
1961–6/68 ...	.00260	.00539	.00279	1.93	.01060	.02081	.01021	2.04

D. *Tests of the S-L Hypothesis*

In the Sharpe-Lintner two-parameter model of market equilibrium one has, in addition to conditions C1–C3, the hypothesis that $E(\tilde{\gamma}_{0t}) = R_{ft}$. The work of Friend and Blume (1970) and Black, Jensen, and Scholes (1972) suggests that the S-L hypothesis is not upheld by the data. At least in the post–World War II period, estimates of $E(\tilde{\gamma}_{0t})$ seem to be significantly greater than R_{ft}.

Each of the four models of table 3 can be used to test the S-L hypothe-

TABLE 5 (*Continued*)

PERIOD	$s^2(\tilde{\gamma}_2)$	$s^2(\hat{\gamma}_2)$	$\overline{s^2(\hat{\phi}_2)}$	F	$s^2(\tilde{\gamma}_3)$	$s^2(\hat{\gamma}_3)$	$\overline{s^2(\hat{\phi}_3)}$	F
Panel A:								
1935–6/68 ...	...	...	...	...	...	...	...	...
1935–45	...	...	...	...	...	...	...	...
1946–55	...	...	...	...	...	...	...	...
1956–6/68 ...	...	...	...	...	...	...	...	...
1935–40	...	...	...	...	...	...	...	...
1941–45	...	...	...	...	...	...	...	...
1946–50	...	...	...	...	...	...	...	...
1951–55	...	...	...	...	...	...	...	...
1956–60	...	...	...	...	...	...	...	...
1961–6/68 ...	...	...	...	...	...	...	...	...
Panel B:								
1935–6/68 ...	.00121	.00318	.00197	1.61	...	...	...	...
1935–45	.00171	.00548	.00377	1.45	...	...	...	...
1946–55	.00063	.00112	.00049	2.29	...	...	...	...
1956–6/68 ...	.00122	.00278	.00156	1.78	...	...	...	...
1935–40	.00041	.00566	.00524	1.08	...	...	...	...
1941–45	.00327	.00527	.00201	2.62	...	...	...	...
1946–50	.00066	.00103	.00037	2.78	...	...	...	...
1951–55	.00058	.00120	.00062	1.94	...	...	...	...
1956–60	.00033	.00083	.00050	1.66	...	...	...	...
1961–6/68 ...	.00182	.00410	.00227	1.81	...	...	...	...
Panel C:								
1935–6/68 ...	...	...	...	...	.341	.753	.412	1.83
1935–45	...	...	...	...	.535	.847	.313	2.71
1946–55	...	...	...	...	.165	.370	.206	1.80
1956–6/68 ...	...	...	...	...	.304	.968	.664	1.46
1935–40	...	...	...	...	.270	.553	.282	1.96
1941–45	...	...	...	...	.840	1.189	.349	3.41
1946–50	...	...	...	...	.118	.254	.136	1.87
1951–55	...	...	...	...	.217	.493	.276	1.79
1956–60	...	...	...	...	.622	1.355	.734	1.85
1961–6/68 ...	...	...	...	...	.105	.722	.617	1.17
Panel D:								
1935–6/68 ...	.00061	.00362	.00301	1.21	.276	.864	.588	1.47
1935–45	...	.00624	.00644	.97	.392	1.001	.613	1.63
1946–55	.00061	.00148	.00087	1.70	.028	.383	.355	1.08
1956–6/68 ...	.00134	.00304	.00169	1.80	.374	1.125	.751	1.50
1935–40	...	.00723	.00886	.82	.120	.682	.562	1.21
1941–45	.00162	.00515	.00353	1.46	.720	1.395	.675	2.07
1946–50	.00083	.00180	.00096	1.87	.023	.348	.325	1.07
1951–55	.00039	.00116	.00077	1.51	.038	.424	.386	1.10
1956–60	.00037	.00103	.00066	1.56	.712	1.654	.941	1.76
1961–6/68 ...	.00202	.00440	.00238	1.85	.163	.787	.624	1.26

sis.[12] The most efficient tests, however, are provided by the one-variable

[12] The least-squares intercepts $\hat{\gamma}_{0t}$ in the four cross-sectional risk-return regressions can always be interpreted as returns for month t on zero-$\hat{\beta}$ portfolios (n. 10). For the three-variable model of panel D, table 3, the unbiasedness of the least-squares coefficients can be shown to imply that in computing $\hat{\gamma}_{0t}$, negative and positive weights are assigned to the 20 portfolios in such a way that the resulting portfolio has not only zero-$\hat{\beta}$ but also zero averages of the 20 $\hat{\beta}_p^2$ and of the 20 $\bar{s}_p(\hat{\epsilon}_i)$. Analogous statements apply to the two-variable models of panels B and C.

Black, Jensen, and Scholes test the S-L hypothesis with a time series of monthly returns on a "minimum variance zero-$\hat{\beta}$ portfolio" which they derive directly. It turns model of panel A, since the values of $s(\hat{\gamma}_0)$ for this model [which are nearly identical with the values of $s(\hat{\gamma}_0 - R_f)$] are substantially smaller than those for other models. Except for the most recent period 1961–6/68, the values of $\overline{\hat{\gamma}_0 - R_f}$ in panel A are all positive and generally greater than 0.4 percent per month. The value of $t(\overline{\hat{\gamma}_0 - R_f})$ for the overall period 1935–6/68 is 2.55, and the t-statistics for the subperiods 1946–55, 1951–55, and 1956–60 are likewise large. Thus, the results in panel A, table 3, support the negative conclusions of Friend and Blume (1970) and Black, Jensen, and Scholes (1972) with respect to the S-L hypothesis.

The S-L hypothesis seems to do somewhat better in the two-variable quadratic model of panel B, table 3 and especially in the three-variable model of panel D. The values of $t(\overline{\hat{\gamma}_0 - R_f})$ are substantially closer to zero for these models than for the model of panel A. This is due to values of $\overline{\hat{\gamma}_0 - R_f}$ that are closer to zero, but it also reflects the fact that $s(\hat{\gamma}_0)$ is substantially higher for the models of panels B and D than for the model of panel A.

But the effects of $\bar{\hat{\beta}}_p^2$ and $\bar{s}_p(\hat{\epsilon}_i)$ on tests of the S-L hypothesis are in fact not at all so clear-cut. Consider the model

$$\tilde{R}_{it} = \tilde{\gamma}'_{0t} + \tilde{\gamma}'_{1t}\beta_i + \tilde{\gamma}_{2t}(1 - \beta_i)^2 + \tilde{\gamma}_{3t}s_i + \tilde{\eta}_{it}. \qquad (13)$$

Equations (7) and (13) are equivalent representations of the stochastic process generating returns, with $\tilde{\gamma}_{1t} = \tilde{\gamma}'_{1t} - 2\tilde{\gamma}_{2t}$ and $\tilde{\gamma}_{0t} = \tilde{\gamma}'_{0t} + \tilde{\gamma}_{2t}$. Moreover, if the steps used to obtain the regression equation (10) from the stochastic model (7) are applied to (13), we get the regression equation,

$$R_{pt} = \hat{\gamma}'_{0t} + \hat{\gamma}'_{1t}\hat{\beta}_p + \hat{\gamma}_{2t}\overline{(1 - \hat{\beta}_p)^2} + \hat{\gamma}_{3t}\bar{s}_p(\hat{\epsilon}_i) + \hat{\eta}_{pt}, \qquad (14)$$

where, just as $\bar{\hat{\beta}}_p^2$ in (10) is the average of $\hat{\beta}_i^2$ for securities i in portfolio p, $\overline{(1 - \hat{\beta}_p)^2}$ is the average of $(1 - \hat{\beta}_i)^2$. The values of the estimates $\hat{\gamma}_{2t}$ and $\hat{\gamma}_{3t}$ are identical in (10) and (14); in addition, $\hat{\gamma}_{1t} = \hat{\gamma}'_{1t} - 2\hat{\gamma}_{2t}$ and $\hat{\gamma}_{0t} = \hat{\gamma}'_{0t} + \hat{\gamma}_{2t}$. But although the regression equations (10) and (14) are statistically indistinguishable, tests of the hypothesis $E(\tilde{\gamma}_{0t}) =$

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out, however, that this portfolio is constructed under what amounts to the assumptions of the Gauss-Markov Theorem on the underlying disturbances of the one-variable risk-return regression (11). With these assumptions the least-squares estimate $\hat{\gamma}_{0t}$, obtained from the cross-sectional risk-return regression of (11) for month t, is precisely the return for month t on the minimum variance zero-$\hat{\beta}$ portfolio that can be constructed from the 20 portfolio $\hat{\beta}_p$. Thus, the tests of the S-L hypothesis in panel A of table 3 are conceptually the same as those of Black, Jensen, and Scholes.

If one makes the assumptions of the Gauss-Markov Theorem on the underlying disturbances of the models of panels B–D of table 3, the regression intercepts for these models can likewise be interpreted as returns on minimum-variance zero-$\hat{\beta}$ portfolios. These portfolios then differ in terms of whether or not they also constrain the averages of the 20 $\bar{\hat{\beta}}_p^2$ and of the 20 $\bar{s}_p(\hat{\epsilon}_i)$ to be zero. Given the collinearity of $\hat{\beta}_p$, $\bar{\hat{\beta}}_p^2$, and $\bar{s}_p(\hat{\epsilon}_i)$, however, the assumptions of the Gauss-Markov Theorem cannot apply to all four of the models.

R_{ft} from (10) do not yield the same results as tests of the hypothesis $E(\tilde{\gamma}'_{0t}) = R_{ft}$ from (14). In panel D of table 3, $\overline{\hat{\gamma}_0 - R_f}$ is never statistically very different from zero, whereas in tests (not shown) from (14), the results are similar to those of panel A, table 3. That is, $\overline{\hat{\gamma}'_0 - R_f}$ is systematically positive for all periods but 1961–6/68 and statistically very different from zero for the overall period 1935–6/68 and for the 1946–55, 1951–55, and 1956–60 subperiods.

Thus, tests of the S-L hypothesis from our three-variable models are ambiguous. Perhaps the ambiguity could be resolved and more efficient tests of the hypothesis could be obtained if the omitted variables for which $\bar{s}_p(\hat{\epsilon}_i)$, $\hat{\beta}_p^2$, or $(1 - \hat{\beta}_p)^2$ are almost surely proxies were identified. As indicated above, however, at the moment the most efficient tests of the S-L hypothesis are provided by the one-variable model of panel A, table 3, and the results for that model support the negative conclusions of others.

Given that the S-L hypothesis is not supported by the data, tests of the market efficiency hypothesis that $\tilde{\gamma}_{0t} - E(\tilde{R}_{0t})$ is a fair game are difficult since we no longer have a specific hypothesis about $E(\tilde{R}_{0t})$. And using the mean of the $\hat{\gamma}_{0t}$ as an estimate of $E(\tilde{R}_{0t})$ does not work as well in this case as it does for the market efficiency tests on γ_{1t}. One should note, however, that although the serial correlations $\rho_M(\hat{\gamma}_0)$ in table 4 are often large relative to estimates of their standard errors, they are small in terms of the proportion of the time series variance of $\hat{\gamma}_{0t}$ that they explain, and the latter is the more important criterion for judging whether market efficiency is a workable representation of reality (see n. 8).

VI. Conclusions

In sum our results support the important testable implications of the two-parameter model. Given that the market portfolio is efficient—or, more specifically, given that our proxy for the market portfolio is at least approximately efficient—we cannot reject the hypothesis that average returns on New York Stock Exchange common stocks reflect the attempts of risk-averse investors to hold efficient portfolios. Specifically, on average there seems to be a positive tradeoff between return and risk, with risk measured from the portfolio viewpoint. In addition, although there are "stochastic nonlinearities" from period to period, we cannot reject the hypothesis that on average their effects are zero and unpredictably different from zero from one period to the next. Thus, we cannot reject the hypothesis that in making a portfolio decision, an investor should assume that the relationship between a security's portfolio risk and its expected return is linear, as implied by the two-parameter model. We also cannot reject the hypothesis of the two-parameter model that no measure of risk, in addition to portfolio risk, systematically affects average returns. Finally, the observed fair game properties of the coefficients and residuals of the risk-return regressions are consistent with an efficient capital market—that is, a market where prices of securities fully reflect available information.

Appendix

Some Related Issues

A1. Market Models and Tests of Market Efficiency

The time series of regression coefficients from (10) are, of course, the inputs for the tests of the two-parameter model. But these coefficients can also be useful in tests of capital market efficiency—that is, tests of the speed of price adjustment to different types of new information. Since the work of Fama et al. (1969), such tests have commonly been based on the "one-factor market model":

$$R_{it} = \hat{a}_i + \hat{\beta}_i R_{mt} + \hat{\epsilon}_{it}. \quad (15)$$

In this regression equation, the term involving R_{mt} is assumed to capture the effects of market-wide factors. The effects on returns of events specific to company *i*, like a stock split or a change in earnings, are then studied through the residuals $\hat{\epsilon}_{it}$.

But given that there is period-to-period variation in $\hat{\gamma}_{0t}$, $\hat{\gamma}_{2t}$, and $\hat{\gamma}_{3t}$ in (10) that is above and beyond pure sampling error, then these coefficients can be interpreted as market factors, (in addition to R_{mt}) that influence the returns on all securities. To see this, substitute (12) into (11) to obtain the "two-factor market model":

$$R_{pt} = \hat{\gamma}_{0t}(1 - \hat{\beta}_p) + \hat{\beta}_p R_{mt} + \hat{\eta}_{pt}. \quad 16$$

In like fashion, from equation (10) itself we easily obtain the "four-factor market model":

$$R_{pt} = \hat{\gamma}_{0t}(1 - \hat{\beta}_p) + \hat{\beta}_p R_{mt} + \hat{\gamma}_{2t}(\hat{\beta}^2{}_p - \hat{\beta}_p \bar{\hat{\beta}}^2) + \hat{\gamma}_{3t} [\bar{s}_p(\hat{\epsilon}_i) - \hat{\beta}_p \bar{\bar{s}}(\hat{\epsilon}_i)] + \hat{\eta}_{pt}, \quad (17)$$

where $\bar{\hat{\beta}}^2$ and $\bar{\bar{s}}(\hat{\epsilon}_i)$ are the averages over *p* of the $\hat{\beta}_p{}^2$ and the $\bar{s}_p(\hat{\epsilon}_i)$.

Comparing equations (15–17) it is clear that the residuals $\hat{\epsilon}_{it}$ from the one-factor market model contain variation in the market factors $\hat{\gamma}_{0t}$, $\hat{\gamma}_{2t}$, and $\hat{\gamma}_{3t}$. Thus, if one is interested in the effect on a security's return of an event specific to the given company, this effect can probably be studied more precisely from the residuals of the two- or even the four-factor market models of (16) and (17) than from the one-factor model of (15). This has in fact already been done in a study of changes in accounting techniques by Ball (1972), in a study of insider trading by Jaffe (1972), and in a study of mergers by Mandelker (1972).

Ball, Jaffe, and Mandelker use the two-factor rather than the four-factor market model, and there is probably some basis for this. First, one can see from table 5 that because of the collinearity of $\hat{\beta}_p$, $\hat{\beta}_p{}^2$, and $\bar{s}_p(\hat{\epsilon}_i)$, the coefficient estimates $\hat{\gamma}_{0t}$ and $\hat{\gamma}_{1t}$ have much smaller standard errors in the two-factor model. Second, we have computed residual variances for each of our 20 portfolios for various time periods from the time series of $\hat{\epsilon}_{pt}$ and $\hat{\eta}_{pt}$ from (15), (16), and (17). The decline in residual variance that is obtained in going from (15) to (16) is as predicted: That is, the decline is noticeable over more or less the entire range of $\hat{\beta}_p$ and it is proportional to $(1 - \hat{\beta}_p)^2$. On the other hand, in going from the two- to the four-factor model, reductions in residual variance are generally noticeable only in the portfolios with the lowest and highest $\hat{\beta}_p$, and the reductions for these two portfolios are generally small. Moreover, including $\bar{s}_p(\hat{\epsilon}_i)$ as an explanatory variable in addition to $\hat{\beta}_p$ and $\hat{\beta}_p^2$ never results in a noticeable reduction in residual variances.

A2. *Multifactor Models and Errors in the* $\hat{\beta}$

If the return-generating process is a multifactor market model, then the usual estimates of β_i from the one-factor model of (15) are not most efficient. For example, if the return-generating process is the population analog of (16), more efficient estimates of β_i could in principle be obtained from a constrained regression applied to

$$\tilde{R}_{it} - \tilde{\gamma}_{0t} = \beta_i(\tilde{R}_{mt} - \tilde{\gamma}_{0t}) + \tilde{\eta}_{it}.$$

But this approach requires the time series of the true $\tilde{\gamma}_{0t}$. All we have are estimates $\hat{\gamma}_{0t}$, themselves obtained from estimates of $\hat{\beta}_p$ from the one-factor model of (15).

It can also be shown that with a multifactor return-generating process the errors in the $\hat{\beta}$ computed from the one-factor market model of (8) and (15) are correlated across securities and portfolios. This results from the fact that if the true process is a multifactor model, the disturbances of the one-factor model are correlated across securities and portfolios. Moreover, the interdependence of the errors in the $\hat{\beta}$ is higher the farther the true β's are from 1.0. This was already noted in the discussion of table 2 where we found that the relative reduction in the standard errors of the $\hat{\beta}$'s obtained by using portfolios rather than individual securities is lower the farther $\hat{\beta}_p$ is from 1.0.

Interdependence of the errors in the $\hat{\beta}_p$ also complicates the formal analysis of the effects of errors-in-the-variables on properties of the estimated coefficients (the $\hat{\gamma}_{jt}$) in the risk-return regressions of (10). This topic is considered in detail in an appendix to an earlier version of this paper that can be made available to the reader on request.

References

Ball, R. "Changes in Accounting Techniques and Stock Prices." Ph.D. dissertation, University of Chicago, 1972.

Black, F. "Capital Market Equilibrium with Restricted Borrowing." *J. Bus.* 45 (July 1972): 444–55.

Black, F.; Jensen, M.; Scholes, M. "The Capital Asset Pricing Model: Some Empirical Results." In *Studies in the Theory of Capital Markets,* edited by Michael Jensen. New York: Praeger, 1972.

Blume, M. E. "Portfolio Theory: A Step toward Its Practical Application." *J. Bus.* 43 (April 1970): 152–73.

Douglas, G. W. "Risk in the Equity Markets: An Empirical Appraisal of Market Efficiency." *Yale Econ. Essays* 9 (Spring 1969): 3–45.

Fama, E. F. "The Behavior of Stock Market Prices." *J. Bus.* 38 (January 1965): 34–105. (*a*)

———. "Portfolio Analysis in a Stable Paretian Market." *Management Sci.* 11 (January 1965): 404–19. (*b*)

———. "Multiperiod Consumption-Investment Decisions." *A.E.R.* 60 (March 1970): 163–74. (*a*)

———. "Efficient Capital Markets: A Review of Theory and Empirical Work." *J. Finance* 25 (May 1970): 383–417. (*b*)

———. "Risk, Return and Equilibrium." *J.P.E.* 79 (January/February 1971): 30–55.

Fama, E. F., and Babiak, H. "Dividend Policy: An Empirical Analysis." *J. American Statis. Assoc.* 48 (December 1968): 1132–61.

Fama, E. F.; Fisher, L.; Jensen, M.; Roll, R. "The Adjustment of Stock Prices to New Information." *Internat. Econ. Rev.* 10 (February 1969): 1–21.

Fama, E. F., and Miller, M. *The Theory of Finance.* New York: Holt, Rinehart & Winston, 1972.

Fama, E. F., and Roll, R. "Some Properties of Symmetric Stable Distributions." *J. American Statis. Assoc.* 48 (September 1968): 817–36.

Fisher, L. "Some New Stock Market Indexes." *J. Bus.* 39 (January 1966): 191–225.

Friend, I., and Blume, M. "Measurement of Portfolio Performance under Uncertainty." *A.E.R.* 60 (September 1970): 561–75.

Gonedes, N. J. "Evidence on the Information Content of Accounting Numbers: Accounting-based and Market-based Estimates of Systematic Risk." *J. Financial and Quantitative Analysis* (1973): in press.

Jaffe, J. "Security Regulation, Special Information, and Insider Trading." Ph.D. dissertation, University of Chicago, 1972.

Jensen, M. "The Foundations and Current State of Capital Market Theory." *Bell J. Econ. and Management Sci.* 3 (Autumn 1972): 357–98.

Lintner, J. "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets." *Rev. Econ. and Statis.* 47 (February 1965): 13–37.

Mandelker, G. "Returns to Stockholders from Mergers." Ph.D. proposal, University of Chicago, 1972.

Markowitz, H. *Portfolio Selection: Efficient Diversification of Investments.* New York: Wiley, 1959.

Miller, K. D. "Futures Trading and Investor Returns: An Investigation of Commodity Market Risk Premiums." Ph.D. dissertation, University of Chicago, 1971.

Miller, M., and Scholes, M. "Rates of Return in Relation to Risk: A Re-Examination of Some Recent Findings." In *Studies in the Theory of Capital Markets,* edited by Michael Jensen. New York: Praeger, 1972.

Mood, A. M., and Graybill, F. A. *Introduction to the Theory of Statistics.* New York: McGraw-Hill, 1963.

Officer, R. R. "A Time Series Examination of the Market Factor of the New York Stock Exchange." Ph.D. dissertation, University of Chicago, 1971.

Roll, R. *The Behavior of Interest Rates.* New York: Basic, 1970.

Sharpe, W. F. "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk." *J. Finance* 19 (September 1964): 425–42.

Tobin, J. "Liquidity Preference as Behavior towards Risk." *Rev. Econ. Studies* 25 (February 1958): 65–86.

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THE EFFECT OF PERSONAL TAXES AND DIVIDENDS ON CAPITAL ASSET PRICES

Theory and Empirical Evidence

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This paper derives an after tax version of the Capital Asset Pricing Model. The model accounts for a progressive tax scheme and for wealth and income related constraints on borrowing. The equilibrium relationship indicates that before-tax expected rates of return are linearly related to systematic risk and to dividend yield. The sample estimates of the variances of observed betas are used to arrive at maximum likelihood estimators of the coefficients. The results indicate that, unlike prior studies, there is a strong positive relationship between dividend yield and expected return for NYSE stocks. Evidence is also presented for a clientele effect.

1. Introduction

The effect of dividend policy on the prices of equity securities has been an issue of interest in financial theory. The traditional view was that investors prefer a current, certain return in the form of dividends to the uncertain prospect of future dividends. Consequently, they bid up the price of high yield securities relative to low yield securities [see Cottle, Dodd and Graham (1962) and Gordon (1963)]. In their now classic paper Miller and Modigliani (1961) argued that in a world without taxes and transactions costs, the dividend policy of a corporation, given its investment policy, has no effect on the price of its shares. In a world where capital gains receive preferential treatment relative to dividends, the Miller-Modigliani 'irrelevance proposition' would seem to break down. They argue, however, that since tax rates vary across investors each corporation would attract to itself a clientele of investors that most desired its dividend policy. Black and Scholes (1974) assert that corporations would adjust their payout policies until in equilib-

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rium the spectrum of policies offered would be such that any one firm is unable to affect the price of its shares by (marginal) changes in its payout policy.

In the absence of taxes, capital asset pricing theory suggests that individuals choose mean-variance efficient portfolios. Under personal income taxes, individuals would be expected to choose portfolios that are mean-variance efficient in after-tax rates of return. However, the tax laws in the United States are such that some economic units (for example, corporations) would seem to prefer dividends relative to capital gains. Other units (for example, non-profit organizations) pay no taxes and would be indifferent to the level of yield for a given level of expected return. The resulting effect of dividend yield on common stock prices seems to be an empirical issue.

Brennan (1973) first proposed an extended form of the single period Capital Asset Pricing Model that accounted for the taxation of dividends. Under the assumption of proportional individual tax rates (not a function of income), certain dividends, and unlimited borrowing at the riskless rate of interest (among others) he derived the following equilibrium relationship:

$$E(\tilde{R}_i)-r_f=b\beta_i+\tau(d_i-r_f), \tag{1}$$

where $\tilde{R}_i$ is the before tax total return to security i, β_i is its systematic risk, $b=[E(R_m)-r_f-\tau(d_m-r_f)]$ is the after-tax excess rate of return on the market portfolio, r_f is the return on a riskless asset, d_i is the dividend yield on security i; and the subscript m denotes the market portfolio. τ is a positive coefficient that accounts for the taxation of dividends and interest as ordinary income and taxation of capital gains at a preferential rate.

In empirical tests [of the form (1)] to date, the evidence has been inconsistent. Black and Scholes (1974, p. 1) conclude that

> '...it is not possible to demonstrate that the expected returns on high yield common stocks differ from the expected returns on low yield common stocks either before or after taxes.'

Alternatively, stated in terms of the Brennan model, their tests were not sufficiently powerful either to reject the hypothesis that $\tau=0$ or to reject the hypothesis that $\tau=0.5$. Rosenberg and Marathe (1978) attribute the lack of power in the Black–Scholes tests to (a) the loss in efficiency from grouping stocks into portfolios and (b) the inefficiency of their estimating procedures, which are equivalent to Ordinary Least Squares. Using an instrumental variables approach to the problem of errors in variables and a more complete specification of the variance-covariance matrix (of disturbances in the regression), Rosenberg and Marathe find that the dividend term is statistically significant. Both the Rosenberg and Marathe and the Black and Scholes studies use an average dividend yield from the prior twelve month period as a surrogate for the expected dividend yield. Since most dividends are paid quarterly, their proxy understates the expected dividend yield in ex-dividend months and overstates it in those months that a stock does not go ex-dividend, thereby reducing the efficiency of the estimated coefficient on the dividend yield term. Both studies (Rosenberg and Marathe in using instrumental variables, and Black–Scholes in grouping) sacrifice efficiency to achieve consistency.

The present paper derives an after-tax version of the Capital Asset Pricing Model that accounts for a progressive tax scheme and both wealth and income related constraints on borrowing. Alternative econometric procedures are used to test the implications of this model. Unlike prior tests of the CAPM, the tests here use the variance of the observed betas to arrive at maximum likelihood estimators of the coefficients. Consistent estimators are obtained without loss of efficiency. Also, for ex-dividend months the expected dividend yield based on prior information is used, and for other months the expected dividend yield is set equal to zero. While the estimate of the coefficient of dividend yield is of the same order of magnitude as that found in Black and Scholes, and lower than that found by Rosenberg and Marathe, the t-value is substantially larger, indicating a substantial increase in efficiency. Furthermore, the tests are consistent with the existence of a clientele effect, indicating that the aversion for dividends relative to capital gains is lower for high yield stocks and higher for low yield stocks. This is consistent with the Elton and Gruber (1970) empirical results on the ex-dividend behavior of common stocks.

2. Theory

This section derives a version of the Capital Asset Pricing Model that accounts for the tax treatment of dividend and interest income under a progressive taxation scheme. Two types of constraints on individual borrowing are imposed. The first constrains the maximum interest on riskless borrowing to be equal to the individual's dividend income, and the second is a margin requirement that restricts the fraction of security holdings that may be financed through borrowing. In previous published work, Brennan (1973) derives an after-tax version of the Capital Asset Pricing Model with unlimited borrowing and with constant tax rates which may vary across individuals.[1] Under his model when interest on borrowing exceeds dividend income the investor would pay a negative tax. The theoretical model

[1]Brennan (1970) also derives a model with a progressive tax scheme. However, he neither considers constraints on borrowing nor the limiting of interest deduction on margin borrowing to dividend income. Consideration of the limit on the interest tax deduction to dividend income combined with a positive capital gains tax would result in a preference for dividends by those individuals whose interest payments exceed their dividend income.

developed here may be viewed as an extension of the Brennan analysis to account for constraints on borrowing along with a progressive tax scheme. Special cases of the model are examined, where the income related constraint and/or the margin constraint on individual borrowing are removed.

The following assumptions are made:

(A.1) Individuals' Von Neumann–Morgenstern utility functions are monotone increasing strictly concave functions of after-tax end of period wealth.

(A.2) Security rates of return have a multivariate normal distribution.

(A.3) There are no transactions costs, and no restrictions on the short sale of securities, and individuals are price takers.

(A.4) Individuals have homogeneous expectations.

(A.5) All assets are marketable.

(A.6) A riskless asset, paying a constant rate r_f, exists.

(A.7) Dividends on securities are paid at the end of the period and are known with certainty at the beginning of the period.

(A.8) Income taxes are progressive and the marginal tax rate is a continuous function of taxable income.

(A.9) There are no taxes on capital gains.

(A.10) Constraints on individuals' borrowing are of the form:

(i) A constraint that the interest on borrowing cannot exceed dividend income, called the income constraint on borrowing, and/or

(ii) a margin constraint that the individual's net worth be at least a given fraction of the market value of his holdings of risky securities.

Assumptions (A.1) through (A.6) are standard assumptions of the Capital Asset Pricing Model. Assumptions (A.1) and (A.2) taken together imply that preferences can be described over the mean and the variance of after-tax end of period wealth. Under these conditions individuals prefer more mean return and are averse to the variance of return. The individual's marginal rate of substitution between the mean and variance of after-tax end of period wealth, at the optimum, can be written as the ratio of his global risk tolerance to his initial period wealth. That is, if $u_k(W_1^k)$ is the kth individual's utility function in terms of after-tax end of period wealth, $f^k(\mu_k, \sigma_k^2)$ is his objective function in terms of the mean and variance of the after-tax portfolio return, and W^k is his initial wealth,

$$f_1^k / -2f_2^k = \theta^k / W^k, \tag{2}$$

where $\theta^k \equiv -E(u')/E(u'')$ is the individual's global risk tolerance at the optimum [see Gonzalez-Gaverra (1973) and Rubinstein (1973)]. (A.7) implies

that dividends are announced at the beginning of the period and paid at its end. Since, firms display relatively stable dividend policies this may be a reasonable approximation for a monthly holding period.

Assumption (A.8) closely resembles the tax treatment of ordinary dividends in the U.S. The $100 dividend exclusion is ignored, since the small magnitude of the exclusion implies that for the majority of stockholders the marginal tax rate applicable to ordinary income is the same as that applied to dividends. Assumption (A.9) abstracts from the effects of capital gains taxes. Since capital gains are taxed only upon realization, their treatment in a single period model is not possible. It is, however, straightforward to model a capital gains tax on an accrual basis [see Brennan (1973)]. Since most capital gains go unrealized for long periods, this would tend to overstate the effect of the actual tax. Noting that the ratio of realizations to accruals is small, and that capital gains are exempt from tax when transferred by inheritance, Bailey (1969) has argued that the effective tax is rather small.

Under assumption (A.8), the kth individual's average tax rate, t^k, is a non-decreasing function of his taxable end of period income Y_1^k,

$$\begin{aligned} t^k &= g(Y_1^k), \\ g(0) = 0, \qquad g'(Y_1^k) &= 0 \quad \text{for} \quad Y_1^k \leq 0, \\ &> 0 \quad \text{for} \quad Y_1^k > 0. \end{aligned} \tag{3}$$

The kth individual's marginal tax rate, written T^k, is the first derivative of taxes paid with respect to taxable income. This is equal to the average tax rate plus the product of taxable income and the derivative of the average tax rate,

$$T^k \equiv d(t^k Y_1^k)/dY_1^k = t^k + Y_1^k g'(Y_1^k). \tag{4}$$

The margin constraint in assumption (A.10-ii) resembles institutional margin restrictions. By (A.10-i), borrowing is constrained up to a point where interest paid equal dividends received. This constraint incorporates the casual empirical observation that loan applications require information on income (which this constraint accounts for) in addition to information on wealth (which the margin constraint accounts for). One or both of the constraints may be binding, for a given individual. This formulation allows the analysis of an equilibrium with both constraints, with only one of them imposed or with no borrowing constraints.

The following notation is employed:

$\tilde{R}_i$ = the total before tax rate of return on security i, equal to the ratio of the value of the security at the end of the period plus dividends over its current value, less one,

d_i = the dividend yield on security i, equal to the dollar dividend divided by the current price,
X_i^k = the fraction of the kth individual's wealth invested in the ith risky asset, $i = 1, 2, \ldots, N$ (a negative value is a short sale),
X_f^k = the fraction of the kth individual's wealth invested in the safe asset (a negative value indicates borrowing),
$\tilde{R}_p^k$ = the before-tax rate of return on the kth individual's portfolio,
W^k = the kth individual's initial wealth, and
$f^k(\mu_k; \sigma_k^2)$ = the kth individual's expected utility function defined over the mean and variance of after-tax portfolio return, μ_k and σ_k^2, respectively.

The kth individual's ordinary income is then

$$Y_1^k = W^k\left(\sum_i X_i^k d_i + X_f^k r_f\right). \tag{5}$$

The mean after-tax return on the individual's portfolio is

$$\mu_k = \sum_i X_i^k E(\tilde{R}_i) + X_f^k r_f - t^k\left(\sum_i X_i^k d_i + X_f^k r_f\right), \tag{6}$$

and under assumption (A.7) the variance of after-tax return is

$$\begin{aligned} \sigma_k^2 &= \sum_i \sum_j X_i^k X_j^k \operatorname{cov}(\tilde{R}_i - d_i t^k, \tilde{R}_j - d_j t^k) \\ &= \sum_i \sum_j X_i^k X_j^k \operatorname{cov}(\tilde{R}_i, \tilde{R}_j). \end{aligned} \tag{7}$$

By assumption (A.10-i) the income constraint on borrowing is

$$W^k\left\{\sum_i X_i^k d_i + X_f^k r_f\right\} \geq 0, \tag{8}$$

and the margin constraint on borrowing is

$$W^k\left\{(1-\alpha)\sum_i X_i^k + X_f^k\right\} \geq 0, \tag{9}$$

where α; $0 < \alpha < 1$, is the margin requirement on the individual. As pointed out earlier, one or both of these constraints may be binding.

The kth individual's optimization problem is stated in terms of the following Lagrangian:

$$\begin{aligned} \mathscr{L}^k \equiv{} & f^k(\mu_k, \sigma_k^2) + \lambda_1^k\left[1 - \sum_i X_i^k - X_f^k\right] \\ & + \lambda_2^k\left[\sum_i X_i^k d_i + X_f^k r_f - S_2^k\right] + \lambda_3^k\left[(1-\alpha)\sum_i X_i^k + X_f^k - S_3^k\right], \end{aligned} \tag{10}$$

where

λ_1^k = the Lagrange multiplier on the kth individual's budget,
λ_2^k, S_2^k = the Lagrange multiplier and non-negative slack variable for the income related constraint on the kth individual's borrowing, respectively (when the constraint is binding $\lambda_2^k > 0$ and $S_2^k = 0$, and when it is not binding $\lambda_2^k = 0$ and $S_2^k \geq 0$), and
λ_3^k, S_3^k = the Lagrange multiplier and non-negative slack variables for the margin constraint on the kth individual's borrowing, respectively; again if the constraint is binding (not binding), $\lambda_3^k > (=)\ 0$ and $S_3^k = (\geq)\ 0$.

The stationary points satisfy the following first order conditions:

$$\begin{aligned} \frac{\partial \mathscr{L}^k}{\partial X_i^k} ={} & f_1^k\{E(\tilde{R}_i) - [t^k + Y_1^k g'(Y_1^k)]d_i\} - \lambda_1^k + \lambda_2^k d_i \\ & + \lambda_3^k(1-\alpha) + 2f_2^k \sum_j X_j^k \operatorname{cov}(\tilde{R}_i, \tilde{R}_j) = 0, \qquad i = 1, 2, \ldots, N, \end{aligned} \tag{11}$$

$$\frac{\partial \mathscr{L}^k}{\partial X_f^k} = f_1^k\{r_f - [t^k + Y_1^k g'(Y_1^k)]r_f\} - \lambda_1^k + \lambda_2^k r_f + \lambda_3^k = 0, \tag{12}$$

where $f_1^k \equiv \partial f^k(\mu_k, \sigma_k^2)/\partial \mu_k$, $f_2^k \equiv \partial f^k(\mu_k, \sigma_k^2)/\partial \sigma_k^2$. The other first order conditions are the constraints and specify the signs of the Lagrangian multipliers and are omitted here. The progressive nature of the tax scheme [assumption (A.8)] ensures that the mean variance efficient frontier in after-tax terms is concave, and this together with risk aversion from assumption (A.8) is sufficient to guarantee the second order conditions for a maximum.

Recall the following relationships: (i) the marginal tax rate, $T^k = [t^k + Y_1^k g'(\cdot)]$, (ii) the covariance $\sum_j X_j^k \operatorname{cov}(\tilde{R}_i, \tilde{R}_j) = \operatorname{cov}(\tilde{R}_i, \tilde{R}_p^k)$, and (iii) the global risk tolerance $\theta^k = W^k(f_1^k / -2f_2^k)$. Subtracting relation (12) from relation (11) and re-arranging terms yields

$$\begin{aligned} [E(\tilde{R}_i) - r_f] ={} & \alpha(\lambda_3^k / f_1^k) + (W^k/\theta^k)\operatorname{cov}(\tilde{R}_i, \tilde{R}_p^k) \\ & + [T^k - (\lambda_2^k / f_1^k)](d_i - r_f). \end{aligned} \tag{13}$$

Relation (13) must be satisfied for the individual's portfolio optimum.

Market equilibrium requires that relation (13) holds for all individuals, and that markets clear. For markets to clear all assets have to be held which implies the conservation relation (14) that requires the value weighted average of all individuals' portfolios be equal to the market portfolio,

$$\sum_k (W^k/W^m)\tilde{R}_p^k = \tilde{R}_m, \tag{14}$$

or

$$\sum_k W^k \tilde{R}_p^k = W^m \tilde{R}_m,$$

where

$$\sum_k W^k \equiv W^m.$$

Multiplying both sides of relation (13) by θ^k, summing over all individuals, using the conservation relation (14) and re-arranging terms yields

$$E(\tilde{R}_i) - r_f = a + b\beta_i + c(d_i - r_f), \tag{15}$$

where

$$\beta_i \equiv \operatorname{cov}(\tilde{R}_i, \tilde{R}_m)/\operatorname{var}(\tilde{R}_m),$$

$$a \equiv \alpha \sum_k (\theta^k/\theta^m)(\lambda_2^k/f_1^k),$$

$$b \equiv \operatorname{var}(\tilde{R}_m)(W^m/\theta^m),$$

$$c \equiv \sum_k (\theta^k/\theta^m)[T^k - (\lambda_2^k/f_1^k)],$$

$$\theta^m \equiv \sum_k \theta^k.$$

The term 'a', the intercept of the implied security market plane, is the fractional margin requirement α times the weighted average of the ratios of individual shadow prices on the margin constraint and the expected marginal utility of mean return. The weights, (θ^k/θ^m), are proportional to individuals' global risk tolerances. When $\alpha>0$ and the constraint is binding for some individuals, $\lambda_2^k>0$ for some k, a is positive. In the absence of margin requirements $(\alpha=0)$ or when the margin constraint is not binding for all individuals, $(\lambda_2^k=0)$ for all k), $a=0$.

Interpreting eq. (15), 'a' is the excess return on a zero beta portfolio (relative to the market) whose dividend yield is equal to the riskless rate, i.e.,

$a=E(\tilde{R}_z)-r_f$. The term 'b', the coefficient on beta is equal to the product of the variance of the rate of return on the market portfolio and global market relative risk aversion, i.e., $b=\operatorname{var}(\tilde{R}_m)(W^m/\theta^m)$. Since relation (15) also holds for the market portfolio, b may be alternatively expressed as $b=[E(\tilde{R}_m)-r_f-c(d_m-r_f)-a]$. If '$c$' is interpreted as a tax rate, b may be viewed as the expected after-tax rate of return on a hedge portfolio which is long the market portfolio and short a portfolio having a zero beta and a dividend yield equal to the riskless rate of interest; i.e., $b=[E(\tilde{R}_m)-E(\tilde{R}_z)-c(d_m-d_z)]$. The term '$c$' is a weighted average of individual's marginal tax rates $(\sum_k(\theta^k/\theta^m)T^k)$, less the weighted average of the individual's ratios of the shadow price on the income related borrowing constraint and the expected marginal utility of mean portfolio return $\sum_k(\theta^k/\theta^m)(\lambda_3^k/f_1^k)$. For the cases where the income related margin constraint is either non-existent or non-binding for all individuals, c is simply the weighted average of marginal tax rates, and is positive. Otherwise, the sign of 'c' depends on the magnitudes of these two terms. Define B as the set of indices of those individuals k for whom the income related constraint is binding: and define N (not B) as the set of indices for which the constraint is non-binding. Now for $k \in B$, $\lambda_3^k>0$, $Y_1^k=0$ and $T^k=t^k=0$. And for $k \in N$, $\lambda_3^k=0$, $Y_1^k \geqq 0$ and $T^k \geqq t^k \geqq 0$. Hence

$$c=\sum_{k \in N}\frac{\theta^k}{\theta^m}T^k-\sum_{k \in B}\frac{\theta^k}{\theta^m}\frac{\lambda_3^k}{f_1^k}. \tag{16}$$

The individuals in N may be viewed as a clientele that prefers capital gains to dividends. The individuals in B may be viewed as a clientele that shows a preference for dividends: in the context of this model, these individuals wish to borrow more than the income related constraint allows them, and increased dividends serve to increase their debt capacity without additional tax obligations. To this point corporate dividend policies have been treated as exogenous in this model.

Now consider supply adjustments by value maximizing firms. If $c>0$ $(c<0)$ firms could increase their market values by decreasing (increasing) cash dividends and increasing (decreasing) share repurchases or decreasing (increasing) external equity flotations. Value maximizing firms (in absence of any restrictions the IRS may impose) would adjust the supply of dividends until an equilibrium was obtained where

$$\sum_{k \in N}(\theta^k/\theta^m)T^k=\sum_{k \in B}(\theta^k/\theta^m)(\lambda_3^k/f_1^k). \tag{17}$$

When condition (17) is satisfied an individual firm's dividend decision does not affect its market value, $c=0$ and dividend yield has no effect on the before tax rate of return on any security.[2]

Under unrestricted supply effects, $c=0$ and the equilibrium relationship (15) reduces to the before tax zero beta version of the Capital Asset Pricing Model:

$$E(\tilde{R}_i)=(a+r_f)(1-\beta_i)+E(\tilde{R}_m)\beta_i. \tag{18}$$

Note that this obtains in the presence of taxes. Long (1975) has studied conditions under which the before tax and after-tax mean variance efficient frontiers are identical for any individual. He does not, however, study the equilibrium as is done here: for even though the before tax and after-tax individual mean variance frontiers are not identical, (18) demonstrates that prices are found as if there is no tax effect.

In the case where there are no margin constraints, $a=0$, and relation (18) reduces to the before tax traditional Sharpe–Lintner version of the Capital Asset Pricing Model,

$$E(\tilde{R}_i)=r_f+[E(\tilde{R}_m)-r_f]\beta_i. \tag{19}$$

Return now to the case where the income related borrowing constraint is absent. Then, in (16), $c=\sum_k T^k(\theta^k/\theta^m)\equiv T^m$, the 'market' marginal tax bracket: and the relation reduces to an after-tax version of the Black (1972), Lintner (1965), Vasicek (1971) zero beta model,

$$E(\tilde{R}_i)-T^m d_i=[r_f(1-T^m)+a](1-\beta_i)+(E(\tilde{R}_m)-T^m d_m)\beta_i. \tag{20}$$

When there is no margin constraint or when it is non-binding for all individuals, $a=0$, and relation (20) reduced to an after-tax version of the Sharpe (1964), Lintner (1965) model.

$$E(\tilde{R}_i)-T^m d_i=[r_f(1-T^m)]+[E(\tilde{R}_m)-T^m d^m-r_f(1-T^m)]\beta_i. \tag{21}$$

However, in none of these cases is T^m a weighted average of individual

[2]Note, however, that this equilibrium, where dividends do not affect before tax returns, may not exist. For example, the income constraint may be binding for no one even when dividends are zero. If all individuals had the same endowments and had the same utility functions this constraint would be non-binding for all individuals.

This argument is in the spirit of the 'supply effect' alluded to in Black and Scholes (1974). Unlike the recent argument in Miller and Scholes (1977) for a zero dividend effect, the present argument does not depend on an artificial segmentation of accumulators and non-accumulators, and the existence of tax-sheltered lending opportunities with zero administrative costs. The major problem with the argument here is that with the existence of two distinct clienteles, one preferring higher dividends and the other preferring lower dividends, shareholders would not agree on the direction in which firms should change their dividend. Thus the assertion of value maximizing behavior by firms does not have a strong theoretical basis.

average tax rates. It is only when taxes are simply proportional to income that $T^k=t^k$, and relation (21) is identical to the equilibrium implied by Brennan (1973), who assumes a constant tax rate that may differ across investors.

3. Empirical tests

From the theory, the equilibrium specification to be tested is

$$E(\tilde{R}_i)-r_f=a+b\beta_i+c(d_i-r_f). \tag{22}$$

The hypotheses are $a>0$, $b>0$ and in the absence of the income related constraint on borrowing $c>0$.

In obtaining econometric estimates of a, b and c, two problems arise. The first is that expectations are not directly observed. The usual procedure is to assume that expectations are rational and that the parameters a, b and c are constant over time; the realized returns are used on the left-hand side

$$\tilde{R}_{it}-r_{ft}=\gamma_0+\gamma_1\beta_{it}+\gamma_2(d_{it}-r_{ft})+\tilde{\varepsilon}_{it}, \qquad i=1,2,\dots,N_t, \quad t=1,2,\dots,T. \tag{23}$$

where $\tilde{R}_{it}$ is the return of security i in period t, β_{it} and d_{it} are the systematic risk and the dividend yield of security i in period t respectively. The disturbance term $\tilde{\varepsilon}_{it}$ is $\tilde{R}_{it}-E(\tilde{R}_{it})$, the deviation of the realized return from its expected value. The coefficients γ_0, γ_1 and γ_2 correspond to a, b and c. The variance of the column vector of disturbance terms, $\tilde{\varepsilon}\equiv\{\tilde{\varepsilon}_{it}: i=1,2,\dots,N_t, t=1,\dots,T\}$, is not proportional to the identity matrix, since contemporaneous covariances between security returns are non-zero, and return variances differ across securities. (Note that in order to conserve space '{ }' is used to denote a column vector.) This means that ordinary least squares (OLS) estimators are inefficient, for either a cross-sectional regression in month t, or a pooled time series and cross-sectional regression. The computed variance of the OLS estimator (based on the assumption that the variance of $\tilde{\varepsilon}$ is proportional to the identity matrix) is not equal to the true variance of the estimator.

The second problem is that the true population β_{it}'s are unobservable. The usual procedure uses an estimate from past data, and this estimate has an associated measurement error. This means that the OLS estimates will be biased and inconsistent. The method used in tackling these problems is discussed in this section.

To fix matters, assume that data exist for rates of return, true betas and for dividend yields in periods t, $i=1,2,\dots,N_t$ securities in each period t, $t=1,\dots,T$. Define the vector of realized excess returns as

$$\tilde R \equiv \{\tilde R_1 \tilde R_2, \ldots, \tilde R_t, \ldots, \tilde R_T\},$$

where

$$\tilde R_t \equiv \{(\tilde R_{1t} - r_{ft})(\tilde R_{2t} - r_{ft})(\tilde R_{it} - r_{ft}), \ldots, (\tilde R_{N_t t} - r_{ft})\},$$

and the matrices X of explanatory variables as

$$X \equiv \{X_1 X_2, \ldots, X_t, \ldots, X_T\},$$

where

$$X_t \equiv \begin{bmatrix} 1 & \beta_{1t} & (d_{1t} - r_{ft}) \\ 1 & \beta_{2t} & (d_{2t} - r_{ft}) \\ \vdots & \vdots & \vdots \\ 1 & \beta_{N_t} & (d_{N_t} - r_{ft}) \end{bmatrix}.$$

By defining the vector of regression coefficients as $\Gamma = \{\gamma_0 \gamma_1 \gamma_2\}$ one can write the pooled time series and cross-sectional regression as

$$\tilde R = X\Gamma + \tilde\varepsilon, \tag{24}$$

where

$$\tilde\varepsilon \equiv \{\tilde\varepsilon_1 \tilde\varepsilon_2, \ldots, \tilde\varepsilon_t, \ldots \tilde\varepsilon_T\},$$

and

$$\tilde\varepsilon_t \equiv \{\tilde\varepsilon_{1t} \tilde\varepsilon_{2t}, \ldots, \tilde\varepsilon_{it}, \ldots, \tilde\varepsilon_{N_t}\}.$$

It is assumed that

$$E(\tilde\varepsilon) = 0,$$

and that

$$E(\tilde\varepsilon_t \tilde\varepsilon_t') = V_t,$$

some symmetric positive definite matrix of order $(N_t \times N_t)$. It is also assumed that security returns are serially uncorrelated, so that

$$E(\tilde\varepsilon_{it} \tilde\varepsilon_{js}) = 0 \quad \text{for} \quad t \neq s.$$

This means that the variance-covariance matrix $V \equiv E(\tilde\varepsilon\tilde\varepsilon')$ is block diagonal, with the off-diagonal blocks being zero. The matrices V_t appears along the diagonal of V.

It is well known that the estimator for Γ which is linear in $\tilde R$, unbiased and has minimum variance is unique, and is given by the Aitken or Generalized Least Squares estimator (GLS),

$$\hat\Gamma = (X'V^{-1}X)^{-1}X'V^{-1}\tilde R. \tag{25}$$

From the block diagonal nature of V, it follows that V^{-1} is also block diagonal. The matrices V_t^{-1}, $t = 1, 2, \ldots, T$, appear along the diagonal of V^{-1}, with the off-diagonal blocks being zero. Assuming that Γ is an intertemporal constant, Γ can be estimated by efficiently pooling T independent GLS estimates of Γ, namely $\hat\Gamma_1, \hat\Gamma_2, \ldots, \hat\Gamma_t, \ldots, \hat\Gamma_T$, obtained by using cross-sectional data in periods $1, 2, \ldots, t, \ldots, T$.

$$\hat\Gamma_t = (X_t'V_t^{-1}X_t)^{-1}X_t'V_t^{-1}\tilde R_t, \quad t = 1, 2, \ldots, T. \tag{26}$$

That is, the monthly estimators $\hat\gamma_{kt}$ for γ_k, $k = 0$, 1 or 2, are serially uncorrelated, and the pooled GLS estimator $\hat\gamma_k$ is found as the weighted mean of the monthly estimates, where the weights are inversely proportional to the variances of these estimates.

$$\hat\gamma_k = \sum_{t=1}^{T} Z_{kt}\hat\gamma_{kt}, \tag{27}$$

$$\text{var}(\hat\gamma_k) = \sum_{t=1}^{T} Z_{kt}^2 \,\text{var}(\hat\gamma_{kt}), \tag{28}$$

$$Z_{kt} = [\text{var}(\hat\gamma_{kt})]^{-1} \Big/ \sum_t [\text{var}(\hat\gamma_{kt})]^{-1}. \tag{29}$$

For some of the results presented in section [illegible] each $\hat\gamma_{kt}$ is assumed to be drawn from a stationary distribution, and the estimates of $\hat\gamma_k$ and its variance are

$$\hat\gamma_k = \sum_{t=1}^{T} (\hat\gamma_{kt}/T), \tag{30}$$

$$\hat\sigma^2(\hat\gamma_k) = \left[\sum_{t=1}^{T} (\hat\gamma_{kt} - \hat\gamma_k)^2 / T(T-1)\right], \quad k = 0, 1, 2. \tag{31}$$

A useful portfolio interpretation can be given to each of the GLS estimators $\hat\Gamma_t$ in (26). Choose any matrix numbers of order $N_t \times N_t$, say W_t^{-1}, such that $(X_t' W_t^{-1} X_t)^{-1}$ exists. Construct an estimator, using cross-sectional data in period t, as

$$(X_t' W_t^{-1} X_t)^{-1} X_t' W_t^{-1} \tilde{R}_t. \tag{32}$$

This estimator is linear in $\tilde{R}_t$ and unbiased for Γ. This estimator is a linear combination of realized security excess returns in period t. From the fact that

$$(X_t' W_t^{-1} X_t)^{-1} X_t' W_t^{-1} X_t = I, \tag{33}$$

where I is the identity matrix, it follows that the estimator for γ_0 in (32) is the realized excess return on a zero beta portfolio having a dividend yield equal to the riskless rate. Similarly, the estimator for γ_1 is the realized excess return on a hedge portfolio that has a beta of one and dividend yield equal to zero; and that for γ_2 is the realized excess return on a hedge portfolio having a zero beta and a dividend yield equal to unity. This interpretation[3] can be given to any estimator of the form (32). When W_t^{-1} (or, equivalently, the portfolio weights discussed above) is chosen so as to minimize the variance of the portfolio return, the resulting estimator is the GLS estimator. This is because portfolio estimates as in (32) are linear and unbiased by construction, and by the Gauss–Markov theorem the GLS estimator is the unique minimum variance estimator among linear unbiased estimators [see Amemiya (1972)].

It is not possible to specify the elements of the variance–covariance matrix V_t a priori. The task of estimating these elements is greatly simplified by assuming that the Sharpe single index model is a correct description of the return generating process. The process that generates returns at the beginning of period t is assumed to be as follows:

$$\tilde{R}_{it} = \alpha_{it} + \beta_{it} \tilde{R}_{mt} + \tilde{\varepsilon}_{it}, \qquad i = 1, 2, \ldots, N_t, \tag{34}$$

$$\begin{aligned} \operatorname{cov}(\tilde{\varepsilon}_{it}, \tilde{\varepsilon}_{jt}) &= 0, \qquad i \neq j, \\ &= s_{jj}, \qquad i = j, \end{aligned} \tag{35}$$

$$\alpha_{it} = E(\tilde{R}_{it} \mid R_{mt} = 0).$$

With this specification the element in the ith row and the jth column of V_t, written as $V_t(i,j)$, is given by

$$\begin{aligned} V_t(i,j) &= \beta_{it}\beta_{jt}\sigma_{mm}, \qquad i \neq j, \\ &= \beta_{it}^2 \sigma_{mm} + s_{ii}, \qquad i = j, \end{aligned} \qquad i, j = 1, 2, \ldots, N_t, \tag{36}$$

[3] For a similar interpretation, see Rosenberg and Marathe (1978).

where

$$\sigma_{mm} \equiv \operatorname{var}(\tilde{R}_{mt}).$$

Under these conditions the GLS estimator of Γ obtained by using data in period t reduces to

$$\hat{\Gamma}_t = (X_t' \Omega_t^{-1} X_t)^{-1} X_t' \Omega_t^{-1} R_t, \tag{37}$$

where Ω_t is a diagonal matrix of order $(N_t \times N_t)$, whose element in the ith row and jth column is given by

$$\begin{aligned} \Omega_t(i,j) &= 0, \qquad i \neq j, \\ &= s_{ii}, \qquad i = j, \end{aligned} \qquad i, j = 1, 2, \ldots, N_t. \tag{38}$$

In appendix A it is shown that this estimator is the GLS estimator for Γ. That is, under the assumptions of the single index model, the estimator minimizes the 'residual risk' of three portfolio returns, subject to the constraint that the expected returns on these portfolios are γ_0, γ_1 and γ_2 respectively. This estimator can be constructed as a heteroscedastic transformation on $\tilde{R}_t$ and X_t. Define the matrix P_t of order $(N_t \times N_t)$ whose elements are given by

$$\begin{aligned} P_t(i,j) &= \phi/s_i \equiv \phi/\sqrt{s_{ii}}, \qquad i = j \\ &= 0, \qquad i \neq j, \end{aligned} \tag{39}$$

where ϕ is a positive scalar. Then $\hat{\Gamma}_t$ can also be arrived at from the OLS regression on the transformed variables,

$$\tilde{R}_t^* = X_t^* \Gamma + \tilde{\varepsilon}_t^*, \tag{40}$$

where

$$\tilde{R}_t^* = P\tilde{R}_t \quad \text{and} \quad X_t^* = PX_t.$$

This is equivalent to deflating the variables in the ith rows of $\tilde{R}_t$ and X_t by a factor proportional to the residual standard error s_i. Note that Black and Scholes (1974), who used the portfolio approach, assumed in addition to the single index model that the 'residual' risks of all securities were equal; that is, they assumed that $s_{ii} = s^2$ for all i. Therefore, the Black–Scholes estimator reduces to OLS on the untransformed variables.

Errors in variables. Since true population β_{it} variables are unobserved, estimates of this variable, $\hat{\beta}_{it}$ are obtained from historical data. The estimated beta is assumed equal to the true beta plus a measurement error $\tilde{v}_{it}$,

$$\hat{\beta}_{it} = \beta_{it} + \tilde{v}_{it}. \tag{41}$$

The presence of measurement error causes a misspecification in OLS and GLS estimators, and the resulting estimates of Γ are biased and inconsistent [see, for example, Johnston (1972), for a discussion of the bias in the coefficients of a variable without error, here dividend yield; see Fisher (1977)]. The estimates $\hat{\beta}_{it}$ are obtained from a regression of $\tilde{R}_{i\tau}$ on the return of the market portfolio $\tilde{R}_{m\tau}$ from data prior to period t,

$$\tilde{R}_{i\tau} = \alpha_{it} + \beta_{it}\tilde{R}_{m\tau} + \tilde{e}_{i\tau}, \qquad \tau = t-60, t-59, \ldots, t-1. \tag{42}$$

Since the single index model is assumed, $\mathrm{cov}(\tilde{e}_{i\tau}, \tilde{e}_{j\tau}) = 0$ and hence $\mathrm{cov}(\tilde{v}_{it}, \tilde{v}_{jt}) = 0$. If the joint probability distribution between security rates of return and market return is stationary, the variance of the measurement error $\mathrm{var}(\tilde{v}_{it})$ is proportional to the variance of the residual risk term $\mathrm{var}(\tilde{e}_{i\tau})$, for each i. Since month t is not used in this time series regression, $\mathrm{cov}(\tilde{\varepsilon}_{it}, \tilde{v}_{it}) = 0$. Note that this time series regression yields a measured beta, $\hat{\beta}_{it}$, its variance $\mathrm{var}(\tilde{v}_{it})$ and the variance of the residual risk term $\mathrm{var}(\tilde{e}_{i\tau}) = s_{it}$.

Consistent with prior empirical studies, the assumption $E(\tilde{\varepsilon}_{it}) = 0$ has been made. However, it is recognized that if the 'market return' used in (42) is not the true market return, then the estimate of β_{it} may be biased, as has been observed by Sharpe (1977), Mayers (1972) and Roll (1977).

Because of errors in variables, most previous empirical tests have grouped stocks into portfolios. Since errors in measurement in betas for different securities are less than perfectly correlated, grouping risky assets into portfolios would reduce the asymptotic bias in OLS estimators. However, grouping results in a reduction of efficiency caused by the loss of information. The efficiency of the OLS estimator of the coefficient of a single independent variable is proportional to the cross sectional variation in that independent variable (beta). For the two independent variables case (dividend yield and beta), Stehle (1976) has shown that the efficiency of the OLS estimator of the coefficient of a given independent variable, using grouped data, is proportional to the cross-sectional variation in that variable unexplained by the variation in the other independent variable. Since the within group variation in dividend yield unexplained by beta is eliminated, the efficiency of the estimate of the dividend yield coefficient using grouped data is lower than that using all the data.[4] For this reason the present study does not use the grouping approach to errors in variables. Instead, use is made of the measurement error in beta to arrive at a consistent estimator for Γ.

In constructing the GLS estimator $\hat{\Gamma}_t$ in (37), each variable has been deflated by a factor proportional to the residual standard deviation. The factor of proportionality was an arbitrary positive scalar. The structure of our problem is such that the standard error of measurement in $\hat{\beta}_{it}$, $\sigma_i = (\mathrm{var}(\tilde{v}_{it}))^{\frac{1}{2}}$, is proportional to the standard deviation of residual risk, $s_i = (\mathrm{var}(\tilde{e}_{i\tau}))^{\frac{1}{2}}$. That is, if the time series regression model satisfies the OLS assumptions,

$$\sigma_i = s_i \bigg/ \left(\sum_{\tau=t-60}^{t-1} (\tilde{R}_{m\tau} - \bar{R}_m)^2 \right)^{\frac{1}{2}}, \tag{43}$$

where $\bar{R}_m$ is the sample mean of the market return in the prior 60 month period.[5] Assume that σ_i is known and let

$$\phi = s_i/\sigma_i, \tag{44}$$

in the definition of P in (39). Thus each variable in the rows of $\tilde{R}_t$ and X_t is now deflated by the standard deviation of the measurement error in β_{it}. If $\hat{\beta}_{it}$ is used in place of β_{it} (unobserved), the measurement error in the deflated independent variable, $\hat{\beta}^* = \hat{\beta}_{it}/\sigma_i$ will now have unit variance.

Call the matrix of regressors used $\hat{X}_t^*$, which is simply X_t^* with $\hat{\beta}_{it}$ replacing β_{it}. Then

$$\hat{X}_t^* = X_t^* + \begin{bmatrix} 0 & \tilde{v}_{1t}/\sigma_1 & 0 \\ 0 & \tilde{v}_{2t}/\sigma_2 & 0 \\ \vdots & \vdots & \vdots \\ 0 & \tilde{v}_{N_t t}/\sigma_{N_t} & 0 \end{bmatrix}, \tag{45}$$

where $\mathrm{var}(\tilde{v}_{it}/\sigma_i) = 1$. Then the computed overall estimator

[4] The variance of the OLS estimator of the second independent variable (dividend yield) is equal to the variance of the error term divided by the portion of its variation that is unexplained by the first independent variable (beta). Therefore, unless the independent variables are uncorrelated sequential grouping procedures as used by Black and Scholes (1974) are inefficient relative to grouping procedures that maximize the between group variation in dividend yield that is unexplained by the between group variation in beta.

[5] In the actual estimation, risk premiums were used. That is, $\tilde{R}_{i\tau} - r_{f\tau}$ was regressed on $\tilde{R}_{m\tau} - r_{f\tau}$ to estimate β_{it} as explained in section 4 below. Thus in the computation in (43), $(\tilde{R}_{m\tau} - r_{f\tau} - \overline{R_m - r_f})^2$ is used in place of $(\tilde{R}_{m\tau} - \bar{R}_m)^2$.

$$\hat{\Gamma} = \sum_{t=1}^{T} (\hat{\Gamma}_t/T), \tag{46}$$

where

$$\hat{\Gamma}_t = (\hat{X}_t^{*\prime} \hat{X}_t^{*})^{-1} \hat{X}_t^{*\prime} \tilde{R}_t^{*} \tag{47}$$

is inconsistent. This is because

$$\operatorname*{plim}_{N_t} \hat{\Gamma}_t = \left(\Sigma_{X_t^* X_t^*} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{-1} \frac{X_t^{*\prime} \tilde{R}_t}{N_t}, \tag{48}$$

where

$$\Sigma_{X_t^* X_t^*} = \operatorname*{plim}_{N_t} \frac{X_t^{*\prime} X_t^*}{N_t}.$$

This says that each cross sectional estimator is biased even in large samples. Hence the overall estimator, being an arithmetic mean of the cross-sectional estimators, is inconsistent.

Consider the following estimator in each cross sectional month:

$$\hat{\Gamma}_t = \left(\frac{\hat{X}_t^{*\prime} \hat{X}_t^{*}}{N_t} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{-1} \frac{\hat{X}_t^{*\prime} R_t^{*}}{N_t}. \tag{49}$$

Then

$$\operatorname*{plim}_{N_t} \hat{\Gamma} = \frac{X_t^{*\prime} \tilde{R}_t}{X_t^{*\prime} X_t^{*}}, \tag{50}$$

and

$$E\left(\operatorname*{plim}_{N_t} \hat{\Gamma}_t \right) = \frac{X_t^{*\prime} E(\tilde{R}_t^{*})}{X_t^{*\prime} X_t^{*}} = \Gamma. \tag{51}$$

Thus each cross-sectional estimator is unbiased, in large samples, for Γ.

Note that a portfolio interpretation can also be given to (47). Since

$$\operatorname*{plim}_{N_t} \left(\frac{\hat{X}_t^{*\prime} \hat{X}_t^{*}}{N_t} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right)^{-1} \frac{\hat{X}_t^{*\prime} X_t^{*}}{N_t} = I, \tag{52}$$

it follows that the estimator for γ_0 in (47) is the realized excess return on a normal portfolio that has, in probability limit, a zero beta and a dividend yield equal to the riskless rate. Similarly the estimator for γ_1 (or γ_2) is the realized excess return on a hedge portfolio that has, in probability limit, a beta of one (or zero) and a dividend yield equal to zero (or unity).

The overall estimator,

$$\hat{\Gamma} = \sum_{t=1}^{T} (\hat{\Gamma}_t/T), \tag{53}$$

combines T independent estimates, and is consistent,

$$\operatorname*{plim}_{T} \left[\operatorname*{plim}_{N_t} \sum_{t=1}^{T} (\hat{\Gamma}_t/T) \right] = \Gamma. \tag{54}$$

It is shown in appendix B that, if $\tilde{\varepsilon}_{it}$ and $\tilde{\varepsilon}_{it}$ are jointly normal and independent, then $\hat{\Gamma}_t$ is the maximum likelihood estimator (MLE) for Γ using data in period t.

4. Data and results

Data on security rates of return (R_{it}) were obtained from the monthly return tapes supplied by the Center for Research in Security Prices (CRSP) at the University of Chicago. The same service provides the monthly return on a value weighted index of all the securities on the tape, and this index was used as the market return (R_{mt}) for the time series regressions. From January 1931 until December 1951, the monthly return on high grade commercial paper was used as the return on the riskless asset (r_{ft}); from January 1952 until December 1977 the return on a Treasury Bill (with one month to maturity) was used for r_{ft}. Estimates of each security's beta, $\hat{\beta}_{it}$, and its associated standard error were obtained from regressions of the security excess return on the market excess return for 60 months prior to t,

$$\tilde{R}_{i\tau} - r_{f\tau} = \alpha_{it} + \beta_{it}(\tilde{R}_{m\tau} - r_{f\tau}) + \tilde{\varepsilon}_{i\tau}, \quad \tau = t-60, t-59, \ldots, t-1. \tag{55}$$

This was repeated for all securities on the CRSP tapes from $t=1$ (January 1936) to $t=T=504$ (December 1977). January 1936 was chosen as the initial month for (subsequent) cross-sectional regressions because that was when dividends first became taxable.

To conduct the cross-sectional regression, the dividend yield variable (d_{it}) was computed from the CRSP monthly master file. This is

$$d_{it} = 0,$$

if in month t, security i did not go ex-dividend; or if it did, it was a non-recurring dividend not announced prior to month t;

$$d_{it} = D_{it}/P_{it-1},$$

if in month t, security i went ex-dividend, and the dollar taxable dividend D_{it} per share was announced prior to month t; and

$$d_{it} = \hat{D}_{it}/P_{it-1},$$

if in month t security i went ex-dividend and this was a recurring dividend not previously announced. Here $\hat{D}_{it}$ was the previous (going back at most 12 months), recurring, taxable dividend per share, adjusted for any changes in the number of shares outstanding in the interim; where P_{it-1} is the closing price in month $t-1$.

This construction assumes that the investor knows at the end of each month whether or not the subsequent month is an ex-dividend month for a recurring dividend. However, the surrogate for the dividend is based only on information that would have been available ex ante to the investor.

The cross-sectional regressions in each month provide a sequence of estimates $\{(\hat{\gamma}_{0t}, \hat{\gamma}_{1t}, \hat{\gamma}_{2t}),\ t = 1, 2, \ldots, 504\}$. Three such sequences are available: the first uses OLS, the second uses GLS and the third uses maximum likelihood estimation. The econometric procedures developed in section 3 apply equally well to the single variable regression, excess returns on beta alone. This corresponds to a test of the two factor Capital Asset Pricing Model, as in Black, Jensen and Scholes (1972) and Fama and MacBeth (1973),

$$\tilde{R}_{it} - r_{ft} = \gamma_0' + \gamma_1' \beta_{it} + \tilde{u}_{it}, \qquad i = 1, 2, \ldots, N_t, \quad t = 1, 2, \ldots, 504, \tag{56}$$

where $\tilde{u}_{it}$ is the deviation of $\tilde{R}_{it}$ from its expected value. These cross sectional regressions provide three sequences $\{(\hat{\gamma}_{0t}', \hat{\gamma}_{1t}'),\ t = 1, 2, \ldots, 504\}$, the first using OLS, the second using GLS and the third using maximum likelihood estimation.

The estimated coefficients were shown to be realized excess rates of return on portfolios (with certain characteristics)[6] in month t. It is assumed that the excess rates of return on these portfolios are stationary and serially uncorrelated. Under these conditions the most efficient estimators of the expected excess return on these portfolios would be the unweighted means of the monthly realized excess returns. The sample variance of the mean is computed as the time series sample variance of the respective portfolio returns divided by the number of months,

[6] See section 3, and also appendix A.

$$\hat{\gamma}_k = \sum_{t=1}^{504} \hat{\gamma}_{kt} \Big/ 504, \qquad k = 0, 1, 2, \tag{57}$$

$$\operatorname{var}(\hat{\gamma}_k) = \sum_{t=1}^{504} (\hat{\gamma}_{kt} - \hat{\gamma}_k)^2 / (504 \cdot 503). \tag{58}$$

A similar computation is made for $\hat{\gamma}_0'$ and $\hat{\gamma}_1'$.

The three sets of estimators of γ_0, γ_1 and γ_2 (and of γ_0' and γ_1') and their respective t-statistics for the overall period January 1936 to December 1977 are provided in Panel A (Panel B) of table 1.

Table 1

Pooled time series and cross section estimates of the after-tax and the before-tax CAPM: 1936–1977.[a]

	Panel A: After-tax model			Panel B: Before-tax model	
Procedure	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_0^*$	$\hat{\gamma}_1^*$
OLS	0.00616 (4.37)	0.00268 (1.51)	0.227 (6.33)	0.00681 (4.84)	0.00228 (1.26)
GLS	0.00446 (3.53)	0.00344 (1.87)	0.234 (8.24)	0.00516 (4.09)	0.00302 (1.63)
MLE	0.00363 (2.63)	0.00421 (1.86)	0.236 (5.62)	0.00443 (3.22)	0.00369 (1.62)

[a] Notes: The after-tax version corresponds to the regression

$$\tilde{R}_{it} - r_{ft} = \gamma_0 + \gamma_1 \beta_{it} + \gamma_2 (d_{it} - r_{ft}) + \tilde{\varepsilon}_{it}, \quad i = 1, 2, \ldots, N_t,\ t = 1, 2, \ldots, T.$$

The before-tax version corresponds to the regression

$$\tilde{R}_{it} - r_{ft} = \gamma_0' + \gamma_1' \beta_{it} + \tilde{u}_{it}, \quad i = 1, 2, \ldots, N_t,\ t = 1, 2, \ldots, T.$$

Each regression above is performed across securities in a given month. This gives estimates $\{\hat{\gamma}_{0t}, \hat{\gamma}_{1t}, \hat{\gamma}_{2t}: t = 1, 2, \ldots, T\}$ and $\{\hat{\gamma}_{0t}', \hat{\gamma}_{1t}': t = 1, 2, \ldots, T\}$. The reported coefficients are arithmetic averages of this time series; for example,

$$\hat{\gamma}_1 = \sum_{t=1}^{T} \frac{\hat{\gamma}_{1t}}{T},$$

where $T = 504$, t-statistics are in parentheses under each coefficient, and they refer to $t(\hat{\gamma}_j)$, where $j = 1, 2, 3$.

The OLS and GLS estimators are biased and inconsistent due to measurement error in beta. The maximum likelihood estimators are consistent: consistency is a large sample property and for this study the monthly cross sectional regressions have between 600 and 1200 firms, and there were 504 months.[7] In Panel A, table 1, the MLE estimator of γ_1 is about 60 percent greater than the corresponding GLS estimator. Consistent with prior studies, the MLE estimator of γ_1 is significantly positive, indicating that investors are risk averse. Also consistent with prior studies, the MLE estimator of γ_0 is significantly positive. In Panel B, tests of the two factor model are presented. Note that in both panels, the GLS procedure results in an increase in the efficiency of the estimator of γ_1, which is $\hat{\gamma}_1$ ($\tilde{\gamma}_1$) in Panel A (Panel B). Consistent with prior tests of the traditional version of the Capital Asset Pricing Model, the null hypothesis that $\tilde{\gamma}_0=0$ is rejected. Consistent with investor risk aversion $\tilde{\gamma}_1$ is significantly positive at the 0.1 level. Explanations for a positive intercept ($\gamma_0>0$) include, in addition to margin constraints on borrowing, misspecification of the market porfolio [see Mayers (1972), Sharpe (1977) and Roll (1977)], or beta serving as a surrogate for systematic skewness [see Kraus and Litzenberger (1976)].

The coefficient of the excess dividend yield variable, $\hat{\gamma}_2$, (Panel A) is highly significant under all the estimating procedures. The standard errors of the GLS and maximum likelihood estimators of γ_2 are about 25 percent smaller than that of the OLS estimator. The magnitude of the coefficient indicates that for every dollar of taxable return investors require between 23 and 24 cents of additional before tax return.

While the finding of a significant dividend coefficient contrasts with the Black-Scholes (1974) finding of an insignificant dividend effect, the magnitude of the coefficient in table 1 is consistent with their study. The dividend yield (independent) variable they used was $(d_i-d_m)/d_m$, where d_m was the average dividend yield on stocks. Since the coefficient they found was 0.0009, and the average annual yield in their period of study (1936–1966) was 0.048, their estimate of γ_2 can be approximated by 0.0009/(0.048/12), or 0.225.

It has been assumed that the variance of the estimator of Γ is constant over time. If, due to the quarterly patterns in the incidence of dividend payments, the variances of the estimators are not constant, the equally weighted estimators in (50) are inefficient relative to an estimator that accounts for any seasonal pattern in the variance. Since dividends are usually paid once every quarter, it is possible to compute three independent estimates of Γ by averaging the coefficients obtained in only the first, only the second and only the third month of each quarter. These three estimates of Γ may be weighted by the inverse of their variances to obtain a more efficient estimator. This is provided in table 2. As can be seen from this table,

[7]Consistency here is with respect to the overall estimator so one takes probability limits with respect to t and with respect to N_t. See section 3.

the overall estimator for γ_2 is very close to the MLE estimate in table 1. The estimate of the standard error of $\hat{\gamma}_2$ is approximately the same for the first two months, but about 30 percent less for the third month.

Table 2

Pooled time series and cross section estimates of the after-tax CAPM: 1936–1977 (based on quarterly dividend patterns).[a]

Month of quarter	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
First	0.00748 (0.00234)	0.00770 (0.00379)	0.28932 (0.05415)
Second	0.00212 (0.00232)	0.00071 (0.00335)	0.23531 (0.05034)
Third	0.00134 (0.00243)	0.00399 (0.00453)	0.18940 (0.03534)
Overall estimate	0.00373 (0.00137)	0.00353 (0.00219)	0.22535 (0.02552)

[a]Notes: The after-tax version corresponds to the regression

$$\tilde{R}_{it}-r_{ft}=\gamma_0+\gamma_1\beta_{it}+\gamma_2(d_{it}-r_{ft}),\quad i=1,2,\ldots,N_t.$$

This regression is performed across securities in a given month t. Maximum likelihood estimation is used. The reported coefficients are arithmetic averages of the coefficients obtained over time (see note to table 1). The first three rows use the estimates from only the first, only the second and only the third months of each quarter. There are 168 months' estimates in each row. Standard errors are in parentheses under each coefficient. The 'overall estimates' use the estimates in each row above, weighted inversely by their variances.

It may be inappropriate to treat γ_2 as an intertemporal constant: in the absence of income related constraints on borrowing, γ_2 is a weighted average of individuals' marginal tax rates, which may have changed over time. Assume that investors have utility functions that display decreasing absolute risk aversion and non-decreasing relative risk aversion. Assume in addition that the distribution of wealth is independent of individual utility functions. Under these conditions the weight of the marginal tax rates of individuals in the higher tax brackets would be greater than that of individuals in lower tax brackets. Holland (1962) has shown that from 1938 to 1960 there was no pronounced upward trend in the marginal tax rates of individuals with taxable income in excess of $25,000. To examine empirically whether there is evidence of an upward trend in γ_2 over time, the maximum likelihood results are presented for six subperiods in table 3. The estimators of γ_2 for the subperiods were consistently positive and, except for the 1/1955 to 12/1961 period, significantly different from zero. There does not appear to be a trend to the estimate.

Table 3

Pooled time series and cross section estimates of the after-tax CAPM (for 6 subperiods).[a]

Period	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$
1/36–12/40	−0.00287 (−0.52)	0.00728 (0.65)	0.335 (2.64)
1/41–12/47	0.00454 (1.44)	0.00703 (1.59)	0.408 (7.35)
1/48–12/54	0.00528 (2.77)	0.00617 (1.45)	0.158 (4.37)
1/55–12/61	0.01355 (5.62)	−0.00316 (−0.78)	0.018 (0.32)
1/62–12/68	−0.00164 (−0.47)	0.01063 (1.95)	0.171 (2.33)
1/69–12/77	0.00166 (0.47)	−0.00045 (−0.09)	0.329 (6.00)

[a]Notes: The after-tax version corresponds to the regression

$$\tilde{R}_{it}-r_{ft}=\gamma_0+\gamma_1\beta_{it}+\gamma_2(d_{it}-r_{ft})+\tilde{\varepsilon}_{it},\quad i=1,2,\ldots,N_t,\ t=1,2,\ldots,T.$$

Maximum likelihood estimation is used for the cross sectional regression. The reported coefficients are arithmetic averages of the coefficients estimated in the months in the period (see note to table 1). t-statistics are in parentheses under each coefficient.

It is possible that the positive coefficient on dividend yield is not a tax effect and that in non-ex-dividend months the effect completely reverses itself. If dividends are paid quarterly there would be twice as many non-ex-dividend months as ex-dividend months. Thus, a complete reversal would require a negative effect on returns in each non-ex-dividend month that is half the absolute size of the effect in an ex-dividend month. It is also possible that a stock's dividend yield is a proxy for the covariance of its return with classes of assets not included in the value weighted index of NYSE stocks used to calculate betas in the present study. If the coefficient on dividend yield is entirely due to the effects of omitted assets, the effect in non-ex-dividend months should be positive and the same size as the effect in ex-dividend months.

In order to test whether there is a reversal effect or a re-inforcing effect in non-ex-dividend months the following cross-sectional regression was estimated:

$$\tilde{R}_{it}-r_{ft}=\gamma_0+\gamma_1\hat{\beta}_{it}+\gamma_2\{\delta_{it}d_{it}^0-r_{ft}\}+\gamma_3\{(1-\delta_{it})d_{it}^0\}+\tilde{\varepsilon}_{it},\quad i=1,2,\ldots,N_t \tag{59}$$

where

$$d_{it}^0=D_{it}/P_{it-1},$$

if a dividend was announced prior to month t, to go ex-dividend in month t;

$$d_{it}^0=\hat{D}_{it}/P_{it-1}$$

otherwise; and

$$\delta_{it}=1,$$

if month t was an ex-dividend month for a recurring dividend;

$$\delta_{it}=0,$$

otherwise.

The variable $(1-\delta_{it})d_{it}^0$ is intended to pick up the effect of a dividend payment in subsequent, non-ex-dividend months. The variable $\delta_{it}d_{it}^0$ is identical to d_{it}, the variable used earlier. If dividends are paid quarterly, and γ_3 is negative and has an absolute value half the size of γ_2, then one can conclude that there is a complete reversal over the course of the quarter so that there is no net tax effect. On the other hand, if there is no reversal, $\hat{\gamma}_3$ should not be significantly negative.

The MLE estimates of the coefficients in (52) are presented in table 4. The estimated value of $\hat{\gamma}_3$ is positive and significantly different from zero: this rejects the hypothesis that there is complete reversal.

The significant positive γ_3 is evidence of a re-inforcing effect in non-ex-dividend months. If the coefficient on dividend yield is entirely attributable

Table 4

Pooled time series and cross section test of the reversal effect of dividend yield: 1936–1977.[a]

$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$
0.00184 (1.29)	0.00493 (2.17)	0.12784 (7.31)	0.10321 (2.87)

[a]Notes: The regression performed in each month is

$$\tilde{R}_{it}-r_{ft}=\gamma_0+\gamma_1\beta_{it}+\gamma_2\{\delta_{it}d_{it}^0-r_{ft}\}+\gamma_3(1-\delta_{it})d_{it}^0+\tilde{\varepsilon}_{it}\quad i=1,2,\ldots,N_t,\ t=1,2,\ldots,T.$$

Maximum likelihood estimation is used for the cross-sectional regression. The reported coefficients are arithmetic averages of the coefficients in each month (see note to table 1). t-statistics are in parentheses under each coefficient.

to the effect of omitted assets γ_3 should be the same order of magnitude as γ_2. If the effect in ex-dividend months exceeds the combined effect in the subsequent two non-ex-dividend months γ_2 should be more than twice as large as γ_3. $\hat{\gamma}_2 - 2\hat{\gamma}_3$ is 0.1214 and has a t-value of 2.79. Thus, the effect in an ex-dividend month is more than twice the size of the effect in a non-ex-dividend month. This evidence suggests that the coefficient on dividend yield in ex-dividend months is not solely attributable to the effects of missing assets and that the effect in an ex-dividend month exceeds the combined effect in the subsequent two non-ex-dividend months. If the effect in non-ex-dividend months is asserted to be entirely due to the effect of missing assets, the difference $\hat{\gamma}_2 - \hat{\gamma}_3 = 0.225$ is an estimate of the tax effect. However, further theoretical work on the combined effects of transaction costs and personal taxes in a multi-period valuation framework is required to be able to understand the cause of a significant yield effect in non-ex-dividend months. For the present it seems reasonable to conclude that 0.225 is a lower bound estimate of the tax effect.[8]

The empirical evidence presented by Elton and Gruber (1970) on the ex-dividend behavior of common stocks suggests that the coefficient on the excess dividend yield term may be a decreasing function of yield. The theoretical rationale for this effect is that investors in low (high) tax brackets invest in high (low) dividend yield stocks: a possible explanation is that institutional restrictions on short sales results in a segmentation of security holdings according to investors' tax brackets. To provide a simple test of this 'clientele' effect, the coefficient c in (22) is hypothesized to be a linear decreasing function of the ith security's dividend yield. That is c, which is now dependent on i, is written c_i and given by

$$c_i = k - hd_i, \tag{60}$$

where $k, h > 0$, and the hypothesized relationship is

$$E(\tilde{R}_i) - r_f = a + b\beta_i + (k - hd_i)(d_i - r_f). \tag{61}$$

The econometric model is

[8] It might be argued that the persistent dividend effect is due to the fact that the dividend variable used incorporates knowledge of the ex-dividend month, which the investor may not have. To test whether this introduces spurious correlations between yields and returns the variable (d_{it}^e/3) was used in the cross-sectional regression (23). The variable does not incorporate knowledge of the ex-dividend month except when it was announced. It is divided by 3 so as to distribute the yield over the three months of every quarter. The overall estimate (1936–1977) of γ_2 is 0.39, with a t-value of 3.57: one cannot attribute the earlier results due to knowledge of ex-dividend months. This is consistent with the Rosenberg and Marathe (1978) study. Note that this estimate is lower than the total effect in table 4, which is $\hat{\gamma}_2 + 2\hat{\gamma}_3 = 0.52$. The lower estimate is attributable to constraining the coefficient on yield to be the same in non-ex-dividend months and ex-dividend months.

$$\tilde{R}_{it} - r_{ft} = \gamma_0 + \gamma_1\beta_{it} + \gamma_2(d_{it} - r_{ft}) + \gamma_4 d_{it}(d_{it} - r_{ft}) + \tilde{\varepsilon}_{it}, \quad i = 1, 2, \ldots, N_t, \tag{62}$$

where the estimate of k is γ_2 and that for $-h$ is γ_4. The maximum likelihood approach is used in each cross sectional regression, and the pooled estimates presented in table 5.

Table 5

Pooled time series and cross section test of the clientele effect: 1936–1977.[a]

$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_4$
0.00365	0.00425	0.336	−6.92
(2.65)	(1.88)	(6.60)	(−1.70)

[a] Notes: This corresponds to the following cross-sectional regression in each month:

$$\tilde{R}_{it} - r_{ft} = \gamma_0 + \gamma_1\beta_{it} + \gamma_2(d_{it} - r_{ft}) + \gamma_4 d_{it}(d_{it} - r_{ft}) + \tilde{\varepsilon}_{it} \qquad i = 1, 2, \ldots, N_t, \quad t = 1, 2, \ldots, T.$$

Maximum likelihood estimation is used for the cross-sectional regression. The reported coefficients are arithmetic averages of the coefficients in each month (see note in table 1). t-statistics are in parentheses under each coefficient.

Consistent with the existence of a clientele effect, the maximum likelihood estimate of γ_2 is significantly positive and that of γ_4 is significantly negative, both at the 0.05 level. The magnitude of $\hat{\gamma}_4$ suggests that for every percentage point in yield the implied tax rate for ex-dividend months declines by 0.069. For example, if the annual yield was 4 percent, the implied tax rate would be approximately 0.336–6.92 (0.04/4) = 0.268, assuming quarterly payments. The empirical evidence supporting a clientele effect suggests the need for further research that rigorously derives an equilibrium model that incorporates institutional restrictions on short sales, along with personal taxes.

5. Conclusion

In this paper, an after-tax version of the Capital Asset Pricing Model is derived. The model extends the Brennan after-tax version of the CAPM to incorporate wealth and income related constraints on borrowing along with a progressive tax scheme. The wealth related constraint on borrowing causes the expected return on a zero-beta portfolio (having a dividend yield equal to the riskless rate) to exceed the riskless rate of interest. The income related constraint tends to offset the effect that personal taxes have on the equilibrium structure of share prices. The equilibrium relationship indicates that the before tax expected return on a security is linearly related to its systematic risk and to its dividend yield. Unrestricted supply adjustments in corporate dividends would result in the before tax version of the CAPM in a world where dividends and interest are taxed as ordinary income. If income related constraints are non-binding and/or corporate supply adjustments are restricted, the before tax return on a security would be an increasing linear function of its dividend yield.

Unlike prior tests of the CAPM that used grouping or instrumental variables to correct for measurement error in beta, this paper uses the sample estimate of the variance of observed betas to arrive at maximum likelihood estimates of the coefficients in the relations tested. Unlike prior studies of the effect of dividend yields on asset prices, which used average monthly yields as a surrogate for the expected yield in both ex-dividend and non-ex-dividend months, the expected dividend yield based on prior information is used for ex-dividend months and is set to zero for other months.

The results indicate that there is a strong positive relationship between before tax expected returns and dividend yields of common stocks. The coefficient of the dividend yield variable was positive, less than unity, and significantly different from zero. The data indicates that for every dollar increase in return in the form of dividends, investors require an additional 23 cents in before tax return. There was no noticeable trend in the coefficient over time. A test was constructed to determine whether the effect of dividend yield reverses itself in non-ex-dividend months, and this hypothesis was rejected. Indeed, the data indicates that the effect of a dividend payment on before tax expected returns is positive in both the ex-dividend month and in the subsequent non-ex-dividend months. However, the combined effect in the subsequent non-ex-dividend months is significantly less than the effect in the ex-dividend month.

Evidence is also presented for a clientele effect: that is, that stockholders in higher tax brackets choose stocks with low yields, and vice versa. Further work is needed to derive a model that implies the existence of such clienteles and to test its implications.

Appendix A

In this appendix it is shown that the estimator for Γ, given by

$$\hat{\Gamma}_t = (X_t' \Omega_t^{-1} X_t)^{-1} X_t' \Omega_t^{-1} \tilde{R}_t,$$

using data in period t, is the Generalized Least Squares (GLS) estimator for Γ under the assumption of the single index model. It was shown in section 3 of the paper that each estimated coefficient corresponds to the realized excess

return of a specific portfolio. Suppose portfolio weights $\{h_{it}, i=1,2,\ldots,N_t\}$ are chosen in each period, for investment in assets $i=1,2,\ldots,N_t$. Using eq. (23) from the text the excess return on such a portfolio is given by

$$\sum_i h_{it}(\tilde{R}_{it}-r_{ft}) = \gamma_0\left(\sum_i h_{it}\right) + \gamma_1\left(\sum_i h_{it}\hat{\beta}_{it}\right) + \gamma_2\left[\sum_i h_{it}(d_{it}-r_{ft})\right] + \sum_i h_{it}\tilde{\varepsilon}_{it}.$$

The expected excess return on this portfolio is

$$\gamma_0 \quad \text{if} \quad \sum_i h_{it}=1, \quad \sum_i h_{it}\beta_{it}=0, \quad \sum_i h_{it}(d_{it}-r_{ft})=0,$$

$$\gamma_1 \quad \text{if} \quad \sum_i h_{it}=0, \quad \sum_i h_{it}\beta_{it}=1, \quad \sum_i h_{it}(d_{it}-r_{ft})=0,$$

$$\gamma_2 \quad \text{if} \quad \sum_i h_{it}=0, \quad \sum_i h_{it}\beta_{it}=0, \quad \sum_i h_{it}(d_{it}-r_{ft})=1.$$

Under the assumption of the single index model, the variance of the return on such a portfolio is, from eq. (36) in the text,

$$\operatorname{var}\left(\sum_i h_{it}(\tilde{R}_{it}-r_{ft})\right) = \left(\sum_i h_{it}\beta_{it}\right)^2 \sigma_{mt} + \sum_i h_{it}^2 s_{it}.$$

Suppose one wishes to minimize the variance of the excess return on such a portfolio subject to the condition that the expected excess return on the portfolio is, in turn, γ_0, γ_1 or γ_2. This condition enforces $\sum_i h_{it}\beta_{it}$ to be either zero or unity. Hence minimizing

$$\left(\sum_i h_{it}\beta_{it}\right)^2 \sigma_{mt} + \sum_i h_{it}^2 s_{it},$$

subject to the unbiasedness condition, is equivalent to minimizing

$$\sum_i h_{it}^2 s_{it},$$

the 'residual risk' of the portfolio subject to the unbiasedness condition. Thus, one is using the residual risk of the portfolio as the minimand and enforcing the unbiasedness condition. By construction, Ω_t is the diagonal matrix of the residual variances s_{it}, and by construction, $\hat{\Gamma}_t$ is linear and unbiased for Γ. The variance of the estimator has been minimized under the single index model. But by the Gauss–Markov theorem, the GLS estimator [using the full matrix V_t in (36) as the variance–covariance matrix] is the unique minimum variance estimator among linear and unbiased estimators. Hence $\hat{\Gamma}_t$ is the GLS estimator for Γ, under the assumption of the single index model.

Appendix B

In this section, it is shown that under certain conditions, $\hat{\hat{\Gamma}}_t$ in (49) is the maximum likelihood estimator for Γ in period t.

First, note that there are no errors in the measurement of β, then if security returns are multivariate normal, then the GLS estimator in (37) is also the maximum likelihood estimator [see Johnston (1972)].

Suppose now there are errors in the measurement of β. Then one can use the transformation P defined in (39), with $\phi = s_d/\sigma_\beta$, to write the model as

$$\tilde{R}^*_{it} = \gamma_0 p^*_{it} + \gamma_1 \beta^*_{it} + \gamma_2 d^*_{it} + \tilde{\varepsilon}^*_{it}, \tag{B.1}$$

and the observed beta as

$$\tilde{\beta}^*_{it} = \beta^*_{it} + \tilde{v}^*_{it}, \tag{B.2}$$

where

$$\tilde{R}^*_{it} = (\tilde{R}_{it} - r_{ft})/\sigma_i, \quad p^*_{it} = 1/\sigma_i, \quad \beta^*_{it} = \beta_{it}/\sigma_i,$$
$$\tilde{\beta}^*_{it} = \tilde{\beta}_{it}/\sigma_i, \quad d^*_{it} = (d_{it} - r_{ft})/\sigma_i, \quad \tilde{\varepsilon}^*_{it} = \tilde{\varepsilon}_{it}/\sigma_i$$

and

$$\tilde{v}^*_{it} = \tilde{v}_{it}/\sigma_i.$$

Define the variable

$$m_{xy} = \sum_{i=1}^{N_t} x_{it} y_{it} / N_t, \tag{B.3}$$

as the raw co-moment for a given sequence $\{(x_{it}, y_{it}),\ i = 1, 2, \ldots, N_t\}$. Then from (B.1) and (B.2),

$$m_{R^* p^*} = \gamma_0 m_{p^* p^*} + \gamma_1 m_{\beta^* p^*} + \gamma_2 m_{d^* p^*} + m_{\varepsilon^* p^*}, \tag{B.4}$$

$$m_{R^* \tilde{\beta}^*} = \gamma_0 [m_{p^* \beta^*} + m_{p^* v^*}] + \gamma_1 [m_{\beta^* \beta^*} + m_{v^* \beta^*}] + \gamma_2 [m_{d^* \beta^*} + m_{d^* v^*}] + m_{\varepsilon^* \beta^*} + m_{\varepsilon^* v^*}, \tag{B.5}$$

$$m_{R^* d^*} = \gamma_0 m_{p^* d^*} + \gamma_1 m_{\beta^* d^*} + \gamma_2 m_{d^* d^*} + m_{\varepsilon^* d^*}, \tag{B.6}$$

$$m_{\tilde{\beta}^* p^*} = m_{\beta^* p^*} + m_{v^* p^*}, \tag{B.7}$$

$$m_{\tilde{\beta}^* \tilde{\beta}^*} = m_{\beta^* \beta^*} + 2 m_{\beta^* v^*} + m_{v^* v^*}, \tag{B.8}$$

$$m_{d^* \tilde{\beta}^*} = m_{d^* \beta^*} + m_{d^* v^*}. \tag{B.9}$$

In these six equations, take expectations and use the fact that

$$E(\tilde{v}^*_{it}) = E(\tilde{\varepsilon}^*_{it}) = 0,$$
$$E(\tilde{v}^*_{it} \tilde{\varepsilon}^*_{it}) = 0, \tag{B.10}$$
$$E(\tilde{v}^*_{it} \tilde{v}^*_{it}) = E[\tilde{v}^2_{it}/\sigma^2_i] = 1.$$

The left-hand side of each of (B.4) through (B.9), after taking expectations, corresponds to the population co-moments of the subscripted variables.

If v_{it} and ε_{it} are independently normally distributed, then the corresponding sample moment is a maximum likelihood estimator of the population parameter. Replace these expected values by their maximum likelihood estimates. There are now six equations for the six unknown parameters γ_0, γ_1, γ_2, $m_{\beta^* \beta^*}$, $m_{\beta^* d^*}$, and $m_{\beta^* p^*}$. They can be solved for the coefficients of interest from the following 'normal' equations, which are in terms of observed sample estimates,

$$m_{R^* p^*} = \gamma_0 m_{p^* p^*} + \gamma_1 m_{\tilde{\beta}^* p^*} + \gamma_2 m_{d^* p^*}, \tag{B.11}$$

$$m_{R^* \tilde{\beta}^*} = \gamma_0 m_{p^* \tilde{\beta}^*} + \gamma_1 (m_{\tilde{\beta}^* \tilde{\beta}^*} - 1) + \gamma_2 m_{d^* \tilde{\beta}^*}, \tag{B.12}$$

$$m_{R^* d^*} = \gamma_0 m_{p^* d^*} + \gamma_1 m_{\tilde{\beta}^* d^*} + \gamma_2 m_{d^* d^*}, \tag{B.13}$$

and are themselves maximum likelihood [see Mood et al. (1974, p. 285)].

The solution to this set gives estimates $\hat{\hat{\gamma}}_{kt}$, $k = 0, 1, 2$, which are embodied in (49). They are functions of maximum likelihood estimates. Note that in addition to (B.4) through (B.9), one could write an equation for $m_{R^* R^*}$,

$$\begin{aligned} m_{R^* R^*} = {} & \gamma_0^2 m_{p^* p^*} + \gamma_1^2 m_{\beta^* \beta^*} + \gamma_2^2 m_{d^* d^*} + 2\gamma_0\gamma_1 m_{p^* \beta^*} \\ & + 2\gamma_0\gamma_2 m_{p^* d^*} + 2\gamma_1\gamma_2 m_{\beta^* d^*} + 2\gamma_0 m_{p^* \varepsilon^*} + 2\gamma_1 m_{\beta^* \varepsilon^*} \\ & + 2\gamma_2 m_{d^* \varepsilon^*} + m_{\varepsilon^* \varepsilon^*}. \end{aligned} \tag{B.14}$$

If we take expectations, using (B.10) and the fact that

$$E(m_{\epsilon^*\epsilon^*})=E\left(\sum_{i=1}^{N_t}\frac{\tilde{\epsilon}_{it}^2}{s_i^2 N_t}\right)$$

$$=\frac{1}{N_t}\sum_{i=1}^{N_t}\frac{E(\tilde{\epsilon}_{it}^2)}{s_i^2}=\frac{1}{N_t}\cdot N_t\phi^2=\phi^2,$$

we have

$$E(m_{g^*g^*})=\gamma_0^2 m_{\rho^*\rho^*}+\gamma_1^2 m_{\beta^*\beta^*}+\gamma_2^2 m_{\delta^*\delta^*}+2\gamma_0\gamma_1 m_{\rho^*\beta^*}$$
$$+2\gamma_0\gamma_2 m_{\rho^*\delta^*}+2\gamma_1\gamma_2 m_{\beta^*\delta^*}+\phi^2, \qquad \text{(B.15)}$$

where ϕ^2 is assumed known.

By writing down the likelihood function and maximizing it for an analogous case, Johnston (1963) demonstrates a maximum likelihood estimator over the parameter space ($\gamma_0, \gamma_1, \gamma_2, \beta_{it}$ for $i=1,2,\ldots,N_t, \phi$). This has the undesirable characteristic that the parameter space grows with the sample size.[9] It turns out in our problem that ϕ is assumed known. If this ϕ satisfies (B.15), when in (B.15) we use the sample co-moment estimates for the population parameters, then Johnston's M.L. procedure coincides with the solution to (B.11) through (B.13). Whereas our estimators are linear in the returns and can be interpreted as portfolios, the expanded parameter space estimator in Johnston is non-linear and has no such analog to theory. Thus conditional on ϕ^2 coinciding with the residual variation in the sample, using our estimates, the estimator in (49) is a maximum likelihood estimator over the parameter space ($\gamma_0, \gamma_1, \gamma_2$).

[9] See Kendall and Stuart (1973, especially pp. 62 and 402).

References

Amemiya, T., 1972, Theory of econometrics: Lecture notes, Unpublished manuscript (Department of Economics, Stanford University, Stanford, CA).

Bailey, M.J., 1969, Capital gains and income taxation, in: A.C. Harberger and M.J. Bailey, The taxation of income from capital (Brookings Institution, Washington, DC) 11–49.

Black, F., 1972, Capital market equilibrium with restricted borrowing, Journal of Business 45, 444–454.

Black, F., M. Jensen and M. Scholes, 1972, The capital asset pricing model: Some empirical tests, in: M. Jensen, ed., Studies in the theory of capital markets (Praeger, New York) 79–121.

Black, F. and M. Scholes, 1974, The effects of dividend yield and dividend policy on common stock prices and returns, Journal of Financial Economics 1, 1–22.

Blume, M. and I. Friend, 1973, A new look at the capital asset pricing model, Journal of Finance 28, 19–33.

Brennan, M.J., 1973, Taxes, market valuation and corporate financial policy, National Tax Journal 23, 417–427.

Brennan, M.J., 1970, Investor taxes, market equilibrium and corporation finance, Unpublished Ph.D. Dissertation (Massachusetts Institute of Technology, Cambridge, MA).

Cottle, S., D.L. Dodd and B. Graham, 1962, Security analysis: Principles and techniques (McGraw-Hill, New York).

Elton, E. and Gruber, 1970, Marginal stockholder tax rates and the clientele effect, Review of Economics and Statistics 52, 68–74.

Fama, E.F. and J.D. MacBeth, 1973, Risk, return and equilibrium: Empirical tests, Journal of Political Economy 71, 607–636.

Fama, E.F. and M.H. Miller, 1972, The theory of finance (Holt, Rinehart and Winston, New York).

Fisher, F.M., 1977, The effect of simple specification error on the coefficients of 'unaffected' variables, Working Paper no. 194 (Department of Economics, Massachusetts Institute of Technology, Cambridge, MA).

Friend, I. and M. Blume, 1970, Measurement of portfolio performance under uncertainty, American Economic Review, 561–575.

Gonzalez-Gaverra, N.G., 1973, Inflation and capital asset market prices: Theory and tests, Unpublished Ph.D. Dissertation (Graduate School of Business, Stanford University, Stanford, CA).

Gordon, M.J., 1963, Optimal investment and financing policy, Journal of Finance 18, 264–272.

Holland, D.M., 1962, Dividends under the income tax (NBER, Princeton, NJ).

Johnston, J., 1963, Econometric methods (McGraw-Hill, New York).

Johnston, J., 1972, Econometric methods (McGraw-Hill, New York).

Kendall, M.G. and A. Stuart, 1973, The advanced theory of statistics (Hafner, New York).

Kraus, A. and R.H. Litzenberger, 1976, Skewness preference and the valuation of risk assets, Journal of Finance 31, 1085–1100.

Lintner, J., 1965, The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, Review of Economics and Statistics 47, 13–37.

Litzenberger, R.H. and J.C. Van Horne, 1978, Elimination of the double taxation of dividends and corporate financial policy, Journal of Finance 34, 737–749.

Long, J., 1975, Efficient portfolio choice with differential taxation of dividends and capital gains, Journal of Financial Economics 5, 25–53.

Mayers, D., 1972, Non-market assets and capital market equilibrium under uncertainty, in: M.C. Jensen, ed., Studies in the theory of capital markets (Praeger, New York).

Miller, M. and M. Modigliani, 1961, Dividend policy growth, and the valuation of shares, Journal of Business 4, 411–433.

Miller, M. and M. Scholes, 1972, Rates of return in relation to risk: A re-examination of some recent findings, in: M. Jensen, ed., Studies in the theory of capital markets (Praeger, New York).

Miller, M. and M. Scholes, 1977, Dividends and taxes, Working Paper no. 5 (University of Chicago, Chicago, IL).

Mood, A., F.S. Graybill and D.C. Boes, 1974, Introduction to the theory of statistics (McGraw-Hill, New York).

Roll, R., 1977, A critique of the asset pricing theory's tests, Journal of Financial Economics 4, 129–176.

Rosenberg, B. and V. Marathe, 1978, Test of capital asset pricing hypotheses, Journal of Financial Research, forthcoming.

Rubinstein, M., 1973, A comparative statics analysis of risk premiums, Journal of Business 46.

Sharpe, W.F., 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, Journal of Finance 19, 425–442.

Sharpe, W.F., 1977, The capital asset pricing model: A multi-beta interpretation, in: H. Levy and M. Sarnat, eds., Financial decision making under uncertainty (Academic Press, New York).

Stehle, R.D., 1976, The valuation of risk assets in an international capital market: Theory and tests, Unpublished Ph.D. Dissertation (Graduate School of Business, Stanford, CA).

Vasicek, O., 1971, Capital market equilibrium with no riskless borrowing, Wells Fargo Bank Memorandum (San Francisco, CA).

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THE RELATIONSHIP BETWEEN RETURN AND MARKET VALUE OF COMMON STOCKS*

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This study examines the empirical relationship between the return and the total market value of NYSE common stocks. It is found that smaller firms have had higher risk adjusted returns, on average, than larger firms. This 'size effect' has been in existence for at least forty years and is evidence that the capital asset pricing model is misspecified. The size effect is not linear in the market value; the main effect occurs for very small firms while there is little difference in return between average sized and large firms. It is not known whether size *per se* is responsible for the effect or whether size is just a proxy for one or more true unknown factors correlated with size.

1. Introduction

The *single-period* capital asset pricing model (henceforth CAPM) postulates a simple linear relationship between the expected return and the market risk of a security. While the results of direct tests have been inconclusive, recent evidence suggests the existence of additional factors which are relevant for asset pricing. Litzenberger and Ramaswamy (1979) show a significant positive relationship between dividend yield and return of common stocks for the 1936-1977 period. Basu (1977) finds that price-earnings ratios and risk adjusted returns are related. He chooses to interpret his findings as evidence of market inefficiency but as Ball (1978) points out, market efficiency tests are often joint tests of the efficient market hypothesis and a particular equilibrium relationship. Thus, some of the anomalies that have been attributed to a lack of market efficiency might well be the result of a misspecification of the pricing model.

This study contributes another piece to the emerging puzzle. It examines the relationship between the total market value of the common stock of a firm and its return. The results show that, in the 1936-1975 period, the common stock of small firms had, on average, higher risk-adjusted returns

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than the common stock of large firms. This result will henceforth be referred to as the 'size effect'. Since the results of the study are not based on a particular theoretical equilibrium model, it is not possible to determine conclusively whether market value *per se* matters or whether it is only a proxy for unknown true additional factors correlated with market value. The last section of this paper will address this question in greater detail.

The various methods currently available for the type of empirical research presented in this study are discussed in section 2. Since there is a considerable amount of confusion about their relative merit, more than one technique is used. Section 3 discusses the data. The empirical results are presented in section 4. A discussion of the relationship between the size effect and other factors, as well as some speculative comments on possible explanations of the results, constitute section 5.

2. Methodologies

The empirical tests are based on a generalized asset pricing model which allows the expected return of a common stock to be a function of risk β and an additional factor ϕ, the market value of the equity.[1] A simple linear relationship of the form

$$E(R_i)=\gamma_0+\gamma_1\beta_i+\gamma_2[(\phi_i-\phi_m)/\phi_m], \tag{1}$$

is assumed, where

$E(R_i)$ = expected return on security i,
γ_0 = expected return on a zero-beta portfolio,
γ_1 = expected market risk premium,
ϕ_i = market value of security i,
ϕ_m = average market value, and
γ_2 = constant measuring the contribution of ϕ_i to the expected return of a security.

If there is no relationship between ϕ_i and the expected return, i.e., $\gamma_2=0$, (1) reduces to the Black (1972) version of the CAPM.

Since expectations are not observable, the parameters in (1) must be estimated from historical data. Several methods are available for this purpose. They all involve the use of pooled cross-sectional and time series regressions to estimate γ_0, γ_1, and γ_2. They differ primarily in (a) the assumption concerning the residual variance of the stock returns (homoscedastic or heteroscedastic in the cross-sectional), and (b) the treatment of the

[1] In the empirical tests, ϕ_i and ϕ_m are defined as the market proportion of security i and average market proportion, respectively. The two specifications are, of course, equivalent.

errors-in-variables problem introduced by the use of estimated betas in (1). All methods use a constrained optimization procedure, described in Fama (1976, ch. 9), to generate minimum variance (m.v.) portfolios with mean returns γ_i, $i=0,\ldots,2$. This imposes certain constraints on the portfolio weights, since from (1)

$$E(R_p) \equiv \gamma_i = \gamma_0 \sum_j w_j + \gamma_1 \sum_j w_j \beta_j + \gamma_2 \left[\left(\sum_j w_j \phi_j - \phi_m \sum_j w_j \right) \Big/ \phi_m \right], \qquad i=0,\ldots,2. \tag{2}$$

where the w_j are the portfolio proportions of each asset j, $j=1,\ldots,N$. An examination of (2) shows that $\hat{\gamma}_0$ is the mean return of a standard m.v. portfolio ($\sum_j w_j = 1$) with zero beta and $\phi_p \equiv \sum_j w_j \phi_j = \phi_m$ [to make the second and third terms of the right-hand side of (2) vanish]. Similarly, $\hat{\gamma}_1$ is the mean return on a zero-investment m.v. portfolio with beta of one and $\phi_p = 0$, and $\hat{\gamma}_2$ is the mean return on a m.v. zero-investment, zero-beta portfolio with $\phi_p = \phi_m$. As shown by Fama (1976, ch. 9), this constrained optimization can be performed by running a cross-sectional regression of the form

$$R_{it} = \gamma_{0t} + \gamma_{1t}\beta_{it} + \gamma_{2t}[(\phi_{it} - \phi_{mt})/\phi_{mt}] + \varepsilon_{it}, \qquad i=1,\ldots,N, \tag{3}$$

on a period-by-period basis, using estimated betas $\hat{\beta}_{it}$ and allowing for either homoscedastic or heteroscedastic error terms. Invoking the usual stationarity arguments the final estimates of the gammas are calculated as the averages of the T estimates.

One basic approach involves grouping individual securities into portfolios on the basis of market value and security beta, reestimating the relevant parameters (beta, residual variance) of the portfolios in a subsequent period, and finally performing either an ordinary least squares (OLS) regression [Fama and MacBeth (1973)] which assumes homoscedastic errors, or a generalized least squares (GLS) regression [Black and Scholes (1974)] which allows for heteroscedastic errors, on the portfolios in each time period.[2] Grouping reduces the errors-in-variables problem, but is not very efficient because it does not make use of all information. The errors-in-variables problem should not be a factor as long as the portfolios contain a reasonable number of securities.[3]

Litzenberger and Ramaswamy (1979) have suggested an alternative method which avoids grouping. They allow for heteroscedastic errors in the cross-section and use the estimates of the standard errors of the security

[2] Black and Scholes (1974) do not take account of heteroscedasticity, even though their method was designed to do so.

[3] Black, Jensen and Scholes (1972, p. 116)

betas as estimates of the measurement errors. As Theil (1971, p. 610) has pointed out, this method leads to unbiased maximum likelihood estimators for the gammas as long as the error in the standard error of beta is small and the standard assumptions of the simple errors-in-variables model are met. Thus, it is very important that the diagonal model is the correct specification of the return-generating process, since the residual variance assumes a critical position in this procedure. The Litzenberger–Ramaswamy method is superior from a theoretical viewpoint; however, preliminary work has shown that it leads to serious problems when applied to the model of this study and is not pursued any further.[4]

Instead of estimating equation (3) with data for all securities, it is also possible to construct arbitrage portfolios containing stocks of very large and very small firms by combining long positions in small firms with short positions in large firms. A simple time series regression is run to determine the difference in risk-adjusted returns between small and large firms. This approach, long familiar in the efficient markets and option pricing literature, has the advantage that no assumptions about the exact functional relationships between market value and expected return need to be made, and it will therefore be used in this study.

3. Data

The sample includes all common stocks quoted on the NYSE for at least five years between 1926 and 1975. Monthly price and return data and the number of shares outstanding at the end of each month are available in the monthly returns file of the Center for Research in Security Prices (CRSP) of the University of Chicago. Three different market indices are used; this is in response to Roll's (1977) critique of empirical tests of the CAPM. Two of the three are pure common stock indices — the CRSP equally- and value-weighted indices. The third is more comprehensive: a value-weighted combination of the CRSP value-weighted index and return data on corporate and government bonds from Ibbotson and Sinquefield (1977) (henceforth 'market index').[5] The weights of the components of this index are derived from information on the total market value of corporate and government bonds, in various issues of the *Survey of Current Business* (updated annually) and from the market value of common stocks in the CRSP monthly index file. The stock indices, made up of riskier assets, have both higher returns

[4] If the diagonal model (or market model) is an incomplete specification of the return generating process, the estimate of the standard error of beta is likely to have an upward bias, since the residual variance estimate is too large. The error in the residual variance estimate appears to be related to the second factor. Therefore, the resulting gamma estimates are biased.

[5] No pretense is made that this index is complete; thus, the use of quotation marks. It ignores real estate, foreign assets, etc.; it should be considered a first step toward a comprehensive index. See Ibbotson and Fall (1979).

and higher risk than the bond indices and the 'market index'.[6] A time series of commercial paper returns is used as the risk-free rate.[7] While not actually constant through time, its variation is very small when compared to that of the other series, and it is not significantly correlated with any of the three indices used as market proxies.

4. Empirical results

4.1. *Results for methods based on grouped data*

The portfolio selection procedure used in this study is identical to the one described at length in Black and Scholes (1974). The securities are assigned to one of twenty-five portfolios containing similar numbers of securities, first to one of five on the basis of the market value of the stock, then the securities in each of those five are in turn assigned to one of five portfolios on the basis of their beta. Five years of data are used for the estimation of the security beta; the next five years' data are used for the reestimation of the portfolio betas. Stock price and number of shares outstanding at the end of the five year periods are used for the calculation of the market proportions. The portfolios are updated every year. The cross-sectional regression (3) is then performed in each month and the means of the resulting time series of the gammas could be (and have been in the past) interpreted as the final estimators. However, having used estimated parameters, it is not certain that the series have the theoretical properties, in particular, the hypothesized beta. Black and Scholes (1974, p. 17) suggest that the time series of the gammas be regressed once more on the excess return of the market index. This correction involves running the time series regression (for $\hat{\gamma}_2$)

$$\hat{\gamma}_{2t} - R_{Ft} = \hat{\alpha}_2 + \hat{\beta}_2 (R_{mt} - R_{Ft}) + \hat{\varepsilon}_{2t}. \tag{4}$$

It has been shows earlier that the theoretical β_2 is zero. (4) removes the effects of a non-zero β_2 on the return estimate $\hat{\gamma}_2$ and $\hat{\alpha}_2$ is used as the final estimator for $\hat{\gamma}_2 - R_F$. Similar corrections are performed for γ_0 and γ_1. The

[6] Mean monthly returns and standard deviations for the 1926–1975 period are:

	Mean return	Standard deviation
'Market index'	0.0046	0.0178
CRSP value-weighted index	0.0085	0.0588
CRSP equally-weighted index	0.0120	0.0830
Government bond index	0.0027	0.0157
Corporate bond index	0.0032	0.0142

[7] I am grateful to Myron Scholes for making this series available. The mean monthly return for the 1926–1975 period is 0.0026 and the standard deviation is 0.0021.

derivations of the β_i, $i=0,\ldots,2$, in (4) from their theoretical values also allow us to check whether the grouping procedure is an effective means to eliminate the errors-in-beta problem.

The results are essentially identical for both OLS and GLS and for all three indices. Thus, only one set of results, those for the 'market index' with GLS, is presented in table 1. For each of the gammas, three numbers are reported: the mean of that time series of returns which is relevant for the test of the hypothesis of interest (i.e., whether or not $\hat{\gamma}_0$ and $\hat{\gamma}_1$ are different from the risk-free rate and the risk premium, respectively), the associated t-statistic, and finally, the estimated beta of the time series of the gamma from (4). Note that the means are corrected for the deviation from the theoretical beta as discussed above.

The table shows a significantly negative estimate for γ_2 for the overall time period. Thus, shares of firms with large market values have had smaller returns, on average, than similar small firms. The CAPM appears to be misspecified. The table also shows that γ_0 is different from the risk-free rate. As both Fama (1976, ch. 9) and Roll (1977) have pointed out, if a test does not use the true market portfolio, the Sharpe–Lintner model might be wrongly rejected. The estimates for γ_0 are of the same magnitude as those reported by Fama and MacBeth (1973) and others. The choice of a market index and the econometric method does not affect the results. Thus, at least within the context of this study, the choice of a proxy for the market portfolio does not seem to affect the results and allowing for heteroscedastic disturbances does not lead to significantly more efficient estimators.

Before looking at the results in more detail, some comments on econometric problems are in order. The results in table 1 are based on the 'market index' which is likely to be superior to pure stock indices from a theoretical viewpoint since it includes more assets [Roll (1977)]. This superiority has its price. The actual betas of the time series of the gammas are reported in table 1 in the columns labeled $\hat{\beta}_i$. Recall that the theoretical values of β_0 and β_1 are zero and one, respectively. The standard zero-beta portfolio with return $\hat{\gamma}_0$ contains high beta stocks in short positions and low beta stocks in long positions, while the opposite is the case for the zero-investment portfolio with return $\hat{\gamma}_1$. The actual betas are all significantly different from the theoretical values. This suggests a regression effect, i.e., the past betas of high beta securities are overestimated and the betas of low beta securities are underestimated.[8] Past beta is not completely uncorrelated with the error of the current beta and the instrumental variable approach to the error-in-variables problem is not entirely successful.[9]

[8] There is no such effect for β_2 because that portfolio has both zero beta and zero investment; i.e., net holdings of both high and low beta securities are, on average, zero.

[9] This result is first documented in Brenner (1976) who examines the original Fama–McBeth (1973) time series of $\hat{\gamma}_0$.

Table 1

Portfolio estimators for γ_0, γ_1 and γ_2 based on the 'market index' with generalized least squares estimation.[a]

$$R_{it} = \hat{\gamma}_{0t} + \hat{\gamma}_{1t}\hat{\beta}_{it} + \hat{\gamma}_{2t}[(\phi_{it} - \phi_{mt})/\phi_{mt}]$$

Period	$\hat{\gamma}_0 - R_F$	$t(\hat{\gamma}_0 - R_F)$	$\hat{\beta}_0$	$\hat{\gamma}_1 - (R_M - R_F)$	$t(\hat{\gamma}_1 - (R_M - R_F))$	$\hat{\beta}_1$	$\hat{\gamma}_2$	$t(\hat{\gamma}_2)$	$\hat{\beta}_2$
1936–1975	0.00450	2.76	0.45	−0.00092	−1.00	0.75	−0.00052	−2.92	0.01
1936–1955	0.00377	1.66	0.43	−0.00060	−0.80	0.80	−0.00043	−2.12	0.01
1956–1975	0.00531	2.22	0.46	−0.00138	−0.82	0.73	−0.00062	−2.09	0.01
1936–1945	0.00123	0.30	0.63	−0.00098	−0.77	0.82	−0.00075	−2.32	−0.01
1946–1955	0.00650	2.89	0.03	−0.00021	−0.26	0.75	−0.00015	−0.65	0.06
1956–1965	0.00494	2.02	0.34	−0.00098	−0.56	0.96	−0.00039	−1.27	−0.01
1966–1975	0.00596	1.43	0.49	−0.00232	−0.80	0.69	−0.00080	−1.55	0.01

[a] $\hat{\gamma}_0 - R_F$ = mean difference between return on zero beta portfolio and risk-free rate. $\hat{\gamma}_1 - (R_M - R_F)$ = mean difference between actual risk premium ($\hat{\gamma}_1$) and risk premium stipulated by Sharpe–Lintner model ($R_M - R_F$). $\hat{\gamma}_2$ = size premium. $\hat{\beta}_i$ = actual estimated market risk of $\hat{\gamma}_i$ (theoretical values: $\beta_0 = 0$, $\beta_1 = 1$, $\beta_2 = 0$); all β_0, β_1 are significantly different from the theoretical values. $t(\cdot)$ = t-statistic.

The deviations from the theoretical betas are largest for the 'market index', smaller for the CRSP value-weighted index, and smallest for the CRSP equally-weighted index. This is due to two factors: first, even if the true covariance structure is stationary, betas with respect to a value-weighted index change whenever the weights change, since the weighted average of the betas is constrained to be equal to one. Second, the betas and their standard errors with respect to the 'market index' are much larger than for the stock indices (a typical stock beta is between two and three), which leads to larger deviations – a kind of 'leverage' effect. Thus, the results in table 1 show that the final correction for the deviation of $\hat{\beta}_0$ and $\hat{\beta}_1$ from their theoretical values is of crucial importance for maket proxies with changing weights.

Estimated portfolio betas and portfolio market proportions are (negatively) correlated. It is therefore possible that the errors in beta induce an error in the coefficient of the market proportion. According to Levi (1973), the probability limit of $\hat{\gamma}_1$ in the standard errors-in-the-variables model is

$$\operatorname{plim} \hat{\gamma}_1 = \gamma_1/(1 + (\sigma_u^2 \cdot \sigma_2^2)/D) < \gamma_1,$$

with

$$D = (\sigma_1^2 + \sigma_u^2) \cdot \sigma_2^2 - \sigma_{12}^2 > 0,$$

where σ_1^2, σ_2^2 are the variances of the true factors β and ϕ respectively, σ_u^2 is the variance of the error in beta and σ_{12} is the covariance of β and ϕ. Thus, the bias in $\hat{\gamma}_1$ is unambiguously towards zero for positive γ_1. The probability limit of $\hat{\gamma}_2 - \gamma_2$ is [Levi (1973)]

$$\operatorname{plim} (\hat{\gamma}_2 - \gamma_2) = (\sigma_u^2 \cdot \sigma_{12} \cdot \gamma_1)/D.$$

We find that the bias in $\hat{\gamma}_2$ depends on the covariance between β and ϕ and the sign of γ_1. If σ_{12} has the same sign as the covariance between β and ϕ, i.e., $\sigma_{12} < 0$, and if $\gamma_1 > 0$, then $\operatorname{plim}(\hat{\gamma}_2 - \gamma_2) < 0$, i.e., $\operatorname{plim} \hat{\gamma}_2 < \gamma_2$. If the grouping procedure is not successful in removing the error in beta, then it is likely that the reported $\hat{\gamma}_2$ overstates the true magnitude of the size effect. If this was a serious problem in this study, the results for the different market indices should reflect the problem. In particular, using the equally-weighted stock index should then lead to the smallest size effect since, as was pointed out earlier, the error in beta problem is apparently less serious for that kind of index. In fact, we find that there is little difference between the estimates.[10]

[10] For the overall time period, $\hat{\gamma}_2$ with the equally-weighted CRSP index is −0.00044, with the value weighted CRSP index −0.00044 as well as opposed to the −0.00052 for the 'market index' reported in table 1. The estimated betas of $\hat{\gamma}_0$ and $\hat{\gamma}_1$ which reflect the degree of the error in beta problems are 0.07 and 0.91, respectively, for the equally-weighted CRSP index and 0.13 and 0.87 for the value-weighted CRSP index.

Thus, it does not appear that the size effect is just a proxy for the unobservable true beta even though the market proportion and the beta of securities are negatively correlated.

The correlation coefficient between the mean market values of the twenty-five portfolios and their betas is significantly negative, which might have introduced a multicollinearity problem. One of its possible consequences is coefficients that are very sensitive to addition or deletion of data. This effect does not appear to occur in this case; the results do not change significantly when five portfolios are dropped from the sample. Revising the grouping procedure — ranking on the basis of beta first, then ranking on the basis of market proportion — also does not lead to substantially different results.

4.2. A closer look at the results

An additional factor relevant for asset pricing — the market value of the equity of a firm — has been found. The results are based on a linear model. Linearity was assumed only for convenience and there is no theoretical reason (since there is no model) why the relationship should be linear. If it is nonlinear, the particular form of the relationship might give us a starting point for the discussion of possible causes of the size effect in the next section. An analysis of the residuals of the twenty-five portfolios is the easiest way to look at the linearity question. For each month t, the estimated residual return

$$\hat{\varepsilon}_{it} = R_{it} - \hat{\gamma}_{0t} - \hat{\gamma}_{1t}\hat{\beta}_{it} - \hat{\gamma}_{2t}[(\phi_{it} - \phi_{mt})/\phi_{mt}], \qquad i = 1, \ldots, 25, \tag{5}$$

is calculated for all portfolios. The mean residuals over the forty-five year sample period are plotted as a function of the mean market proportion in fig. 1. Since the distribution of the market proportions is very skewed, a logarithmic scale is used. The solid line connects the mean residual returns of each size group. The numbers identify the individual portfolios within each group according to beta, '1' being the one with the largest beta, '5' being the one with the smallest beta.

The figure shows clearly that the linear model is misspecified.[11] The residuals are not randomly distributed around zero. The residuals of the portfolios containing the smallest firms are all positive; the remaining ones are close to zero. As a consequence, it is impossible to use $\hat{\gamma}_2$ as a simple size premium in the cross-section. The plot also shows, however, that the misspecification is not responsible for the significance of $\hat{\gamma}_2$ since the linear model underestimates the true size effect present for very small firms. To illustrate this point, the five portfolios containing the smaller firms are

[11] The nonlinearity cannot be eliminated by defining ϕ_i as the log of the market proportion.

deleted from the sample and the parameters reestimated. The results, summarized in table 2, show that the $\hat{\gamma}_2$ remain essentially the same. The relationship is still not linear: the new $\hat{\gamma}_2$ still cannot be used as a size premium.

Fig. 1 suggests that the main effect occurs for very small firms. Further support for this conclusion can be obtained from a simple test. We can regress the returns of the twenty-five portfolios in each result on beta alone and examine the residuals. The regression is misspecified and the residuals contain information about the size effect. Fig. 2 shows the plot of those residuals in the same format as fig. 1. The smallest firms have, on average, very large unexplained mean returns. There is no significant difference between the residuals of the remaining portfolios.

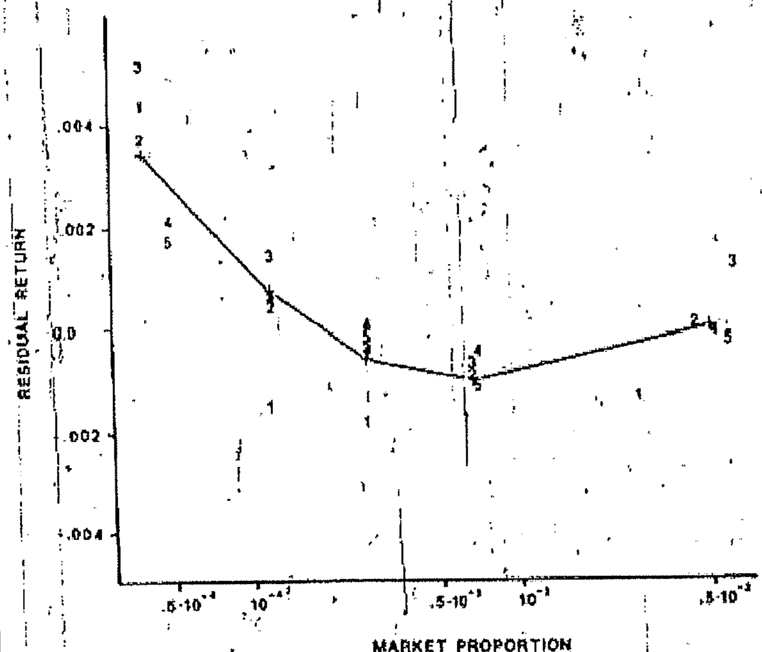

Fig. 1. Mean residual returns of portfolios (1936-1975) with equally-weighted CRSP index as market proxy. The residual is calculated with the three-factor model [eq. (3)]. The numbers 1,...,5 represent the mean residual return for the five portfolios within each size group (1: portfolio with largest beta,..., 5: portfolio with smallest beta). + represents the mean of the mean residuals of the five portfolios with similar market values.

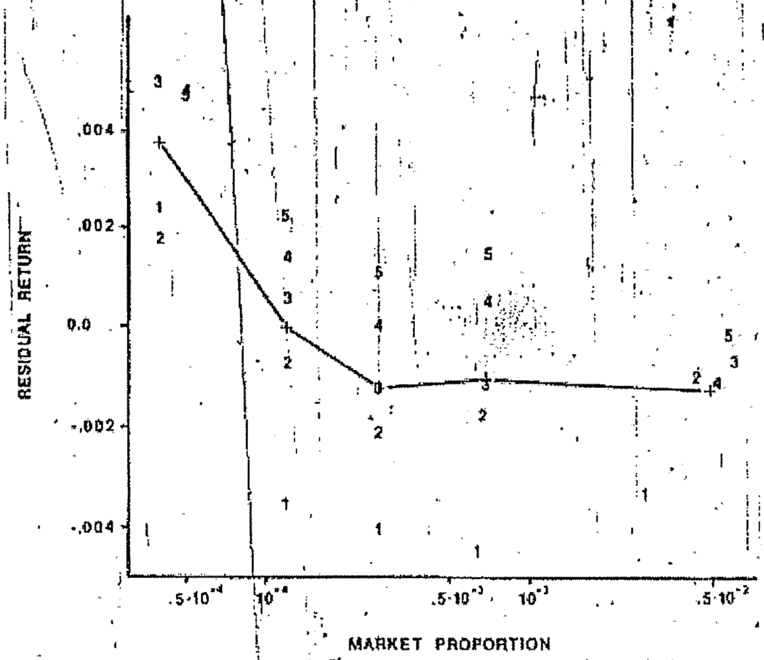

Fig. 2. Mean residual returns of portfolios (1936–1975) with equally-weighted CRSP index as market proxy. The residual is calculated with the two-factor model ($\hat{\varepsilon}_{it} = R_{it} - \hat{\gamma}_{0t} - \hat{\gamma}_{1t}\hat{\beta}_{it}$). The symbols are as defined for fig. 1.

4.3. 'Arbitrage' portfolio returns

One important empirical question still remains: How important is the size effect from a practical point of view? Fig. 2 suggests that the difference in returns between the smallest firms and the remaining ones is, on average, about 0.4 percent per month. A more dramatic result can be obtained when the securities are chosen solely on the basis of their market value.

As an illustration, consider putting equal dollar amounts into portfolios containing the smallest, largest and median-sized firms at the beginning of a year. These portfolios are to be equally weighted and contain, say, ten, twenty or fifty securities. They are to be held for five years and are rebalanced every month. They are levered or unlevered to have the same beta. We are then interested in the differences in their returns,

$$R_{1t} = R_{st} - R_{lt}, \qquad R_{2t} = R_{st} - R_{mt}, \qquad R_{3t} = R_{mt} - R_{lt}, \tag{6}$$

Table 2

Portfolio estimators for γ_2 for all 25 portfolios and for 20 portfolios (portfolios containing smallest firms deleted) based on CRSP equally weighted index with generalized least-squares estimation.[a]

	Size premium $\hat{\gamma}_2$ with	
Period	25 portfolios	20 portfolios
1936–1975	−0.00044 (−2.42)	−0.00043 (−2.54)
1936–1955	−0.00037 (−1.72)	−0.00041 (−1.88)
1956–1975	−0.00056 (−1.91)	−0.00050 (−1.91)
1936–1945	−0.00085 (−2.81)	−0.00083 (−2.48)
1946–1955	0.00003 (0.12)	−0.00003 (−0.13)
1956–1965	−0.00023 (−0.83)	−0.00017 (−0.65)
1966–1975	−0.00091 (−1.78)	−0.00085 (−1.84)

[a] t-statistic in parentheses.

where R_{1t}, R_{2t} and $\tilde{R}_{3t}$ are the returns on the portfolios containing the smallest, median-sized and largest firms at portfolio formation time (and $R_{1t} = R_{2t} + R_{3t}$). The procedure involves (a) the calculation of the three differences in raw returns in each month and (b) running time series regressions of the differences on the excess returns of the market proxy. The intercept terms of these regressions are then interpreted as the $\bar{R}_i$, $i=1,\ldots,3$. Thus, the differences can be interpreted as 'arbitrage' returns, since, e.g., R_{1t} is the return obtained from holding the smallest firms long and the largest firms short, representing zero net investment in a zero-beta portfolio.[12] Simple equally weighted portfolios are used rather than more sophisticated minimum variance portfolios to demonstrate that the size effect is not due to some quirk in the covariance matrix.

Table 3 shows that the results of the earlier tests are fully confirmed. $\bar{R}_2$, the difference in returns between very small firms and median-size firms, is typically considerably larger than $\bar{R}_3$, the difference in returns between median-sized and very large firms. The average excess return from holding very small firms long and very large firms short is, on average, 1.52 percent

[12] No *ex post* sample bias is introduced, since monthly rebalancing includes stocks delisted during the five years. Thus, the portfolio size is generally accurate only for the first month of each period.

Table 3
Mean monthly returns on 'arbitrage' portfolios.[a]
$R_j - R_k = \hat{\alpha}_i + \hat{\beta}_i(R_m - R_F)$

	$\hat{\alpha}_1$[b]			$\hat{\alpha}_2$[c]			$\hat{\alpha}_3$[d]		
	$n=10$	$n=20$	$n=50$	$n=10$	$n=20$	$n=50$	$n=10$	$n=20$	$n=50$
Overall period									
1931–1975	0.0152 (2.99)	0.0148 (3.53)	0.0101 (3.07)	0.0130 (2.90)	0.0124 (3.56)	0.0089 (3.64)	0.0021 (1.06)	0.0024 (1.41)	0.0012 (0.85)
Five-year subperiods									
1931–1935	0.0559 (2.25)	0.0597 (2.81)	0.0427 (2.35)	0.0462 (1.92)	0.0462 (2.55)	0.0326 (2.46)	0.0127 (1.09)	0.0134 (1.49)	0.0101 (1.42)
1936–1940	0.0201 (0.82)	0.0182 (0.97)	0.0089 (0.67)	0.0118 (0.55)	0.0145 (0.90)	0.0064 (0.65)	0.0084 (1.20)	0.0037 (0.62)	0.0025 (0.49)
1941–1945	0.0430 (2.29)	0.0408 (2.46)	0.0269 (2.17)	0.0381 (2.29)	0.0367 (2.54)	0.0228 (2.02)	0.0049 (1.25)	0.0038 (1.09)	0.0041 (1.68)
1946–1950	−0.0060 (−1.17)	−0.0046 (−0.97)	−0.0036 (−0.97)	−0.0058 (−1.03)	−0.0059 (−1.29)	−0.0029 (−0.83)	−0.0002 (−0.07)	−0.0104 (−0.50)	−0.0007 (−0.38)
1951–1955	−0.0067 (−0.89)	−0.0011 (−0.21)	0.0013 (0.32)	−0.0004 (−0.07)	0.0026 (0.72)	0.0010 (0.39)	−0.0062 (−1.29)	−0.0037 (−0.99)	0.0003 (0.11)
1956–1960	0.0039 (0.67)	0.0008 (0.15)	0.0037 (0.89)	0.0007 (0.14)	−0.0027 (−0.64)	0.0011 (0.45)	0.0031 (0.88)	0.0035 (1.16)	0.0026 (0.97)
1961–1965	0.0131 (1.38)	0.0060 (0.67)	0.0024 (0.31)	0.0096 (1.11)	0.0046 (0.72)	0.0036 (0.77)	0.0035 (0.59)	0.0014 (0.24)	−0.0012 (−0.24)
1966–1970	0.0121 (1.64)	0.0117 (2.26)	0.0077 (1.91)	0.0129 (1.93)	0.0110 (2.71)	0.0071 (2.43)	0.0008 (0.23)	0.0007 (0.22)	0.0006 (0.27)
1971–1975	0.0063 (0.60)	0.0108 (1.23)	0.0098 (1.45)	0.0033 (0.39)	0.0077 (1.18)	0.0083 (1.79)	0.0030 (0.64)	0.0031 (0.72)	0.0015 (0.43)

[a]Equally-weighted portfolios with n securities, adjusted for differences in market risk with respect to CRSP value-weighted index, t-statistics in parentheses.
[b]Small firms held long, large firms held short.
[c]Small firms held long, median-size firms held short.
[d]Median-size firms held long, large firms held short.

per month or 19.8 percent on an annualized basis. This strategy, which suggests very large 'profit opportunities', leaves the investor with a poorly diversified portfolio. A portfolio of small firms has typically much larger residual risk with respect to a value-weighted index than a portfolio of very large firms with the same number of securities [Banz (1978, ch. 3)]. Since the fifty largest firms make up more than 25 percent of the total market value of NYSE stocks, it is not surprising that a larger part of the variation of the return of a portfolio of those large firms can be explained by its relation with the value-weighted market index. Table 3 also shows that the strategy would not have been successful in every five year subperiod. Nevertheless, the magnitude of the size effect during the past forty-five years is such that it is of more than just academic interest.

5. Conclusions

The evidence presented in this study suggests that the CAPM is misspecified. On average, small NYSE firms have had significantly larger risk adjusted returns than large NYSE firms over a forty year period. This size effect is not linear in the market proportion (or the log of the market proportion) but is most pronounced for the smallest firms in the sample. The effect is also not very stable through time. An analysis of the ten year subperiods show substantial differences in the magnitude of the coefficient of the size factor (table 1).

There is no theoretical foundation for such an effect. We do not even know whether the factor is size itself or whether size is just a proxy for one or more true but unknown factors correlated with size. It is possible, however, to offer some conjectures and even discuss some factors for which size is suspected to proxy. Recent work by Reinganum (1980) has eliminated one obvious candidate: the price-earnings (P/E) ratio.[13] He finds that the P/E-effect, as reported by Basu (1977), disappears for both NYSE and AMEX stocks when he controls for size but that there is a significant size effect even when he controls for the P/E-ratio, i.e., the P/E-ratio effect is a proxy for the size effect and not vice versa. Stattman (1980), who found a significant negative relationship between the ratio of book value and market value of equity and its return, also reports that this relationship is just a proxy for the size effect. Naturally, a large number of possible factors remain to be tested.[14] But the Reinganum results point out a potential problem with some of the existing negative evidence of the efficient market hypothesis. Basu believed to have identified a market inefficiency but his P/E-effect is

[13] The average correlation coefficient between P/E-ratio and market value is only 0.16 for individual stocks for thirty-eight quarters ending in 1978. But for the portfolios formed on the basis of P/E-ratio, it rises to 0.82. Recall that Basu (1977) used ten portfolios in his study.

[14] E.g., debt equity ratios, skewness of the return distribution [Kraus and Litzenberger (1976)].

just a proxy for the size effect. Given its longevity, it is not likely that it is due to a market inefficiency but it is rather evidence of a pricing model misspecification. To the extent that tests of market efficiency use data of firms of different sizes and are based on the CAPM, their results might be at least contaminated by the size effect.

One possible explanation involving the size of the firm directly is based on a model by Klein and Bawa (1977). They find that if insufficient information is available about a subset of securities, investors will not hold these securities because of estimation risk, i.e., uncertainty about the true parameters of the return distribution. If investors differ in the amount of information available, they will limit their diversification to different subsets of all securities in the market.[15] It is likely that the amount of information generated is related to the size of the firm. Therefore, many investors would not desire to hold the common stock of very small firms. I have shown elsewhere [Banz (1978, ch. 2)] that securities sought by only a subset of the investors have higher risk-adjusted returns than those considered by all investors. Thus, lack of information about small firms leads to limited diversification and therefore to higher returns for the 'undesirable' stocks of small firms.[16] While this informal model is consistent with the empirical results, it is, nevertheless, just conjecture.

To summarize, the size effect exists but it is not at all clear why it exists. Until we find an answer, it should be interpreted with caution. It might be tempting to use the size effect, e.g., as the basis for a theory of mergers — large firms are able to pay a premium for the stock of small firms since they will be able to discount the same cash flows at a smaller discount rate. Naturally, this might turn out to be complete nonsense if size were to be shown to be just a proxy.

The preceding discussion suggests that the results of this study leave many questions unanswered. Further research should consider the relationship between size and other factors such as the dividend yield effect, and the tests should be expanded to include OTC stocks as well.

[15]Klein and Bawa (1977, p. 102).

[16]A similar result can be obtained with the introduction of fixed holding costs which lead to limited diversification as well. See Brennan (1975), Banz (1978, ch. 2) and Mayshar (1979).

References

Ball, Ray, 1978, Anomalies in relationships between securities' yields and yield surrogates, Journal of Financial Economics 6, 103–126.

Banz, Rolf W., 1978, Limited diversification and market equilibrium: An empirical analysis, Ph.D. dissertation (University of Chicago, Chicago, IL).

Basu, S., 1977, Investment performance of common stocks in relation to their price-earnings ratios: A test of market efficiency, Journal of Finance 32, June, 663–682.

Black, Fischer, 1972, Capital market equilibrium with restricted borrowing, Journal of Business 45, July, 444–454.
Black, Fischer, and Myron Scholes, 1974, The effects of dividend yield and dividend policy on common stock prices and returns, Journal of Financial Economics 1, May, 1–22.
Black, Fischer, Michael C. Jensen and Myron Scholes, 1972, The capital asset pricing model: Some empirical tests, in: M.C. Jensen, ed., Studies in the theory of capital markets (Praeger, New York) 79–121.
Brennan, Michael J., 1975, The optimal number of securities in a risky asset portfolio when there are fixed costs of transacting: Theory and some empirical evidence, Journal of Financial and Quantitative Analysis 10, Sept., 483–496.
Brenner, Menachem, 1976, A note on risk, return and equilibrium: Empirical tests, Journal of Political Economy 84, 407–409.
Fama, Eugene F., 1976, Foundations of finance (Basic Books, New York).
Fama, Eugene F. and James D. MacBeth, 1973, Risk return and equilibrium: Some empirical tests, Journal of Political Economy 71, May–June, 607–636.
Ibbotson, Roger G. and Carol L. Fall, 1979, The United States market wealth portfolio, Journal of Portfolio Management 6, 82–92.
Ibbotson, Roger G. and Rex A. Sinquefield, 1977, Stocks, bonds, bills and inflation: The past (1926–1976) and the future (1977–2000) (Financial Analysis Research Foundation).
Klein, Roger W. and Vijay S. Bawa, 1977, The effect of limited information and estimation risk on optimal portfolio diversification, Journal of Financial Economics 5, Aug., 89–111.
Kraus, Alan and Robert H. Litzenberger, 1976, Skewness preference and the valuation of risk assets, Journal of Finance 31, 1085–1100.
Levi, Maurice D., 1973, Errors in the variables bias in the presence of correctly measured variables, Econometrica 41, Sept., 985–986.
Litzenberger, Robert H. and Krishna Ramaswamy, 1979, The effect of personal taxes and dividends on capital asset prices: Theory and empirical evidence, Journal of Financial Economics 7, June, 163–195.
Mayshar, Joram, 1979, Transaction costs in a model of capital market equilibrium, Journal of Political Economy 87, 673–700.
Reinganum, Marc R., 1980, Misspecification of capital asset pricing: Empirical anomalies based on earnings yields and market values, Journal of Financial Economics, this issue.
Roll, Richard, 1977, A critique of the asset pricing theory's tests: Part I, Journal of Financial Economics 4, Jan., 129–176.
Stattman, Dennis, 1980, Book values and expected stock returns, Unpublished M.B.A. honors paper (University of Chicago, Chicago, IL).
Theil, Henri, 1971, Principles of econometrics (Wiley, New York).
U.S. Department of Commerce, Office of Business Economics, 1969, 1970, Survey of current business 49, May, 11–12; 50, May, 14.

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The Cross-Section of Expected Stock Returns

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ABSTRACT

Two easily measured variables, size and book-to-market equity, combine to capture the cross-sectional variation in average stock returns associated with market β, size, leverage, book-to-market equity, and earnings-price ratios. Moreover, when the tests allow for variation in β that is unrelated to size, the relation between market β and average return is flat, even when β is the only explanatory variable.

THE ASSET-PRICING MODEL OF Sharpe (1964), Lintner (1965), and Black (1972) has long shaped the way academics and practitioners think about average returns and risk. The central prediction of the model is that the market portfolio of invested wealth is mean-variance efficient in the sense of Markowitz (1959). The efficiency of the market portfolio implies that (a) expected returns on securities are a positive linear function of their market βs (the slope in the regression of a security's return on the market's return), and (b) market βs suffice to describe the cross-section of expected returns.

There are several empirical contradictions of the Sharpe-Lintner-Black (SLB) model. The most prominent is the size effect of Banz (1981). He finds that market equity, ME (a stock's price times shares outstanding), adds to the explanation of the cross-section of average returns provided by market βs. Average returns on small (low ME) stocks are too high given their β estimates, and average returns on large stocks are too low.

Another contradiction of the SLB model is the positive relation between leverage and average return documented by Bhandari (1988). It is plausible that leverage is associated with risk and expected return, but in the SLB model, leverage risk should be captured by market β. Bhandari finds, however, that leverage helps explain the cross-section of average stock returns in tests that include size (ME) as well as β.

Stattman (1980) and Rosenberg, Reid, and Lanstein (1985) find that average returns on U.S. stocks are positively related to the ratio of a firm's book value of common equity, BE, to its market value, ME. Chan, Hamao, and Lakonishok (1991) find that book-to-market equity, BE/ME, also has a strong role in explaining the cross-section of average returns on Japanese stocks.

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Finally, Basu (1983) shows that earnings-price ratios (E/P) help explain the cross-section of average returns on U.S. stocks in tests that also include size and market β. Ball (1978) argues that E/P is a catch-all proxy for unnamed factors in expected returns; E/P is likely to be higher (prices are lower relative to earnings) for stocks with higher risks and expected returns, whatever the unnamed sources of risk.

Ball's proxy argument for E/P might also apply to size (ME), leverage, and book-to-market equity. All these variables can be regarded as different ways to scale stock prices, to extract the information in prices about risk and expected returns (Keim (1988)). Moreover, since E/P, ME, leverage, and BE/ME are all scaled versions of price, it is reasonable to expect that some of them are redundant for describing average returns. Our goal is to evaluate the joint roles of market β, size, E/P, leverage, and book-to-market equity in the cross-section of average returns on NYSE, AMEX, and NASDAQ stocks.

Black, Jensen, and Scholes (1972) and Fama and MacBeth (1973) find that, as predicted by the SLB model, there is a positive simple relation between average stock returns and β during the pre-1969 period. Like Reinganum (1981) and Lakonishok and Shapiro (1986), we find that the relation between β and average return disappears during the more recent 1963–1990 period, even when β is used alone to explain average returns. The appendix shows that the simple relation between β and average return is also weak in the 50-year 1941–1990 period. In short, our tests do not support the most basic prediction of the SLB model, that average stock returns are positively related to market βs.

Unlike the simple relation between β and average return, the univariate relations between average return and size, leverage, E/P, and book-to-market equity are strong. In multivariate tests, the negative relation between size and average return is robust to the inclusion of other variables. The positive relation between book-to-market equity and average return also persists in competition with other variables. Moreover, although the size effect has attracted more attention, book-to-market equity has a consistently stronger role in average returns. Our bottom-line results are: (a) β does not seem to help explain the cross-section of average stock returns, and (b) the combination of size and book-to-market equity seems to absorb the roles of leverage and E/P in average stock returns, at least during our 1963–1990 sample period.

If assets are priced rationally, our results suggest that stock risks are multidimensional. One dimension of risk is proxied by size, ME. Another dimension of risk is proxied by BE/ME, the ratio of the book value of common equity to its market value.

It is possible that the risk captured by BE/ME is the relative distress factor of Chan and Chen (1991). They postulate that the earning prospects of firms are associated with a risk factor in returns. Firms that the market judges to have poor prospects, signaled here by low stock prices and high ratios of book-to-market equity, have higher expected stock returns (they are penalized with higher costs of capital) than firms with strong prospects. It is also possible, however, that BE/ME just captures the unraveling (regression toward the mean) of irrational market whims about the prospects of firms.

Whatever the underlying economic causes, our main result is straightforward. Two easily measured variables, size (ME) and book-to-market equity (BE/ME), provide a simple and powerful characterization of the cross-section of average stock returns for the 1963–1990 period.

In the next section we discuss the data and our approach to estimating β. Section II examines the relations between average return and β and between average return and size. Section III examines the roles of E/P, leverage, and book-to-market equity in average returns. In sections IV and V, we summarize, interpret, and discuss applications of the results.

I. Preliminaries

A. Data

We use all nonfinancial firms in the intersection of (a) the NYSE, AMEX, and NASDAQ return files from the Center for Research in Security Prices (CRSP) and (b) the merged COMPUSTAT annual industrial files of income-statement and balance-sheet data, also maintained by CRSP. We exclude financial firms because the high leverage that is normal for these firms probably does not have the same meaning as for nonfinancial firms, where high leverage more likely indicates distress. The CRSP returns cover NYSE and AMEX stocks until 1973 when NASDAQ returns also come on line. The COMPUSTAT data are for 1962–1989. The 1962 start date reflects the fact that book value of common equity (COMPUSTAT item 60), is not generally available prior to 1962. More important, COMPUSTAT data for earlier years have a serious selection bias; the pre-1962 data are tilted toward big historically successful firms.

To ensure that the accounting variables are known before the returns they are used to explain, we match the accounting data for all fiscal yearends in calendar year $t-1$ (1962–1989) with the returns for July of year t to June of $t+1$. The 6-month (minimum) gap between fiscal yearend and the return tests is conservative. Earlier work (e.g., Basu (1983)) often assumes that accounting data are available within three months of fiscal yearends. Firms are indeed required to file their 10-K reports with the SEC within 90 days of their fiscal yearends, but on average 19.8% do not comply. In addition, more than 40% of the December fiscal yearend firms that do comply with the 90-day rule file on March 31, and their reports are not made public until April. (See Alford, Jones, and Zmijewski (1992).)

We use a firm's market equity at the end of December of year $t-1$ to compute its book-to-market, leverage, and earnings-price ratios for $t-1$, and we use its market equity for June of year t to measure its size. Thus, to be included in the return tests for July of year t, a firm must have a CRSP stock price for December of year $t-1$ and June of year t. It must also have monthly returns for at least 24 of the 60 months preceding July of year t (for "pre-ranking" β estimates, discussed below). And the firm must have COMPUSTAT data on total book assets (A), book equity (BE), and earnings (E), for its fiscal year ending in (any month of) calendar year $t - 1$.

Our use of December market equity in the E/P, BE/ME, and leverage ratios is objectionable for firms that do not have December fiscal yearends because the accounting variable in the numerator of a ratio is not aligned with the market value in the denominator. Using ME at fiscal yearends is also problematic; then part of the cross-sectional variation of a ratio for a given year is due to market-wide variation in the ratio during the year. For example, if there is a general fall in stock prices during the year, ratios measured early in the year will tend to be lower than ratios measured later. We can report, however, that the use of fiscal-yearend MEs, rather than December MEs, in the accounting ratios has little impact on our return tests.

Finally, the tests mix firms with different fiscal yearends. Since we match accounting data for all fiscal yearends in calendar year $t - 1$ with returns for July of t to June of $t + 1$, the gap between the accounting data and the matching returns varies across firms. We have done the tests using the smaller sample of firms with December fiscal yearends with similar results.

B. Estimating Market βs

Our asset-pricing tests use the cross-sectional regression approach of Fama and MacBeth (1973). Each month the cross-section of returns on stocks is regressed on variables hypothesized to explain expected returns. The time-series means of the monthly regression slopes then provide standard tests of whether different explanatory variables are on average priced.

Since size, E/P, leverage, and BE/ME are measured precisely for individual stocks, there is no reason to smear the information in these variables by using portfolios in the Fama-MacBeth (FM) regressions. Most previous tests use portfolios because estimates of market βs are more precise for portfolios. Our approach is to estimate βs for portfolios and then assign a portfolio's β to each stock in the portfolio. This allows us to use individual stocks in the FM asset-pricing tests.

B.1. β Estimation: Details

In June of each year, all NYSE stocks on CRSP are sorted by size (ME) to determine the NYSE decile breakpoints for ME. NYSE, AMEX, and NASDAQ stocks that have the required CRSP-COMPUSTAT data are then allocated to 10 size portfolios based on the NYSE breakpoints. (If we used stocks from all three exchanges to determine the ME breakpoints, most portfolios would include only small stocks after 1973, when NASDAQ stocks are added to the sample.)

We form portfolios on size because of the evidence of Chan and Chen (1988) and others that size produces a wide spread of average returns and βs. Chan and Chen use only size portfolios. The problem this creates is that size and the βs of size portfolios are highly correlated (-0.988 in their data), so asset-pricing tests lack power to separate size from β effects in average returns.

To allow for variation in β that is unrelated to size, we subdivide each size decile into 10 portfolios on the basis of pre-ranking βs for individual stocks. The pre-ranking βs are estimated on 24 to 60 monthly returns (as available) in the 5 years before July of year t. We set the β breakpoints for each size decile using only NYSE stocks that satisfy our COMPUSTAT-CRSP data requirements for year $t-1$. Using NYSE stocks ensures that the β breakpoints are not dominated after 1973 by the many small stocks on NASDAQ. Setting β breakpoints with stocks that satisfy our COMPUSTAT-CRSP data requirements guarantees that there are firms in each of the 100 size-β portfolios.

After assigning firms to the size-β portfolios in June, we calculate the equal-weighted monthly returns on the portfolios for the next 12 months, from July to June. In the end, we have post-ranking monthly returns for July 1963 to December 1990 on 100 portfolios formed on size and pre-ranking βs. We then estimate βs using the full sample (330 months) of post-ranking returns on each of the 100 portfolios, with the CRSP value-weighted portfolio of NYSE, AMEX, and (after 1972) NASDAQ stocks used as the proxy for the market. We have also estimated βs using the value-weighted or the equal-weighted portfolio of NYSE stocks as the proxy for the market. These βs produce inferences on the role of β in average returns like those reported below.

We estimate β as the sum of the slopes in the regression of the return on a portfolio on the current and prior month's market return. (An additional lead and lag of the market have little effect on these sum βs.) The sum βs are meant to adjust for nonsynchronous trading (Dimson (1979)). Fowler and Rorke (1983) show that sum βs are biased when the market return is autocorrelated. The 1st- and 2nd-order autocorrelations of the monthly market returns for July 1963 to December 1990 are 0.06 and -0.05, both about 1 standard error from 0. If the Fowler-Rorke corrections are used, they lead to trivial changes in the βs. We stick with the simpler sum βs. Appendix Table AI shows that using sum βs produces large increases in the βs of the smallest ME portfolios and small declines in the βs of the largest ME portfolios.

Chan and Chen (1988) show that full-period β estimates for portfolios can work well in tests of the SLB model, even if the true βs of the portfolios vary through time, if the variation in the βs is proportional,

$$\beta_{jt} - \beta_j = k_t(\beta_j - \beta), \tag{1}$$

where β_{jt} is the true β for portfolio j at time t, β_j is the mean of β_{jt} across t, and β is the mean of the β_j. The Appendix argues that (1) is a good approximation for the variation through time in the true βs of portfolios (j) formed on size and β. For diehard β fans, sure to be skeptical of our results on the weak role of β in average stock returns, we can also report that the results stand up to robustness checks that use 5-year pre-ranking βs, or 5-year post-ranking βs, instead of the full-period post-ranking βs.

We allocate the full-period post-ranking β of a size-β portfolio to each stock in the portfolio. These are the βs that will be used in the Fama-MacBeth cross-sectional regressions for individual stocks. We judge that the precision of the full-period post-ranking portfolio βs, relative to the imprecise β estimates that would be obtained for individual stocks, more than makes up for the fact that true βs are not the same for all stocks in a portfolio. And note that assigning full-period portfolio βs to stocks does not mean that a stock's β is constant. A stock can move across portfolios with year-to-year changes in the stock's size (ME) and in the estimates of its β for the preceding 5 years.

B.2. β Estimates

Table I shows that forming portfolios on size and pre-ranking βs, rather than on size alone, magnifies the range of full-period post-ranking βs. Sorted on size alone, the post-ranking βs range from 1.44 for the smallest ME portfolio to 0.92 for the largest. This spread of βs across the 10 size deciles is smaller than the spread of post-ranking βs produced by the β sort of *any* size decile. For example, the post-ranking βs for the 10 portfolios in the smallest size decile range from 1.05 to 1.79. Across all 100 size-β portfolios, the post-ranking βs range from 0.53 to 1.79, a spread 2.4 times the spread, 0.52, obtained with size portfolios alone.

Two other facts about the βs are important. First, in each size decile the post-ranking βs closely reproduce the ordering of the pre-ranking βs. We take this to be evidence that the pre-ranking β sort captures the ordering of true post-ranking βs. (The appendix gives more evidence on this important issue.) Second, the β sort is not a refined size sort. In any size decile, the average values of ln(ME) are similar across the β-sorted portfolios. Thus the pre-ranking β sort achieves its goal. It produces strong variation in post-ranking βs that is unrelated to size. This is important in allowing our tests to distinguish between β and size effects in average returns.

II. β and Size

The Sharpe-Lintner-Black (SLB) model plays an important role in the way academics and practitioners think about risk and the relation between risk and expected return. We show next that when common stock portfolios are formed on size alone, there seems to be evidence for the model's central prediction: average return is positively related to β. The βs of size portfolios are, however, almost perfectly correlated with size, so tests on size portfolios are unable to disentangle β and size effects in average returns. Allowing for variation in β that is unrelated to size breaks the logjam, but at the expense of β. Thus, when we subdivide size portfolios on the basis of pre-ranking βs, we find a strong relation between average return and size, but no relation between average return and β.

A. Informal Tests

Table II shows post-ranking average returns for July 1963 to December 1990 for portfolios formed from one-dimensional sorts of stocks on size or β. The portfolios are formed at the end of June each year and their equal-weighted returns are calculated for the next 12 months. We use returns for July to June to match the returns in later tests that use the accounting data. When we sort on just size or 5-year pre-ranking βs, we form 12 portfolios. The middle 8 cover deciles of size or β. The 4 extreme portfolios (1A, 1B, 10A, and 10B) split the bottom and top deciles in half.

Table II shows that when portfolios are formed on size alone, we observe the familiar strong negative relation between size and average return (Banz (1981)), and a strong positive relation between average return and β. Average returns fall from 1.64% per month for the smallest ME portfolio to 0.90% for the largest. Post-ranking βs also decline across the 12 size portfolios, from 1.44 for portfolio 1A to 0.90 for portfolio 10B. Thus, a simple size sort seems to support the SLB prediction of a positive relation between β and average return. But the evidence is muddied by the tight relation between size and the βs of size portfolios.

The portfolios formed on the basis of the ranked market βs of stocks in Table II produce a wider range of βs (from 0.81 for portfolio 1A to 1.73 for 10B) than the portfolios formed on size. Unlike the size portfolios, the β-sorted portfolios do not support the SLB model. There is little spread in average returns across the β portfolios, and there is no obvious relation between β and average returns. For example, although the two extreme portfolios, 1A and 10B, have much different βs, they have nearly identical average returns (1.20% and 1.18% per month). These results for 1963–1990 confirm Reinganum's (1981) evidence that for β-sorted portfolios, there is no relation between average return and β during the 1964–1979 period.

The 100 portfolios formed on size and then pre-ranking β in Table I clarify the contradictory evidence on the relation between β and average return produced by portfolios formed on size or β alone. Specifically, the two-pass sort gives a clearer picture of the separate roles of size and β in average returns. Contrary to the central prediction of the SLB model, the second-pass β sort produces little variation in average returns. Although the post-ranking βs in Table I increase strongly in each size decile, average returns are flat or show a slight tendency to decline. In contrast, within the columns of the average return and β matrices of Table I, average returns and βs decrease with increasing size.

The two-pass sort on size and β in Table I says that variation in β that is tied to size is positively related to average return, but variation in β unrelated to size is not compensated in the average returns of 1963–1990. The proper inference seems to be that there is a relation between size and average return, but controlling for size, there is no relation between β and average return. The regressions that follow confirm this conclusion, and they produce another that is stronger. The regressions show that when one allows

Table I

Average Returns, Post-Ranking βs and Average Size For Portfolios Formed on Size and then β: Stocks Sorted on ME (Down) then Pre-Ranking β (Across): July 1963 to December 1990

Portfolios are formed yearly. The breakpoints for the size (ME, price times shares outstanding) deciles are determined in June of year t (t = 1963–1990) using all NYSE stocks on CRSP. All NYSE, AMEX, and NASDAQ stocks that meet the CRSP-COMPUSTAT data requirements are allocated to the 10 size portfolios using the NYSE breakpoints. Each size decile is subdivided into 10 β portfolios using pre-ranking βs of individual stocks, estimated with 2 to 5 years of monthly returns (as available) ending in June of year t. We use only NYSE stocks that meet the CRSP-COMPUSTAT data requirements to establish the β breakpoints. The equal-weighted monthly returns on the resulting 100 portfolios are then calculated for July of year t to June of year $t + 1$.

The post-ranking βs use the full (July 1963 to December 1990) sample of post-ranking returns for each portfolio. The pre- and post-ranking βs (here and in all other tables) are the sum of the slopes from a regression of monthly returns on the current and prior month's returns on the value-weighted portfolio of NYSE, AMEX, and (after 1972) NASDAQ stocks. The average return is the time-series average of the monthly equal-weighted portfolio returns, in percent. The average size of a portfolio is the time-series average of monthly averages of ln(ME) for stocks in the portfolio at the end of June of each year, with ME denominated in millions of dollars

The average number of stocks per month for the size-β portfolios in the smallest size decile varies from 70 to 177. The average number of stocks for the size-β portfolios in size deciles 2 and 3 is between 15 and 41, and the average number for the largest 7 size deciles is between 11 and 22.

The All column shows statistics for equal-weighted size-decile (ME) portfolios. The All row shows statistics for equal-weighted portfolios of the stocks in each β group.

	All	Low-β	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	High-β
Panel A: Average Monthly Returns (in Percent)											
All	1.25	1.34	1.29	1.36	1.31	1.33	1.28	1.24	1.21	1.25	1.14
Small-ME	1.52	1.71	1.57	1.79	1.61	1 50	1.50	1.37	1.63	1.50	1.42
ME-2	1.29	1.25	1.42	1.36	1.39	1.65	1.61	1.37	1 31	1.34	1.11
ME-3	1.24	1.12	1.31	1.17	1.70	1.29	1.10	1.31	1.36	1.26	0.76
ME-4	1.25	1.27	1.13	1.54	1.06	1.34	1.06	1.41	1.17	1.35	0.98
ME-5	1.29	1.34	1.42	1.39	1.48	1.42	1.18	1.13	1.27	1.18	1.08
ME-6	1.17	1.08	1.53	1.27	1.15	1.20	1.21	1.18	1.04	1.07	1.02
ME-7	1.07	0.95	1.21	1.26	1.09	1.18	1.11	1.24	0.62	1.32	0.76
ME-8	1.10	1.09	1.05	1.37	1.20	1.27	0.98	1.18	1.02	1.01	0.94
ME-9	0.95	0.98	0.88	1.02	1.14	1.07	1.23	0.94	0.82	0.88	0.59
Large-ME	0.89	1.01	0.93	1.10	0.94	0.93	0.89	1.03	0.71	0.74	0.56

Table I—*Continued*

	All	Low-β	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	High-β
Panel B: Post-Ranking βs											
All		0.87	0.99	1.09	1.16	1.26	1.29	1.35	1.45	1.52	1.72
Small-ME	1.44	1.05	1.18	1.28	1.32	1.40	1.40	1.49	1.61	1.64	1.79
ME-2	1.39	0.91	1.15	1.17	1.24	1.36	1.41	1.43	1.50	1.66	1.76
ME-3	1.35	0.97	1.13	1.13	1.21	1.26	1.28	1.39	1.50	1.51	1.75
ME-4	1.34	0.78	1.03	1.17	1.16	1.29	1.37	1.46	1.51	1.64	1.71
ME-5	1.25	0.66	0.85	1.12	1.15	1.16	1.26	1.30	1.43	1.59	1.68
ME-6	1.23	0.61	0.78	1.05	1.16	1.22	1.28	1.36	1.46	1.49	1.70
ME-7	1.17	0.57	0.92	1.01	1.11	1.14	1.26	1.24	1.39	1.34	1.60
ME-8	1.09	0.53	0.74	0.94	1.02	1.13	1.12	1.18	1.26	1.35	1.52
ME-9	1.03	0.58	0.74	0.80	0.95	1.06	1.15	1.14	1.21	1.22	1.42
Large-ME	0.92	0.57	0.71	0.78	0.89	0.95	0.92	1.02	1.01	1.11	1.32
Panel C: Average Size (ln(ME))											
All	4.11	3.86	4.26	4.33	4.41	4.27	4.32	4.26	4.19	4.03	3.77
Small-ME	2.24	2.12	2.27	2.30	2.30	2.28	2.29	2.30	2.32	2.25	2.15
ME-2	3.63	3.65	3.68	3.70	3.72	3.69	3.70	3.69	3.69	3.70	3.68
ME-3	4.10	4.14	4.18	4.12	4.15	4.16	4.16	4.18	4.14	4.15	4.15
ME-4	4.50	4.53	4.53	4.57	4.54	4.56	4.55	4.52	4.58	4.52	4.56
ME-5	4.89	4.91	4.91	4.93	4.95	4.93	4.92	4.93	4.92	4.92	4.95
ME-6	5.30	5.30	5.33	5.34	5.34	5.33	5.33	5.33	5.33	5.34	5.36
ME-7	5.73	5.73	5.75	5.77	5.76	5.73	5.77	5.77	5.76	5.72	5.76
ME-8	6.24	6.26	6.27	6.26	6.24	6.24	6.27	6.24	6.24	6.24	6.26
ME-9	6.82	6.82	6.84	6.82	6.82	6.81	6.81	6.81	6.81	6.80	6.83
Large-ME	7.93	7.94	8.04	8.10	8.04	8.02	8.02	7.94	7.80	7.75	7.62

Table II

Properties of Portfolios Formed on Size or Pre-Ranking β: July 1963 to December 1990

At the end of June of each year *t*, 12 portfolios are formed on the basis of ranked values of size (ME) or pre-ranking β. The pre-ranking βs use 2 to 5 years (as available) of monthly returns ending in June of *t*. Portfolios 2–9 cover deciles of the ranking variables. The bottom and top 2 portfolios (1A, 1B, 10A, and 10B) split the bottom and top deciles in half. The breakpoints for the ME portfolios are based on ranked values of ME for all NYSE stocks on CRSP. NYSE breakpoints for pre-ranking βs are also used to form the β portfolios. NYSE, AMEX, and NASDAQ stocks are then allocated to the size or β portfolios using the NYSE breakpoints. We calculate each portfolio's monthly equal-weighted return for July of year *t* to June of year $t + 1$, and then reform the portfolios in June of $t + 1$.

BE is the book value of common equity plus balance-sheet deferred taxes, A is total book assets, and E is earnings (income before extraordinary items, plus income-statement deferred taxes, minus preferred dividends). BE, A, and E are for each firm's latest fiscal year ending in calendar year $t - 1$. The accounting ratios are measured using market equity ME in December of year $t - 1$. Firm size ln(ME) is measured in June of year *t*, with ME denominated in millions of dollars.

The average return is the time-series average of the monthly equal-weighted portfolio returns, in percent. ln(ME), ln(BE/ME), ln(A/ME), ln(A/BE), E/P, and E/P dummy are the time-series averages of the monthly average values of these variables in each portfolio. Since the E/P dummy is 0 when earnings are positive, and 1 when earnings are negative, E/P dummy gives the average proportion of stocks with negative earnings in each portfolio.

β is the time-series average of the monthly portfolio βs. Stocks are assigned the post-ranking β of the size-β portfolio they are in at the end of June of year *t* (Table I). These individual-firm βs are averaged to compute the monthly βs for each portfolio for July of year *t* to June of year $t + 1$.

Firms is the average number of stocks in the portfolio each month.

	1A	1B	2	3	4	5	6	7	8	9	10A	10B
						Panel A: Portfolios Formed on Size						
Return	1.64	1.16	1.29	1.24	1.25	1.29	1.17	1.07	1.10	0.95	0.88	0.90
β	1.44	1.44	1.39	1.34	1.33	1.24	1.22	1.16	1.08	1.02	0.95	0.90
ln(ME)	1.98	3.18	3.63	4.10	4.50	4.89	5.30	5.73	6.24	6.82	7.39	8.44
ln(BE/ME)	−0.01	−0.21	−0.23	−0.26	−0.32	−0.36	−0.36	−0.44	−0.40	−0.42	−0.51	−0.65
ln(A/ME)	0.73	0.50	0.46	0.43	0.37	0.32	0.32	0.24	0.29	0.27	0.17	−0.03
ln(A/BE)	0.75	0.71	0.69	0.69	0.68	0.67	0.68	0.67	0.69	0.70	0.68	0.62
E/P dummy	0.26	0.14	0.11	0.09	0.06	0.04	0.04	0.03	0.03	0.02	0.02	0.01
E(+)/P	0.09	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0 10	0.10	0.09	0.09
Firms	772	189	236	170	144	140	128	125	119	114	60	64

Table II—*Continued*

	1A	1B	2	3	4	5	6	7	8	9	10A	10B
						Panel B: Portfolios Formed on Pre-Ranking β						
Return	1.20	1 20	1.32	1.26	1.31	1.30	1.30	1.23	1.23	1.33	1.34	1.18
β	0.81	0.79	0.92	1.04	1.13	1.19	1.26	1.32	1.41	1.52	1 63	1.73
ln(ME)	4.21	4.86	4.75	4.68	4.59	4.48	4.36	4.25	3.97	3.78	3.52	3.15
ln(BE/ME)	−0.18	−0.13	−0.22	−0.21	−0.23	−0.22	−0.22	−0.25	−0.23	−0.27	−0.31	−0.50
ln(A/ME)	0.60	0.66	0.49	0.45	0.42	0.42	0.45	0.42	0.47	0 46	0.46	0.31
ln(A/BE)	0.78	0.79	0.71	0.66	0.64	0.65	0.67	0.67	0.70	0.73	0.77	0 81
E/P dummy	0.12	0.06	0.09	0.09	0.08	0.09	0.10	0.12	0.12	0.14	0.17	0.23
E(+)/P	0.11	0.12	0.10	0.10	0.10	0.10	0.10	0.09	0.10	0.09	0 09	0.08
Firms	116	80	185	181	179	182	185	205	227	267	165	291

for variation in β that is unrelated to size, the relation between β and average return is flat, even when β is the only explanatory variable.

B. Fama-MacBeth Regressions

Table III shows time-series averages of the slopes from the month-by-month Fama-MacBeth (FM) regressions of the cross-section of stock returns on size, β, and the other variables (leverage, E/P, and book-to-market equity) used to explain average returns. The average slopes provide standard FM tests for determining which explanatory variables on average have non-zero expected premiums during the July 1963 to December 1990 period.

Like the average returns in Tables I and II, the regressions in Table III say that size, ln(ME), helps explain the cross-section of average stock returns. The average slope from the monthly regressions of returns on size alone is -0.15%, with a t-statistic of -2.58. This reliable negative relation persists no matter which other explanatory variables are in the regressions; the average slopes on ln(ME) are always close to or more than 2 standard errors from 0. The size effect (smaller stocks have higher average returns) is thus robust in the 1963–1990 returns on NYSE, AMEX, and NASDAQ stocks.

In contrast to the consistent explanatory power of size, the FM regressions show that market β does not help explain average stock returns for 1963–1990. In a shot straight at the heart of the SLB model, the average slope from the regressions of returns on β alone in Table III is 0.15% per month and only 0.46 standard errors from 0. In the regressions of returns on size and β, size has explanatory power (an average slope -3.41 standard errors from 0), but the average slope for β is negative and only 1.21 standard errors from 0. Lakonishok and Shapiro (1986) get similar results for NYSE stocks for 1962–1981. We can also report that β shows no power to explain average returns (the average slopes are typically less than 1 standard error from 0) in FM regressions that use various combinations of β with size, book-to-market equity, leverage, and E/P.

C. Can β Be Saved?

What explains the poor results for β? One possibility is that other explanatory variables are correlated with true βs, and this obscures the relation between average returns and measured βs. But this line of attack cannot explain why β has no power when used alone to explain average returns. Moreover, leverage, book-to-market equity, and E/P do not seem to be good proxies for β. The averages of the monthly cross-sectional correlations between β and the values of these variables for individual stocks are all within 0.15 of 0.

Another hypothesis is that, as predicted by the SLB model, there is a positive relation between β and average return, but the relation is obscured by noise in the β estimates. However, our full-period post-ranking βs do not seem to be imprecise. Most of the standard errors of the βs (not shown) are

Table III

Average Slopes (t-Statistics) from Month-by-Month Regressions of Stock Returns on β, Size, Book-to-Market Equity, Leverage, and E/P: July 1963 to December 1990

Stocks are assigned the post-ranking β of the size-β portfolio they are in at the end of June of year t (Table I). BE is the book value of common equity plus balance-sheet deferred taxes, A is total book assets, and E is earnings (income before extraordinary items, plus income-statement deferred taxes, minus preferred dividends) BE, A, and E are for each firm's latest fiscal year ending in calendar year $t - 1$. The accounting ratios are measured using market equity ME in December of year $t - 1$. Firm size ln(ME) is measured in June of year t. In the regressions, these values of the explanatory variables for individual stocks are matched with CRSP returns for the months from July of year t to June of year $t + 1$. The gap between the accounting data and the returns ensures that the accounting data are available prior to the returns. If earnings are positive, E(+)/P is the ratio of total earnings to market equity and E/P dummy is 0. If earnings are negative, E(+)/P is 0 and E/P dummy is 1.

The average slope is the time-series average of the monthly regression slopes for July 1963 to December 1990, and the t-statistic is the average slope divided by its time-series standard error.

On average, there are 2267 stocks in the monthly regressions. To avoid giving extreme observations heavy weight in the regressions, the smallest and largest 0.5% of the observations on E(+)/P, BE/ME, A/ME, and A/BE are set equal to the next largest or smallest values of the ratios (the 0.005 and 0.995 fractiles). This has no effect on inferences.

β	ln(ME)	ln(BE/ME)	ln(A/ME)	ln(A/BE)	E/P Dummy	E(+)/P
0.15 (0.46)						
	−0.15 (−2 58)					
−0.37 (−1.21)	−0.17 (−3.41)					
		0.50 (5.71)				
			0.50 (5 69)	−0.57 (−5.34)		
					0.57 (2.28)	4.72 (4.57)
	−0.11 (−1.99)	0.35 (4.44)				
	−0 11 (−2 06)		0.35 (4.32)	−0 50 (−4.56)		
	−0.16 (−3.06)				0.06 (0 38)	2.99 (3.04)
	−0.13 (−2.47)	0.33 (4.46)			−0.14 (−0.90)	0.87 (1.23)
	−0.13 (−2.47)		0.32 (4.28)	−0.46 (−4.45)	−0.08 (−0.56)	1.15 (1.57)

0.05 or less, only 1 is greater than 0.1, and the standard errors are small relative to the range of the βs (0.53 to 1.79).

The β-sorted portfolios in Tables I and II also provide strong evidence against the β-measurement-error story. When portfolios are formed on pre-ranking βs alone (Table II), the post-ranking βs for the portfolios almost perfectly reproduce the ordering of the pre-ranking βs. Only the β for portfolio 1B is out of line, and only by 0.02. Similarly, when portfolios are formed on size and then pre-ranking βs (Table I), the post-ranking βs in each size decile closely reproduce the ordering of the pre-ranking βs.

The correspondence between the ordering of the pre-ranking and post-ranking βs for the β-sorted portfolios in Tables I and II is evidence that the post-ranking βs are informative about the ordering of the true βs. The problem for the SLB model is that there is no similar ordering in the average returns on the β-sorted portfolios. Whether one looks at portfolios sorted on β alone (Table II) or on size and then β (Table I), average returns are flat (Table II) or decline slightly (Table I) as the post-ranking βs increase.

Our evidence on the robustness of the size effect and the absence of a relation between β and average return is so contrary to the SLB model that it behooves us to examine whether the results are special to 1963–1990. The appendix shows that NYSE returns for 1941–1990 behave like the NYSE, AMEX, and NASDAQ returns for 1963–1990; there is a reliable size effect over the full 50-year period, but little relation between β and average return. Interestingly, there is a reliable simple relation between β and average return during the 1941–1965 period. These 25 years are a major part of the samples in the early studies of the SLB model of Black, Jensen, and Scholes (1972) and Fama and MacBeth (1973). Even for the 1941–1965 period, however, the relation between β and average return disappears when we control for size.

III. Book-to-Market Equity, E/P, and Leverage

Tables I to III say that there is a strong relation between the average returns on stocks and size, but there is no reliable relation between average returns and β. In this section we show that there is also a strong cross-sectional relation between average returns and book-to-market equity. If anything, this book-to-market effect is more powerful than the size effect. We also find that the combination of size and book-to-market equity absorbs the apparent roles of leverage and E/P in average stock returns.

A. Average Returns

Table IV shows average returns for July 1963 to December 1990 for portfolios formed on ranked values of book-to-market equity (BE/ME) or earnings-price ratio (E/P). The BE/ME and E/P portfolios in Table IV are formed in the same general way (one-dimensional yearly sorts) as the size and β portfolios in Table II. (See the tables for details.)

The relation between average return and E/P has a familiar U-shape (e.g., Jaffe, Keim, and Westerfield (1989) for U.S. data, and Chan, Hamao, and Lakonishok (1991) for Japan). Average returns decline from 1.46% per month for the negative E/P portfolio to 0.93% for the firms in portfolio 1B that have low but positive E/P. Average returns then increase monotonically, reaching 1.72% per month for the highest E/P portfolio.

The more striking evidence in Table IV is the strong positive relation between average return and book-to-market equity. Average returns rise from 0.30% for the lowest BE/ME portfolio to 1.83% for the highest, a difference of 1.53% per month. This spread is twice as large as the difference of 0.74% between the average monthly returns on the smallest and largest size portfolios in Table II. Note also that the strong relation between book-to-market equity and average return is unlikely to be a β effect in disguise; Table IV shows that post-ranking market βs vary little across portfolios formed on ranked values of BE/ME.

On average, only about 50 (out of 2317) firms per year have negative book equity, BE. The negative BE firms are mostly concentrated in the last 14 years of the sample, 1976–1989, and we do not include them in the tests. We can report, however, that average returns for negative BE firms are high, like the average returns of high BE/ME firms. Negative BE (which results from persistently negative earnings) and high BE/ME (which typically means that stock prices have fallen) are both signals of poor earning prospects. The similar average returns of negative and high BE/ME firms are thus consistent with the hypothesis that book-to-market equity captures cross-sectional variation in average returns that is related to relative distress.

B. Fama-MacBeth Regressions

B.1. BE/ME

The FM regressions in Table III confirm the importance of book-to-market equity in explaining the cross-section of average stock returns. The average slope from the monthly regressions of returns on ln(BE/ME) alone is 0.50%, with a t-statistic of 5.71. This book-to-market relation is stronger than the size effect, which produces a t-statistic of -2.58 in the regressions of returns on ln(ME) alone. But book-to-market equity does not replace size in explaining average returns. When both ln(ME) and ln(BE/ME) are included in the regressions, the average size slope is still -1.99 standard errors from 0; the book-to-market slope is an impressive 4.44 standard errors from 0.

B.2. Leverage

The FM regressions that explain returns with leverage variables provide interesting insight into the relation between book-to-market equity and average return. We use two leverage variables, the ratio of book assets to market equity, A/ME, and the ratio of book assets to book equity, A/BE. We interpret A/ME as a measure of market leverage, while A/BE is a measure

Table IV

Properties of Portfolios Formed on Book-to-Market Equity (BE/ME) and Earnings-Price Ratio (E/P): July 1963 to December 1990

At the end of each year $t-1$, 12 portfolios are formed on the basis of ranked values of BE/ME or E/P. Portfolios 2–9 cover deciles of the ranking variables. The bottom and top 2 portfolios (1A, 1B, 10A, and 10B) split the bottom and top deciles in half. For E/P, there are 13 portfolios; portfolio 0 is stocks with negative E/P. Since BE/ME and E/P are not strongly related to exchange listing, their portfolio breakpoints are determined on the basis of the ranked values of the variables for all stocks that satisfy the CRSP-COMPUSTAT data requirements. BE is the book value of common equity plus balance-sheet deferred taxes, A is total book assets, and E is earnings (income before extraordinary items, plus income-statement deferred taxes, minus preferred dividends). BE, A, and E are for each firm's latest fiscal year ending in calendar year $t-1$. The accounting ratios are measured using market equity ME in December of year $t-1$. Firm size ln(ME) is measured in June of year t, with ME denominated in millions of dollars. We calculate each portfolio's monthly equal-weighted return for July of year t to June of year $t+1$, and then reform the portfolios at the end of year t.

Return is the time-series average of the monthly equal-weighted portfolio returns (in percent). ln(ME), ln(BE/ME), ln(A/ME), ln(A/BE), E(+)/P, and E/P dummy are the time-series averages of the monthly average values of these variables in each portfolio Since the E/P dummy is 0 when earnings are positive, and 1 when earnings are negative, E/P dummy gives the average proportion of stocks with negative earnings in each portfolio.

β is the time-series average of the monthly portfolio βs. Stocks are assigned the post-ranking β of the size-β portfolio they are in at the end of June of year t (Table I). These individual-firm βs are averaged to compute the monthly βs for each portfolio for July of year t to June of year $t+1$.

Firms is the average number of stocks in the portfolio each month.

Portfolio	0	1A	1B	2	3	4	5	6	7	8	9	10A	10B
Panel A: Stocks Sorted on Book-to-Market Equity (BE/ME)													
Return		0.30	0.67	0.87	0.97	1.04	1.17	1.30	1.44	1.50	1.59	1.92	1.83
β		1.36	1.34	1.32	1.30	1.28	1.27	1.27	1.27	1.27	1.29	1.33	1.35
ln(ME)		4.53	4.67	4.69	4.56	4.47	4.38	4.23	4.06	3.85	3.51	3.06	2.65
ln(BE/ME)		−2.22	−1.51	−1.09	−0.75	−0.51	−0 32	−0.14	0.03	0.21	0.42	0.66	1.02
ln(A/ME)		−1.24	−0.79	−0.40	−0.05	0.20	0.40	0.56	0.71	0.91	1.12	1.35	1.75
ln(A/BE)		0.94	0.71	0.68	0.70	0.71	0.71	0.70	0.68	0.70	0.70	0.70	0.73
E/P dummy		0.29	0.15	0.10	0.08	0.08	0.08	0.09	0.09	0.11	0.15	0.22	0.36
E(+)/P		0.03	0.04	0.06	0.08	0.09	0.10	0.11	0.11	0.12	0.12	0.11	0.10
Firms		89	98	209	222	226	230	235	237	239	239	120	117

Table IV—*Continued*

Portfolio	0	1A	1B	2	3	4	5	6	7	8	9	10A	10B
Panel B: Stocks Sorted on Earnings-Price Ratio (E/P)													
Return	1.46	1.04	0.93	0.94	1.03	1.18	1.22	1.33	1.42	1.46	1.57	1.74	1.72
β	1.47	1.40	1.35	1.31	1.28	1.26	1.25	1.26	1.24	1.23	1.24	1.28	1.31
ln(ME)	2.48	3.64	4.33	4.61	4.64	4.63	4.58	4.49	4.37	4.28	4.07	3.82	3.52
ln(BE/ME)	−0.10	−0.76	−0.91	−0.79	−0.61	−0.47	−0.33	−0.21	−0.08	0.02	0.15	0.26	0.40
ln(A/ME)	0.90	−0.05	−0.27	−0.16	0.03	0.18	0.31	0.44	0.58	0.70	0.85	1.01	1.25
ln(A/BE)	0.99	0.70	0.63	0.63	0.64	0.65	0.64	0.65	0.66	0.68	0 71	0.75	0.86
E/P dummy	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
E(+)/P	0.00	0.01	0.03	0.05	0.06	0.08	0.09	0.11	0.12	0.14	0.16	0.20	0.28
Firms	355	88	90	182	190	193	196	194	197	195	195	95	91

of book leverage. The regressions use the natural logs of the leverage ratios, ln(A/ME) and ln(A/BE), because preliminary tests indicated that logs are a good functional form for capturing leverage effects in average returns. Using logs also leads to a simple interpretation of the relation between the roles of leverage and book-to-market equity in average returns.

The FM regressions of returns on the leverage variables (Table III) pose a bit of a puzzle. The two leverage variables are related to average returns, but with opposite signs. As in Bhandari (1988), higher market leverage is associated with higher average returns; the average slopes for ln(A/ME) are always positive and more than 4 standard errors from 0. But higher book leverage is associated with lower average returns; the average slopes for ln(A/BE) are always negative and more than 4 standard errors from 0.

The puzzle of the opposite slopes on ln(A/ME) and ln(A/BE) has a simple solution. The average slopes for the two leverage variables are opposite in sign but close in absolute value, e.g., 0.50 and −0.57. Thus it is the difference between market and book leverage that helps explain average returns. But the difference between market and book leverage is book-to-market equity, ln(BE/ME) = ln(A/ME) − ln(A/BE). Table III shows that the average book-to-market slopes in the FM regressions are indeed close in absolute value to the slopes for the two leverage variables.

The close links between the leverage and book-to-market results suggest that there are two equivalent ways to interpret the book-to-market effect in average returns. A high ratio of book equity to market equity (a low stock price relative to book value) says that the market judges the prospects of a firm to be poor relative to firms with low BE/ME. Thus BE/ME may capture the relative-distress effect postulated by Chan and Chen (1991). A high book-to-market ratio also says that a firm's market leverage is high relative to its book leverage; the firm has a large amount of market-imposed leverage because the market judges that its prospects are poor and discounts its stock price relative to book value. In short, our tests suggest that the relative-distress effect, captured by BE/ME, can also be interpreted as an involuntary leverage effect, which is captured by the difference between A/ME and A/BE.

B.3. E/P

Ball (1978) posits that the earnings-price ratio is a catch-all for omitted risk factors in expected returns. If current earnings proxy for expected future earnings, high-risk stocks with high expected returns will have low prices relative to their earnings. Thus, E/P should be related to expected returns, whatever the omitted sources of risk. This argument only makes sense, however, for firms with positive earnings. When current earnings are negative, they are not a proxy for the earnings forecasts embedded in the stock price, and E/P is not a proxy for expected returns. Thus, the slope for E/P in the FM regressions is based on positive values; we use a dummy variable for E/P when earnings are negative.

The U-shaped relation between average return and E/P observed in Table IV is also apparent when the E/P variables are used alone in the FM regressions in Table III. The average slope on the E/P dummy variable (0.57% per month, 2.28 standard errors from 0) confirms that firms with negative earnings have higher average returns. The average slope for stocks with positive E/P (4.72% per month, 4.57 standard errors from 0) shows that average returns increase with E/P when it is positive.

Adding size to the regressions kills the explanatory power of the E/P dummy. Thus the high average returns of negative E/P stocks are better captured by their size, which Table IV says is on average small. Adding both size and book-to-market equity to the E/P regressions kills the E/P dummy and lowers the average slope on E/P from 4.72 to 0.87 (t = 1.23). In contrast, the average slopes for ln(ME) and ln(BE/ME) in the regressions that include E/P are similar to those in the regressions that explain average returns with only size and book-to-market equity. The results suggest that most of the relation between (positive) E/P and average return is due to the positive correlation between E/P and ln(BE/ME), illustrated in Table IV; firms with high E/P tend to have high book-to-market equity ratios.

IV. A Parsimonious Model for Average Returns

The results to here are easily summarized:

(1) When we allow for variation in β that is unrelated to size, there is no reliable relation between β and average return.
(2) The opposite roles of market leverage and book leverage in average returns are captured well by book-to-market equity.
(3) The relation between E/P and average return seems to be absorbed by the combination of size and book-to-market equity.

In a nutshell, market β seems to have no role in explaining the average returns on NYSE, AMEX, and NASDAQ stocks for 1963–1990, while size and book-to-market equity capture the cross-sectional variation in average stock returns that is related to leverage and E/P.

A. Average Returns, Size and Book-to-Market Equity

The average return matrix in Table V gives a simple picture of the two-dimensional variation in average returns that results when the 10 size deciles are each subdivided into 10 portfolios based on ranked values of BE/ME for individual stocks. Within a size decile (across a row of the average return matrix), returns typically increase strongly with BE/ME: on average, the returns on the lowest and highest BE/ME portfolios in a size decile differ by 0.99% (1.63% – 0.64%) per month. Similarly, looking down the columns of the average return matrix shows that there is a negative relation between average return and size: on average, the spread of returns across the size portfolios in a BE/ME group is 0.58% per month. The average return matrix gives life to the conclusion from the regressions that,

Table V

Average Monthly Returns on Portfolios Formed on Size and Book-to-Market Equity; Stocks Sorted by ME (Down) and then BE/ME (Across): July 1963 to December 1990

In June of each year t, the NYSE, AMEX, and NASDAQ stocks that meet the CRSP-COMPUSTAT data requirements are allocated to 10 size portfolios using the NYSE size (ME) breakpoints. The NYSE, AMEX, and NASDAQ stocks in each size decile are then sorted into 10 BE/ME portfolios using the book-to-market ratios for year $t-1$. BE/ME is the book value of common equity plus balance-sheet deferred taxes for fiscal year $t-1$, over market equity for December of year $t-1$. The equal-weighted monthly portfolio returns are then calculated for July of year t to June of year $t+1$.

Average monthly return is the time-series average of the monthly equal-weighted portfolio returns (in percent).

The All column shows average returns for equal-weighted size decile portfolios. The All row shows average returns for equal-weighted portfolios of the stocks in each BE/ME group.

		Book-to-Market Portfolios									
	All	Low	2	3	4	5	6	7	8	9	High
All	1.23	0.64	0.98	1.06	1.17	1.24	1.26	1.39	1.40	1.50	1.63
Small-ME	1.47	0.70	1.14	1.20	1.43	1.56	1.51	1.70	1.71	1.82	1.92
ME-2	1.22	0.43	1.05	0.96	1.19	1.33	1.19	1.58	1.28	1.43	1.79
ME-3	1.22	0.56	0.88	1.23	0.95	1.36	1.30	1.30	1.40	1.54	1.60
ME-4	1.19	0.39	0.72	1.06	1.36	1.13	1.21	1.34	1.59	1.51	1.47
ME-5	1.24	0.88	0.65	1.08	1.47	1.13	1.43	1.44	1.26	1.52	1.49
ME-6	1.15	0.70	0.98	1.14	1.23	0.94	1.27	1.19	1.19	1.24	1.50
ME-7	1.07	0.95	1.00	0.99	0.83	0.99	1.13	0.99	1.16	1.10	1.47
ME-8	1.08	0.66	1.13	0.91	0.95	0.99	1.01	1.15	1.05	1.29	1.55
ME-9	0.95	0.44	0.89	0.92	1.00	1.05	0.93	0.82	1.11	1.04	1.22
Large-ME	0.89	0.93	0.88	0.84	0.71	0.79	0.83	0.81	0.96	0.97	1.18

controlling for size, book-to-market equity captures strong variation in average returns, and controlling for book-to-market equity leaves a size effect in average returns.

B. The Interaction between Size and Book-to-Market Equity

The average of the monthly correlations between the cross-sections of ln(ME) and ln(BE/ME) for individual stocks is -0.26. The negative correlation is also apparent in the average values of ln(ME) and ln(BE/ME) for the portfolios sorted on ME or BE/ME in Tables II and IV. Thus, firms with low market equity are more likely to have poor prospects, resulting in low stock prices and high book-to-market equity. Conversely, large stocks are more likely to be firms with stronger prospects, higher stock prices, lower book-to-market equity, and lower average stock returns.

The correlation between size and book-to-market equity affects the regressions in Table III. Including ln(BE/ME) moves the average slope on ln(ME) from -0.15 ($t = -2.58$) in the univariate regressions to -0.11 ($t = -1.99$) in the bivariate regressions. Similarly, including ln(ME) in the regressions lowers the average slope on ln(BE/ME) from 0.50 to 0.35 (still a healthy 4.44 standard errors from 0). Thus, part of the size effect in the simple regressions is due to the fact that small ME stocks are more likely to have high book-to-market ratios, and part of the simple book-to-market effect is due to the fact that high BE/ME stocks tend to be small (they have low ME).

We should not, however, exaggerate the links between size and book-to-market equity. The correlation (−0.26) between ln(ME) and ln(BE/ME) is not extreme, and the average slopes in the bivariate regressions in Table III show that ln(ME) and ln(BE/ME) are both needed to explain the cross-section of average returns. Finally, the 10 × 10 average return matrix in Table V provides concrete evidence that, (a) controlling for size, book-to-market equity captures substantial variation in the cross-section of average returns, and (b) within BE/ME groups average returns are related to size.

C. Subperiod Averages of the FM Slopes

The message from the average FM slopes for 1963–1990 (Table III) is that size on average has a negative premium in the cross-section of stock returns, book-to-market equity has a positive premium, and the average premium for market β is essentially 0. Table VI shows the average FM slopes for two roughly equal subperiods (July 1963–December 1976 and January 1977–December 1990) from two regressions: (a) the cross-section of stock returns on size, ln(ME), and book-to-market equity, ln(BE/ME), and (b) returns on β, ln(ME), and ln(BE/ME). For perspective, average returns on the value-weighted and equal-weighted (VW and EW) portfolios of NYSE stocks are also shown.

In FM regressions, the intercept is the return on a standard portfolio (the weights on stocks sum to 1) in which the weighted averages of the explanatory variables are 0 (Fama (1976), chapter 9). In our tests, the intercept is weighted toward small stocks (ME is in millions of dollars so ln(ME) = 0 implies ME = $1 million) and toward stocks with relatively high book-to-market ratios (Table IV says that ln(BE/ME) is negative for the typical firm, so ln(BE/ME) = 0 is toward the high end of the sample ratios). Thus it is not surprising that the average intercepts are always large relative to their standard errors and relative to the returns on the NYSE VW and EW portfolios.

Like the overall period, the subperiods do not offer much hope that the average premium for β is economically important. The average FM slope for β is only slightly positive for 1963–1976 (0.10% per month, $t = 0.25$), and it is negative for 1977–1990 (−0.44% per month, $t = -1.17$). There is a hint that the size effect is weaker in the 1977–1990 period, but inferences about the average size slopes for the subperiods lack power.

Unlike the size effect, the relation between book-to-market equity and average return is so strong that it shows up reliably in both the 1963–1976 and the 1977–1990 subperiods. The average slopes for ln(BE/ME) are all more than 2.95 standard errors from 0, and the average slopes for the

Table VI

Subperiod Average Monthly Returns on the NYSE Equal-Weighted and Value-Weighted Portfolios and Subperiod Means of the Intercepts and Slopes from the Monthly FM Cross-Sectional Regressions of Returns on (a) Size (ln(ME)) and Book-to-Market Equity (ln(BE/ME)), and (b) β, ln(ME), and ln(BE/ME)

Mean is the time-series mean of a monthly return, Std is its time-series standard deviation, and t(Mn) is Mean divided by its time-series standard error.

	7/63–12/90 (330 Mos.)			7/63–12/76 (162 Mos.)			1/77–12/90 (168 Mos.)		
Variable	Mean	Std	t(Mn)	Mean	Std	t(Mn)	Mean	Std	t(Mn)
NYSE Value-Weighted (VW) and Equal-Weighted (EW) Portfolio Returns									
VW	0.81	4.47	3.27	0.56	4.26	1.67	1.04	4.66	2.89
EW	0.97	5.49	3.19	0.77	5.70	1.72	1.15	5.28	2.82
$R_{it} = a + b_{2t}\ln(ME_{it}) + b_{3t}\ln(BE/ME_{it}) + e_{it}$									
a	1.77	8.51	3.77	1.86	10.10	2.33	1.69	6.67	3.27
b_2	−0.11	1.02	−1.99	−0.16	1.25	−1.62	−0.07	0.73	−1.16
b_3	0.35	1.45	4.43	0.36	1.53	2.96	0.35	1.37	3.30
$R_{it} = a + b_{1t}\beta_{it} + b_{2t}\ln(ME_{it}) + b_{3t}\ln(BE/ME_{it}) + e_{it}$									
a	2.07	5.75	6.55	1.73	6.22	3.54	2.40	5.25	5.92
b_1	−0.17	5.12	−0.62	0.10	5.33	0.25	−0.44	4.91	−1.17
b_2	−0.12	0.89	−2.52	−0.15	1.03	−1.91	−0.09	0.74	−1.64
b_3	0.33	1.24	4.80	0.34	1.36	3.17	0.31	1.10	3.67

subperiods (0.36 and 0.35) are close to the average slope (0.35) for the overall period. The subperiod results thus support the conclusion that, among the variables considered here, book-to-market equity is consistently the most powerful for explaining the cross-section of average stock returns.

Finally, Roll (1983) and Keim (1983) show that the size effect is stronger in January. We have examined the monthly slopes from the FM regressions in Table VI for evidence of a January seasonal in the relation between book-to-market equity and average return. The average January slopes for ln(BE/ME) are about twice those for February to December. Unlike the size effect, however, the strong relation between book-to-market equity and average return is not special to January. The average monthly February-to-December slopes for ln(BE/ME) are about 4 standard errors from 0, and they are close to (within 0.05 of) the average slopes for the whole year. Thus, there is a January seasonal in the book-to-market equity effect, but the positive relation between BE/ME and average return is strong throughout the year.

D. β and the Market Factor: Caveats

Some caveats about the negative evidence on the role of β in average returns are in order. The average premiums for β, size, and book-to-market equity depend on the definitions of the variables used in the regressions. For example, suppose we replace book-to-market equity (ln(BE/ME)) with book equity (ln(BE)). As long as size (ln(ME)) is also in the regression, this change will not affect the intercept, the fitted values or the R^2. But the change, in variables increases the average slope (and the t-statistic) on ln(ME). In other words, it increases the risk premium associated with size. Other redefinitions of the β, size, and book-to-market variables will produce different regression slopes and perhaps different inferences about average premiums, including possible resuscitation of a role for β. And, of course, at the moment, we have no theoretical basis for choosing among different versions of the variables.

Moreover, the tests here are restricted to stocks. It is possible that including other assets will change the inferences about the average premiums for β, size, and book-to-market equity. For example, the large average intercepts for the FM regressions in Table VI suggest that the regressions will not do a good job on Treasury bills, which have low average returns and are likely to have small loadings on the underlying market, size, and book-to-market factors in returns. Extending the tests to bills and other bonds may well change our inferences about average risk premiums, including the revival of a role for market β.

We emphasize, however, that different approaches to the tests are not likely to revive the Sharpe-Lintner-Black model. Resuscitation of the SLB model requires that a better proxy for the market portfolio (a) overturns our evidence that the simple relation between β and average stock returns is flat and (b) leaves β as the only variable relevant for explaining average returns. Such results seem unlikely, given Stambaugh's (1982) evidence that tests of the SLB model do not seem to be sensitive to the choice of a market proxy. Thus, if there is a role for β in average returns, it is likely to be found in a multi-factor model that transforms the flat simple relation between average return and β into a positively sloped conditional relation.

V. Conclusions and Implications

The Sharpe-Lintner-Black model has long shaped the way academics and practitioners think about average return and risk. Black, Jensen, and Scholes (1972) and Fama and MacBeth (1973) find that, as predicted by the model, there is a positive simple relation between average return and market β during the early years (1926–1968) of the CRSP NYSE returns file. Like Reinganum (1981) and Lakonishok and Shapiro (1986), we find that this simple relation between β and average return disappears during the more recent 1963–1990 period. The appendix that follows shows that the relation between β and average return is also weak in the last half century (1941–1990) of returns on NYSE stocks. In short, our tests do not support the central prediction of the SLB model, that average stock returns are positively related to market β.

Banz (1981) documents a strong negative relation between average return and firm size. Bhandari (1988) finds that average return is positively related to leverage, and Basu (1983) finds a positive relation between average return and E/P. Stattman (1980) and Rosenberg, Reid, and Lanstein (1985) document a positive relation between average return and book-to-market equity for U.S. stocks, and Chan, Hamao, and Lakonishok (1992) find that BE/ME is also a powerful variable for explaining average returns on Japanese stocks.

Variables like size, E/P, leverage, and book-to-market equity are all scaled versions of a firm's stock price. They can be regarded as different ways of extracting information from stock prices about the cross-section of expected stock returns (Ball (1978), Keim (1988)). Since all these variables are scaled versions of price, it is reasonable to expect that some of them are redundant for explaining average returns. Our main result is that for the 1963–1990 period, size and book-to-market equity capture the cross-sectional variation in average stock returns associated with size, E/P, book-to-market equity, and leverage.

A. Rational Asset-Pricing Stories

Are our results consistent with asset-pricing theory? Since the FM intercept is constrained to be the same for all stocks, FM regressions always impose a linear factor structure on returns and expected returns that is consistent with the multifactor asset-pricing models of Merton (1973) and Ross (1976). Thus our tests impose a rational asset-pricing framework on the relation between average return and size and book-to-market equity.

Even if our results are consistent with asset-pricing theory, they are not economically satisfying. What is the economic explanation for the roles of size and book-to-market equity in average returns? We suggest several paths of inquiry.

(a) The intercepts and slopes in the monthly FM regressions of returns on ln(ME) and ln(BE/ME) are returns on portfolios that mimic the underlying common risk factors in returns proxied by size and book-to-market equity (Fama (1976), chapter 9). Examining the relations between the returns on these portfolios and economic variables that measure variation in business conditions might help expose the nature of the economic risks captured by size and book-to-market equity.

(b) Chan, Chen, and Hsieh (1985) argue that the relation between size and average return proxies for a more fundamental relation between expected returns and economic risk factors. Their most powerful factor in explaining the size effect is the difference between the monthly returns on low- and high-grade corporate bonds, which in principle captures a kind of default risk in returns that is priced. It would be interesting to test whether loadings on this or other economic factors, such as those of Chen, Roll, and Ross (1986), can explain the roles of size and book-to-market equity in our tests.

(c) In a similar vein, Chan and Chen (1991) argue that the relation between size and average return is a relative-prospects effect. The earning prospects of distressed firms are more sensitive to economic conditions. This results in a distress factor in returns that is priced in expected returns. Chan and Chen construct two mimicking portfolios for the distress factor, based on dividend changes and leverage. It would be interesting to check whether loadings on their distress factors absorb the size and book-to-market equity effects in average returns that are documented here.

(d) In fact, if stock prices are rational, BE/ME, the ratio of the book value of a stock to the market's assessment of its value, should be a direct indicator of the relative prospects of firms. For example, we expect that high BE/ME firms have low earnings on assets relative to low BE/ME firms. Our work (in progress) suggests that there is indeed a clean separation between high and low BE/ME firms on various measures of economic fundamentals. Low BE/ME firms are persistently strong performers, while the economic performance of high BE/ME firms is persistently weak.

B. Irrational Asset-Pricing Stories

The discussion above assumes that the asset-pricing effects captured by size and book-to-market equity are rational. For BE/ME, our most powerful expected-return variable, there is an obvious alternative. The cross-section of book-to-market ratios might result from market overreaction to the relative prospects of firms. If overreaction tends to be corrected, BE/ME will predict the cross-section of stock returns.

Simple tests do not confirm that the size and book-to-market effects in average returns are due to market overreaction, at least of the type posited by DeBondt and Thaler (1985). One overreaction measure used by DeBondt and Thaler is a stock's most recent 3-year return. Their overreaction story predicts that 3-year losers have strong post-ranking returns relative to 3-year winners. In FM regressions (not shown) for individual stocks, the 3-year lagged return shows no power even when used alone to explain average returns. The univariate average slope for the lagged return is negative, -6 basis points per month, but less than 0.5 standard errors from 0.

C. Applications

Our main result is that two easily measured variables, size and book-to-market equity, seem to describe the cross-section of average stock returns. Prescriptions for using this evidence depend on (a) whether it will persist, and (b) whether it results from rational or irrational asset-pricing.

It is possible that, by chance, size and book-to-market equity happen to describe the cross-section of average returns in our sample, but they were and are unrelated to expected returns. We put little weight on this possibility, especially for book-to-market equity. First, although BE/ME has long been touted as a measure of the return prospects of stocks, there is no evidence that its explanatory power deteriorates through time. The 1963–1990 relation between BE/ME and average return is strong, and remarkably similar for the 1963–1976 and 1977–1990 subperiods. Second, our preliminary work on economic fundamentals suggests that high-BE/ME firms tend to be persistently poor earners relative to low-BE/ME firms. Similarly, small firms have a long period of poor earnings during the 1980s not shared with big firms. The systematic patterns in fundamentals give us some hope that size and book-to-market equity proxy for risk factors in returns, related to relative earning prospects, that are rationally priced in expected returns.

If our results are more than chance, they have practical implications for portfolio formation and performance evaluation by investors whose primary concern is long-term average returns. If asset-pricing is rational, size and BE/ME must proxy for risk. Our results then imply that the performance of managed portfolios (e.g., pension funds and mutual funds) can be evaluated by comparing their average returns with the average returns of benchmark portfolios with similar size and BE/ME characteristics. Likewise, the expected returns for different portfolio strategies can be estimated from the historical average returns of portfolios with matching size and BE/ME properties.

If asset-pricing is irrational and size and BE/ME do not proxy for risk, our results might still be used to evaluate portfolio performance and measure the expected returns from alternative investment strategies. If stock prices are irrational, however, the likely persistence of the results is more suspect.

Appendix
Size Versus β: 1941–1990

Our results on the absence of a relation between β and average stock returns for 1963–1990 are so contrary to the tests of the Sharpe-Lintner-Black model by Black, Jensen, and Scholes (1972), Fama and MacBeth (1973), and (more recently) Chan and Chen (1988), that further tests are appropriate. We examine the roles of size and β in the average returns on NYSE stocks for the half-century 1941–1990, the longest available period that avoids the high volatility of returns in the Great Depression. We do not include the accounting variables in the tests because of the strong selection bias (toward successful firms) in the COMPUSTAT data prior to 1962.

We first replicate the results of Chan and Chen (1988). Like them, we find that when portfolios are formed on size alone, there are strong relations between average return and either size or β; average return increases with β and decreases with size. For size portfolios, however, size (ln(ME)) and β are almost perfectly correlated (-0.98), so it is difficult to distinguish between the roles of size and β in average returns.

One way to generate strong variation in β that is unrelated to size is to form portfolios on size and then on β. As in Tables I to III, we find that the resulting independent variation in β just about washes out the positive simple relation between average return and β observed when portfolios are formed on size alone. The results for NYSE stocks for 1941–1990, are thus much like those for NYSE, AMEX, and NASDAQ stocks for 1963–1990.

This appendix also has methodological goals. For example, the FM regressions in Table III use returns on individual stocks as the dependent variable. Since we allocate portfolio βs to individual stocks but use firm-specific values of other variables like size, β may be at a disadvantage in the regressions for individual stocks. This appendix shows, however, that regressions for portfolios, which put β and size on equal footing, produce results comparable to those for individual stocks.

A. Size Portfolios

Table AI shows average monthly returns and market βs for 12 portfolios of NYSE stocks formed on the basis of size (ME) at the end of each year from 1940 to 1989. For these size portfolios, there is a strong positive relation between average return and β. Average returns fall from 1.96% per month for the smallest ME portfolio (1A) to 0.93% for the largest (10B) and β falls from 1.60 to 0.95. (Note also that, as claimed earlier, estimating β as the sum of the slopes in the regression of a portfolio's return on the current and prior month's NYSE value-weighted return produces much larger βs for the smallest ME portfolios and slightly smaller βs for the largest ME portfolios.)

The FM regressions in Table AI confirm the positive simple relation between average return and β for size portfolios. In the regressions of the size-portfolio returns on β alone, the average premium for a unit of β is 1.45% per month. In the regressions of individual stock returns on β (where stocks are assigned the β of their size portfolio), the premium for a unit of β is 1.39%. Both estimates are about 3 standard errors from 0. Moreover, the βs of size portfolios do not leave a residual size effect; the average residuals from the simple regressions of returns on β in Table AI show no relation to size. These positive SLB results for 1941–1990 are like those obtained by Chan and Chen (1988) in tests on size portfolios for 1954–1983.

There is, however, evidence in Table AI that all is not well with the βs of the size portfolios. They do a fine job on the relation between size and average return, but they do a lousy job on their main task, the relation between β and average return. When the residuals from the regressions of returns on β are grouped using the pre-ranking βs of individual stocks, the average residuals are strongly positive for low-β stocks (0.51% per month for group 1A) and negative for high-β stocks (-1.05% for 10B). Thus the market lines estimated with size-portfolio βs exaggerate the tradeoff of average return for β; they underestimate average returns on low-β stocks and overestimate average returns on high-β stocks. This pattern in the β-sorted average residuals for individual stocks suggests that (a) there is variation in β across stocks that is lost in the size portfolios, and (b) this variation in β is not rewarded as well as the variation in β that is related to size.

B. Two-Pass Size-β Portfolios

Like Table I, Table AII shows that subdividing size deciles using the (pre-ranking) βs of individual stocks results in strong variation in β that is

Table AI

Average Returns, Post-Ranking βs and Fama-MacBeth Regression Slopes for Size Portfolios of NYSE Stocks: 1941–1990

At the end of each year $t-1$, stocks are assigned to 12 portfolios using ranked values of ME. Included are all NYSE stocks that have a CRSP price and shares for December of year $t-1$ and returns for at least 24 of the 60 months ending in December of year $t-1$ (for pre-ranking β estimates). The middle 8 portfolios cover size deciles 2 to 9. The 4 extreme portfolios (1A, 1B, 10A, and 10B) split the smallest and largest deciles in half. We compute equal-weighted returns on the portfolios for the 12 months of year t using all surviving stocks. Average Return is the time-series average of the monthly portfolio returns for 1941–1990, in percent. Average firms is the average number of stocks in the portfolios each month. The simple βs are estimated by regressing the 1941–1990 sample of post-ranking monthly returns for a size portfolio on the current month's value-weighted NYSE portfolio return. The sum βs are the sum of the slopes from a regression of the post-ranking monthly returns on the current and prior month's VW NYSE returns.

The independent variables in the Fama-MacBeth regressions are defined for each firm at the end of December of each year $t-1$. Stocks are assigned the post-ranking (sum) β of the size portfolio they are in at the end of year $t-1$. ME is price times shares outstanding at the end of year $t-1$. In the individual-stock regressions, these values of the explanatory variables are matched with CRSP returns for each of the 12 months of year t. The portfolio regressions match the equal-weighted portfolio returns with the equal-weighted averages of β and ln(ME) for the surviving stocks in each month of year t. Slope is the average of the (600) monthly FM regression slopes and SE is the standard error of the average slope. The residuals from the monthly regressions for year t are grouped into 12 portfolios on the basis of size (ME) or pre-ranking β (estimated with 24 to 60 months of data, as available) at the end of year $t-1$. The average residuals are the time-series averages of the monthly equal-weighted portfolio residuals, in percent. The average residuals for regressions (1) and (2) (not shown) are quite similar to those for regressions (4) and (5) (shown).

	Portfolios Formed on Size											
	1A	1B	2	3	4	5	6	7	8	9	10A	10B
Ave. return	1.96	1.59	1.44	1.36	1.28	1.24	1.23	1.17	1.15	1.13	0.97	0.93
Ave. firms	57	56	110	107	107	108	111	113	115	118	59	59
Simple β	1.29	1.24	1.21	1.19	1.16	1 13	1.13	1.12	1.09	1.05	1.00	0.98
Standard error	0.07	0.05	0.04	0.03	0.02	0 02	0.02	0.02	0.01	0.01	0.01	0.01
Sum β	1.60	1.44	1.37	1.32	1.26	1.23	1.19	1.17	1.12	1.06	0.99	0.95
Standard error	0.10	0.06	0.05	0.04	0.03	0.03	0.03	0.02	0.02	0.01	0.01	0.01

Table AI—*Continued*

	Portfolio Regressions				Individual Stock Regressions			
	(1) β	(2) ln(ME)	(3) β and ln(ME)		(4) β	(5) ln(ME)	(6) β and ln(ME)	
Slope	1.45	−0.137	3 05	0.149	1.39	−0.133	0.71	−0 060
SE	0.47	0.044	1.51	0 115	0.46	0.043	0.81	0 062

Average Residuals for Stocks Grouped on Size

	1A	1B	2	3	4	5	6	7	8	9	10A	10B
Regression (4)	0.17	0.00	−0.04	−0.06	−0.05	−0.04	0 00	−0 03	0.03	0 08	0.01	0 04
Standard error	0.11	0.06	0.04	0.04	0.04	0.04	0.03	0.03	0.03	0.03	0.05	0.06
Regression (5)	0.30	0.02	−0.05	−0 06	−0.08	−0.07	−0.03	−0.04	0.02	0.08	0.01	0.13
Standard error	0.14	0.07	0 04	0.04	0 04	0 04	0.04	0.03	0.03	0.03	0.04	0 07
Regression (6)	0.20	0.02	−0.05	−0.07	−0.08	−0 06	−0 01	−0 02	0.04	0 09	0.00	0.06
Standard error	0.10	0.06	0.04	0 04	0 04	0.04	0.03	0.03	0.03	0.03	0 05	0 05

Average Residuals for Stocks Grouped on Pre-Ranking β

	1A	1B	2	3	4	5	6	7	8	9	10A	10B
Regression (4)	0.51	0.61	0.38	0.32	0.16	0.12	0.03	−0.10	−0.27	−0.31	−0.66	−1.05
Standard error	0.21	0.19	0.13	0.08	0 04	0.03	0.04	0 05	0.09	0 11	0 18	0.23
Regression (5)	−0.10	0.00	0.02	0.09	0.05	0.07	0.05	0.00	−0.03	−0.01	−0.11	−0 33
Standard error	0.11	0.10	0.07	0.05	0.04	0.03	0.03	0.04	0.05	0.07	0 10	0 13
Regression (6)	0.09	0.25	0.13	0.19	0.11	0.14	0.09	0.01	−0.11	−0.12	−0.38	−0 70
Standard error	0.41	0.37	0.24	0.14	0 07	0.04	0.04	0.09	0.16	0.21	0 34	0.43

Table AII

Properties of Portfolios Formed on Size and Pre-Ranking β: NYSE Stocks Sorted by ME (Down) then Pre-Ranking β (Across): 1941–1990

At the end of year $t-1$, the NYSE stocks on CRSP are assigned to 10 size (ME) portfolios Each size decile is subdivided into 10 β portfolios using pre-ranking βs of individual stocks, estimated with 24 to 60 monthly returns (as available) ending in December of year $t-1$. The equal-weighted monthly returns on the resulting 100 portfolios are then calculated for year t. The average returns are the time-series averages of the monthly returns, in percent. The post-ranking βs use the full 1941–1990 sample of post-ranking returns for each portfolio. The pre- and post-ranking βs are the sum of the slopes from a regression of monthly returns on the current and prior month's NYSE value-weighted market return. The average size for a portfolio is the time-series average of each month's average value of ln(ME) for stocks in the portfolio. ME is denominated in millions of dollars. There are, on average, about 10 stocks in each size-β portfolio each month. The All column shows parameter values for equal-weighted size-decile (ME) portfolios. The All rows show parameter values for equal-weighted portfolios of the stocks in each β group.

	All	Low-β	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	High-β
Panel A: Average Monthly Return (in Percent)											
All		1.22	1.30	1.32	1.35	1.36	1.34	1.29	1.34	1.14	1.10
Small-ME	1.78	1.74	1.76	2.08	1.91	1.92	1.72	1.77	1.91	1.56	1 46
ME-2	1.44	1.41	1.35	1.33	1.61	1.72	1.59	1.40	1.62	1.24	1.11
ME-3	1.36	1.21	1.40	1.22	1.47	1.34	1.51	1.33	1.57	1.33	1 21
ME-4	1.28	1.26	1.29	1.19	1.27	1.51	1.30	1.19	1.56	1.18	1.00
ME-5	1.24	1.22	1.30	1.28	1.33	1.21	1.37	1.41	1.31	0.92	1.06
ME-6	1.23	1.21	1.32	1 37	1.09	1.34	1.10	1.40	1.21	1.22	1.08
ME-7	1.17	1.08	1.23	1.37	1.27	1.19	1.34	1.10	1.11	0.87	1.17
ME-8	1.15	1.06	1.18	1.26	1.25	1.26	1.17	1.16	1.05	1.08	1.04
ME-9	1.13	0.99	1.13	1.00	1.24	1.28	1.31	1.15	1.11	1.09	1.05
Large-ME	0.95	0.99	1.01	1.12	1.01	0.89	0.95	0.95	1.00	0.90	0.68

Table AII—*Continued*

	All	Low-β	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	High-β
					Panel B: Post-Ranking β						
All		0.76	0.95	1.05	1.14	1 22	1.26	1 34	1.38	1.49	1 69
Small-ME	1 52	1.17	1.40	1.31	1.50	1.46	1.50	1 69	1.60	1.75	1.92
ME-2	1.37	0.86	1.09	1.12	1.24	1 39	1.42	1.48	1.60	1.69	1.91
ME-3	1.32	0.88	0.96	1.18	1.19	1.33	1.40	1.43	1.56	1 64	1.74
ME-4	1.26	0.69	0.95	1.06	1.15	1 24	1 29	1 46	1.43	1.64	1.83
ME-5	1.23	0.70	0 95	1.04	1.10	1.22	1.32	1.34	1.41	1.56	1.72
ME-6	1.19	0.68	0.86	1.04	1.13	1.20	1.20	1 35	1.36	1.48	1.70
ME-7	1.17	0 67	0.88	0.95	1 14	1.18	1.26	1.27	1 32	1.44	1.68
ME-8	1.12	0.64	0.83	0 99	1.06	1.14	1.14	1.21	1.26	1.39	1.58
ME-9	1.06	0.68	0.81	0.94	0.96	1.06	1.11	1.18	1.22	1.25	1.46
Large-ME	0 97	0 65	0.73	0 90	0.91	0.97	1.01	1.01	1.07	1.12	1.38
					Panel C: Average Size (ln(ME))						
All		4.39	4.39	4.40	4 40	4.39	4.40	4 38	4.37	4 37	4.34
Small-ME	1 93	2.04	1.99	2.00	1 96	1.92	1.92	1 91	1.90	1 87	1.80
ME-2	2.80	2.81	2.79	2.81	2.83	2 80	2 79	2.80	2.80	2.79	2.79
ME-3	3.27	3.28	3.27	3 28	3.27	3.27	3.28	3.29	3.27	3.27	3.26
ME-4	3.67	3.67	3.67	3.67	3.68	3.68	3.67	3.68	3.66	3.67	3.67
ME-5	4.06	4.07	4.06	4.05	4.06	4.07	4.06	4 05	4.05	4.06	4.06
ME-6	4 45	4.45	4.44	4.46	4.45	4.45	4 45	4.45	4.44	4.45	4.45
ME-7	4.87	4.86	4.87	4.86	4 87	4.87	4.88	4.87	4.87	4 85	4.87
ME-8	5.36	5.38	5.38	5.38	5.35	5.36	5.37	5.37	5.36	5 35	5.34
ME-9	5.98	5.96	5.98	5.99	6 00	5.98	5.98	5.97	5.95	5 96	5.96
Large-ME	7.12	7.10	7.12	7.16	7.17	7 20	7.29	7.14	7.09	7 04	6.83

independent of size. The β sort of a size decile always produces portfolios with similar average ln(ME) but much different (post-ranking) βs. Table AII also shows, however, that investors are not compensated for the variation in β that is independent of size. Despite the wide range of βs in each size decile, average returns show no tendency to increase with β. AII

The FM regressions in Table AIII formalize the roles of size and β in NYSE average returns for 1941–1990. The regressions of returns on β alone show that using the βs of the portfolios formed on size and β, rather than size alone, causes the average slope on β to fall from about 1.4% per month (Table AI) to about 0.23% (about 1 standard error from 0). Thus, allowing for variation in β that is unrelated to size flattens the relation between average return and β, to the point where it is indistinguishable from no relation at all.

The flatter market lines in Table AIII succeed, however, in erasing the negative relation between β and average residuals observed in the regressions of returns on β alone in Table AI. Thus, forming portfolios on size and β (Table AIII) produces a better description of the simple relation between average return and β than forming portfolios on size alone (Table AI). This improved description of the relation between average return and β is evidence that the β estimates for the two-pass size-β portfolios capture variation in true βs that is missed when portfolios are formed on size alone.

Unfortunately, the flatter market lines in Table AIII have a cost, the emergence of a residual size effect. Grouped on the basis of ME for individual stocks, the average residuals from the univariate regressions of returns on the βs of the 100 size-β portfolios are strongly positive for small stocks and negative for large stocks (0.60% per month for the smallest ME group, 1A, and −0.27% for the largest, 10B). Thus, when we allow for variation in β that is independent of size, the resulting βs leave a large size effect in average returns. This residual size effect is much like that observed by Banz (1981) with the βs of portfolios formed on size and β.

The correlation between size and β is −0.98 for portfolios formed on size alone. The independent variation in β obtained with the second-pass sort on β lowers the correlation to −0.50. The lower correlation means that bivariate regressions of returns on β and ln(ME) are more likely to distinguish true size effects from true β effects in average returns.

The bivariate regressions (Table AIII) that use the βs of the size-β portfolios are more bad news for β. The average slopes for ln(ME) are close to the values in the univariate size regressions, and almost 4 standard errors from 0, but the average slopes for β are negative and less than 1 standard error from 0. The message from the bivariate regressions is that there is a strong relation between size and average return. But like the regressions in Table AIII that explain average returns with β alone, the bivariate regressions say that there is no reliable relation between β and average returns when the tests use βs that are not close substitutes for size. These uncomfortable SLB results for NYSE stocks for 1941–1990 are much like those for NYSE, AMEX, and NASDAQ stocks for 1963–1990 in Table III.

C. Subperiod Diagnostics

Our results for 1941–1990 seem to contradict the evidence in Black, Jensen, and Scholes (BJS) (1972) and Fama and MacBeth (FM) (1973) that there is a reliable positive relation between average return and β. The βs in BJS and FM are from portfolios formed on β alone, and the market proxy is the NYSE equal-weighted portfolio. We use the βs of portfolios formed on size and β, and our market is the value-weighted NYSE portfolio. We can report, however, that our inference that there isn't much relation between β and average return is unchanged when (a) the market proxy is the NYSE EW portfolio, (b) portfolios are formed on just (pre-ranking) βs, or (c) the order of forming the size-β portfolios is changed from size then β to β then size.

A more important difference between our results and the earlier studies is the sample periods. The tests in BJS and FM end in the 1960s. Table AIV shows that when we split the 50-year 1941–1990 period in half, the univariate FM regressions of returns on β produce an average slope for 1941–1965 (0.50% per month, $t = 1.82$) more like that of the earlier studies. In contrast, the average slope on β for 1966–1990 is close to 0 (-0.02, $t = 0.06$).

But Table AIV also shows that drawing a distinction between the results for 1941–1965 and 1966–1990 is misleading. The stronger tradeoff of average return for β in the simple regressions for 1941–1965 is due to the first 10 years, 1941–1950. This is the only period in Table AIV that produces an average premium for β (1.26% per month) that is both positive and more than 2 standard errors from 0. Conversely, the weak relation between β and average return for 1966–1990 is largely due to 1981–1990. The strong negative average slope in the univariate regressions of returns on β for 1981–1990 (-1.01, $t = -2.10$) offsets a positive slope for 1971–1980 (0.82, $t = 1.27$).

The subperiod variation in the average slopes from the FM regressions of returns on β alone seems moot, however, given the evidence in Table AIV that adding size always kills any positive tradeoff of average return for β in the subperiods. Adding size to the regressions for 1941–1965 causes the average slope for β to drop from 0.50 ($t = 1.82$) to 0.07 ($t = 0.28$). In contrast, the average slope on size in the bivariate regressions (-0.16, $t = -2.97$) is close to its value (-0.17, $t = -2.88$) in the regressions of returns on ln(ME) alone. Similar comments hold for 1941–1950. In short, any evidence of a positive average premium for β in the subperiods seems to be a size effect in disguise.

D. Can the SLB Model Be Saved?

Before concluding that β has no explanatory power, it is appropriate to consider other explanations for our results. One possibility is that the variation in β produced by the β sorts of size deciles in just sampling error. If so, it is not surprising that the variation in β within a size decile is unrelated to average return, or that size dominates β in bivariate tests. The standard errors of the βs suggest, however, that this explanation cannot save the SLB

Table AIII

Average Slopes, Their Standard Errors (SE), and Average Residuals from Monthly FM Regressions for Individual NYSE Stocks and for Portfolios Formed on Size and Pre-Ranking β: 1941–1990

Stocks are assigned the post-ranking β of the size-β portfolio they are in at the end of year $t-1$ (Table AII). ln(ME) is the natural log of price times shares outstanding at the end of year $t-1$. In the individual-stock regressions, these values of the explanatory variables are matched with CRSP returns for each of the 12 months in year t. The portfolio regressions match the equal-weighted portfolio returns for the size-β portfolios (Table AII) with the equal-weighted averages of β and ln(ME) for the surviving stocks in each month of year t. Slope is the time-series average of the monthly regression slopes from 1941–1990 (600 months); SE is the time-series standard error of the average slope.

The residuals from the monthly regressions in year t are grouped into 12 portfolios on the basis of size or pre-ranking β (estimated with 24 to 60 months of returns, as available) as of the end of year $t-1$. The average residuals are the time-series averages of the monthly equal-weighted averages of the residuals in percent. The average residuals (not shown) from the FM regressions (1) to (3) that use the returns on the 100 size-β portfolios as the dependent variable are always within 0.01 of those from the regressions for individual stock returns. This is not surprising given that the correlation between the time-series of 1941–1990 monthly FM slopes on β or ln(ME) for the comparable portfolio and individual stock regressions is always greater than 0.99.

	Portfolio Regressions				Individual Stock Regressions			
	(1) β	(2) ln(ME)	(3) β and ln(ME)		(4) β	(5) ln(ME)	(6) β and ln(ME)	
Slope	0.22	−0.128	−0.13	−0.143	0.24	−0.133	−0.14	−0.147
SE	0.24	0.043	0.21	0.039	0.23	0.043	0.21	0.039

	Average Residuals for Stocks Grouped on Size											
	1A	1B	2	3	4	5	6	7	8	9	10A	10B
Regression (4)	0.60	0.26	0.13	0.06	−0.01	−0.03	−0.03	−0.09	−0.10	−0.11	−0.25	−0.27
Standard error	0.21	0.10	0.06	0.04	0.04	0.04	0.04	0.04	0.04	0.05	0.06	0.08
Regression (5)	0.30	0.02	−0.05	−0.06	−0.08	−0.07	−0.03	−0.04	0.02	0.08	0.01	0.13
Standard error	0.14	0.07	0.04	0.04	0.04	0.04	0.04	0.03	0.03	0.03	0.04	0.07
Regression (6)	0.31	0.02	−0.05	−0.06	−0.09	−0.07	−0.03	−0.04	0.02	0.08	0.01	0.13
Standard error	0.14	0.07	0.04	0.04	0.04	0.04	0.04	0.03	0.03	0.03	0.04	0.07

Table AIII—*Continued*

	Portfolio Regressions						Individual Stock Regressions					
	(1) β		(2) ln(ME)		(3) β and ln(ME)		(4) β		(5) ln(ME)		(6) β and ln(ME)	
	Average Residuals for Stocks Grouped on Pre-Ranking β											
	1A	1B	2	3	4	5	6	7	8	9	10A	10B
Regression (4)	−0.08	0.03	−0.01	0.08	0.04	0.08	0.04	0.02	−0.03	0.02	−0.11	−0.32
Standard error	0.07	0.05	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.06	0.07
Regression (5)	−0.10	0.00	0.02	0.09	0.05	0.07	0.05	0.00	−0.03	−0.01	−0.11	−0.33
Standard error	0.11	0.10	0.07	0.05	0.04	0.03	0.03	0.04	0.05	0.07	0.10	0.13
Regression (6)	−0.17	−0.07	−0.02	0.07	0.04	0.06	0.05	0.03	0.00	0.04	−0.04	−0.23
Standard error	0.05	0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.04	0.04	0.06	0.07

Table AIV

Subperiod Average Returns on the NYSE Value-Weighted and Equal-Weighted Portfolios and Average Values of the Intercepts and Slopes for the FM Cross-Sectional Regressions of Individual Stock Returns on β and Size (ln(ME))

Mean is the average VW or EW return or an average slope from the monthly cross-sectional regressions of individual stock returns on β and/or ln(ME). Std is the standard deviation of the time-series of returns or slopes, and t(Mn) is Mean over its time-series standard error. The average slopes (not shown) from the FM regressions that use the returns on the 100 size-β portfolios of Table AII as the dependent variable are quite close to those for individual stock returns. (The correlation between the 1941–1990 month-by-month slopes on β or ln(ME) for the comparable portfolio and individual stock regressions is always greater than 0.99.)

Panel A

	1941–1990 (600 Mos.)			1941–1965 (300 Mos.)			1966–1990 (300 Mos.)		
Variable	Mean	Std	t(Mn)	Mean	Std	t(Mn)	Mean	Std	t(Mn)
NYSE Value-Weighted (VW) and Equal-Weighted (EW) Portfolio Returns									
VW	0.93	4.15	5 49	1.10	3.58	5.30	0.76	4.64	2.85
EW	1.12	5.10	5.37	1.33	4.42	5.18	0.91	5.70	2.77
$R_{it} = a + b_{1t}\beta_{it} + e_{it}$									
a	0.98	3.93	6.11	0.84	3.18	4.56	1.13	4.57	4.26
b_1	0.24	5.52	1.07	0.50	4.75	1.82	−0.02	6.19	−0 06
$R_{it} = a + b_{2t}\ln(ME_{it}) + e_{it}$									
a	1.70	8.24	5.04	1.88	6.43	5.06	1.51	9.72	2.69
b_2	−0.13	1.06	−3.07	−0.17	1.01	−2.88	−0.10	1.11	−1.54
$R_{it} = a + b_{1t}\beta_{it} + b_{2t}\ln(ME_{it}) + e_{it}$									
a	1.97	6.16	7.84	1.80	4.77	6.52	2.14	7.29	5.09
b_1	−0.14	5.05	−0.66	0.07	4.15	0.28	−0.34	5.80	−1.01
b_2	−0.15	0.96	−3.75	−0.16	0.94	−2.97	−0.13	0.99	−2.34

Table AIV—*Continued*

Panel B:

Return	1941–1950 Mean	1941–1950 t(Mn)	1951–1960 Mean	1951–1960 t(Mn)	1961–1970 Mean	1961–1970 t(Mn)	1971–1980 Mean	1971–1980 t(Mn)	1981–1990 Mean	1981–1990 t(Mn)
			NYSE Value-Weighted (VW) and Equal-Weighted (EW) Portfolio Returns							
VW	1.05	2.88	1.18	3.95	0.66	1.84	0.72	1.67	1.04	2.40
EW	1.59	3.16	1.13	3.76	0.88	1.96	1.04	1.82	0.95	2.01
				$R_{it} = a + b_{1t}\beta_{it} + e_{it}$						
a	0.24	0.66	1.41	6.36	0.64	1.94	0.27	0.62	2.35	5.99
b_1	1.26	2.20	−0.19	−0.63	0.32	0.72	0.82	1.27	−1.01	−2.10
				$R_{it} = a + b_{2t}\ln(ME_{it}) + e_{it}$						
a	2.63	3.47	1.08	2.73	1.78	2.50	2.18	2.03	0.82	1.20
b_2	−0.37	−2.90	0.03	0.53	−0.17	−2.19	−0.20	−1.57	0.04	0.57
				$R_{it} = a + b_{1t}\beta_{it} + b_{2t}\ln(ME_{it}) + e_{it}$						
a	2.14	3.93	1.38	4.03	2.01	4.16	1.50	2.12	2.84	4.25
b_1	0.34	0.75	−0.17	−0.53	−0.11	−0.27	0.41	0.75	−1.14	−2.16
b_2	−0.34	−2.92	0.01	0.20	−0.18	−2.89	−0.16	−1.50	−0.07	−0.84

model. The standard errors for portfolios formed on size and β are only slightly larger (0.02 to 0.11) than those for portfolios formed on size alone (0.01 to 0.10, Table AI). And the range of the post-ranking βs within a size decile is always large relative to the standard errors of the βs.

Another possibility is that the proportionality condition (1) for the variation through time in true βs, that justifies the use of full-period post-ranking βs in the FM tests, does not work well for portfolios formed on size and β. If this is a problem, post-ranking βs for the size-β portfolios should not be highly correlated across subperiods. The correlation between the half-period (1941–1965 and 1966–1990) βs of the size-β portfolios is 0.91, which we take to be good evidence that the full-period β estimates for these portfolios are informative about true βs. We can also report that using 5-year βs (pre- or post-ranking) in the FM regressions does not change our negative conclusions about the role of β in average returns, as long as portfolios are formed on β as well as size, or on β alone.

Any attempt to salvage the simple positive relation between β and average return predicted by the SLB model runs into three damaging facts, clear in Table AII. (a) Forming portfolios on size and pre-ranking βs produces a wide range of post-ranking βs in every size decile. (b) The post-ranking βs closely reproduce (in deciles 2 to 10 they exactly reproduce) the ordering of the pre-ranking βs used to form the β-sorted portfolios. It seems safe to conclude that the increasing pattern of the post-ranking βs in every size decile captures the ordering of the true βs. (c) Contrary to the SLB model, the β sorts do not produce a similar ordering of average returns. Within the rows (size deciles) of the average return matrix in Table AII, the high-β portfolios have average returns that are close to or less than the low-β portfolios.

But the most damaging evidence against the SLB model comes from the univariate regressions of returns on β in Table AIII. They say that when the tests allow for variation in β that is unrelated to size, the relation between β and average return for 1941–1990 is weak, perhaps nonexistent, even when β is the only explanatory variable. We are forced to conclude that the SLB model does not describe the last 50 years of average stock returns.

REFERENCES

Alford, Andrew, Jennifer J. Jones, and Mark E. Zmijewski, 1992, Extensions and violations of the statutory SEC Form 10-K filing date, Unpublished manuscript, University of Chicago, Chicago, IL.

Ball, Ray, 1978, Anomalies in relationships between securities' yields and yield-surrogates, *Journal of Financial Economics* 6, 103–126.

Banz, Rolf W., 1981, The relationship between return and market value of common stocks, *Journal of Financial Economics* 9, 3–18.

Basu, Sanjoy, 1983, The relationship between earnings yield, market value, and return for NYSE common stocks: Further evidence, *Journal of Financial Economics* 12, 129–156.

Bhandari, Laxmi Chand, 1988, Debt/Equity ratio and expected common stock returns: Empirical evidence, *Journal of Finance* 43, 507–528.

Black, Fischer, 1972, Capital market equilibrium with restricted borrowing, *Journal of Business* 45, 444–455.

———, Michael C. Jensen, and Myron Scholes, 1972, The capital asset pricing model: some empirical tests, in M. Jensen, ed.: *Studies in the Theory of Capital Markets* (Praeger).

Chan, Louis K., Yasushi Hamao, and Josef Lakonishok, 1991, Fundamentals and stock returns in Japan, *Journal of Finance* 46, 1739–1789

Chan, K. C. and Nai-fu Chen, 1988, An unconditional asset-pricing test and the role of firm size as an instrumental variable for risk, *Journal of Finance* 43, 309–325.

———, and Nai-fu Chen, 1991, Structural and return characteristics of small and large firms, *Journal of Finance* 46, 1467–1484.

———, Nai-fu Chen, and David A. Hsieh, 1985, An exploratory investigation of the firm size effect, *Journal of Financial Economics* 14, 451–471.

Chen, Nai-fu, Richard Roll, and Stephen A. Ross, 1986, Economic forces and the stock market, *Journal of Business* 56, 383–403.

DeBondt, Werner F. M., and Richard H. Thaler, 1985, Does the stock market overreact, *Journal of Finance* 40, 557–581.

Dimson, Elroy, 1979, Risk measurement when shares are subject to infrequent trading, *Journal of Financial Economics* 7, 197–226.

Fama, Eugene F., 1976, *Foundations of Finance* (Basic Books, New York).

———, and James MacBeth, 1973, Risk, return and equilibrium: Empirical tests, *Journal of Political Economy* 81, 607–636.

Fowler, David J. and C. Harvey Rorke, 1983, Risk measurement when shares are subject to infrequent trading: Comment, *Journal of Financial Economics* 12, 279–283.

Jaffe, Jeffrey, Donald B. Keim, and Randolph Westerfield, 1989, Earnings yields, market values, and stock returns, *Journal of Finance* 44, 135–148.

Keim, Donald B., 1983, Size-related anomalies and stock return seasonality, *Journal of Financial Economics* 12, 13–32.

———, 1988, Stock market regularities: A synthesis of the evidence and explanations, in Elroy Dimson, ed.: *Stock Market Anomalies* (Cambridge University Press, Cambridge).

Lakonishok, Josef, and Alan C. Shapiro, 1986, Systematic risk, total risk and size as determinants of stock market returns, *Journal of Banking and Finance* 10, 115–132.

Lintner, John, 1965, The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics* 47, 13–37.

Markowitz, Harry, 1959, *Portfolio Selection: Efficient Diversification of Investments* (Wiley, New York).

Merton, Robert C., 1973, An intertemporal capital asset pricing model, *Econometrica* 41, 867–887.

Reinganum, Marc R., 1981, A new empirical perspective on the CAPM, *Journal of Financial and Quantitative Analysis* 16, 439–462.

Roll, Richard, 1983, Vas ist Das? The turn-of-the-year effect and the return premia of small firms, *Journal of Portfolio Management* 9, 18–28.

Rosenberg, Barr, Kenneth Reid, and Ronald Lanstein, 1985, Persuasive evidence of market inefficiency, *Journal of Portfolio Management* 11, 9–17.

Ross, Stephen A., 1976, The arbitrage theory of capital asset pricing, *Journal of Economic Theory* 13, 341–360.

Sharpe, William F., 1964, Capital asset prices: a theory of market equilibrium under conditions of risk, *Journal of Finance* 19, 425–442.

Stambaugh, Robert F., 1982, On the exclusion of assets from tests of the two-parameter model: A sensitivity analysis, *Journal of Financial Economics* 10, 237–268.

Stattman, Dennis, 1980, Book values and stock returns, *The Chicago MBA: A Journal of Selected Papers* 4, 25–45.

REQUEST:

[Rate of Return] - Please provide copies of all workpapers, spreadsheets, etc. used, referenced or generated by Dr. Vander Weide in the preparation of his analysis. Please provide the aforementioned copies in both hard copy and electronic (Microsoft Excel) formats, with all data and formulae intact.

RESPONSE:

Please see Attachment 1 for Dr. Vander Weide's workpapers.

ATTACHMENT:

ATTACHMENT 1 - Atmos Energy Corporation, Dr. James Vander Weide Workpapers, 68 Pages.

Respondent: Dr. James Vander Weide

Table 1
Cost of Equity Model Results

Method	Model Result
Discounted Cash Flow	11.90%
Ex Ante Risk Premium	10.90%
Ex Post Risk Premium	10.60%
Historical CAPM	10.20%
DCF CAPM	11.50%
Average	11.00%

ATMOS ENERGY
SCHEDULE 1
SUMMARY OF DISCOUNTED CASH FLOW ANALYSIS
FOR NATURAL GAS COMPANIES

Line No.	Company	d_0	D_0	P_0	Growth	Cost of Equity
1	AGL Resources	0.430	1.72	31.017	4.25%	10.5%
2	Atmos Energy	0.330	1.32	25.230	5.00%	11.0%
3	EQT Corp.	0.220	0.88	35.962	9.00%	11.9%
4	National Fuel Gas	0.325	1.34	35.078	8.50%	12.9%
5	Nicor Inc.	0.465	1.86	33.610	4.33%	10.6%
6	NiSource Inc.	0.230	0.92	11.570	3.00%	12.0%
7	Northwest Nat. Gas	0.395	1.58	43.398	4.75%	8.9%
8	ONEOK Inc.	0.400	1.68	29.035	7.25%	13.8%
9	Piedmont Natural Gas	0.270	1.08	23.733	6.93%	12.2%
10	South Jersey Inds.	0.298	1.19	34.848	9.67%	13.7%
11	Southwest Gas	0.238	0.95	21.663	6.00%	10.9%
12	Market-Weighted Average					11.9%

ATMOS ENERGY

SCHEDULE 1 (continued)

,UE LINE SAFETY RANKS AND STANDARD & POOR'S BOND RATINGS

FOR PROXY GAS COMPANIES

Line No.	Company	Safety Rank	S&P BOND RATING	S&P BOND RATING (Numerical)	Market Cap $ (Mil)
1	AGL Resources	2	A-	5	2,598
2	Atmos Energy	2	BBB+	6	2,499
3	EQT Corp.	3	BBB	7	5,024
4	National Fuel Gas	2	BBB	7	3,227
5	Nicor Inc.	3	AA	1	1,648
6	NiSource Inc.	3	BBB-	8	3,539
7	Northwest Nat. Gas	1	AA-	2	1,183
8	ONEOK Inc.	3	BBB	7	3,485
9	Piedmont Natural Gas	2	A	4	1,796
10	South Jersey Inds.	2	BBB+	6	1,099
11	Southwest Gas	3	BBB	7	1,083
12	Market-Weighted Average	2.5	BBB+	6	
13	Simple Average	2.4	A- to BBB+	5.5	

Line No.	Date	DCF	Bond Yield	Risk Premium
1	Jun-98	0.1154	0.0703	0.0451
2	Jul-98	0.1186	0.0703	0.0483
3	Aug-98	0.1234	0.0700	0.0534
4	Sep-98	0.1273	0.0693	0.0580
5	Oct-98	0.1260	0.0696	0.0564
6	Nov-98	0.1211	0.0703	0.0508
7	Dec-98	0.1185	0.0691	0.0494
8	Jan-99	0.1195	0.0697	0.0498
9	Feb-99	0.1243	0.0709	0.0534
10	Mar-99	0.1257	0.0726	0.0531
11	Apr-99	0.1260	0.0722	0.0538
12	May-99	0.1221	0.0747	0.0474
13	Jun-99	0.1208	0.0774	0.0434
14	Jul-99	0.1222	0.0771	0.0451
15	Aug-99	0.1220	0.0791	0.0429
16	Sep-99	0.1226	0.0793	0.0433
17	Oct-99	0.1233	0.0806	0.0427
18	Nov-99	0.1240	0.0794	0.0446
19	Dec-99	0.1280	0.0814	0.0466
20	Jan-00	0.1301	0.0835	0.0466
21	Feb-00	0.1344	0.0825	0.0519
22	Mar-00	0.1344	0.0828	0.0516
23	Apr-00	0.1316	0.0829	0.0487
24	May-00	0.1292	0.0870	0.0422
25	Jun-00	0.1295	0.0836	0.0459
26	Jul-00	0.1317	0.0825	0.0492
27	Aug-00	0.1290	0.0813	0.0477
28	Sep-00	0.1257	0.0823	0.0434
29	Oct-00	0.1260	0.0814	0.0446
30	Nov-00	0.1251	0.0811	0.0440
31	Dec-00	0.1239	0.0784	0.0455
32	Jan-01	0.1261	0.0780	0.0481
33	Feb-01	0.1261	0.0774	0.0487
34	Mar-01	0.1275	0.0768	0.0507
35	Apr-01	0.1227	0.0794	0.0433
36	May-01	0.1302	0.0799	0.0503
37	Jun-01	0.1304	0.0785	0.0519
38	Jul-01	0.1338	0.0778	0.0560
39	Aug-01	0.1327	0.0759	0.0568
40	Sep-01	0.1268	0.0775	0.0493
41	Oct-01	0.1268	0.0763	0.0505
42	Nov-01	0.1268	0.0757	0.0511
43	Dec-01	0.1254	0.0783	0.0471
44	Jan-02	0.1236	0.0766	0.0470
45	Feb-02	0.1241	0.0754	0.0487
46	Mar-02	0.1189	0.0776	0.0413
47	Apr-02	0.1159	0.0757	0.0402
48	May-02	0.1162	0.0752	0.0410
49	Jun-02	0.1170	0.0741	0.0429
50	Jul-02	0.1242	0.0731	0.0511
51	Aug-02	0.1234	0.0717	0.0517
52	Sep-02	0.1260	0.0708	0.0552
53	Oct-02	0.1250	0.0723	0.0527
54	Nov-02	0.1221	0.0714	0.0507
55	Dec-02	0.1216	0.0707	0.0509
56	Jan-03	0.1219	0.0706	0.0513
57	Feb-03	0.1232	0.0693	0.0539
58	Mar-03	0.1195	0.0679	0.0516
59	Apr-03	0.1162	0.0664	0.0498
60	May-03	0.1126	0.0636	0.0490
61	Jun-03	0.1114	0.0621	0.0493
62	Jul-03	0.1127	0.0657	0.0470
63	Aug-03	0.1139	0.0678	0.0461
64	Sep-03	0.1127	0.0656	0.0471
65	Oct-03	0.1123	0.0643	0.0480
66	Nov-03	0.1089	0.0637	0.0452
67	Dec-03	0.1071	0.0627	0.0444

68	Jan-04	0.1059	0.0615	0.0444
69	Feb-04	0.1039	0.0615	0.0424
70	Mar-04	0.1037	0.0597	0.0440
71	Apr-04	0.1041	0.0635	0.0406
72	May-04	0.1045	0.0662	0.0383
73	Jun-04	0.1036	0.0646	0.0390
74	Jul-04	0.1011	0.0627	0.0384
75	Aug-04	0.1008	0.0614	0.0394
76	Sep-04	0.0976	0.0598	0.0378
77	Oct-04	0.0974	0.0594	0.0380
78	Nov-04	0.0962	0.0597	0.0365
79	Dec-04	0.0970	0.0592	0.0378
80	Jan-05	0.0990	0.0578	0.0412
81	Feb-05	0.0979	0.0561	0.0418
82	Mar-05	0.0979	0.0583	0.0396
83	Apr-05	0.0988	0.0564	0.0424
84	May-05	0.0981	0.0553	0.0427
85	Jun-05	0.0976	0.0540	0.0436
86	Jul-05	0.0966	0.0551	0.0415
87	Aug-05	0.0969	0.0550	0.0419
88	Sep-05	0.0980	0.0552	0.0428
89	Oct-05	0.0990	0.0579	0.0411
90	Nov-05	0.1049	0.0588	0.0461
91	Dec-05	0.1045	0.0580	0.0465
92	Jan-06	0.0982	0.0575	0.0407
93	Feb-06	0.1124	0.0582	0.0542
94	Mar-06	0.1127	0.0598	0.0529
95	Apr-06	0.1100	0.0629	0.0471
96	May-06	0.1056	0.0642	0.0414
97	Jun-06	0.1049	0.0640	0.0409
98	Jul-06	0.1087	0.0637	0.0450
99	Aug-06	0.1041	0.0620	0.0421
100	Sep-06	0.1053	0.0600	0.0453
101	Oct-06	0.1030	0.0598	0.0432
102	Nov-06	0.1033	0.0580	0.0453
103	Dec-06	0.1035	0.0581	0.0454
104	Jan-07	0.1013	0.0596	0.0417
105	Feb-07	0.1018	0.0590	0.0428
106	Mar-07	0.1018	0.0585	0.0433
107	Apr-07	0.1007	0.0597	0.0410
108	May-07	0.0967	0.0599	0.0368
109	Jun-07	0.0970	0.0630	0.0340
110	Jul-07	0.1006	0.0625	0.0381
111	Aug-07	0.1021	0.0624	0.0397
112	Sep-07	0.1014	0.0618	0.0396
113	Oct-07	0.1080	0.0611	0.0469
114	Nov-07	0.1083	0.0597	0.0486
115	Dec-07	0.1084	0.0616	0.0468
116	Jan-08	0.1113	0.0602	0.0511
117	Feb-08	0.1139	0.0621	0.0518
118	Mar-08	0.1147	0.0621	0.0526
119	Apr-08	0.1167	0.0629	0.0538
120	May-08	0.1069	0.0627	0.0442
121	Jun-08	0.1062	0.0638	0.0424
122	Jul-08	0.1086	0.0640	0.0446
123	Aug-08	0.1123	0.0637	0.0486
124	Sep-08	0.1130	0.0649	0.0481
125	Oct-08	0.1213	0.0756	0.0457
126	Nov-08	0.1221	0.0760	0.0461
127	Dec-08	0.1162	0.0654	0.0508
128	Jan-09	0.1131	0.0639	0.0492
129	Feb-09	0.1155	0.0630	0.0524
130	Mar-09	0.1198	0.0642	0.0556
131	Apr-09	0.1146	0.0648	0.0498
132	May-09	0.1225	0.0649	0.0576
133	Jun-09	0.1208	0.0620	0.0588
134	Jul-09	0.1166	0.0597	0.0569
135	Average	0.1145	0.0679	0.0466

Month Ending	AGL High	AGL Low	AGL Average	AGL Dividend	AGL Growth	AGL DCF	AGL DCF	ATO High	ATO Low	ATO Average	ATO Dividend	ATO Growth	ATO DCF	ATO DCF	Cascade High	Cascade Low	Cascade Average	Cascade Dividend
Jun-98	20.00	19.38	19.69	1.08	4.32%	10.48%	0.0073	30.50	29.25	29.88	1.06	8.53%	12.64%	0.0070	15.88	15.50	15.69	0.96
Jul-98	20.56	18.56	19.56	1.08	4.36%	10.56%	0.0076	30.94	28.25	29.59	1.06	8.53%	12.68%	0.0075	15.81	14.69	15.25	0.96
Aug-98	19.44	17.94	18.69	1.08	4.43%	10.93%	0.0061	30.50	27.63	29.06	1.06	8.53%	12.76%	0.0058	16.31	14.63	15.47	0.96
Sep-98	19.56	17.69	18.63	1.08	4.54%	11.07%	0.0060	28.88	24.75	26.81	1.06	8.53%	13.12%	0.0058	16.50	15.19	15.84	0.96
Oct-98	21.19	18.81	20.00	1.08	4.54%	10.61%	0.0059	30.94	28.13	29.53	1.10	8.53%	12.85%	0.0055	17.56	16.00	16.78	0.96
Nov-98	22.00	20.31	21.16	1.08	4.54%	10.27%	0.0058	32.25	29.19	30.72	1.10	8.45%	12.60%	0.0054	18.31	16.13	17.22	0.96
Dec-98	23.38	21.19	22.28	1.08	4.54%	9.98%	0.0056	32.25	27.63	29.94	1.10	8.95%	13.23%	0.0057	18.69	17.31	18.00	0.96
Jan-99	23.38	19.81	21.59	1.08	4.54%	10.15%	0.0067	33.00	28.88	30.94	1.10	8.95%	13.09%	0.0060	18.13	15.88	17.00	0.96
Feb-99	20.06	18.31	19.19	1.08	4.59%	10.93%	0.0072	29.69	23.25	26.47	1.10	8.95%	13.79%	0.0063	16.75	15.06	15.91	0.96
Mar-99	20.00	17.50	18.75	1.08	4.66%	11.15%	0.0071	26.25	22.75	24.50	1.10	8.95%	14.19%	0.0062	16.63	14.88	15.75	0.96
Apr-99	18.94	16.81	17.88	1.08	4.66%	11.48%	0.0065	27.38	23.88	25.63	1.10	8.12%	13.09%	0.0047	16.25	14.38	15.31	0.96
May-99	19.06	17.88	18.47	1.08	4.66%	11.25%	0.0062	25.94	23.75	24.84	1.10	8.12%	13.25%	0.0047	16.94	15.63	16.28	0.96
Jun-99	19.44	18.44	18.94	1.08	4.66%	11.09%	0.0060	26.31	24.38	25.34	1.10	8.12%	13.15%	0.0046	19.75	16.38	18.06	0.96
Jul-99	20.75	18.50	19.63	1.08	4.61%	10.80%	0.0057	26.25	24.13	25.19	1.10	8.12%	13.18%	0.0051	18.88	17.13	18.00	0.96
Aug-99	19.19	17.88	18.53	1.08	4.66%	11.23%	0.0059	26.38	24.25	25.31	1.10	8.12%	13.15%	0.0051	18.44	16.19	17.31	0.96
Sep-99	18.88	15.63	17.25	1.08	4.66%	11.73%	0.0062	25.50	23.75	24.63	1.10	8.12%	13.29%	0.0051	18.69	17.44	18.06	0.96
Oct-99	17.88	15.56	16.72	1.08	4.89%	12.21%	0.0058	25.00	22.50	23.75	1.14	7.96%	13.52%	0.0050	18.38	16.81	17.59	0.96
Nov-99	19.19	17.19	18.19	1.08	5.16%	11.89%	0.0057	23.63	22.00	22.81	1.14	7.39%	13.15%	0.0049	18.06	16.44	17.25	0.96
Dec-99	19.00	16.56	17.78	1.08	5.16%	12.05%	0.0059	22.69	19.63	21.16	1.14	7.39%	13.61%	0.0051	17.81	15.38	16.59	0.96
Jan-00	18.00	16.00	17.00	1.08	5.16%	12.37%	0.0065	20.50	16.75	18.63	1.14	7.39%	14.48%	0.0052	16.44	14.19	15.31	0.96
Feb-00	17.44	16.00	16.72	1.08	5.24%	12.58%	0.0067	18.25	15.69	16.97	1.14	7.34%	15.13%	0.0055	15.50	13.38	14.44	0.96
Mar-00	18.38	16.75	17.56	1.08	5.24%	12.22%	0.0062	18.88	15.25	17.06	1.14	7.09%	14.82%	0.0052	16.13	13.50	14.81	0.96
Apr-00	18.31	16.88	17.59	1.08	5.24%	12.21%	0.0067	16.88	14.25	15.56	1.14	7.09%	15.59%	0.0049	16.38	14.94	15.66	0.96
May-00	18.44	15.75	17.09	1.08	5.36%	12.54%	0.0072	18.38	14.94	16.66	1.14	6.59%	14.48%	0.0048	17.75	15.94	16.84	0.96
Jun-00	17.31	15.50	16.41	1.08	5.36%	12.85%	0.0073	20.56	17.50	19.03	1.14	7.09%	14.00%	0.0046	18.13	15.31	16.72	0.96
Jul-00	18.19	16.06	17.13	1.08	5.96%	13.17%	0.0055	20.63	17.75	19.19	1.14	6.84%	13.68%	0.0039	17.06	15.81	16.44	0.96
Aug-00	19.56	17.91	18.73	1.08	5.96%	12.54%	0.0052	23.25	20.00	21.63	1.14	6.84%	12.89%	0.0036	17.94	16.38	17.16	0.96
Sep-00	20.50	18.75	19.63	1.08	5.96%	12.23%	0.0048	22.38	19.50	20.94	1.14	6.67%	12.92%	0.0034	17.88	15.50	16.69	0.96
Oct-00	20.94	18.81	19.88	1.08	5.96%	12.15%	0.0056	23.13	19.19	21.16	1.16	6.67%	12.96%	0.0037	18.63	16.75	17.69	0.96
Nov-00	23.00	19.88	21.44	1.08	5.95%	11.68%	0.0055	25.44	23.00	24.22	1.16	6.95%	12.44%	0.0036	20.50	17.31	18.91	0.96
Dec-00	23.19	21.44	22.31	1.08	5.95%	11.45%	0.0055	26.25	21.56	23.91	1.16	6.34%	11.88%	0.0035	20.88	17.38	19.13	0.96
Jan-01	22.31	19.50	20.91	1.08	5.95%	11.83%	0.0060	25.75	23.25	24.50	1.16	6.95%	12.38%	0.0036	20.69	17.38	19.03	0.96
Feb-01	21.94	20.00	20.97	1.08	5.95%	11.81%	0.0060	24.70	22.51	23.61	1.16	6.95%	12.59%	0.0037	19.21	17.85	18.53	0.96
Mar-01	21.99	20.01	21.00	1.08	5.95%	11.80%	0.0059	23.99	20.85	22.42	1.16	6.95%	12.89%	0.0037	21.00	18.81	19.90	0.96
Apr-01	22.86	20.90	21.88	1.08	5.51%	11.10%	0.0048	24.05	21.15	22.60	1.16	6.93%	12.83%	0.0045	20.60	18.70	19.65	0.96
May-01	24.25	22.10	23.18	1.08	6.59%	11.92%	0.0049	23.98	22.45	23.22	1.16	7.36%	13.12%	0.0044	20.97	19.00	19.98	0.96

Month Ending	Cascade Growth	Cascade DCF	Cascade DCF	EGN High	EGN Low	EGN Average	EGN Dividend	EGN Growth	EGN DCF	EGN DCF	EQT High	EQT Low	EQT Average	EQT Dividend	EQT Growth	EQT DCF	EQT DCF
Jun-98	3.38%	10.20%	0.0011	20.44	19.75	20.09	0.62	8.05%	11.60%	0.0042	28.45	25.13	26.79	1.18	8.56%	13.68%	0.0092
Jul-98	3.38%	10.40%	0.0012	20.75	17.06	18.91	0.64	8.05%	11.95%	0.0045	28.22	22.97	25.60	1.18	9.00%	14.39%	0.0081
Aug-98	3.38%	10.30%	0.0009	19.25	15.25	17.25	0.64	8.05%	12.33%	0.0036	23.49	20.30	21.90	1.18	8.44%	14.72%	0.0057
Sep-98	3.38%	10.13%	0.0009	19.13	15.13	17.13	0.64	8.05%	12.36%	0.0035	24.43	19.42	21.93	1.18	8.44%	14.72%	0.0064
Oct-98	3.38%	9.75%	0.0009	19.13	17.69	18.41	0.64	8.05%	12.06%	0.0032	27.50	23.61	25.56	1.18	7.42%	12.74%	0.0063
Nov-98	3.38%	9.58%	0.0009	19.44	17.44	18.44	0.64	8.05%	12.05%	0.0032	28.15	25.49	26.82	1.18	7.42%	12.48%	0.0066
Dec-98	3.38%	9.31%	0.0008	19.50	17.81	18.66	0.64	8.05%	12.00%	0.0032	28.57	25.52	27.05	1.18	7.58%	12.61%	0.0066
Jan-99	3.38%	9.66%	0.0010	19.75	16.38	18.06	0.64	8.05%	12.14%	0.0032	28.39	24.33	26.36	1.18	7.58%	12.74%	0.0059
Feb-99	3.38%	10.11%	0.0010	17.25	13.25	15.25	0.64	7.55%	12.38%	0.0033	25.39	23.40	24.40	1.18	7.93%	13.53%	0.0063
Mar-99	3.38%	10.17%	0.0010	15.75	13.13	14.44	0.64	7.24%	12.33%	0.0032	25.94	24.67	25.31	1.18	7.56%	12.94%	0.0059
Apr-99	3.38%	10.37%	0.0009	17.44	14.50	15.97	0.64	7.24%	11.84%	0.0027	26.00	22.44	24.22	1.18	7.21%	12.81%	0.0060
May-99	3.45%	10.02%	0.0009	19.81	17.00	18.41	0.64	7.24%	11.22%	0.0025	31.15	25.51	28.33	1.18	6.79%	11.55%	0.0061
Jun-99	3.45%	9.36%	0.0008	19.94	18.13	19.03	0.64	7.24%	11.09%	0.0025	36.82	30.54	33.68	1.18	6.79%	10.78%	0.0067
Jul-99	3.45%	9.38%	0.0009	19.31	18.38	18.84	0.66	7.24%	11.25%	0.0031	38.04	35.78	36.91	1.18	7.58%	11.25%	0.0067
Aug-99	3.45%	9.62%	0.0009	19.31	17.50	18.41	0.66	7.20%	11.30%	0.0031	37.61	35.84	36.73	1.18	7.44%	11.12%	0.0065
Sep-99	3.45%	9.36%	0.0009	20.38	18.81	19.59	0.66	7.20%	11.05%	0.0031	37.30	35.33	36.32	1.18	7.61%	11.34%	0.0069
Oct-99	3.45%	9.52%	0.0009	21.25	18.13	19.69	0.66	7.20%	11.03%	0.0032	37.73	34.72	36.23	1.18	7.61%	11.35%	0.0066
Nov-99	4.20%	10.44%	0.0010	19.75	18.31	19.03	0.66	7.20%	11.17%	0.0032	36.68	34.32	35.50	1.18	7.83%	11.65%	0.0064
Dec-99	4.20%	10.69%	0.0010	19.25	15.75	17.50	0.66	7.20%	11.52%	0.0034	35.75	32.28	34.02	1.18	7.83%	11.82%	0.0063
Jan-00	4.20%	11.25%	0.0011	18.94	16.13	17.53	0.66	7.20%	11.51%	0.0029	36.74	32.28	34.51	1.18	7.83%	11.76%	0.0070
Feb-00	4.20%	11.69%	0.0012	17.75	14.75	16.25	0.66	7.20%	11.86%	0.0030	37.88	32.25	35.07	1.18	8.39%	12.28%	0.0081
Mar-00	4.20%	11.49%	0.0011	18.69	14.69	16.69	0.66	7.20%	11.73%	0.0029	46.00	35.81	40.91	1.18	11.17%	14.58%	0.0110
Apr-00	4.20%	11.09%	0.0009	18.88	16.00	17.44	0.66	7.89%	12.25%	0.0030	47.25	41.62	44.44	1.18	11.17%	14.31%	0.0120
May-00	4.27%	10.67%	0.0009	23.69	17.06	20.38	0.66	7.89%	11.62%	0.0030	50.88	46.06	48.47	1.18	12.22%	15.12%	0.0143
Jun-00	4.27%	10.72%	0.0009	22.50	19.50	21.00	0.66	7.89%	11.50%	0.0029	51.38	45.62	48.50	1.18	12.11%	15.01%	0.0135
Jul-00	4.27%	10.83%	0.0010	24.50	21.00	22.75	0.68	7.89%	11.32%	0.0034	54.44	46.81	50.63	1.18	12.38%	15.16%	0.0125
Aug-00	4.27%	10.55%	0.0010	26.50	21.50	24.00	0.68	9.46%	12.76%	0.0038	59.75	52.31	56.03	1.18	12.25%	14.76%	0.0127
Sep-00	4.27%	10.73%	0.0010	30.38	25.25	27.81	0.68	9.70%	12.55%	0.0035	63.44	56.38	59.91	1.18	12.25%	14.60%	0.0134
Oct-00	4.27%	10.36%	0.0009	33.56	26.94	30.25	0.68	9.70%	12.32%	0.0044	64.38	56.50	60.44	1.18	11.69%	14.00%	0.0111
Nov-00	4.20%	9.88%	0.0008	31.81	28.00	29.91	0.68	9.70%	12.35%	0.0045	60.00	55.75	57.88	1.18	13.08%	15.53%	0.0121
Dec-00	4.20%	9.82%	0.0009	33.50	26.06	29.78	0.68	11.75%	14.46%	0.0053	66.75	55.75	61.25	1.18	13.08%	15.39%	0.0145
Jan-01	4.20%	9.84%	0.0009	32.44	27.50	29.97	0.68	11.75%	14.44%	0.0052	66.69	55.38	61.04	1.18	13.92%	16.26%	0.0131
Feb-01	4.27%	10.07%	0.0010	32.06	27.50	29.78	0.68	11.75%	14.46%	0.0052	63.34	57.04	60.19	1.18	12.64%	14.98%	0.0118
Mar-01	4.27%	9.67%	0.0009	35.30	27.75	31.52	0.68	11.75%	14.31%	0.0050	70.50	57.55	64.03	1.28	12.94%	15.34%	0.0142
Apr-01				38.10	32.70	35.40	0.68	11.40%	13.67%	0.0050	40.00	39.26	39.63	0.64	11.44%	13.35%	0.0068
May-01				40.25	31.70	35.98	0.68	11.00%	13.23%	0.0046	40.50	38.04	39.27	0.64	12.34%	14.28%	0.0130

Month Ending	Keyspan High	Keyspan Low	Keyspan Average	Keyspan Dividend	Keyspan Growth	Keyspan DCF	Keyspan DCF	LG High	LG Low	LG Average	LG Dividend	LG Growth	LG DCF	LG DCF	NJR High	NJR Low
Jun-98	30.69	29.25	29.97	1.50				24.69	24.25	24.47	1.32				36.00	34.81
Jul-98	30.75	26.50	28.63	1.20				25.00	23.06	24.03	1.32				36.50	33.63
Aug-98	30.25	26.69	28.47	1.20	7.88%	12.75%	0.0315	23.81	22.38	23.09	1.32				34.44	31.50
Sep-98	29.25	25.38	27.31	1.20	9.50%	14.65%	0.0351	24.31	22.38	23.34	1.32				37.19	33.38
Oct-98	32.25	28.69	30.47	1.78	9.50%	16.39%	0.0342	26.00	23.00	24.50	1.32				40.25	35.75
Nov-98	30.75	29.56	30.16	1.78	7.88%	14.74%	0.0308	26.06	24.44	25.25	1.32				40.00	38.00
Dec-98	31.25	29.19	30.22	1.78	7.88%	14.73%	0.0308	27.00	24.63	25.81	1.32				39.63	37.63
Jan-99	31.31	27.06	29.19	1.78	7.88%	14.97%	0.0326	27.00	23.44	25.22	1.32				40.13	36.00
Feb-99	28.94	26.50	27.72	1.78	7.88%	15.36%	0.0333	24.19	22.38	23.28	1.32				36.38	33.63
Mar-99	28.13	25.13	26.63	1.78	7.88%	15.67%	0.0329	23.69	20.63	22.16	1.32				37.88	34.69
Apr-99	26.88	24.63	25.75	1.78	7.88%	15.95%	0.0310	21.31	20.00	20.66	1.34				38.00	35.44
May-99	27.63	26.50	27.06	1.78	7.88%	15.55%	0.0296	22.38	20.13	21.25	1.34				38.19	35.00
Jun-99	27.69	25.88	26.78	1.78	7.88%	15.63%	0.0294	23.63	21.50	22.56	1.34				39.50	37.00
Jul-99	27.94	26.38	27.16	1.78	8.88%	16.59%	0.0303	23.75	23.00	23.38	1.34				39.94	37.50
Aug-99	30.00	26.56	28.28	1.78	8.88%	16.27%	0.0297	23.75	21.56	22.66	1.34				40.13	38.56
Sep-99	31.06	28.31	29.69	1.78	8.88%	15.92%	0.0291	23.38	21.25	22.31	1.34				40.13	37.50
Oct-99	29.69	27.00	28.34	1.78	8.88%	16.26%	0.0310	23.44	21.00	22.22	1.34				41.13	39.44
Nov-99	29.69	24.88	27.28	1.78	9.08%	16.77%	0.0321	23.00	21.13	22.06	1.34				41.13	39.00
Dec-99	26.06	22.50	24.28	1.78	9.08%	17.74%	0.0345	23.00	20.00	21.50	1.34				40.50	38.88
Jan-00	24.25	22.06	23.16	1.78	9.08%	18.18%	0.0307	21.88	18.88	20.38	1.34				39.75	36.50
Feb-00	23.63	20.31	21.97	1.78	9.16%	18.77%	0.0319	20.00	17.50	18.75	1.34				39.31	36.19
Mar-00	27.88	20.19	24.03	1.78	9.16%	17.92%	0.0293	21.38	18.63	20.00	1.34				42.88	36.50
Apr-00	30.13	26.00	28.06	1.78	9.44%	16.93%	0.0271	20.63	19.25	19.94	1.34				42.75	38.50
May-00	30.88	28.50	29.69	1.78	9.64%	16.73%	0.0280	20.50	19.13	19.81	1.34				41.00	38.63
Jun-00	32.69	30.13	31.41	1.78	9.64%	16.33%	0.0269	19.94	18.75	19.34	1.34				41.38	37.88
Jul-00	33.19	30.94	32.06	1.78	9.64%	16.19%	0.0326	20.13	19.19	19.66	1.34				40.69	37.63
Aug-00	36.94	31.88	34.41	1.78	9.64%	15.73%	0.0311	21.88	19.63	20.75	1.34				43.13	39.13
Sep-00	40.14	34.19	37.16	1.78	9.68%	15.32%	0.0286	22.69	20.88	21.78	1.34	3.67%	10.55%	0.0016	41.75	38.94
Oct-00	40.63	34.94	37.78	1.78	9.68%	15.22%	0.0306	22.94	21.38	22.16	1.34	3.67%	10.43%	0.0019	41.44	37.63
Nov-00	38.63	33.50	36.06	1.78	9.64%	15.45%	0.0317	23.63	21.75	22.69	1.34	3.67%	10.27%	0.0019	41.63	37.50
Dec-00	43.63	38.00	40.81	1.78	9.64%	14.76%	0.0307	24.75	22.13	23.44	1.34	3.67%	10.05%	0.0019	44.63	40.13
Jan-01	41.94	35.19	38.56	1.78	9.64%	15.07%	0.0341	24.63	21.25	22.94	1.34	3.67%	10.19%	0.0018	43.25	37.26
Feb-01	40.80	37.15	38.98	1.78	9.64%	15.01%	0.0342	24.15	21.26	22.70	1.34	3.67%	10.26%	0.0018	39.09	37.26
Mar-01	38.90	34.20	36.55	1.78	9.64%	15.37%	0.0343	24.48	22.28	23.38	1.34	3.67%	10.07%	0.0017	41.15	38.00
Apr-01	41.10	38.15	39.63	1.78	9.64%	14.92%	0.0317	24.48	23.10	23.79	1.34	3.67%	9.95%	0.0018	43.40	40.20
May-01	40.50	37.85	39.17	1.78	11.07%	16.48%	0.0335	25.30	23.10	24.20	1.34	3.33%	9.49%	0.0016	46.00	42.53

Month Ending	NJR Average	NJR Dividend	NJR Growth	NJR DCF	NJR DCF	GAS High	GAS Low	GAS Average	GAS Dividend	GAS Growth	GAS DCF	GAS DCF	NI High	NI Low	NI Average	NI Dividend
Jun-98	35.41	1.64	5.83%	11.09%	0.0046	40.31	39.81	40.06	1.48	7.26%	11.49%	0.0146				
Jul-98	35.06	1.64	5.83%	11.14%	0.0046	41.00	37.13	39.06	1.48	7.80%	12.16%	0.0152				
Aug-98	32.97	1.64	5.83%	11.48%	0.0036	39.75	37.38	38.56	1.48	7.26%	11.66%	0.0112				
Sep-98	35.28	1.64	5.83%	11.10%	0.0034	41.94	37.13	39.53	1.48	7.26%	11.55%	0.0107				
Oct-98	38.00	1.64	5.88%	10.77%	0.0034	44.25	40.44	42.34	1.48	6.83%	10.82%	0.0105				
Nov-98	39.00	1.64	5.88%	10.65%	0.0034	44.44	42.00	43.22	1.48	6.83%	10.73%	0.0104				
Dec-98	38.63	1.64	5.88%	10.69%	0.0034	42.94	40.38	41.66	1.48	5.48%	9.48%	0.0092				
Jan-99	38.06	1.68	5.88%	10.89%	0.0037	42.94	38.13	40.53	1.48	5.48%	9.59%	0.0092				
Feb-99	35.00	1.68	6.00%	11.46%	0.0039	38.63	36.50	37.56	1.48	5.48%	9.92%	0.0095				
Mar-99	36.28	1.68	6.00%	11.26%	0.0037	38.81	34.69	36.75	1.48	5.70%	10.25%	0.0095				
Apr-99	36.72	1.68	6.00%	11.20%	0.0037	37.63	34.13	35.88	1.56	5.70%	10.62%	0.0098				
May-99	36.59	1.68	6.00%	11.22%	0.0037	38.63	36.38	37.50	1.56	5.70%	10.41%	0.0094				
Jun-99	38.25	1.68	6.00%	10.99%	0.0035	39.50	36.81	38.16	1.56	5.70%	10.32%	0.0092				
Jul-99	38.72	1.68	6.00%	10.92%	0.0037	39.44	36.75	38.09	1.56	5.82%	10.46%	0.0091				
Aug-99	39.34	1.68	6.00%	10.85%	0.0037	39.50	37.63	38.56	1.56	6.13%	10.72%	0.0093				
Sep-99	38.81	1.68	6.00%	10.91%	0.0037	40.00	35.69	37.84	1.56	6.13%	10.81%	0.0094				
Oct-99	40.28	1.68	6.00%	10.73%	0.0037	39.38	36.56	37.97	1.56	6.56%	11.24%	0.0096				
Nov-99	40.06	1.68	5.90%	10.65%	0.0036	38.69	34.38	36.53	1.56	6.49%	11.36%	0.0098				
Dec-99	39.69	1.68	5.90%	10.70%	0.0037	34.94	31.19	33.06	1.56	6.49%	11.88%	0.0104				
Jan-00	38.13	1.72	5.90%	11.02%	0.0041	36.38	31.31	33.84	1.56	6.49%	11.75%	0.0093				
Feb-00	37.75	1.72	6.10%	11.28%	0.0042	35.69	29.69	32.69	1.56	6.21%	11.65%	0.0093				
Mar-00	39.69	1.72	6.10%	11.02%	0.0040	33.31	29.38	31.34	1.56	6.21%	11.88%	0.0091				
Apr-00	40.63	1.72	6.10%	10.91%	0.0039	34.88	32.06	33.47	1.66	6.21%	11.86%	0.0098				
May-00	39.81	1.72	6.38%	11.30%	0.0042	37.13	32.75	34.94	1.66	6.24%	11.65%	0.0101				
Jun-00	39.63	1.72	6.38%	11.32%	0.0042	37.50	32.38	34.94	1.66	6.24%	11.65%	0.0099				
Jul-00	39.16	1.72	6.38%	11.38%	0.0037	35.50	32.13	33.81	1.66	6.24%	11.84%	0.0091				
Aug-00	41.13	1.72	6.38%	11.14%	0.0035	40.06	34.81	37.44	1.66	6.24%	11.29%	0.0085				
Sep-00	40.34	1.72	6.38%	11.24%	0.0034	39.38	35.25	37.31	1.66	6.24%	11.30%	0.0080				
Oct-00	39.53	1.72	6.38%	11.34%	0.0034	36.38	32.19	34.28	1.66	6.24%	11.76%	0.0079				
Nov-00	39.56	1.72	6.50%	11.46%	0.0036	40.00	34.81	37.41	1.66	6.13%	11.18%	0.0077				
Dec-00	42.38	1.72	6.50%	11.12%	0.0035	43.88	38.00	40.94	1.66	6.13%	10.73%	0.0075				
Jan-01	40.25	1.76	6.50%	11.49%	0.0035	42.38	35.21	38.79	1.66	6.13%	10.99%	0.0083				
Feb-01	38.17	1.76	6.83%	12.11%	0.0037	39.20	35.95	37.58	1.66	6.13%	11.15%	0.0085				
Mar-01	39.58	1.76	6.83%	11.92%	0.0036	38.49	35.12	36.81	1.66	6.13%	11.26%	0.0084				
Apr-01	41.80	1.76	6.83%	11.64%	0.0032	39.90	35.95	37.93	1.76	5.93%	11.20%	0.0075				
May-01	44.26	1.76	6.83%	11.37%	0.0030	39.47	37.20	38.34	1.76	5.94%	11.15%	0.0071				

Month Ending	NI Growth	NI DCF	NI DCF	NWN High	NWN Low	NWN Average	NWN Dividend	NWN Growth	NWN DCF	NWN DCF	NUI High	NUI Low	NUI Average	NUI Dividend	NUI Growth	NUI DCF
Jun-98				28.06	26.38	27.22	1.22	5.26%	10.31%	0.0043	25.63	25.00	25.31	0.98	10.60%	15.18%
Jul-98				28.00	26.00	27.00	1.22	5.16%	10.25%	0.0042	25.94	22.13	24.03	0.98	10.60%	15.42%
Aug-98				27.25	24.25	25.75	1.22	5.16%	10.50%	0.0033	22.44	20.31	21.38	0.98	10.60%	16.04%
Sep-98				27.75	24.50	26.13	1.22	5.16%	10.43%	0.0032	23.44	20.63	22.03	0.98	10.60%	15.87%
Oct-98				29.25	26.25	27.75	1.22	5.18%	10.13%	0.0033	23.44	21.56	22.50	0.98	10.27%	15.41%
Nov-98				29.63	27.13	28.38	1.22	4.42%	9.23%	0.0030	25.94	23.31	24.63	0.98	10.27%	14.96%
Dec-98				30.25	25.75	28.00	1.22	4.42%	9.29%	0.0031	27.00	23.81	25.41	0.98	10.27%	14.82%
Jan-99				27.00	23.38	25.19	1.22	4.42%	9.85%	0.0034	27.06	22.19	24.63	0.98	10.27%	14.96%
Feb-99				24.81	22.13	23.47	1.22	4.42%	10.25%	0.0035	23.25	20.38	21.81	0.98	10.27%	15.58%
Mar-99				25.50	21.00	23.25	1.22	4.42%	10.31%	0.0034	22.94	20.75	21.84	0.98	9.70%	14.97%
Apr-99				23.44	19.50	21.47	1.22	4.42%	10.81%	0.0033	22.75	20.81	21.78	0.98	9.70%	14.99%
May-99				27.00	21.31	24.16	1.22	4.42%	10.08%	0.0030	25.00	21.69	23.34	0.98	9.70%	14.63%
Jun-99				26.38	22.63	24.50	1.22	4.42%	10.00%	0.0030	25.63	23.25	24.44	0.98	9.70%	14.40%
Jul-99				27.88	24.00	25.94	1.22	4.42%	9.69%	0.0030	28.06	24.88	26.47	0.98	9.70%	14.04%
Aug-99				27.69	25.00	26.34	1.22	4.42%	9.60%	0.0030	27.00	24.75	25.88	0.98	9.70%	14.14%
Sep-99				27.44	23.31	25.38	1.22	4.42%	9.81%	0.0031	26.56	24.63	25.59	0.98	9.70%	14.19%
Oct-99				26.38	23.75	25.06	1.24	4.42%	9.97%	0.0031	25.63	23.44	24.53	0.98	9.70%	14.39%
Nov-99				27.00	23.00	25.00	1.24	4.28%	9.83%	0.0031	27.16	24.00	25.58	0.98	9.70%	14.19%
Dec-99				25.13	21.13	23.13	1.24	4.02%	10.02%	0.0032	28.19	24.75	26.47	0.98	9.70%	14.04%
Jan-00				22.25	19.19	20.72	1.24	4.02%	10.73%	0.0034	30.75	25.06	27.91	0.98	9.70%	13.81%
Feb-00				22.50	18.50	20.50	1.24	4.02%	10.80%	0.0034	27.94	22.94	25.44	0.98	12.20%	16.82%
Mar-00				19.88	17.75	18.81	1.24	4.02%	11.43%	0.0035	26.25	23.25	24.75	0.98	12.20%	16.95%
Apr-00				22.00	18.88	20.44	1.24	4.02%	10.82%	0.0028	27.81	25.25	26.53	0.98	12.20%	16.63%
May-00				22.50	20.00	21.25	1.24	3.70%	10.22%	0.0028	28.19	25.94	27.06	0.98	12.20%	16.54%
Jun-00				23.88	21.50	22.69	1.24	4.03%	10.15%	0.0027	28.19	26.56	27.38	0.98	12.20%	16.49%
Jul-00				24.00	21.63	22.81	1.24	4.53%	10.64%	0.0029	28.69	26.19	27.44	0.98	13.16%	17.47%
Aug-00				23.94	22.13	23.03	1.24	4.53%	10.58%	0.0029	30.31	27.63	28.97	0.98	13.16%	17.24%
Sep-00				24.63	22.19	23.41	1.24	4.53%	10.48%	0.0027	32.44	28.63	30.53	0.98	13.16%	17.03%
Oct-00				23.44	21.88	22.66	1.24	4.53%	10.68%	0.0027	31.19	27.88	29.53	0.98	13.16%	17.16%
Nov-00				24.94	22.56	23.75	1.24	4.42%	10.28%	0.0026	31.00	28.88	29.94	0.98	11.95%	15.86%
Dec-00				27.50	23.88	25.69	1.24	4.42%	9.83%	0.0026	33.94	28.00	30.97	0.98	11.95%	15.73%
Jan-01				26.75	24.00	25.38	1.24	4.42%	9.90%	0.0026	32.31	25.31	28.81	0.98	11.95%	16.01%
Feb-01				26.65	23.62	25.14	1.24	4.50%	10.03%	0.0026	28.28	26.35	27.32	0.98	11.95%	16.24%
Mar-01				24.45	23.05	23.75	1.24	4.50%	10.36%	0.0027	28.40	25.44	26.92	0.98	11.95%	16.30%
Apr-01				24.10	22.00	23.05	1.24	4.33%	10.36%	0.0024	27.03	21.95	24.49	0.98	10.92%	15.67%
May-01				24.25	21.65	22.95	1.24	4.25%	10.31%	0.0023	22.92	20.62	21.77	0.98	10.95%	16.30%

Month Ending	NUI DCF	OKE High	OKE Low	OKE Average	OKE Dividend	OKE Growth	OKE DCF	OKE DCF	Peoples High	Peoples Low	Peoples Average	Peoples Dividend	Peoples Growth	Peoples DCF	Peoples DCF	PNY High
Jun-98	0.0034	20.16	19.31	19.73	0.60	7.00%	10.47%	0.0080	39.00	36.75	37.88	1.92	5.61%	11.36%	0.0094	33.81
Jul-98	0.0033	20.47	17.00	18.73	0.60	7.00%	10.65%	0.0084	38.63	33.88	36.25	1.92	5.61%	11.62%	0.0099	34.63
Aug-98	0.0026	17.59	14.88	16.23	0.60	7.00%	11.22%	0.0068	37.00	33.06	35.03	1.92	5.61%	11.84%	0.0078	30.88
Sep-98	0.0025	18.63	14.97	16.80	0.60	7.00%	11.08%	0.0065	38.00	32.13	35.06	1.92	5.61%	11.83%	0.0075	34.50
Oct-98	0.0024	18.16	16.50	17.33	0.60	7.67%	11.65%	0.0065	38.19	35.50	36.84	1.92	4.81%	10.68%	0.0071	35.44
Nov-98	0.0023	18.97	17.13	18.05	0.60	7.67%	11.49%	0.0064	39.50	37.13	38.31	1.92	4.81%	10.45%	0.0069	35.50
Dec-98	0.0023	18.47	16.19	17.33	0.60	7.00%	10.95%	0.0061	40.13	37.19	38.66	1.92	4.36%	9.92%	0.0066	36.13
Jan-99	0.0023	18.59	14.25	16.42	0.62	7.00%	11.32%	0.0057	40.25	33.56	36.91	1.92	4.36%	10.19%	0.0067	36.63
Feb-99	0.0024	15.44	13.00	14.22	0.62	7.00%	12.00%	0.0061	34.75	31.75	33.25	1.92	4.36%	10.85%	0.0071	34.81
Mar-99	0.0022	14.94	12.25	13.59	0.62	6.50%	11.71%	0.0057	36.00	32.06	34.03	1.92	4.64%	10.99%	0.0070	35.00
Apr-99	0.0023	14.53	12.25	13.39	0.62	6.50%	11.79%	0.0053	38.44	32.13	35.28	1.96	4.64%	10.89%	0.0072	35.88
May-99	0.0022	15.25	13.72	14.48	0.62	6.50%	11.38%	0.0050	39.88	37.00	38.44	1.96	4.64%	10.37%	0.0068	33.94
Jun-99	0.0021	16.06	14.50	15.28	0.62	6.50%	11.12%	0.0048	39.94	37.63	38.78	1.96	4.64%	10.32%	0.0066	33.88
Jul-99	0.0022	16.56	15.41	15.98	0.62	6.67%	11.09%	0.0052	39.50	36.56	38.03	1.96	4.64%	10.43%	0.0070	34.38
Aug-99	0.0022	16.34	15.00	15.67	0.62	6.67%	11.18%	0.0052	38.00	35.81	36.91	1.96	4.64%	10.61%	0.0071	34.19
Sep-99	0.0022	15.78	14.81	15.30	0.62	6.67%	11.29%	0.0053	37.88	34.00	35.94	1.96	4.64%	10.78%	0.0073	34.00
Oct-99	0.0022	15.38	14.00	14.69	0.62	6.67%	11.49%	0.0053	39.00	34.50	36.75	1.96	4.44%	10.43%	0.0065	32.50
Nov-99	0.0022	15.06	13.13	14.09	0.62	6.67%	11.70%	0.0055	39.44	35.63	37.53	1.96	5.13%	11.03%	0.0069	33.19
Dec-99	0.0022	14.63	12.50	13.56	0.62	6.67%	11.90%	0.0056	38.00	33.25	35.63	1.96	5.13%	11.35%	0.0072	32.88
Jan-00	0.0024	14.13	12.19	13.16	0.62	6.67%	12.06%	0.0055	33.69	30.38	32.03	2.00	5.13%	12.21%	0.0077	30.69
Feb-00	0.0029	13.34	10.88	12.11	0.62	6.67%	12.54%	0.0058	32.88	27.44	30.16	2.00	5.13%	12.66%	0.0081	29.69
Mar-00	0.0028	12.78	11.16	11.97	0.62	6.67%	12.61%	0.0056	29.50	26.19	27.84	2.00	5.13%	13.31%	0.0082	26.75
Apr-00	0.0030	13.19	12.06	12.63	0.62	6.67%	12.29%	0.0046	32.19	26.63	29.41	2.00	5.19%	12.93%	0.0071	28.25
May-00	0.0031	14.72	12.34	13.53	0.62				34.38	29.88	32.13	2.00	5.19%	12.25%	0.0071	30.38
Jun-00	0.0030	15.63	12.88	14.25	0.62				35.06	32.00	33.53	2.00	5.19%	11.95%	0.0068	31.31
Jul-00	0.0029	13.89	12.63	13.26	0.62	6.67%	12.02%	0.0049	33.50	31.25	32.38	2.00	5.44%	12.47%	0.0072	29.13
Aug-00	0.0028	16.28	13.31	14.80	0.62	6.67%	11.45%	0.0046	35.13	31.63	33.38	2.00	6.06%	12.91%	0.0073	29.94
Sep-00	0.0027	20.00	15.91	17.95	0.62	6.67%	10.60%	0.0040	35.38	31.50	33.44	2.00	6.06%	12.90%	0.0069	31.19
Oct-00	0.0029	22.38	19.00	20.69	0.62	6.67%	10.08%	0.0046	34.88	31.75	33.31	2.00	6.06%	12.92%	0.0065	30.63
Nov-00	0.0027	21.56	19.81	20.69	0.62	6.67%	10.08%	0.0047	43.00	34.00	38.50	2.00	6.25%	12.18%	0.0063	34.38
Dec-00	0.0027	25.31	20.41	22.86	0.62				46.94	41.13	44.03	2.00	6.25%	11.42%	0.0059	39.44
Jan-01	0.0025	24.34	21.44	22.89	0.62				44.63	35.88	40.25	2.04	6.25%	12.03%	0.0081	38.00
Feb-01	0.0026	23.98	21.33	22.65	0.62				40.40	36.74	38.57	2.04	6.25%	12.29%	0.0083	34.19
Mar-01	0.0025	22.71	18.13	20.42	0.62				41.95	37.01	39.48	2.04	6.25%	12.15%	0.0080	35.50
Apr-01	0.0022	22.50	19.31	20.90	0.62	7.67%	11.07%	0.0057	41.12	37.80	39.46	2.04	6.25%	12.15%	0.0076	36.55
May-01	0.0022	21.95	20.38	21.16	0.62	11.60%	15.08%	0.0074	41.15	38.45	39.80	2.04	5.57%	11.38%	0.0069	36.00

Month Ending	PNY Low	PNY Average	PNY Dividend	PNY Growth	PNY DCF	PNY DCF	Semco High	Semco Low	Semco Average	Semco Dividend	Semco Growth	Semco DCF	Semco DCF	SJI High	SJI Low
Jun-98	32.94	33.38	1.30	7.33%	11.80%	0.0071	17.75	17.38	17.56	0.76				27.88	27.25
Jul-98	28.88	31.75	1.30	7.33%	12.03%	0.0077	18.00	17.00	17.50	0.80				27.88	25.50
Aug-98	27.88	29.38	1.30	7.33%	12.42%	0.0061	17.25	13.50	15.38	0.80				26.38	22.75
Sep-98	28.06	31.28	1.30	7.33%	12.10%	0.0058	15.75	13.13	14.44	0.80				26.31	22.00
Oct-98	32.38	33.91	1.30	7.33%	11.73%	0.0060	17.25	14.50	15.88	0.80				27.00	25.44
Nov-98	32.75	34.13	1.30	6.75%	11.10%	0.0057	17.50	15.75	16.63	0.80				26.13	25.00
Dec-98	33.75	34.94	1.30	6.75%	10.99%	0.0056	17.25	15.50	16.38	0.80				26.25	25.06
Jan-99	30.00	33.31	1.30	6.75%	11.20%	0.0057	17.50	15.88	16.69	0.80				26.69	25.50
Feb-99	28.63	31.72	1.30	6.75%	11.43%	0.0058	16.38	14.75	15.56	0.80				25.50	21.50
Mar-99	32.88	33.94	1.30	6.60%	10.96%	0.0054	15.94	14.25	15.09	0.80				25.00	21.63
Apr-99	31.13	33.50	1.38	6.10%	10.78%	0.0061	16.88	14.00	15.44	0.80	12.17%	18.42%	0.0024	24.81	21.63
May-99	31.06	32.50	1.38	6.10%	10.92%	0.0060	15.00	13.25	14.13	0.80	12.17%	19.01%	0.0024	30.00	23.06
Jun-99	30.75	32.31	1.38	6.10%	10.95%	0.0060	15.56	13.25	14.41	0.80	12.17%	18.87%	0.0023	28.69	26.81
Jul-99	30.69	32.53	1.38	6.10%	10.92%	0.0053	16.00	15.13	15.56	0.82	12.17%	18.52%	0.0022	30.75	28.19
Aug-99	32.75	33.47	1.38	6.10%	10.78%	0.0052	16.00	14.00	15.00	0.82	12.17%	18.77%	0.0023	30.75	28.38
Sep-99	30.31	32.16	1.38	6.10%	10.97%	0.0053	14.75	13.00	13.88	0.82	12.17%	19.31%	0.0023	30.13	26.06
Oct-99	30.25	31.38	1.38	6.10%	11.10%	0.0053	15.38	13.63	14.50	0.82				27.38	25.50
Nov-99	30.50	31.84	1.38	6.10%	11.02%	0.0053	14.25	13.13	13.69	0.82				30.25	26.13
Dec-99	28.94	30.91	1.38	6.07%	11.14%	0.0054	13.88	10.94	12.41	0.82				29.50	28.00
Jan-00	28.25	29.47	1.38	6.07%	11.40%	0.0059	12.88	11.25	12.06	0.82	8.25%	16.21%	0.0019	29.50	28.38
Feb-00	23.69	26.69	1.38	6.07%	11.96%	0.0062	14.00	11.00	12.50	0.82	8.60%	16.30%	0.0019	29.63	28.75
Mar-00	24.00	25.38	1.38	6.00%	12.20%	0.0061	12.25	10.75	11.50	0.82	8.60%	16.98%	0.0020	29.44	27.56
Apr-00	25.19	26.72	1.46	6.00%	12.23%	0.0054	13.13	11.13	12.13	0.84	8.60%	16.74%	0.0018	28.81	26.56
May-00	27.00	28.69	1.46	5.67%	11.45%	0.0053	15.00	11.81	13.41	0.84	8.60%	15.94%	0.0018	27.00	25.94
Jun-00	26.56	28.94	1.46	5.67%	11.39%	0.0052	13.94	11.50	12.72	0.84	8.60%	16.35%	0.0019	27.63	24.50
Jul-00	26.88	28.00	1.46	5.67%	11.59%	0.0053	15.19	12.25	13.72	0.84	8.60%	15.77%	0.0017	27.56	26.06
Aug-00	26.50	28.22	1.46	5.67%	11.54%	0.0052	16.06	14.25	15.16	0.84	8.60%	15.08%	0.0016	27.75	26.38
Sep-00	27.13	29.16	1.46	5.67%	11.35%	0.0048	16.94	14.50	15.72	0.84	8.60%	14.84%	0.0015	29.25	26.94
Oct-00	28.25	29.44	1.46	5.67%	11.30%	0.0045	15.94	13.75	14.84	0.84	8.60%	15.22%	0.0019	30.13	28.25
Nov-00	29.19	31.78	1.46	5.67%	10.87%	0.0044	16.63	14.81	15.72	0.84	8.60%	14.84%	0.0019	29.75	28.56
Dec-00	32.50	35.97	1.46	5.67%	10.26%	0.0042	16.13	14.50	15.31	0.84	8.60%	15.01%	0.0020	29.81	29.00
Jan-01	33.00	35.50	1.46	5.43%	10.07%	0.0046	15.44	13.19	14.31	0.84	8.25%	15.09%	0.0017	32.25	29.19
Feb-01	31.75	32.97	1.46	5.43%	10.43%	0.0048	15.10	13.81	14.46	0.84	7.75%	14.49%	0.0017	32.00	29.00
Mar-01	31.82	33.66	1.46	5.43%	10.33%	0.0047	14.50	13.53	14.01	0.84	7.75%	14.71%	0.0017	31.85	27.60
Apr-01	34.20	35.38	1.54	5.43%	10.34%	0.0045	15.05	13.85	14.45	0.84	7.75%	14.50%	0.0014	30.95	29.05
May-01	34.01	35.00	1.54	5.43%	10.40%	0.0043	15.20	14.00	14.60	0.84	7.28%	13.93%	0.0013	31.55	29.95

Month Ending	SJI Average	SJI Dividend	SJI Growth	SJI DCF	SJI DCF	SWX High	SWX Low	SWX Average	SWX Dividend	SWX Growth	SWX DCF	SWX DCF	UGI High	UGI Low
Jun-98	27.56	1.44				25.00	23.88	24.44	0.82	4.53%	8.27%	0.0029	25.13	23.75
Jul-98	26.69	1.44				24.50	22.69	23.59	0.82	4.53%	8.41%	0.0035	25.81	23.44
Aug-98	24.56	1.44				23.94	17.38	20.66	0.82	8.18%	12.77%	0.0040	24.06	21.88
Sep-98	24.16	1.44				20.69	18.06	19.38	0.82	8.18%	13.08%	0.0040	23.75	20.50
Oct-98	26.22	1.44				24.13	20.19	22.16	0.82	8.18%	12.46%	0.0036	23.88	22.19
Nov-98	25.56	1.44				24.94	22.00	23.47	0.82	8.18%	12.21%	0.0036	25.75	22.88
Dec-98	25.66	1.44				26.88	23.19	25.03	0.82	4.83%	8.49%	0.0025	24.63	21.63
Jan-99	26.09	1.44				26.69	25.81	26.25	0.82	4.83%	8.32%	0.0034	24.38	21.38
Feb-99	23.50	1.44				29.00	25.25	27.13	0.82	4.53%	7.90%	0.0032	22.44	19.56
Mar-99	23.31	1.44				28.94	26.50	27.72	0.82	4.90%	8.20%	0.0032	20.38	15.00
Apr-99	23.22	1.44				29.50	26.88	28.19	0.82	4.90%	8.15%	0.0034	17.94	16.56
May-99	26.53	1.44				29.50	28.13	28.81	0.82	4.90%	8.08%	0.0033	21.00	17.06
Jun-99	27.75	1.44				28.69	28.00	28.34	0.82	4.90%	8.13%	0.0033	20.44	19.31
Jul-99	29.47	1.44				29.13	28.63	28.88	0.82	4.90%	8.07%	0.0033	23.88	19.75
Aug-99	29.56	1.44				28.94	27.75	28.34	0.82	4.90%	8.13%	0.0033	24.69	22.81
Sep-99	28.09	1.44				28.75	26.88	27.81	0.82	4.90%	8.19%	0.0033	24.19	22.38
Oct-99	26.44	1.44	3.67%	9.74%	0.0015	27.31	22.88	25.09	0.82	5.00%	8.66%	0.0035	24.00	22.25
Nov-99	28.19	1.44	3.67%	9.36%	0.0015	24.81	22.88	23.84	0.82	5.00%	8.85%	0.0036	23.94	19.13
Dec-99	28.75	1.44	3.67%	9.24%	0.0015	23.63	20.38	22.00	0.82	5.00%	9.18%	0.0038	22.13	19.31
Jan-00	28.94	1.46	4.33%	9.98%	0.0017	23.00	19.00	21.00	0.82	5.00%	9.38%	0.0033	22.25	19.88
Feb-00	29.19	1.46	4.33%	9.93%	0.0017	19.94	17.06	18.50	0.82	5.00%	9.99%	0.0036	21.25	18.56
Mar-00	28.50	1.46	4.50%	10.25%	0.0017	19.94	17.50	18.72	0.82	5.00%	9.93%	0.0034	22.31	18.19
Apr-00	27.69	1.46	5.00%	10.95%	0.0020	19.38	17.75	18.56	0.82	5.00%	9.97%	0.0033	22.44	19.75
May-00	26.47	1.46	5.00%	11.23%	0.0021	20.19	18.00	19.09	0.82	5.00%	9.83%	0.0034	22.63	20.56
Jun-00	26.06	1.46	5.00%	11.33%	0.0021	19.69	17.50	18.59	0.82	5.00%	9.96%	0.0034	22.25	20.13
Jul-00	26.81	1.46	5.00%	11.15%	0.0016	18.56	16.88	17.72	0.82	5.00%	10.21%	0.0029	22.44	20.56
Aug-00	27.06	1.46	5.00%	11.09%	0.0016	19.25	17.13	18.19	0.82	4.75%	9.81%	0.0028	23.31	21.44
Sep-00	28.09	1.46	5.00%	10.86%	0.0015	21.25	18.94	20.09	0.82	4.75%	9.32%	0.0025	24.31	22.31
Oct-00	29.19	1.46	5.20%	10.85%	0.0015	22.50	20.06	21.28	0.82	4.75%	9.06%	0.0024	24.69	21.38
Nov-00	29.16	1.46	5.25%	10.91%	0.0015	20.88	19.38	20.13	0.82	4.75%	9.32%	0.0025	24.00	22.13
Dec-00	29.41	1.46	5.25%	10.86%	0.0015	22.44	19.31	20.88	0.82	4.75%	9.15%	0.0025	26.31	22.38
Jan-01	30.72	1.46	5.25%	10.62%	0.0015	22.44	19.50	20.97	0.82	4.75%	9.13%	0.0026	25.38	22.50
Feb-01	30.50	1.46	5.25%	10.65%	0.0015	23.10	20.79	21.95	0.82	4.75%	8.93%	0.0025	25.20	23.18
Mar-01	29.73	1.46	5.25%	10.80%	0.0015	21.15	19.16	20.15	0.82	4.75%	9.31%	0.0026	25.10	23.13
Apr-01	30.00	1.48	6.00%	11.61%	0.0016	21.20	19.90	20.55	0.82	4.75%	9.22%	0.0024	26.98	24.20
May-01	30.75	1.48	6.00%	11.47%	0.0015	23.45	20.46	21.95	0.82	4.67%	8.85%	0.0022	27.90	25.50

Month Ending	UGI Average	UGI Dividend	UGI Growth	UGI DCF	UGI DCF	WGL High	WGL Low	WGL Average	WGL Dividend	WGL Growth	WGL DCF	WGL DCF	NFG High	NFG Low	NFG Average	NFG Dividend	NFG Growth	NFG DCF
Jun-98	24.44	1.44				27.88	26.19	27.03	1.20	4.63%	9.61%	0.0073	41.22	38.51	39.87	1.800	7.42%	12.62%
Jul-98	24.63	1.46				27.69	23.63	25.66	1.20	4.71%	9.96%	0.0072	41.22	37.51	39.37	1.800	8.10%	13.40%
Aug-98	22.97	1.46				25.56	23.06	24.31	1.20	4.71%	10.26%	0.0057	42.22	37.87	40.05	1.800	8.10%	13.31%
Sep-98	22.13	1.46				27.88	23.75	25.81	1.20	4.71%	9.93%	0.0053	44.72	38.45	41.59	1.800	7.70%	12.69%
Oct-98	23.03	1.46				28.75	26.13	27.44	1.20	4.71%	9.61%	0.0059	47.22	44.06	45.64	1.800	7.70%	12.24%
Nov-98	24.31	1.46				26.63	24.94	25.78	1.20	4.83%	10.06%	0.0062	46.86	42.70	44.78	1.800	7.50%	12.12%
Dec-98	23.13	1.46				27.13	25.13	26.13	1.20	4.83%	9.99%	0.0061	44.66	42.94	43.80	1.800	7.50%	12.23%
Jan-99	22.88	1.46				27.38	23.44	25.41	1.20	4.71%	10.01%	0.0056	44.68	40.60	42.64	1.800	7.50%	12.36%
Feb-99	21.00	1.46				24.75	22.25	23.50	1.20	4.71%	10.45%	0.0058	41.20	38.55	39.88	1.800	8.07%	13.30%
Mar-99	17.69	1.46	6.00%	15.51%	0.0057	25.00	21.31	23.16	1.20	4.75%	10.58%	0.0057	42.52	38.12	40.32	1.800	7.81%	12.97%
Apr-99	17.25	1.46	6.00%	15.76%	0.0044	24.44	21.00	22.72	1.22	4.75%	10.80%	0.0061	42.74	36.42	39.58	1.800	7.50%	12.74%
May-99	19.03	1.46	6.00%	14.82%	0.0041	25.38	23.25	24.31	1.22	4.75%	10.39%	0.0057	46.93	42.43	44.68	1.800	7.36%	11.99%
Jun-99	19.88	1.46	6.00%	14.44%	0.0039	27.06	24.06	25.56	1.22	4.75%	10.11%	0.0055	48.57	45.29	46.93	1.860	7.36%	11.91%
Jul-99	21.81	1.46	6.00%	13.67%	0.0041	28.69	25.00	26.84	1.22	4.71%	9.81%	0.0057	48.78	45.35	47.07	1.860	7.58%	12.13%
Aug-99	23.75	1.46	6.00%	13.03%	0.0039	28.88	26.50	27.69	1.22	4.71%	9.65%	0.0056	47.19	44.68	45.94	1.860	7.58%	12.24%
Sep-99	23.28	1.46	6.67%	13.89%	0.0042	28.13	25.38	26.75	1.22	4.71%	9.83%	0.0057	48.11	44.19	46.15	1.860	7.58%	12.22%
Oct-99	23.13	1.50	6.67%	14.14%	0.0042	27.31	25.00	26.16	1.22	4.71%	9.95%	0.0062	48.59	45.56	47.08	1.860	7.58%	12.12%
Nov-99	21.53	1.50	6.67%	14.71%	0.0044	29.44	26.50	27.97	1.22	4.71%	9.60%	0.0060	52.43	48.47	50.45	1.860	7.19%	11.41%
Dec-99	20.72	1.50	6.67%	15.03%	0.0046	29.25	27.06	28.16	1.22	4.63%	9.48%	0.0060	50.26	46.19	48.23	1.860	7.19%	11.61%
Jan-00	21.06	1.50	6.67%	14.89%	0.0047	27.56	24.50	26.03	1.22	4.63%	9.89%	0.0068	46.75	43.12	44.94	1.860	7.19%	11.94%
Feb-00	19.91	1.50	6.67%	15.39%	0.0049	26.00	21.75	23.88	1.22	4.63%	10.37%	0.0072	45.13	39.38	42.26	1.860	7.19%	12.24%
Mar-00	20.25	1.50	6.67%	15.23%	0.0047	27.63	23.00	25.31	1.22	4.63%	10.04%	0.0067	44.75	39.69	42.22	1.860	7.19%	12.25%
Apr-00	21.09	1.50	6.67%	14.88%	0.0043	26.94	24.88	25.91	1.24	4.63%	10.00%	0.0061	48.06	43.12	45.59	1.860	7.19%	11.87%
May-00	21.59	1.50				27.63	25.63	26.63	1.24	4.57%	9.79%	0.0062	51.88	46.25	49.07	1.860	6.79%	11.12%
Jun-00	21.19	1.50	6.67%	14.84%	0.0044	27.44	24.06	25.75	1.24	4.63%	10.04%	0.0063	51.88	48.00	49.94	1.920	7.19%	11.59%
Jul-00	21.50	1.55	6.67%	15.00%	0.0043	25.50	23.94	24.72	1.24	4.63%	10.27%	0.0059	52.38	48.12	50.25	1.920	7.19%	11.57%
Aug-00	22.38	1.55	6.67%	14.66%	0.0041	27.06	24.50	25.78	1.24	4.63%	10.03%	0.0057	53.69	49.50	51.60	1.920	7.19%	11.45%
Sep-00	23.31	1.55	6.67%	14.33%	0.0038	27.75	24.94	26.34	1.24	4.63%	9.91%	0.0053	58.81	52.38	55.60	1.920	7.19%	11.14%
Oct-00	23.03	1.55	6.67%	14.43%	0.0039	27.50	24.81	26.16	1.24	4.63%	9.95%	0.0050	59.62	51.12	55.37	1.920	7.44%	11.42%
Nov-00	23.06	1.55				28.50	25.38	26.94	1.24	4.43%	9.58%	0.0049	59.19	53.50	56.35	1.920	7.58%	11.49%
Dec-00	24.34	1.55				31.50	27.44	29.47	1.24	4.43%	9.13%	0.0048	64.50	53.38	58.94	1.920	7.58%	11.32%
Jan-01	23.94	1.55				30.50	27.06	28.78	1.24	4.43%	9.25%	0.0050	63.19	52.43	57.81	1.920	7.58%	11.39%
Feb-01	24.19	1.55				28.70	26.37	27.54	1.24	4.43%	9.47%	0.0052	55.40	50.97	53.19	1.920	7.50%	11.64%
Mar-01	24.11	1.55				27.95	25.82	26.89	1.24	4.43%	9.59%	0.0052	56.00	50.01	53.01	1.920	7.50%	11.66%
Apr-01	25.59	1.55	6.00%	12.92%	0.0034	29.10	26.30	27.70	1.26	4.43%	9.52%	0.0049	57.61	53.59	55.60	1.920	7.50%	11.46%
May-01	26.70	1.55	7.00%	13.69%	0.0035	29.40	27.90	28.65	1.26	4.43%	9.35%	0.0046	57.97	53.07	55.52	1.920	8.51%	12.51%

Month Ending	NFG DCF	STR High	STR Low	STR Average	STR Dividend	STR Growth	STR DCF	STR DCF	AGL	ATO	CGC	EGN	EQT	KSE	LG	NJR	GAS	NI	NWN
Jun-98	0.0126	20.50	18.69	19.59	0.660	8.75%	12.66%	0.0122	1.10	0.88	0.18	0.58	1.06			0.65	2.00		0.65
Jul-98	0.0132	18.55	16.85	17.70	0.660	8.95%	13.29%	0.0125	1.10	0.90	0.18	0.58	0.85			0.63	1.90		0.62
Aug-98	0.0100	17.49	15.35	16.42	0.660	9.15%	13.84%	0.0088	1.10	0.90	0.18	0.58	0.77	4.90		0.63	1.90		0.62
Sep-98	0.0107	18.54	14.94	16.74	0.660	9.06%	13.66%	0.0100	1.10	0.90	0.18	0.58	0.89	4.90		0.63	1.90		0.62
Oct-98	0.0108	19.25	17.83	18.54	0.660	9.11%	13.26%	0.0103	1.10	0.85	0.18	0.53	0.97	4.10		0.63	1.90		0.65
Nov-98	0.0104	19.17	18.01	18.59	0.660	9.11%	13.25%	0.0102	1.10	0.85	0.18	0.53	1.04	4.10		0.63	1.90		0.65
Dec-98	0.0104	18.69	16.55	17.62	0.660	9.11%	13.48%	0.0105	1.10	0.85	0.18	0.53	1.03	4.10		0.63	1.90		0.65
Jan-99	0.0098	18.46	15.84	17.15	0.660	8.83%	13.31%	0.0088	1.30	0.90	0.20	0.53	0.92	4.30		0.68	1.90		0.68
Feb-99	0.0101	17.31	15.36	16.34	0.660	8.83%	13.53%	0.0097	1.30	0.90	0.20	0.53	0.93	4.30		0.68	1.90		0.68
Mar-99	0.0093	17.79	16.11	16.95	0.660	8.61%	13.13%	0.0086	1.30	0.90	0.20	0.53	0.93	4.30		0.68	1.90		0.68
Apr-99	0.0107	18.09	15.21	16.65	0.660	8.55%	13.15%	0.0097	1.10	0.70	0.18	0.45	0.91	3.80		0.65	1.80		0.60
May-99	0.0107	18.82	17.31	18.07	0.660	8.55%	12.79%	0.0098	1.10	0.70	0.18	0.45	1.05	3.80		0.65	1.80		0.60
Jun-99	0.0108	19.35	17.89	18.62	0.660	8.55%	12.66%	0.0096	1.10	0.70	0.18	0.45	1.26	3.80		0.65	1.80		0.60
Jul-99	0.0104	18.86	17.59	18.23	0.660	8.50%	12.70%	0.0092	1.10	0.80	0.20	0.58	1.23	3.80		0.70	1.80		0.65
Aug-99	0.0105	19.03	18.01	18.52	0.680	8.50%	12.75%	0.0094	1.10	0.80	0.20	0.58	1.22	3.80		0.70	1.80		0.65
Sep-99	0.0107	19.22	17.50	18.36	0.680	8.50%	12.79%	0.0090	1.10	0.80	0.20	0.58	1.26	3.80		0.70	1.80		0.65
Oct-99	0.0108	18.73	17.14	17.94	0.680	8.50%	12.90%	0.0090	1.00	0.78	0.20	0.60	1.22	4.00		0.72	1.80		0.65
Nov-99	0.0105	17.75	16.19	16.97	0.680	8.21%	12.85%	0.0086	1.00	0.78	0.20	0.60	1.14	4.00		0.72	1.80		0.65
Dec-99	0.0102	17.05	14.58	15.82	0.680	8.50%	13.49%	0.0080	1.00	0.78	0.20	0.60	1.09	4.00		0.72	1.80		0.65
Jan-00	0.0109	16.56	14.40	15.48	0.680	8.50%	13.60%	0.0090	1.00	0.69	0.19	0.48	1.13	3.20		0.71	1.50		0.60
Feb-00	0.0104	15.63	13.56	14.60	0.680	8.50%	13.92%	0.0085	1.00	0.69	0.19	0.48	1.25	3.20		0.71	1.50		0.60
Mar-00	0.0109	19.00	14.00	16.50	0.680	8.50%	13.28%	0.0102	1.00	0.69	0.19	0.48	1.47	3.20		0.71	1.50		0.60
Apr-00	0.0121	19.31	17.12	18.22	0.680	8.50%	12.83%	0.0107	1.00	0.58	0.15	0.45	1.52	2.90		0.65	1.50		0.48
May-00	0.0130	20.56	18.00	19.28	0.680	8.55%	12.64%	0.0118	1.00	0.58	0.15	0.45	1.64	2.90		0.65	1.50		0.48
Jun-00	0.0126	20.62	18.88	19.75	0.680	8.55%	12.54%	0.0110	1.00	0.58	0.15	0.45	1.59	2.90		0.65	1.50		0.48
Jul-00	0.0107	20.19	18.88	19.54	0.680	8.55%	12.58%	0.0094	0.88	0.60	0.20	0.63	1.71	4.20		0.68	1.60		0.58
Aug-00	0.0111	22.00	19.56	20.78	0.680	8.55%	12.34%	0.0101	0.88	0.60	0.20	0.63	1.83	4.20		0.68	1.60		0.58
Sep-00	0.0109	28.00	21.38	24.69	0.680	8.39%	11.57%	0.0115	0.88	0.60	0.20	0.63	2.07	4.20	0.35	0.68	1.60		0.58
Oct-00	0.0101	29.50	26.00	27.75	0.680	8.94%	11.78%	0.0107	1.10	0.68	0.20	0.85	1.89	4.80	0.43	0.73	1.60		0.60
Nov-00	0.0110	31.88	27.00	29.44	0.700	8.93%	11.68%	0.0112	1.10	0.68	0.20	0.85	1.82	4.80	0.43	0.73	1.60		0.60
Dec-00	0.0121	31.25	26.38	28.82	0.700	8.92%	11.73%	0.0123	1.10	0.68	0.20	0.85	2.18	4.80	0.43	0.73	1.60		0.60
Jan-01	0.0099	29.94	27.12	28.53	0.700	8.92%	11.76%	0.0111	1.20	0.70	0.23	0.85	1.92	5.40	0.42	0.73	1.80		0.63
Feb-01	0.0100	28.45	26.70	27.58	0.700	8.92%	11.86%	0.0110	1.20	0.70	0.23	0.85	1.87	5.40	0.42	0.73	1.80		0.63
Mar-01	0.0102	29.95	26.35	28.15	0.700	8.79%	11.67%	0.0107	1.20	0.70	0.23	0.85	2.24	5.40	0.42	0.73	1.80		0.63
Apr-01	0.0099	33.17	26.80	29.99	0.700	8.58%	11.27%	0.0114	1.10	0.90		0.93	1.30	5.40	0.45	0.70	1.70		0.60
May-01	0.0107	33.75	30.05	31.90	0.700	9.27%	11.82%	0.0111	1.10	0.90		0.93	2.42	5.40	0.45	0.70	1.70		0.60

	NUI	OKE	PGL	PNY	SEN	SJI	SWX	UGI	WGL	NFG	STR	Mkt Cap	Ave. DCF
Month Ending													
Jun-98	0.35	1.20	1.30	0.95			0.55		1.20	1.57	1.51	15.72	11.54%
Jul-98	0.33	1.20	1.30	0.98			0.63		1.10	1.49	1.43	15.20	11.86%
Aug-98	0.33	1.20	1.30	0.98			0.63		1.10	1.49	1.26	19.84	12.34%
Sep-98	0.33	1.20	1.30	0.98			0.63		1.10	1.72	1.50	20.43	12.73%
Oct-98	0.30	1.10	1.30	1.00			0.58		1.20	1.73	1.53	19.62	12.60%
Nov-98	0.30	1.10	1.30	1.00			0.58		1.20	1.68	1.51	19.62	12.11%
Dec-98	0.30	1.10	1.30	1.00			0.58		1.20	1.67	1.52	19.62	11.85%
Jan-99	0.30	1.00	1.30	1.00			0.80		1.10	1.57	1.31	19.78	11.95%
Feb-99	0.30	1.00	1.30	1.00			0.80		1.10	1.50	1.42	19.82	12.43%
Mar-99	0.30	1.00	1.30	1.00			0.80	0.75	1.10	1.47	1.34	20.47	12.57%
Apr-99	0.30	0.88	1.30	1.10	0.25		0.83	0.55	1.10	1.64	1.45	19.57	12.60%
May-99	0.30	0.88	1.30	1.10	0.25		0.83	0.55	1.10	1.79	1.53	19.94	12.21%
Jun-99	0.30	0.88	1.30	1.10	0.25		0.83	0.55	1.10	1.84	1.53	20.20	12.08%
Jul-99	0.33	0.98	1.40	1.00	0.25		0.85	0.63	1.20	1.78	1.51	20.77	12.22%
Aug-99	0.33	0.98	1.40	1.00	0.25		0.85	0.63	1.20	1.79	1.53	20.79	12.20%
Sep-99	0.33	0.98	1.40	1.00	0.25		0.85	0.63	1.20	1.81	1.47	20.79	12.26%
Oct-99	0.33	0.98	1.30	1.00		0.33	0.85	0.63	1.30	1.88	1.46	20.99	12.33%
Nov-99	0.33	0.98	1.30	1.00		0.33	0.85	0.63	1.30	1.92	1.40	20.90	12.40%
Dec-99	0.33	0.98	1.30	1.00		0.33	0.85	0.63	1.30	1.80	1.22	20.56	12.80%
Jan-00	0.33	0.87	1.20	0.98	0.23	0.33	0.68	0.60	1.30	1.74	1.25	18.97	13.01%
Feb-00	0.33	0.87	1.20	0.98	0.23	0.33	0.68	0.60	1.30	1.60	1.15	18.85	13.44%
Mar-00	0.33	0.87	1.20	0.98	0.23	0.33	0.68	0.60	1.30	1.74	1.50	19.56	13.44%
Apr-00	0.33	0.68	1.00	0.80	0.20	0.33	0.60	0.53	1.10	1.85	1.52	18.14	13.16%
May-00	0.33		1.00	0.80	0.20	0.33	0.60		1.10	2.03	1.62	17.34	12.92%
Jun-00	0.33		1.00	0.80	0.20	0.33	0.60	0.53	1.10	1.91	1.55	17.62	12.95%
Jul-00	0.35	0.85	1.20	0.95	0.23	0.30	0.60	0.60	1.20	1.93	1.56	20.83	13.17%
Aug-00	0.35	0.85	1.20	0.95	0.23	0.30	0.60	0.60	1.20	2.06	1.74	21.26	12.90%
Sep-00	0.35	0.85	1.20	0.95	0.23	0.30	0.60	0.60	1.20	2.20	2.23	22.47	12.57%
Oct-00	0.40	1.10	1.20	0.95	0.30	0.33	0.63	0.65	1.20	2.11	2.17	23.89	12.60%
Nov-00	0.40	1.10	1.20	0.95	0.30	0.33	0.63		1.20	2.24	2.23	23.36	12.51%
Dec-00	0.40		1.20	0.95	0.30	0.33	0.63		1.20	2.47	2.42	23.05	12.39%
Jan-01	0.38		1.60	1.10	0.28	0.34	0.68		1.30	2.07	2.24	23.83	12.61%
Feb-01	0.38		1.60	1.10	0.28	0.34	0.68		1.30	2.05	2.21	23.73	12.61%
Mar-01	0.38		1.60	1.10	0.28	0.34	0.68		1.30	2.12	2.21	24.17	12.75%
Apr-01	0.37	1.30	1.60	1.10	0.25	0.35	0.65	0.68	1.30	2.20	2.58	25.44	12.27%
May-01	0.37	1.30	1.60	1.10	0.25	0.35	0.65	0.68	1.30	2.26	2.50	26.55	13.02%

Month Ending	AGL High	AGL Low	AGL Average	AGL Dividend	AGL Growth	AGL DCF	AGL DCF	ATO High	ATO Low	ATO Average	ATO Dividend	ATO Growth	ATO DCF	ATO DCF	Cascade High	Cascade Low	Cascade Average	Cascade Dividend
Jun-01	24.09	22.50	23.30	1.08	6.59%	11.89%	0.0051	24.00	22.49	23.24	1.16	7.36%	13.11%	0.0046	20.50	19.05	19.77	0.96
Jul-01	24.22	22.18	23.20	1.08	7.16%	12.51%	0.0062	24.55	19.60	22.07	1.16	8.00%	14.10%	0.0052	21.30	19.10		
Aug-01	24.50	21.10	22.80	1.08	7.16%	12.60%	0.0063	22.84	19.85	21.35	1.16	7.50%	13.78%	0.0052	22.00	19.35		
Sep-01	22.05	18.95	20.50	1.08	6.59%	12.63%	0.0067	22.35	20.66	21.51	1.16	6.33%	12.50%	0.0050	22.50	19.50		
Oct-01	21.49	19.50	20.49	1.08	6.75%	12.80%	0.0068	22.21	20.30	21.25	1.16	6.33%	12.57%	0.0050	21.60	19.62		
Nov-01	22.19	20.55	21.37	1.08	6.75%	12.54%	0.0066	21.94	19.46	20.70	1.16	6.33%	12.74%	0.0051	22.80	19.66		
Dec-01	23.24	21.08	22.16	1.08	7.00%	12.60%	0.0067	21.70	19.45	20.58	1.18	6.00%	12.55%	0.0048				
Jan-02	23.02	20.60	21.81	1.08	7.00%	12.69%	0.0069	21.99	20.54	21.27	1.18	6.00%	12.33%	0.0048				
Feb-02	22.78	20.95	21.87	1.08	8.43%	14.18%	0.0081	22.65	20.26	21.46	1.18	6.00%	12.27%	0.0050				
Mar-02	23.69	22.16	22.93	1.08	8.43%	13.91%	0.0078	24.50	22.13	23.31	1.18	6.00%	11.76%	0.0048				
Apr-02	24.34	22.80	23.57	1.08	7.00%	12.25%	0.0068	24.55	23.44	23.99	1.18	6.00%	11.59%	0.0047				
May-02	24.17	22.80	23.48	1.08	7.00%	12.27%	0.0066	24.29	22.73	23.51	1.18	7.60%	13.40%	0.0053				
Jun-02	23.50	21.51	22.51	1.08	7.00%	12.51%	0.0064	23.65	21.00	22.32	1.18	7.33%	13.43%	0.0048				
Jul-02	23.35	17.25	20.30	1.08	7.00%	13.12%	0.0068	23.47	17.56	20.51	1.18	7.14%	13.78%	0.0050				
Aug-02	23.28	20.50	21.89	1.08	7.13%	12.80%	0.0064	22.95	20.41	21.68	1.18	7.14%	13.41%	0.0050				
Sep-02	23.70	21.52	22.61	1.08	7.13%	12.62%	0.0066	22.35	20.70	21.53	1.18	7.71%	14.06%	0.0054				
Oct-02	24.09	20.50	22.30	1.08	7.00%	12.56%	0.0068	22.30	20.62	21.46	1.18	7.71%	14.08%	0.0054	20.20	18.10		
Nov-02	24.50	22.70	23.60	1.08	7.00%	12.25%	0.0066	23.15	21.27	22.21	1.18	6.57%	12.66%	0.0049	20.33	18.75		
Dec-02	25.00	23.75	24.38	1.08	7.00%	12.08%	0.0069	23.88	22.38	23.13	1.20	6.71%	12.66%	0.0049	20.44	17.70		
Jan-03	25.41	22.71	24.06	1.08	7.00%	12.15%	0.0070	24.31	21.40	22.85	1.20	6.71%	12.73%	0.0050	20.24	18.05		
Feb-03	23.14	21.90	22.52	1.08	7.00%	12.50%	0.0072	22.47	21.01	21.74	1.20	6.43%	12.75%	0.0050	19.69	18.50		
Mar-03	23.70	22.03	22.87	1.08	6.47%	11.86%	0.0065	21.90	20.85	21.38	1.20	6.29%	12.71%	0.0051	19.63	18.20		
Apr-03	25.87	23.30	24.59	1.08	6.23%	11.23%	0.0061	22.94	21.05	21.99	1.20	6.09%	12.32%	0.0049	19.54	18.20		
May-03	26.98	24.50	25.74	1.08	5.59%	10.33%	0.0055	24.98	22.37	23.68	1.20	6.09%	11.86%	0.0046	19.80	18.36		
Jun-03	26.98	25.28	26.13	1.12	5.59%	10.44%	0.0061	25.50	23.60	24.55	1.20	6.09%	11.65%	0.0048				
Jul-03	27.67	25.35	26.51	1.12	5.53%	10.30%	0.0061	25.14	24.05	24.60	1.20	6.09%	11.64%	0.0049				
Aug-03	27.92	26.82	27.37	1.12	5.53%	10.15%	0.0060	24.84	23.00	23.92	1.20	6.09%	11.80%	0.0049				
Sep-03	28.49	27.77	28.13	1.12	5.43%	9.92%	0.0064	24.98	23.81	24.40	1.20	6.09%	11.69%	0.0051				
Oct-03	29.04	27.24	28.14	1.12	5.43%	9.92%	0.0064	24.95	24.05	24.50	1.20	6.09%	11.67%	0.0050				
Nov-03	28.72	27.50	28.11	1.12	4.71%	9.17%	0.0059	24.89	24.27	24.58	1.20	5.67%	11.21%	0.0048				
Dec-03	29.35	28.25	28.80	1.12	4.71%	9.06%	0.0059	25.00	23.92	24.46	1.22	5.67%	11.33%	0.0051				
Jan-04	30.63	28.60	29.62	1.12	4.71%	8.94%	0.0062	25.96	24.30	25.13	1.22	5.67%	11.17%	0.0053				
Feb-04	29.39	27.87	28.63	1.12	4.31%	8.67%	0.0057	26.70	24.80	25.75	1.22	5.67%	11.04%	0.0050				
Mar-04	29.02	28.01	28.52	1.16	4.03%	8.56%	0.0055	26.99	25.04	26.02	1.22	5.60%	10.91%	0.0054				
Apr-04	29.41	27.53	28.47	1.16	4.40%	8.95%	0.0057	26.16	24.10	25.13	1.22	5.60%	11.10%	0.0055				
May-04	28.99	26.51	27.75	1.16	4.80%	9.49%	0.0065	25.10	23.40	24.25	1.22	6.67%	12.43%	0.0066				

Month Ending	Cascade Growth	Cascade DCF	Cascade DCF	EGN High	EGN Low	EGN Average	EGN Dividend	EGN Growth	EGN DCF	EGN DCF	EQT High	EQT Low	EQT Average	EQT Dividend	EQT Growth	EQT DCF	EQT DCF
Jun-01				34.80	28.80	31.80	0.68	11.00%	13.52%	0.0049	37.88	31.80	34.84	0.64	11.75%	13.93%	0.0118
Jul-01				28.21	23.95	26.08	0.68	11.00%	14.08%	0.0048	36.60	31.35	33.98	0.64	11.75%	13.98%	0.0134
Aug-01				27.20	24.70	25.95	0.68	11.00%	14.09%	0.0048	36.05	31.83	33.94	0.64	10.50%	12.71%	0.0109
Sep-01				27.28	21.50	24.39	0.70	11.50%	14.91%	0.0055	32.32	26.00	29.16	0.64	10.44%	13.01%	0.0112
Oct-01				25.20	21.50	23.35	0.70	11.50%	15.06%	0.0055	33.80	29.15	31.48	0.64	10.44%	12.82%	0.0120
Nov-01				25.05	22.00	23.52	0.70	11.50%	15.03%	0.0055	34.69	31.00	32.85	0.64	10.44%	12.72%	0.0117
Dec-01				25.09	22.17	23.63	0.70	11.50%	15.02%	0.0048	34.38	31.00	32.69	0.64	10.44%	12.73%	0.0123
Jan-02				24.68	22.16	23.42	0.70	11.50%	15.05%	0.0050	33.92	29.32	31.62	0.64	10.25%	12.62%	0.0113
Feb-02				23.60	21.69	22.65	0.70	11.25%	14.91%	0.0049	33.00	29.50	31.25	0.64	10.00%	12.39%	0.0115
Mar-02				26.49	22.50	24.49	0.70	9.75%	13.09%	0.0042	35.66	32.68	34.17	0.64	10.00%	12.18%	0.0114
Apr-02				29.25	26.45	27.85	0.70	7.40%	10.27%	0.0032	37.55	34.00	35.78	0.64	9.81%	11.89%	0.0116
May-02				29.20	26.00	27.60	0.70	7.20%	10.09%	0.0031	37.27	35.45	36.36	0.68	10.19%	12.38%	0.0117
Jun-02				27.81	24.70	26.26	0.70	7.20%	10.24%	0.0035	36.22	33.54	34.88	0.68	10.19%	12.47%	0.0107
Jul-02				27.53	21.65	24.59	0.70	7.40%	10.65%	0.0037	34.72	28.67	31.70	0.68	10.19%	12.70%	0.0110
Aug-02				27.20	24.29	25.75	0.70	7.40%	10.51%	0.0039	36.49	32.82	34.66	0.68	10.19%	12.48%	0.0119
Sep-02				26.49	23.84	25.17	0.72	7.40%	10.67%	0.0040	36.25	33.17	34.71	0.68	10.19%	12.48%	0.0116
Oct-02				28.21	22.50	25.35	0.72	7.00%	10.23%	0.0037	36.00	32.09	34.05	0.68	10.19%	12.53%	0.0115
Nov-02				28.60	26.72	27.66	0.72	7.00%	9.96%	0.0036	36.55	34.10	35.33	0.68	10.19%	12.44%	0.0114
Dec-02				29.99	26.72	28.35	0.72	7.00%	9.89%	0.0039	36.89	34.62	35.76	0.68	10.17%	12.39%	0.0112
Jan-03				30.95	28.08	29.52	0.72	7.00%	9.77%	0.0038	37.30	34.83	36.07	0.68	10.17%	12.37%	0.0118
Feb-03				30.85	28.47	29.66	0.72	7.20%	9.97%	0.0039	37.84	34.44	36.14	0.68	10.17%	12.37%	0.0115
Mar-03				32.06	30.32	31.19	0.72	7.20%	9.83%	0.0046	37.90	36.05	36.98	0.68	9.56%	11.70%	0.0115
Apr-03				33.19	31.72	32.45	0.72	7.20%	9.73%	0.0045	39.00	37.08	38.04	0.68	9.56%	11.64%	0.0116
May-03				33.95	31.60	32.78	0.72	7.25%	9.75%	0.0044	40.27	37.72	39.00	0.80	9.56%	11.95%	0.0123
Jun-03				34.29	32.35	33.32	0.72	7.25%	9.71%	0.0040	42.00	40.02	41.01	0.80	9.56%	11.83%	0.0103
Jul-03				34.80	31.35	33.08	0.72	7.00%	9.47%	0.0039	41.27	38.37	39.82	1.20	9.56%	13.08%	0.0110
Aug-03				35.99	32.96	34.48	0.72	7.00%	9.37%	0.0039	39.80	37.85	38.83	1.20	9.44%	13.04%	0.0111
Sep-03				37.09	35.30	36.20	0.74	7.00%	9.32%	0.0044	41.65	39.29	40.47	1.20	9.44%	12.90%	0.0119
Oct-03				38.93	36.14	37.54	0.74	7.00%	9.24%	0.0043	41.97	40.68	41.33	1.20	9.50%	12.89%	0.0119
Nov-03				39.04	36.62	37.83	0.74	7.00%	9.22%	0.0043	41.60	39.95	40.78	1.20	9.78%	13.22%	0.0121
Dec-03				42.00	38.55	40.28	0.74	7.00%	9.08%	0.0044	43.42	41.34	42.38	1.20	9.78%	13.09%	0.0121
Jan-04				44.72	40.72	42.72	0.74	7.00%	8.96%	0.0046	44.92	42.34	43.63	1.20	9.75%	12.96%	0.0128
Feb-04				43.49	40.89	42.19	0.74	7.00%	8.98%	0.0044	44.86	42.50	43.68	1.20	9.75%	12.96%	0.0121
Mar-04				43.20	39.87	41.54					44.45	42.10	43.28	1.20	9.75%	12.99%	0.0125
Apr-04				42.61	40.41	41.51					47.80	43.99	45.90	1.20	9.71%	12.76%	0.0123
May-04				44.95	40.12	42.54					48.70	45.16	46.93	1.20	9.40%	12.37%	0.0127

Month Ending	Keyspan High	Keyspan Low	Keyspan Average	Keyspan Dividend	Keyspan Growth	Keyspan DCF	Keyspan DCF	LG High	LG Low	LG Average	LG Dividend	LG Growth	LG DCF	LG DCF	NJR High	NJR Low
Jun-01	40.05	36.37	38.21	1.78	11.07%	16.62%	0.0351	25.30	23.58	24.44	1.34	3.33%	9.42%	0.0017	45.96	42.27
Jul-01	37.20	29.10	33.15	1.78	11.39%	17.82%	0.0329	25.40	21.75	23.57	1.34	3.33%	9.65%	0.0018	45.33	41.00
Aug-01	32.86	29.85	31.36	1.78	11.39%	18.20%	0.0341	25.35	21.95	23.65	1.34	3.33%	9.63%	0.0018	45.81	42.85
Sep-01	33.40	31.50	32.45	1.78	8.38%	14.77%	0.0296	24.87	22.40						45.50	42.24
Oct-01	35.35	31.86	33.60	1.78	8.38%	14.55%	0.0290	25.30	22.60						46.95	43.45
Nov-01	34.44	32.52	33.48	1.78	8.38%	14.57%	0.0291	25.10	22.70						48.80	44.91
Dec-01	34.98	31.53	33.26	1.78	7.83%	14.04%	0.0298								47.35	44.82
Jan-02	35.55	31.25	33.40	1.78	7.17%	13.31%	0.0281								46.86	44.20
Feb-02	32.59	30.01	31.30	1.78	6.83%	13.37%	0.0280								31.16	29.23
Mar-02	36.72	31.98	34.35	1.78	6.83%	12.78%	0.0259								32.00	30.06
Apr-02	37.45	34.35	35.90	1.78	6.71%	12.39%	0.0248								32.90	30.29
May-02	38.20	35.15	36.68	1.78	6.71%	12.27%	0.0239								32.59	30.20
Jun-02	38.00	35.60	36.80	1.78	6.71%	12.25%	0.0251								30.70	28.45
Jul-02	38.19	27.41	32.80	1.78	6.71%	12.94%	0.0270								31.10	24.35
Aug-02	36.68	33.78	35.23	1.78	6.71%	12.50%	0.0252								32.87	29.50
Sep-02	34.85	31.96	33.41	1.78	7.75%	13.92%	0.0288								33.29	30.65
Oct-02	36.98	30.75	33.86	1.78	7.75%	13.84%	0.0282	24.35	21.79						33.20	29.52
Nov-02	37.15	33.80	35.48	1.78	7.75%	13.55%	0.0275	24.50	22.75						32.03	29.86
Dec-02	36.16	34.20	35.18	1.78	7.88%	13.74%	0.0282	24.84	23.00						33.60	31.20
Jan-03	38.14	33.01	35.57	1.78	8.00%	13.80%	0.0282	24.90	23.00						33.60	30.01
Feb-03	34.19	31.02	32.60	1.78	7.78%	14.11%	0.0290	23.80	21.85						32.67	30.42
Mar-03	33.44	31.07	32.25	1.78	7.10%	13.46%	0.0255	23.96	21.90						33.70	31.70
Apr-03	34.25	31.87	33.06	1.78	7.10%	13.30%	0.0249	24.29	23.10						34.79	32.25
May-03	37.51	33.28	35.39	1.78	6.64%	12.40%	0.0228	26.92	23.80						35.49	32.60
Jun-03	36.70	35.12	35.91	1.78	6.64%	12.31%	0.0240								36.60	35.12
Jul-03	35.80	33.52	34.66	1.78	6.64%	12.52%	0.0248								36.87	34.50
Aug-03	34.47	32.30	33.39	1.78	6.64%	12.75%	0.0252								36.39	33.70
Sep-03	35.83	33.83	34.83	1.78	6.55%	12.40%	0.0246								37.36	35.81
Oct-03	36.28	34.37	35.33	1.78	6.55%	12.31%	0.0244								38.00	35.76
Nov-03	35.45	33.64	34.55	1.78	5.88%	11.74%	0.0230								39.25	36.45
Dec-03	37.09	34.86	35.98	1.78	5.88%	11.50%	0.0218								39.54	37.55
Jan-04	37.26	35.72	36.49	1.78	5.88%	11.42%	0.0229								39.49	37.75
Feb-04	38.00	36.16	37.08	1.78	5.29%	10.71%	0.0204								40.00	37.63
Mar-04	38.60	36.87	37.74	1.78	5.29%	10.62%	0.0227								39.20	36.81
Apr-04	38.99	35.41	37.20	1.78	5.14%	10.54%	0.0225								38.90	36.55
May-04	36.90	33.87	35.39	1.78	4.89%	10.56%	0.0241								41.98	38.51

Month Ending	NJR Average	NJR Dividend	NJR Growth	NJR DCF	NJR DCF	GAS High	GAS Low	GAS Average	GAS Dividend	GAS Growth	GAS DCF	GAS DCF	NI High	NI Low	NI Average	NI Dividend
Jun-01	44.11	1.76	6.83%	11.39%	0.0031	39.20	37.98	38.59	1.76	5.94%	11.12%	0.0074				
Jul-01	43.17	1.76	6.83%	11.49%	0.0037	39.40	34.00	36.70	1.76	5.79%	11.23%	0.0083				
Aug-01	44.33	1.76	6.38%	10.90%	0.0035	39.23	36.70	37.97	1.76	5.79%	11.05%	0.0083				
Sep-01	43.87	1.76	6.38%	10.94%	0.0038	39.74	37.00	38.37	1.76	5.90%	11.11%	0.0089				
Oct-01	45.20	1.76	6.38%	10.81%	0.0037	40.90	38.14	39.52	1.76	5.90%	10.95%	0.0087				
Nov-01	46.85	1.76	6.38%	10.65%	0.0037	39.84	37.52	38.68	1.76	5.90%	11.06%	0.0088				
Dec-01	46.09	1.76	6.38%	10.72%	0.0039	42.00	38.20	40.10	1.76	6.00%	10.98%	0.0089				
Jan-02	45.53	1.76	6.33%	10.72%	0.0040	41.90	39.55	40.73	1.76	6.00%	10.90%	0.0088				
Feb-02	30.20	1.20	6.33%	10.85%	0.0041	42.69	39.67	41.18	1.76	6.00%	10.85%	0.0094				
Mar-02	31.03	1.20	6.33%	10.72%	0.0038	46.20	41.69	43.94	1.76	6.00%	10.54%	0.0091				
Apr-02	31.60	1.20	6.33%	10.65%	0.0037	49.00	44.99	47.00	1.76	5.80%	10.03%	0.0086				
May-02	31.40	1.20	6.67%	11.03%	0.0038	49.00	46.05	47.52	1.84	5.80%	10.18%	0.0084				
Jun-02	29.58	1.20	6.67%	11.30%	0.0036	48.70	45.75	47.23	1.84	5.80%	10.21%	0.0084				
Jul-02	27.73	1.20	6.67%	11.61%	0.0037	47.83	18.09	32.96	1.84	6.00%	12.37%	0.0104				
Aug-02	31.18	1.20	6.67%	11.06%	0.0036	31.50	23.80	27.65	1.84	6.00%	13.62%	0.0087				
Sep-02	31.97	1.20	6.67%	10.95%	0.0039	29.39	17.25	23.32	1.84	6.00%	15.08%	0.0081				
Oct-02	31.36	1.20	6.67%	11.03%	0.0040	31.77	24.25	28.01	1.84	5.50%	12.99%	0.0070				
Nov-02	30.94	1.20	6.67%	11.09%	0.0040	33.29	29.72	31.51	1.84	5.17%	11.79%	0.0063				
Dec-02	32.40	1.20	6.67%	10.89%	0.0039	35.39	30.55	32.97	1.84	5.17%	11.49%	0.0066				
Jan-03	31.80	1.24	6.67%	11.12%	0.0040	35.62	30.65	33.13	1.84	5.17%	11.45%	0.0066				
Feb-03	31.54	1.24	7.00%	11.50%	0.0041	32.30	29.75	31.02	1.84	5.17%	11.89%	0.0068				
Mar-03	32.70	1.24	7.00%	11.34%	0.0041	31.85	23.70	27.78	1.84	5.17%	12.70%	0.0059				
Apr-03	33.52	1.24	7.00%	11.23%	0.0040	30.47	27.05	28.76	1.84	5.17%	12.43%	0.0057				
May-03	34.05	1.24	6.50%	10.64%	0.0037	36.30	29.07	32.68	1.86	5.17%	11.61%	0.0052				
Jun-03	35.86	1.24	6.50%	10.43%	0.0035	39.30	35.29	37.30	1.86	5.17%	10.80%	0.0063				
Jul-03	35.69	1.24	6.50%	10.45%	0.0035	37.70	35.35	36.53	1.86	4.38%	10.09%	0.0060				
Aug-03	35.05	1.24	6.50%	10.52%	0.0036	36.40	33.51	34.96	1.86	4.38%	10.35%	0.0061				
Sep-03	36.59	1.24	6.50%	10.35%	0.0036	36.05	34.00	35.03	1.86	4.38%	10.34%	0.0056				
Oct-03	36.88	1.24	6.50%	10.32%	0.0036	36.62	32.75	34.69	1.86	4.33%	10.35%	0.0056				
Nov-03	37.85	1.30	6.50%	10.40%	0.0036	34.45	32.03	33.24	1.86	4.04%	10.30%	0.0055				
Dec-03	38.55	1.30	6.00%	9.81%	0.0034	34.65	32.86	33.76	1.86	3.86%	10.02%	0.0052				
Jan-04	38.62	1.30	6.00%	9.81%	0.0036	34.24	32.49	33.37	1.86	3.83%	10.06%	0.0055				
Feb-04	38.82	1.30	6.00%	9.79%	0.0034	36.25	32.55	34.40	1.86	3.73%	9.76%	0.0051				
Mar-04	38.01	1.30	6.25%	10.13%	0.0036	37.43	34.76	36.10	1.86	3.68%	9.42%	0.0054				
Apr-04	37.73	1.30	6.33%	10.24%	0.0036	35.65	33.31	34.48	1.86	3.68%	9.69%	0.0055				
May-04						34.50	32.04	33.27	1.86	3.67%	9.91%	0.0060				

Month Ending	NI Growth	NI DCF	NI DCF	NWN High	NWN Low	NWN Average	NWN Dividend	NWN Growth	NWN DCF	NWN DCF	NUI High	NUI Low	NUI Average	NUI Dividend	NUI Growth	NUI DCF
Jun-01				25.00	23.90	24.45	1.24	4.25%	9.93%	0.0023	22.40	20.60	21.50	0.98	10.95%	16.37%
Jul-01				25.15	23.58	24.36	1.24	4.24%	9.94%	0.0025	23.60	21.40	22.50	0.98	10.95%	16.12%
Aug-01				25.50	23.81	24.65	1.24	4.55%	10.20%	0.0027	23.95	22.30	23.13	0.98	10.95%	15.98%
Sep-01				25.85	22.39	24.12	1.24	4.64%	10.42%	0.0029	22.70	20.08				
Oct-01				26.00	22.00	24.00	1.24	4.64%	10.45%	0.0029	22.19	20.18				
Nov-01				25.00	23.39	24.19	1.24	4.64%	10.40%	0.0029	23.15	20.45				
Dec-01																
Jan-02																
Feb-02																
Mar-02																
Apr-02											26.91	24.50	25.70	0.98	7.33%	11.70%
May-02											27.25	25.35	26.30	0.98	7.33%	11.60%
Jun-02				30.10	27.60	28.85	1.26	5.30%	10.23%	0.0029	27.50	24.24	25.87	0.98	7.33%	11.67%
Jul-02				30.20	23.46	26.83	1.26	5.30%	10.60%	0.0031	27.45	15.87	21.66	0.98	7.67%	12.89%
Aug-02				29.70	27.53	28.62	1.26	5.30%	10.27%	0.0031	20.60	17.85	19.23	0.98	7.67%	13.56%
Sep-02				29.99	27.00	28.50	1.26	5.30%	10.29%	0.0032	22.25	18.84	20.55	0.98	7.67%	13.18%
Oct-02				30.70	28.54	29.62	1.26	5.30%	10.09%	0.0030	22.25	9.65	15.95	0.98	7.67%	14.80%
Nov-02				30.18	25.50	27.84	1.26	5.30%	10.41%	0.0031	15.27	12.40	13.84	0.98	5.33%	13.41%
Dec-02				27.84	25.63	26.73	1.26	5.67%	11.01%	0.0031	17.50	15.25	16.38	0.98	5.33%	12.12%
Jan-03				28.47	25.49	26.98	1.26	5.67%	10.96%	0.0030	17.40	15.20	16.30	0.98	5.33%	12.16%
Feb-03				26.26	24.05	25.15	1.26	4.67%	10.30%	0.0029	16.03	14.90	15.47	0.98	5.33%	12.53%
Mar-03				25.72	24.13	24.92	1.26	4.67%	10.35%	0.0029	15.74	13.13				
Apr-03				26.00	24.77	25.39	1.26	4.67%	10.25%	0.0029	15.83	14.00				
May-03				28.52	25.52	27.02	1.26	4.67%	9.90%	0.0027	16.05	13.20				
Jun-03				28.88	27.20	28.04	1.26	4.67%	9.71%	0.0032						
Jul-03				28.65	27.03	27.84	1.26	4.67%	9.75%	0.0033						
Aug-03				29.00	27.02	28.01	1.26	4.67%	9.71%	0.0033						
Sep-03				30.10	28.40	29.25	1.26	4.67%	9.50%	0.0026						
Oct-03				30.50	28.51	29.51	1.26	4.67%	9.46%	0.0026						
Nov-03				30.85	28.91	29.88	1.30	4.17%	9.02%	0.0024						
Dec-03				31.30	29.50	30.40	1.30	4.17%	8.94%	0.0024						
Jan-04				31.97	29.95	30.96	1.30	4.17%	8.85%	0.0025						
Feb-04				32.00	30.07	31.04	1.30	4.88%	9.58%	0.0026						
Mar-04				33.00	30.90	31.95	1.30	4.88%	9.44%	0.0028						
Apr-04				31.65	29.15	30.40	1.30	4.88%	9.68%	0.0028						
May-04				29.84	27.46	28.65	1.30	4.88%	9.98%	0.0031						

Month Ending	NUI DCF	OKE High	OKE Low	OKE Average	OKE Dividend	OKE Growth	OKE DCF	OKE DCF	Peoples High	Peoples Low	Peoples Average	Peoples Dividend	Peoples Growth	Peoples DCF	Peoples DCF	PNY High
Jun-01	0.0023	21.80	19.01	20.40	0.62	11.60%	15.21%	0.0077	42.30	38.65	40.48	2.04	5.57%	11.28%	0.0071	35.90
Jul-01	0.0020	20.48	17.45	18.97	0.62	11.60%	15.49%	0.0060	40.75	34.35	37.55	2.04	5.43%	11.59%	0.0067	35.80
Aug-01	0.0020	18.45	15.80	17.13	0.62	11.60%	15.91%	0.0063	39.91	36.56	38.24	2.04	5.57%	11.63%	0.0068	34.11
Sep-01		16.95	14.17	15.56	0.62	10.00%	14.69%	0.0062	39.98	36.81	38.40	2.04	5.57%	11.60%	0.0072	35.10
Oct-01		18.40	16.15	17.27	0.62	10.00%	14.21%	0.0060	42.94	37.70	40.32	2.04	5.57%	11.31%	0.0070	32.15
Nov-01		18.30	16.70	17.50	0.62	10.00%	14.16%	0.0060	40.35	37.54	38.95	2.04	5.57%	11.51%	0.0071	34.80
Dec-01		18.22	16.40	17.31	0.62	10.00%	14.21%	0.0064	38.68	35.40	37.04	2.04	5.58%	11.84%	0.0068	36.60
Jan-02		17.99	16.82	17.41	0.62	10.00%	14.18%	0.0067	38.99	35.50	37.25	2.04	5.58%	11.80%	0.0072	35.89
Feb-02		18.70	16.34	17.52	0.62	8.67%	12.77%	0.0067	37.40	35.25	36.33	2.08	5.58%	12.09%	0.0074	34.05
Mar-02		20.92	18.11	19.52	0.62	9.20%	12.90%	0.0067	39.98	37.06	38.52	2.08	5.58%	11.71%	0.0071	36.25
Apr-02	0.0015	21.95	20.28	21.12	0.62	9.20%	12.61%	0.0065	40.18	38.01	39.09	2.08	5.58%	11.62%	0.0069	37.95
May-02	0.0014	23.14	20.77	21.95	0.62	9.20%	12.48%	0.0062	40.45	38.00	39.23	2.08	5.58%	11.60%	0.0067	38.00
Jun-02	0.0018	22.00	19.70	20.85	0.62	9.20%	12.66%	0.0065	39.40	36.05	37.73	2.08	5.75%	12.02%	0.0066	37.94
Jul-02	0.0021	22.19	14.62	18.41	0.62	9.20%	13.12%	0.0068	37.97	29.07	33.52	2.08	5.69%	12.76%	0.0072	37.70
Aug-02	0.0020	21.00	17.21	19.10	0.62	8.92%	12.69%	0.0062	33.95	27.80	30.88	2.08	5.75%	13.45%	0.0070	37.21
Sep-02	0.0019	19.95	17.85	18.90	0.62	8.58%	12.38%	0.0064	35.32	32.51	33.92	2.08	5.75%	12.74%	0.0064	37.00
Oct-02	0.0022	19.36	16.67	18.02	0.62	8.58%	12.57%	0.0063	36.60	31.06	33.83	2.08	5.50%	12.50%	0.0062	36.45
Nov-02	0.0019	19.55	17.43	18.49	0.62	8.58%	12.46%	0.0062	37.25	33.69	35.47	2.08	5.50%	12.16%	0.0060	36.50
Dec-02	0.0014	19.71	18.56	19.13	0.62	8.58%	12.33%	0.0061	38.99	35.41	37.20	2.08	5.50%	11.85%	0.0063	36.77
Jan-03	0.0014	20.20	16.75	18.48	0.68	8.58%	12.85%	0.0063	40.35	36.14	38.24	2.12	5.50%	11.79%	0.0063	36.87
Feb-03	0.0014	17.55	16.00	16.77	0.68	8.50%	13.20%	0.0065	37.56	35.31	36.44	2.12	5.00%	11.58%	0.0062	35.40
Mar-03		18.58	17.13	17.85	0.68	8.80%	13.23%	0.0072	36.43	34.93	35.68	2.12	5.25%	11.99%	0.0066	35.88
Apr-03		19.45	18.14	18.80	0.68	8.80%	13.00%	0.0070	39.34	35.16	37.25	2.12	4.74%	11.16%	0.0060	37.65
May-03		20.58	18.50	19.54	0.68	8.80%	12.84%	0.0068	44.60	38.46	41.53	2.12	4.99%	10.75%	0.0057	39.69
Jun-03		20.99	19.28	20.14	0.68	8.80%	12.72%	0.0083	45.25	42.45	43.85	2.12	4.99%	10.44%	0.0057	41.50
Jul-03		21.28	19.56	20.42	0.68	8.80%	12.66%	0.0084	44.30	40.89	42.60	2.12	4.99%	10.60%	0.0059	39.74
Aug-03		21.20	18.75	19.98	0.68	8.00%	11.92%	0.0079	41.36	39.53	40.45	2.12	5.14%	11.06%	0.0061	39.32
Sep-03		21.68	20.17	20.93	0.68	8.67%	12.44%	0.0072	42.56	40.06	41.31	2.12	5.14%	10.94%	0.0059	39.95
Oct-03		21.24	19.45	20.35	0.68	8.65%	12.52%	0.0072	42.72	40.03	41.38	2.12	5.14%	10.93%	0.0059	39.98
Nov-03		20.16	19.20	19.68	0.72	7.98%	12.20%	0.0070	40.90	38.82	39.86	2.12	4.80%	10.79%	0.0058	41.13
Dec-03		22.44	19.65	21.05	0.72	7.98%	11.92%	0.0070	42.64	40.06	41.35	2.12	4.80%	10.57%	0.0055	43.95
Jan-04		23.32	21.64	22.48	0.72	7.98%	11.67%	0.0072	43.26	41.37	42.32	2.12	5.00%	10.65%	0.0058	
Feb-04		22.78	21.65	22.22	0.76	7.98%	11.92%	0.0070	44.70	42.47	43.59	2.16	5.00%	10.59%	0.0055	41.86
Mar-04		23.47	21.66	22.57	0.76	7.73%	11.60%	0.0099	46.03	43.52	44.78	2.16	5.00%	10.43%	0.0063	43.06
Apr-04		23.04	20.75	21.90	0.76	7.73%	11.72%	0.0100	45.19	41.15	43.17	2.16	5.00%	10.64%	0.0064	43.03
May-04		21.45	19.69	20.57	0.84	6.50%	11.15%	0.0102	42.01	38.91	40.46	2.16	4.50%	10.50%	0.0068	41.05

Month Ending	PNY Low	PNY Average	PNY Dividend	PNY Growth	PNY DCF	PNY DCF	Semco High	Semco Low	Semco Average	Semco Dividend	Semco Growth	Semco DCF	Semco DCF	SJI High	SJI Low
Jun-01	33.56	34.73	1.54	5.43%	10.44%	0.0045	14.98	13.61	14.29	0.84	7.28%	14.07%	0.0014	31.12	29.95
Jul-01	32.15	33.98	1.54	5.33%	10.45%	0.0047	15.24	14.05	14.64	0.84	7.28%	13.91%	0.0015	31.95	30.65
Aug-01	31.93	33.02	1.54	5.00%	10.25%	0.0047	15.75	14.10	14.93	0.84	7.14%	13.63%	0.0015	32.65	30.75
Sep-01	29.19	32.14	1.54	4.75%	10.13%	0.0050	14.70	13.90	14.30	0.84	6.54%	13.28%	0.0016	32.96	29.30
Oct-01	30.01	31.08	1.54	4.75%	10.32%	0.0050	14.85	11.43	13.14	0.84	6.54%	13.89%	0.0016	34.00	30.41
Nov-01	30.55	32.67	1.54	4.75%	10.04%	0.0049	12.90	11.25	12.07	0.84	6.54%	14.56%	0.0017	34.08	32.57
Dec-01	32.60	34.60	1.54	4.67%	9.66%	0.0048	12.12	8.88	10.50	0.84	6.40%	15.65%	0.0014	34.10	32.50
Jan-02	32.90	34.40	1.54	4.67%	9.69%	0.0048	11.40	9.91	10.66	0.84	5.50%	14.53%	0.0013	32.79	31.40
Feb-02	31.79	32.92	1.60	4.67%	10.13%	0.0050								31.65	29.95
Mar-02	32.01	34.13	1.60	4.50%	9.75%	0.0046								32.70	30.30
Apr-02	35.00	36.48	1.60	4.50%	9.41%	0.0044								35.50	31.70
May-02	35.00	36.50	1.60	4.50%	9.41%	0.0043								36.65	34.19
Jun-02	33.68	35.81	1.60	4.50%	9.50%	0.0041								35.05	32.30
Jul-02	27.35	32.53	1.60	4.50%	10.02%	0.0044								36.05	28.20
Aug-02	32.80	35.00	1.60	4.50%	9.62%	0.0045								33.60	31.80
Sep-02	33.62	35.31	1.60	4.50%	9.57%	0.0047								33.10	31.01
Oct-02	31.55	34.00	1.60	4.50%	9.77%	0.0049	8.15	6.85						33.30	31.40
Nov-02	32.76	34.63	1.60	4.50%	9.68%	0.0048	7.37	6.40						32.60	31.50
Dec-02	34.25	35.51	1.60	4.50%	9.55%	0.0047	7.25	5.60						33.65	32.24
Jan-03	33.95	35.41	1.66	4.50%	9.75%	0.0048	6.20	4.49						33.75	31.75
Feb-03	33.22	34.31	1.66	4.50%	9.92%	0.0049	4.94	3.15						32.41	30.55
Mar-03	33.53	34.70	1.66	4.50%	9.86%	0.0046	4.26	3.52						32.05	30.94
Apr-03	35.15	36.40	1.66	4.50%	9.61%	0.0044	5.65	3.51						35.15	31.54
May-03	36.53	38.11	1.66	4.67%	9.55%	0.0043	7.34	5.03						37.75	34.80
Jun-03	38.78	40.14	1.66	5.00%	9.65%	0.0043									
Jul-03	37.38	38.56	1.66	5.00%	9.84%	0.0044									
Aug-03	37.23	38.28	1.66	5.20%	10.09%	0.0046									
Sep-03	38.69	39.32	1.66	5.20%	9.95%	0.0047									
Oct-03	38.85	39.42	1.66	5.20%	9.94%	0.0047									
Nov-03	39.41	40.27	1.66	5.20%	9.84%	0.0046									
Dec-03	40.71	42.33	1.66	5.20%	9.61%	0.0046									
Jan-04															
Feb-04	40.39	41.13	1.72	5.00%	9.70%	0.0047									
Mar-04	40.70	41.88	1.72	4.75%	9.35%	0.0047									
Apr-04	39.80	41.42	1.72	4.75%	9.40%	0.0047									
May-04	38.32	39.69	1.72	4.80%	9.66%	0.0051									

Month Ending	SJI Average	SJI Dividend	SJI Growth	SJI DCF	SJI DCF	SWX High	SWX Low	SWX Average	SWX Dividend	SWX Growth	SWX DCF	SWX DCF	UGI High	UGI Low
Jun-01	30.54	1.48	6.00%	11.51%	0.0016	24.67	23.08	23.88	0.82	4.67%	8.51%	0.0022	27.26	25.42
Jul-01	31.30	1.48	6.00%	11.38%	0.0018	24.24	22.75	23.49	0.82	4.67%	8.57%	0.0024	27.30	25.30
Aug-01	31.70	1.48	5.67%	10.96%	0.0017	24.40	22.52	23.46	0.82	4.67%	8.57%	0.0024	29.48	26.50
Sep-01	31.13	1.48	5.83%	11.23%	0.0019	23.23	18.61						29.10	25.12
Oct-01	32.20	1.48	5.83%	11.04%	0.0018	22.56	20.31						29.40	26.69
Nov-01	33.33	1.48	5.83%	10.86%	0.0018	21.60	20.48						30.42	28.93
Dec-01	33.30	1.48	6.20%	11.26%	0.0019								31.53	29.33
Jan-02	32.10	1.48	6.20%	11.45%	0.0020								31.15	27.77
Feb-02	30.80	1.48	6.20%	11.67%	0.0019								29.35	27.09
Mar-02	31.50	1.48	5.33%	10.64%	0.0017								31.49	28.45
Apr-02	33.60	1.50	5.33%	10.37%	0.0017								33.21	30.99
May-02	35.42	1.50	5.33%	10.10%	0.0016	24.75	23.40	24.07	0.82	5.00%	8.82%	0.0028	32.95	31.00
Jun-02	33.67	1.50	5.33%	10.36%	0.0016	24.75	23.01	23.88	0.82	5.00%	8.85%	0.0027	32.47	29.40
Jul-02	32.13	1.50	5.33%	10.60%	0.0017	24.75	18.10	21.43	0.82	5.00%	9.29%	0.0029	33.08	25.67
Aug-02	32.70	1.50	5.33%	10.51%	0.0018	23.65	21.15	22.40	0.82	5.75%	9.88%	0.0030	36.48	30.90
Sep-02	32.06	1.50	5.33%	10.61%	0.0018	22.50	20.60	21.55	0.82	5.00%	9.27%	0.0029	36.76	33.58
Oct-02	32.35	1.50	5.33%	10.57%	0.0016	22.75	19.82	21.28	0.82	5.00%	9.32%	0.0028	26.23	23.27
Nov-02	32.05	1.50	5.33%	10.62%	0.0016	22.90	21.40	22.15	0.82	5.00%	9.15%	0.0027	26.99	24.53
Dec-02						23.63	22.00	22.81	0.82	5.00%	9.03%	0.0029	25.43	24.47
Jan-03						23.64	21.11	22.38	0.82	5.00%	9.11%	0.0029	27.89	24.93
Feb-03						21.96	19.92	20.94	0.82	4.75%	9.14%	0.0029	28.37	26.03
Mar-03						20.89	19.30	20.09	0.82	5.00%	9.58%	0.0026	30.57	27.41
Apr-03						21.28	19.74	20.51	0.82	5.00%	9.49%	0.0026	32.55	29.00
May-03						21.77	20.05	20.91	0.82	5.25%	9.66%	0.0026	34.49	30.60
Jun-03						22.45	20.78	21.62	0.82	5.25%	9.52%	0.0024	35.05	30.70
Jul-03						21.72	20.14	20.93	0.82	5.25%	9.66%	0.0025	33.45	30.90
Aug-03						22.83	20.80	21.82	0.82	5.25%	9.48%	0.0025	31.45	28.95
Sep-03						23.49	22.25	22.87	0.82	5.45%	9.49%	0.0026	31.09	28.86
Oct-03						23.48	22.28	22.88	0.82	5.45%	9.48%	0.0026	31.44	28.85
Nov-03						23.15	22.01	22.58	0.82	5.50%	9.59%	0.0026	32.69	30.57
Dec-03						23.18	22.05	22.62	0.82	5.50%	9.58%	0.0025	34.20	32.10
Jan-04						24.05	22.39	23.22	0.82	5.33%	9.30%	0.0025	34.35	31.40
Feb-04						23.99	22.68	23.34	0.82	5.33%	9.28%	0.0024	33.10	31.90
Mar-04						23.57	22.81	23.19	0.82	5.33%	9.31%	0.0026	33.47	31.80
Apr-04						24.06	22.75	23.41	0.82	5.33%	9.27%	0.0026	33.40	31.29
May-04						23.36	21.50						32.14	29.85

Month Ending	UGI Average	UGI Dividend	UGI Growth	UGI DCF	UGI DCF	WGL High	WGL Low	WGL Average	WGL Dividend	WGL Growth	WGL DCF	WGL DCF	NFG High	NFG Low	NFG Average	NFG Dividend	NFG Growth	NFG DCF
Jun-01	26.34	1.55	7.00%	13.78%	0.0036	28.65	26.00	27.32	1.26	4.43%	9.59%	0.0049	57.94	51.79	54.87	2.020	8.51%	12.78%
Jul-01	26.30	1.60	7.00%	14.02%	0.0045	28.40	25.26	26.83	1.26	4.40%	9.66%	0.0052	52.76	44.85	48.81	2.020	8.51%	13.32%
Aug-01	27.99	1.60	6.50%	13.05%	0.0042	28.10	26.60	27.35	1.26	4.40%	9.56%	0.0052	50.30	46.94	48.62	2.020	7.78%	12.57%
Sep-01	27.11	1.60	7.00%	13.80%	0.0048	27.64	25.30	26.47	1.26	4.40%	9.73%	0.0056	24.20	21.96	23.08	1.010	8.00%	13.06%
Oct-01	28.05	1.60	7.00%	13.57%	0.0047	28.53	26.00	27.27	1.26	4.40%	9.57%	0.0055	24.90	22.30	23.60	1.010	8.00%	12.95%
Nov-01	29.68	1.60	7.00%	13.20%	0.0047	28.17	26.80	27.48	1.26	4.40%	9.53%	0.0055	23.89	21.95	22.92	1.010	7.57%	12.65%
Dec-01	30.43	1.60	7.00%	13.05%	0.0047	29.75	27.00	28.38	1.26	4.20%	9.16%	0.0054	24.95	22.06	23.51	1.010	7.57%	12.52%
Jan-02	29.46	1.60	7.00%	13.25%	0.0049	29.48	25.85	27.67	1.26	4.20%	9.29%	0.0057	25.00	22.16	23.58	1.010	7.57%	12.50%
Feb-02	28.22	1.60	6.50%	13.00%	0.0047	27.13	25.71	26.42	1.26	4.20%	9.53%	0.0056	24.90	22.00	23.45	1.010	7.57%	12.53%
Mar-02	29.97	1.60	6.50%	12.61%	0.0045	27.54	26.31	26.93	1.26	4.17%	9.40%	0.0052	25.70	23.90	24.80	1.010	7.57%	12.26%
Apr-02	32.10	1.60	6.17%	11.85%	0.0042	27.95	26.25	27.10	1.26	3.80%	8.97%	0.0049	24.98	23.10	24.04	1.010	7.57%	12.41%
May-02	31.98	1.60	6.17%	11.87%	0.0041	27.40	25.68	26.54	1.27	3.80%	9.14%	0.0049	23.90	22.02	22.96	1.010	7.57%	12.64%
Jun-02	30.94	1.60	6.17%	12.07%	0.0039	26.70	24.46	25.58	1.27	3.80%	9.34%	0.0047	23.25	21.38	22.32	1.040	7.57%	12.95%
Jul-02	29.38	1.65	6.17%	12.59%	0.0042	26.22	19.25	22.73	1.27	4.40%	10.69%	0.0055	22.84	15.61	19.23	1.040	7.57%	13.83%
Aug-02	33.69	1.65	6.88%	12.50%	0.0044	25.15	23.50	24.32	1.27	4.40%	10.27%	0.0050	21.00	18.60	19.80	1.040	7.57%	13.64%
Sep-02	35.17	1.65	6.88%	12.26%	0.0048	24.62	22.75	23.69	1.27	4.40%	10.43%	0.0052	20.91	19.58	20.25	1.040	7.57%	13.51%
Oct-02	24.75	1.10	6.88%	11.97%	0.0048	25.15	21.94	23.55	1.27	4.40%	10.46%	0.0052	20.48	17.95	19.22	1.040	7.57%	13.83%
Nov-02	25.76	1.10	6.88%	11.77%	0.0048	24.45	22.18	23.32	1.27	4.40%	10.53%	0.0052	21.00	19.76	20.38	1.040	7.57%	13.47%
Dec-02	24.95	1.10	6.88%	11.93%	0.0049	24.49	22.65	23.57	1.27	4.40%	10.46%	0.0052	21.86	20.54	21.20	1.040	7.57%	13.23%
Jan-03	26.41	1.10	6.88%	11.64%	0.0048	25.69	23.15	24.42	1.27	4.40%	10.24%	0.0050	21.54	20.02	20.78	1.040	7.57%	13.35%
Feb-03	27.20	1.10	7.25%	11.89%	0.0049	26.10	24.38	25.24	1.27	4.20%	9.84%	0.0049	20.75	18.97	19.86	1.040	7.17%	13.20%
Mar-03	28.99	1.10	6.33%	10.64%	0.0054	26.96	25.00	25.98	1.27	4.33%	9.81%	0.0050	22.25	19.63	20.94	1.040	6.50%	12.18%
Apr-03	30.77	1.14	6.33%	10.54%	0.0053	27.50	26.30	26.90	1.27	4.33%	9.62%	0.0048	23.62	21.60	22.61	1.040	6.50%	11.75%
May-03	32.55	1.14	6.33%	10.31%	0.0051	28.14	25.97	27.05	1.28	4.43%	9.73%	0.0048	25.75	23.15	24.45	1.040	6.33%	11.17%
Jun-03	32.88	1.14	6.33%	10.26%	0.0049	28.79	26.62	27.71	1.28	4.43%	9.60%	0.0046	26.90	25.60	26.25	1.08	6.33%	11.01%
Jul-03	32.18	1.14	6.33%	10.35%	0.0050	27.62	25.21	26.42	1.28	4.43%	9.86%	0.0048	27.51	24.13	25.82	1.08	6.33%	11.09%
Aug-03	30.20	1.14	6.33%	10.62%	0.0052	26.90	25.28	26.09	1.28	4.64%	10.15%	0.0049	23.95	22.51	23.23	1.08	6.17%	11.46%
Sep-03	29.98	1.14	6.33%	10.65%	0.0050	27.97	26.90	27.44	1.28	4.64%	9.87%	0.0046	24.10	22.64	23.37	1.08	6.17%	11.43%
Oct-03	30.15	1.14	6.33%	10.63%	0.0050	28.47	27.37	27.92	1.28	4.64%	9.78%	0.0046	23.85	21.71	22.78	1.08	6.17%	11.57%
Nov-03	31.63	1.14	6.33%	10.42%	0.0048	28.16	26.20	27.18	1.28	4.14%	9.40%	0.0044	23.90	22.76	23.33	1.08	4.80%	10.00%
Dec-03	33.15	1.14	6.33%	10.23%	0.0049	28.55	26.63	27.59	1.28	4.14%	9.32%	0.0042	25.01	23.16	24.09	1.08	4.80%	9.83%
Jan-04	32.88	1.14	6.00%	9.92%	0.0051	28.70	27.15	27.93	1.28	3.86%	8.96%	0.0042	25.74	24.40	25.07	1.08	4.25%	9.06%
Feb-04	32.50	1.14	6.33%	10.31%	0.0050	28.98	27.74	28.36	1.28	3.86%	8.88%	0.0040	26.48	24.75	25.62	1.08	4.25%	8.95%
Mar-04						30.18	28.88	29.53	1.28	3.86%	8.68%	0.0043	26.25	24.26	25.26	1.08	4.25%	9.02%
Apr-04						30.39	27.75	29.07	1.28	3.86%	8.76%	0.0044	25.20	23.75	24.48	1.08	4.00%	8.92%
May-04						29.15	26.66	27.91	1.30	3.93%	9.12%	0.0049	25.57	23.90	24.74	1.08	3.00%	7.82%

Month Ending	NFG DCF	STR High	STR Low	STR Average	STR Dividend	STR Growth	STR DCF	STR DCF	AGL	ATO	CGC	EGN	EQT	KSE	LG	NJR	GAS	NI	NWN
Jun-01	0.0103	31.34	24.00	27.67	0.700	8.56%	11.48%	0.0090	1.10	0.90		0.93	2.17	5.40	0.45	0.70	1.70		0.60
Jul-01	0.0104	25.12	21.33	23.23	0.700	9.29%	12.80%	0.0100	1.20	0.90		0.83	2.34	4.50	0.45	0.78	1.80		0.63
Aug-01	0.0099	24.40	22.35	23.38	0.700	10.11%	13.62%	0.0104	1.20	0.90		0.83	2.05	4.50	0.45	0.78	1.80		0.63
Sep-01	0.0106	23.10	18.58	20.84	0.700	10.38%	14.33%	0.0104	1.20	0.90		0.83	1.93	4.50		0.78	1.80		0.63
Oct-01	0.0107	23.56	19.60	21.58	0.700	9.75%	13.55%	0.0107	1.20	0.90		0.83	2.11	4.50		0.78	1.80		0.63
Nov-01	0.0100	24.47	21.65	23.06	0.720	10.38%	14.05%	0.0118	1.19	0.90		0.83	2.08	4.50		0.78	1.80		0.63
Dec-01	0.0109	25.48	23.08	24.28	0.720	9.75%	13.22%	0.0120	1.20	0.86		0.73	2.18	4.78		0.81	1.82		
Jan-02	0.0103	25.55	23.10	24.33	0.720	9.75%	13.21%	0.0117	1.20	0.85		0.73	1.97	4.64		0.82	1.78		
Feb-02	0.0108	24.13	21.40	22.77	0.720	9.75%	13.45%	0.0109	1.29	0.92		0.74	2.09	4.72		0.84	1.95		
Mar-02	0.0102	25.84	22.29	24.07	0.720	9.75%	13.25%	0.0119	1.30	0.95		0.74	2.18	4.70		0.83	2.00		
Apr-02	0.0100	29.45	25.68	27.57	0.720	9.75%	12.80%	0.0124	1.30	0.95		0.74	2.28	4.70		0.83	2.00		
May-02	0.0097	29.10	26.80	27.95	0.720	9.75%	12.76%	0.0119	1.30	0.95		0.74	2.28	4.70		0.83	2.00		
Jun-02	0.0092	27.50	23.65	25.58	0.720	9.75%	13.04%	0.0104	1.30	0.90		0.88	2.17	5.20		0.80	2.10		0.73
Jul-02	0.0086	25.04	18.01	21.53	0.720	9.75%	13.67%	0.0101	1.30	0.90		0.88	2.17	5.20		0.80	2.10		0.73
Aug-02	0.0097	25.61	21.30	23.46	0.720	10.13%	13.73%	0.0122	1.14	0.85		0.85	2.18	4.62		0.75	1.46		0.68
Sep-02	0.0093	25.10	22.55	23.83	0.720	10.13%	13.68%	0.0111	1.21	0.88		0.87	2.14	4.78		0.81	1.24		0.71
Oct-02	0.0093	26.15	21.41	23.78	0.720	10.13%	13.68%	0.0120	1.30	0.93		0.88	2.21	4.90		0.88	1.30		0.73
Nov-02	0.0093	27.01	24.50	25.76	0.740	10.13%	13.50%	0.0119	1.30	0.93		0.88	2.20	4.90		0.88	1.30		0.73
Dec-02	0.0091	28.39	25.74	27.07	0.740	10.00%	13.20%	0.0124	1.40	0.95		0.95	2.19	5.00		0.88	1.40		0.68
Jan-03	0.0090	28.97	26.50	27.74	0.740	10.00%	13.12%	0.0121	1.40	0.95		0.95	2.32	5.00		0.88	1.40		0.68
Feb-03	0.0085	28.64	26.04	27.34	0.740	10.20%	13.37%	0.0126	1.40	0.95		0.95	2.27	5.00		0.88	1.40		0.68
Mar-03	0.0090	29.85	27.92	28.89	0.740	9.82%	12.81%	0.0131	1.30	0.95		1.10	2.34	4.50		0.85	1.10		0.68
Apr-03	0.0093	31.75	29.35	30.55	0.740	9.00%	11.81%	0.0122	1.30	0.95		1.10	2.39	4.50		0.85	1.10		0.68
May-03	0.0094	33.00	29.72	31.36	0.740	9.00%	11.73%	0.0128	1.30	0.95		1.10	2.51	4.50		0.85	1.10		0.68
Jun-03	0.0079	34.12	32.34	33.23	0.740	9.09%	11.67%	0.0111	1.70	1.20		1.20	2.54	5.70		0.98	1.70		0.98
Jul-03	0.0075	33.99	31.35	32.67	0.740	9.09%	11.71%	0.0107	1.70	1.20		1.20	2.41	5.70		0.98	1.70		0.98
Aug-03	0.0075	32.70	30.11	31.41	0.82	9.00%	12.03%	0.0111	1.70	1.20		1.20	2.46	5.70		0.98	1.70		0.98
Sep-03	0.0077	33.00	30.68	31.84	0.82	9.00%	11.99%	0.0110	1.80	1.20		1.30	2.56	5.50		0.98	1.50		0.75
Oct-03	0.0076	33.35	30.75	32.05	0.82	8.73%	11.69%	0.0111	1.80	1.20		1.30	2.56	5.50		0.98	1.50		0.75
Nov-03	0.0067	34.22	31.80	33.01	0.82	8.60%	11.47%	0.0116	1.80	1.20		1.30	2.56	5.50		0.98	1.50		0.75
Dec-03	0.0068	35.50	33.57	34.54	0.82	8.60%	11.34%	0.0114	1.90	1.30		1.40	2.67	5.50		1.00	1.50		0.78
Jan-04	0.0063	37.08	34.76	35.92	0.82	8.56%	11.19%	0.0114	1.90	1.30		1.40	2.70	5.50		1.00	1.50		0.78
Feb-04	0.0059	36.89	34.40	35.65	0.82	8.33%	10.98%	0.0107	1.90	1.30		1.40	2.70	5.50		1.00	1.50		0.78
Mar-04	0.0067	36.50	33.82	35.16	0.82	8.00%	10.68%	0.0114	1.80	1.40			2.70	6.00		1.00	1.60		0.83
Apr-04	0.0067	37.00	34.51	35.76	0.82	8.00%	10.63%	0.0113	1.80	1.40			2.70	6.00		1.00	1.60		0.83
May-04	0.0062	37.05	34.26	35.66	0.82	8.17%	10.81%	0.0123	1.80	1.40			2.70	6.00			1.60		0.83

Month Ending	NUI	OKE	PGL	PNY	SEN	SJI	SWX	UGI	WGL	NFG	STR	Mkt Cap	Ave. DCF
Jun-01	0.37	1.30	1.60	1.10	0.25	0.35	0.65	0.68	1.30	2.06	2.00	25.60	13.04%
Jul-01	0.30	0.95	1.40	1.10	0.27	0.38	0.68	0.78	1.30	1.90	1.90	24.36	13.38%
Aug-01	0.30	0.95	1.40	1.10	0.27	0.38	0.68	0.78	1.30	1.90	1.83	24.00	13.27%
Sep-01	0.30	0.95	1.40	1.10	0.27	0.38		0.78	1.30	1.83	1.63	22.48	12.68%
Oct-01		0.95	1.40	1.10	0.27	0.38		0.78	1.30	1.86	1.78	22.54	12.68%
Nov-01		0.95	1.40	1.10	0.27	0.38		0.80	1.30	1.78	1.89	22.55	12.68%
Dec-01		1.02	1.30	1.11	0.20	0.38		0.81	1.33	1.96	2.04	22.53	12.54%
Jan-02		1.04	1.34	1.10	0.20	0.39		0.82	1.35	1.82	1.95	22.00	12.36%
Feb-02		1.18	1.39	1.11		0.37		0.82	1.34	1.95	1.82	22.52	12.41%
Mar-02	0.30	1.20	1.40	1.10		0.38		0.83	1.29	1.94	2.09	23.22	11.89%
Apr-02	0.30	1.20	1.40	1.10		0.38		0.83	1.29	1.90	2.28	23.46	11.59%
May-02	0.30	1.20	1.40	1.10		0.38	0.78	0.83	1.29	1.86	2.25	24.17	11.62%
Jun-02	0.40	1.30	1.40	1.10		0.40	0.78	0.83	1.29	1.80	2.02	25.38	11.70%
Jul-02	0.40	1.30	1.40	1.10		0.40	0.78	0.83	1.29	1.55	1.85	24.95	12.42%
Aug-02	0.34	1.11	1.19	1.07		0.38	0.70	0.81	1.10	1.63	2.04	22.89	12.34%
Sep-02	0.33	1.20	1.16	1.13		0.39	0.72	0.90	1.15	1.59	1.87	23.09	12.60%
Oct-02	0.35	1.20	1.20	1.20		0.38	0.73	0.98	1.20	1.62	2.11	24.06	12.50%
Nov-02	0.35	1.20	1.20	1.20		0.38	0.73	0.98	1.20	1.66	2.14	24.13	12.21%
Dec-02	0.28	1.20	1.30	1.20			0.78	1.00	1.20	1.67	2.28	24.34	12.16%
Jan-03	0.28	1.20	1.30	1.20			0.78	1.00	1.20	1.66	2.25	24.43	12.19%
Feb-03	0.28	1.20	1.30	1.20			0.78	1.00	1.20	1.57	2.28	24.33	12.32%
Mar-03		1.30	1.30	1.10			0.65	1.20	1.20	1.76	2.43	23.76	11.95%
Apr-03		1.30	1.30	1.10			0.65	1.20	1.20	1.89	2.48	23.99	11.62%
May-03		1.30	1.30	1.10			0.65	1.20	1.20	2.07	2.67	24.47	11.26%
Jun-03		1.90	1.60	1.30			0.75	1.40	1.40	2.10	2.77	29.22	11.14%
Jul-03		1.90	1.60	1.30			0.75	1.40	1.40	1.95	2.64	28.81	11.27%
Aug-03		1.90	1.60	1.30			0.75	1.40	1.40	1.88	2.66	28.80	11.39%
Sep-03		1.60	1.50	1.30			0.75	1.30	1.30	1.86	2.55	27.74	11.27%
Oct-03		1.60	1.50	1.30			0.75	1.30	1.30	1.82	2.63	27.79	11.23%
Nov-03		1.60	1.50	1.30			0.75	1.30	1.30	1.88	2.83	28.05	10.89%
Dec-03		1.70	1.50	1.40			0.75	1.40	1.30	1.99	2.92	29.01	10.71%
Jan-04		1.70	1.50				0.75	1.40	1.30	1.90	2.80	27.43	10.59%
Feb-04		1.70	1.50	1.40			0.75	1.40	1.30	1.90	2.80	28.83	10.39%
Mar-04		2.40	1.70	1.40			0.78		1.40	2.10	3.00	28.10	10.37%
Apr-04		2.40	1.70	1.40			0.78		1.40	2.10	3.00	28.10	10.41%
May-04		2.40	1.70	1.40					1.40	2.10	3.00	26.33	10.45%

	AGL	AGL	AGL	AGL	AGL	AGL	AGL	ATO	ATO	ATO	ATO	ATO	ATO	ATO	Cascade	Cascade	Cascade	Cascade
Month Ending	High	Low	Average	Dividend	Growth	DCF	DCF	High	Low	Average	Dividend	Growth	DCF	DCF	High	Low	Average	Dividend
Jun-04	29.20	27.92	28.56	1.16	4.83%	9.38%	0.0067	25.60	24.20	24.90	1.22	4.40%	9.89%	0.0051				
Jul-04	29.75	28.60	29.18	1.16	4.83%	9.29%	0.0057	26.18	24.40	25.29	1.22	4.07%	9.46%	0.0042				
Aug-04	30.50	28.82	29.66	1.16	4.33%	8.69%	0.0056	25.55	24.45	25.00	1.22	3.80%	9.24%	0.0047				
Sep-04	31.27	30.20						25.87	24.70									
Oct-04	31.26	30.11						25.90	24.60									
Nov-04	33.26	30.64						27.06	25.15									
Dec-04																		
Jan-05	34.80	32.00	33.40	1.16	4.33%	8.20%	0.0050	27.70	25.90	26.80	1.24	4.40%	9.58%	0.0042				
Feb-05	36.09	33.91	35.00	1.16	4.32%	8.01%	0.0058	29.15	27.20	28.18	1.24	4.40%	9.32%	0.0066				
Mar-05	35.84	34.07	34.96	1.24	4.32%	8.27%	0.0058	28.45	26.70	27.58	1.24	4.40%	9.43%	0.0066				
Apr-05	27.75	25.50	26.63	1.24	4.32%	9.53%	0.0079	36.30	33.80	35.05	1.24	5.54%	9.53%	0.0066				
May-05	28.29	26.15	27.22	1.24	3.93%	9.00%	0.0075	35.29	33.40	34.35	1.24	5.54%	9.61%	0.0066				
Jun-05	28.99	28.03	28.51	1.24	4.58%	9.45%	0.0076	38.89	35.15	37.02	1.24	5.92%	9.70%	0.0064				
Jul-05	29.59	28.53	29.06	1.24	4.58%	9.36%	0.0073	39.32	37.42	38.37	1.24	5.92%	9.57%	0.0062				
Aug-05	29.97	28.26	29.12	1.24	4.58%	9.35%	0.0073	39.09	35.29	37.19	1.24	5.92%	9.69%	0.0062				
Sep-05	29.74	28.10	28.92	1.24	4.64%	9.44%	0.0069	37.95	35.93	36.94	1.24	5.92%	9.71%	0.0055				
Oct-05	28.62	25.55	27.09	1.48	4.64%	10.79%	0.0078	37.54	32.25	34.90	1.24	5.92%	9.94%	0.0056				
Nov-05	27.20	25.85	26.53	1.48	4.64%	10.92%	0.0083	36.68	34.55	35.62	1.24	5.92%	9.86%	0.0058				
Dec-05	35.99	33.74	34.87	1.48	4.63%	9.38%	0.0069	26.90	25.83	26.37	1.24	5.70%	11.03%	0.0063				
Jan-06	36.28	34.83	35.56	1.48	4.63%	9.29%	0.0065	27.08	26.02	26.55	1.26	6.40%	11.82%	0.0065				
Feb-06	36.48	34.40	35.44	1.48	4.63%	9.31%	0.0130	27.01	25.97	26.49	1.26	5.32%	10.69%	0.0116				
Mar-06	36.28	34.75	35.52	1.48	4.63%	9.30%	0.0130	26.95	25.98	26.47	1.26	5.32%	10.70%	0.0117				
Apr-06	36.37	34.43	35.40	1.48	4.43%	9.10%	0.0091	26.80	26.09	26.45	1.26	5.40%	10.79%	0.0085				
May-06	36.67	34.63	35.65	1.48	4.25%	8.88%	0.0075	27.73	25.55	26.64	1.26	6.17%	11.56%	0.0074				
Jun-06	38.13	35.36	36.75	1.48	4.25%	8.74%	0.0077	28.03	26.01	27.02	1.26	6.17%	11.48%	0.0076				
Jul-06	39.40	37.16	38.28	1.48	4.25%	8.56%	0.0069	29.25	27.75	28.50	1.26	6.17%	11.20%	0.0074				
Aug-06	40.00	34.97	37.49	1.48	4.28%	8.68%	0.0070	29.15	27.63	28.39	1.26	6.17%	11.22%	0.0074				
Sep-06	36.85	34.76	35.81	1.48	4.28%	8.89%	0.0074	28.97	27.80	28.39	1.26	6.17%	11.22%	0.0077				
Oct-06	38.66	36.04	37.35	1.48	4.21%	8.63%	0.0072	30.96	28.40	29.68	1.26	6.17%	10.99%	0.0080				
Nov-06	38.83	37.18	38.01	1.48	4.21%	8.55%	0.0075	33.09	30.73	31.91	1.26	6.17%	10.65%	0.0080				
Dec-06	40.09	38.11	39.10	1.48	4.25%	8.47%	0.0074	32.87	31.50	32.19	1.26	6.17%	10.61%	0.0079				
Jan-07	40.21	38.20	39.21	1.48	4.50%	8.71%	0.0071	32.30	30.36	31.33	1.28	6.15%	10.79%	0.0072				
Feb-07	42.90	39.53	41.22	1.64	4.10%	8.53%	0.0074	33.07	31.23	32.15	1.28	6.15%	10.67%	0.0075				
Mar-07	42.99	39.62	41.31	1.64	4.50%	8.94%	0.0076	42.99	39.62	41.31	1.28	6.17%	9.68%	0.0069				
Apr-07	44.67	42.67	43.67	1.64	4.30%	8.48%	0.0072	32.71	30.66	31.69	1.28	6.17%	10.76%	0.0076				
May-07	44.01	41.50	42.76	1.64	4.50%	8.78%	0.0069	33.47	31.59	32.53	1.28	5.75%	10.20%	0.0068				

Month Ending	Cascade Growth	Cascade DCF	Cascade DCF	EGN High	EGN Low	EGN Average	EGN Dividend	EGN Growth	EGN DCF	EGN DCF	EQT High	EQT Low	EQT Average	EQT Dividend	EQT Growth	EQT DCF	EQT DCF
Jun-04				48.56	43.45	46.01					51.72	47.34	49.53	1.52	9.33%	12.90%	0.0169
Jul-04				49.40	45.87	47.64	0.74	7.00%	8.75%	0.0053	52.58	49.89	51.24	1.52	9.33%	12.78%	0.0143
Aug-04				47.56	45.95	46.76	0.74	7.00%	8.78%	0.0051	52.51	49.92	51.22	1.52	9.43%	12.89%	0.0137
Sep-04				51.93	47.29	49.61	0.74	7.00%	8.68%	0.0056	54.49	52.15	53.32	1.52	9.43%	12.75%	0.0151
Oct-04				53.90	50.87	52.39					55.80	53.36	54.58	1.52	9.43%	12.67%	0.0166
Nov-04				58.69	53.73	56.21					59.85	55.01	57.43	1.52	9.43%	12.51%	0.0164
Dec-04											61.18	56.54	58.86	1.52	9.49%	12.50%	0.0173
Jan-05							0.38				61.18	55.78	58.48	1.52	9.49%	12.52%	0.0152
Feb-05							0.38				60.06	56.96	58.51	1.52	9.50%	12.53%	0.0144
Mar-05				34.09	32.10	33.10	0.40	6.50%			61.24	56.01	58.63	1.52	9.50%	12.52%	0.0144
Apr-05				33.66	30.40	32.03	0.40	6.50%	7.91%	0.0057	29.52	28.16	28.84	0.76	9.40%	12.47%	0.0153
May-05				32.65	28.75	30.70	0.40	6.50%	7.97%	0.0057	31.87	28.77	30.32	0.76	9.40%	12.32%	0.0151
Jun-05				35.64	31.70	33.67	0.40	6.50%	7.84%	0.0061	34.42	31.78	33.10	0.84	9.40%	12.35%	0.0157
Jul-05				37.81	34.16	35.99	0.40	6.50%	7.75%	0.0059	36.30	34.01	35.16	0.84	9.40%	12.18%	0.0151
Aug-05				38.32	33.90	36.11	0.40	6.50%	7.75%	0.0059	37.71	34.02	35.87	0.84	9.40%	12.12%	0.0150
Sep-05				43.56	38.17	40.87	0.40	6.50%	7.60%	0.0056	39.90	37.62	38.76	0.84	9.40%	11.92%	0.0148
Oct-05				44.31	36.12	40.22	0.40	6.50%	7.62%	0.0056	41.15	34.51	37.83	0.84	9.40%	11.98%	0.0149
Nov-05				38.73	34.50	36.62	0.40	6.50%	7.73%	0.0060	38.98	36.08	37.53	0.84	9.40%	12.00%	0.0157
Dec-05				38.89	36.03	37.46	0.40	6.50%	7.70%	0.0058	39.51	36.01	37.76	0.84	9.40%	11.98%	0.0142
Jan-06				39.49	36.35	37.92	0.40	6.50%	7.69%	0.0055	39.02	35.82	37.42	0.84	9.40%	12.01%	0.0136
Feb-06											37.19	34.05	35.62	0.84	9.50%	12.24%	0.0275
Mar-06											37.87	35.22	36.55	0.84	9.50%	12.17%	0.0265
Apr-06											37.00	34.92	35.96	0.84	9.80%	12.52%	0.0196
May-06				35.76	32.16	33.96	0.44	7.33%	8.80%	0.0065	35.85	32.20	34.03	0.88	9.80%	12.82%	0.0150
Jun-06				38.42	32.90	35.66	0.44	7.33%	8.73%	0.0067	34.78	31.59	33.19	0.88	9.80%	12.90%	0.0157
Jul-06				43.14	36.95	40.05	0.44	7.33%	8.58%	0.0074	36.29	32.55	34.42	0.88	9.80%	12.78%	0.0154
Aug-06				44.48	41.04	42.76	0.44	7.33%	8.50%	0.0073	36.91	34.85	35.88	0.88	9.80%	12.66%	0.0153
Sep-06				44.02	39.78	41.90	0.44	7.33%	8.52%	0.0077	37.48	34.12	35.80	0.88	9.80%	12.67%	0.0159
Oct-06				42.90	38.50	40.70	0.44	6.00%	7.21%	0.0064	42.35	34.83	38.59	0.88	9.75%	12.41%	0.0173
Nov-06				45.37	42.40	43.89	0.44	5.67%	6.79%	0.0065	44.48	40.06	42.27	0.88	9.75%	12.17%	0.0182
Dec-06				47.60	44.99	46.30	0.44	5.67%	6.73%	0.0064	44.10	41.58	42.84	0.88	9.75%	12.14%	0.0183
Jan-07				46.95	43.78	45.37	0.45	5.00%	6.09%	0.0053	43.69	39.26	41.48	0.88	9.80%	12.27%	0.0167
Feb-07				49.35	45.75	47.55	0.46	5.00%	6.07%	0.0057	44.55	42.00	43.28	0.88	9.80%	12.17%	0.0171
Mar-07				51.43	46.55	48.99	0.46	5.00%	6.04%	0.0060	50.50	41.19	45.85	0.88	9.75%	11.98%	0.0190
Apr-07				57.00	51.05	54.03	0.46	5.00%	5.94%	0.0059	53.39	47.96	50.68	0.88	9.83%	11.85%	0.0188
May-07				60.49	55.86	58.18	0.46	5.00%	5.88%	0.0060	52.77	49.75	51.26	0.88	9.75%	11.75%	0.0178

Month Ending	Keyspan High	Keyspan Low	Keyspan Average	Keyspan Dividend	Keyspan Growth	Keyspan DCF	Keyspan DCF	LG High	LG Low	LG Average	LG Dividend	LG Growth	LG DCF	LG DCF	NJR High	NJR Low
Jun-04	36.78	34.67	35.73	1.78	4.89%	10.50%	0.0250								42.40	40.24
Jul-04	37.38	35.19	36.29	1.78	4.89%	10.41%	0.0218								42.40	40.24
Aug-04	38.10	35.74	36.92	1.78	4.89%	10.32%	0.0205								40.97	39.54
Sep-04	39.49	38.06	38.78	1.78	5.04%	10.21%	0.0227								42.35	40.38
Oct-04	39.99	38.22	39.11	1.78	4.89%	10.01%	0.0222								42.49	40.54
Nov-04	41.53	39.46	40.50	1.78	4.71%	9.64%	0.0214								44.55	40.95
Dec-04	39.87	37.57	38.72	1.78	4.33%	9.47%	0.0222								44.43	42.35
Jan-05	39.79	38.04	38.92	1.82	4.33%	9.56%	0.0203								44.09	41.20
Feb-05	40.61	39.02	39.82	1.82	4.20%	9.31%	0.0193								44.66	42.98
Mar-05	40.90	38.21	39.56	1.82	4.20%	9.34%	0.0188								45.50	42.69
Apr-05	39.89	36.83	38.36	1.82	4.30%	9.61%	0.0187	30.75	26.90						44.57	42.63
May-05	39.98	37.62	38.80	1.82	4.30%	9.55%	0.0186	30.35	27.26						46.02	43.45
Jun-05	40.88	39.45	40.17	1.82	3.63%	8.66%	0.0160	32.00	29.86						48.42	45.20
Jul-05	41.03	39.33	40.18	1.82	3.63%	8.66%	0.0156	33.59	31.25						49.34	46.51
Aug-05	40.79	36.68	38.74	1.82	3.63%	8.85%	0.0159	33.10	30.40						47.54	44.43
Sep-05	38.79	36.35	37.57	1.82	3.70%	9.09%	0.0147	34.31	31.44						47.26	44.78
Oct-05	37.09	32.66	34.88	1.82	3.70%	9.51%	0.0153	33.19	28.60						46.95	40.80
Nov-05	35.84	33.55	34.70	1.82	3.70%	9.55%	0.0162	30.69	28.86						43.32	41.37
Dec-05	36.42	33.27	34.85	1.86	3.58%	9.52%	0.0159								44.58	41.51
Jan-06	36.92	35.38	36.15	1.86	3.42%	9.14%	0.0146								45.55	41.49
Feb-06															45.96	42.99
Mar-06															45.32	42.70
Apr-06															46.43	43.70
May-06															45.72	42.85
Jun-06															47.38	43.95
Jul-06															50.90	46.34
Aug-06															51.39	47.41
Sep-06																
Oct-06															52.11	48.49
Nov-06															53.16	50.53
Dec-06															52.54	48.46
Jan-07															48.70	46.30
Feb-07															51.10	46.73
Mar-07															50.60	48.19
Apr-07															54.83	50.05
May-07															56.45	53.69

Month Ending	NJR Average	NJR Dividend	NJR Growth	NJR DCF	NJR DCF	GAS High	GAS Low	GAS Average	GAS Dividend	GAS Growth	GAS DCF	GAS DCF	NI High	NI Low	NI Average	NI Dividend
Jun-04						35.18	33.04	34.11	1.86							
Jul-04	41.32	1.30	5.45%	8.99%	0.0033	34.29	32.37	33.33	1.86							
Aug-04	40.26	1.30	5.45%	9.08%	0.0035	36.00	32.65	34.33	1.86	3.10%	9.11%	0.0047				
Sep-04	41.37	1.30	5.45%	8.98%	0.0039	37.36	35.72	36.54	1.86	2.63%	8.24%	0.0047				
Oct-04	41.52	1.36	5.45%	9.13%	0.0040	37.80	36.30	37.05	1.86	2.72%	8.26%	0.0048				
Nov-04	42.75	1.36	5.50%	9.08%	0.0040	39.65	36.89	38.27	1.86	2.72%	8.08%	0.0047				
Dec-04	43.39	1.36	5.60%	9.13%	0.0042	38.00	35.89	36.95	1.86	2.15%	7.67%	0.0047				
Jan-05	42.65	1.36	5.60%	9.19%	0.0037	37.30	35.50	36.40	1.86	2.15%	7.76%	0.0042				
Feb-05	43.82	1.36	5.86%	9.36%	0.0034	38.33	36.37	37.35	1.86	1.83%	7.27%	0.0038				
Mar-05	44.10	1.36	5.86%	9.34%	0.0036	38.13	36.10	37.12	1.86	1.83%	7.31%	0.0037				
Apr-05	43.60	1.36	5.86%	9.38%	0.0036	37.81	35.76	36.79	1.86	1.83%	7.36%	0.0039				
May-05	44.74	1.36	5.86%	9.29%	0.0035	39.82	36.81	38.32	1.86	1.83%	7.13%	0.0038				
Jun-05	46.81	1.36	5.30%	8.56%	0.0031	41.87	39.38	40.63	1.86	2.17%	7.18%	0.0036				
Jul-05	47.93	1.36	5.30%	8.48%	0.0030	42.15	40.01	41.08	1.86	2.17%	7.13%	0.0035				
Aug-05	45.99	1.36	5.30%	8.62%	0.0030	41.98	39.10	40.54	1.86	2.17%	7.19%	0.0036				
Sep-05	46.02	1.36	5.30%	8.61%	0.0027	42.59	40.53	41.56	1.86	2.17%	7.07%	0.0033				
Oct-05	43.88	1.36	5.30%	8.78%	0.0027	42.97	37.45	40.21	1.86	2.17%	7.24%	0.0034				
Nov-05	42.35	1.36	5.30%	8.91%	0.0029	41.15	38.72	39.94	1.86	3.00%	8.14%	0.0040				
Dec-05	43.05	1.36	5.33%	8.88%	0.0029	42.09	39.03	40.56	1.86	3.00%	8.06%	0.0039				
Jan-06	43.52	1.44	5.33%	9.05%	0.0028	42.83	39.25	41.04	1.86	3.00%	8.00%	0.0037				
Feb-06	44.48	1.44	5.25%	8.88%	0.0054											
Mar-06	44.01	1.44	5.25%	8.92%	0.0056	42.93	39.25									
Apr-06	45.07	1.44	5.25%	8.84%	0.0039	40.69	38.72	39.71	1.86	3.10%	8.28%	0.0053				
May-06	44.29	1.44	5.67%	9.33%	0.0034	42.29	39.26	40.78	1.86	3.10%	8.14%	0.0043				
Jun-06	45.67	1.44	5.67%	9.22%	0.0036	41.87	39.58	40.73	1.86	3.10%	8.15%	0.0045				
Jul-06	48.62	1.44	6.00%	9.34%	0.0036	44.40	41.01	42.71	1.86	2.67%	7.46%	0.0040				
Aug-06	49.40	1.44	6.00%	9.29%	0.0036	44.39	42.29	43.34	1.86	2.67%	7.39%	0.0040				
Sep-06						43.89	42.15	43.02	1.86	2.67%	7.42%	0.0041				
Oct-06	50.30	1.44	5.25%	8.46%	0.0034	46.54	42.38	44.46	1.86	2.67%	7.27%	0.0039				
Nov-06	51.85	1.44	5.67%	8.79%	0.0036											
Dec-06	50.50	1.52	5.25%	8.62%	0.0035											
Jan-07	47.50	1.52	5.33%	8.92%	0.0030	47.38	44.46	45.92	1.86	3.37%	7.85%	0.0042				
Feb-07	48.92	1.52	5.33%	8.82%	0.0033											
Mar-07	49.40	1.52	5.33%	8.78%	0.0033											
Apr-07	52.44	1.52	5.33%	8.58%	0.0032	53.66	48.47									
May-07						51.74	46.80	49.27	1.86	4.60%	8.82%	0.0041				

Month Ending	NI Growth	NI DCF	NI DCF	NWN High	NWN Low	NWN Average	NWN Dividend	NWN Growth	NWN DCF	NWN DCF	NUI High	NUI Low	NUI Average	NUI Dividend	NUI Growth	NUI DCF
Jun-04				30.75	28.89	29.82	1.30	4.88%	9.78%	0.0029						
Jul-04				31.55	29.13	30.34	1.30	4.17%	8.95%	0.0023						
Aug-04				30.90	28.84											
Sep-04				32.37	30.48											
Oct-04				32.35	30.77											
Nov-04				34.13	31.34											
Dec-04				34.06	32.04	33.05	1.30	5.50%	9.94%	0.0033						
Jan-05				34.02	32.42	33.22	1.30	5.50%	9.91%	0.0030						
Feb-05				37.24	33.73	35.49	1.30	5.50%	9.63%	0.0028						
Mar-05				37.17	35.04	36.11	1.30	5.50%	9.56%	0.0029						
Apr-05				36.50	34.36											
May-05				37.71	35.04											
Jun-05				38.67	36.14	37.41	1.30									
Jul-05				39.20	37.67	38.44	1.30	5.60%	9.41%	0.0026						
Aug-05				39.63	35.62	37.63	1.30	5.60%	9.49%	0.0026						
Sep-05				37.74	35.60	36.67	1.30	5.60%	9.60%	0.0025						
Oct-05				37.77	33.25	35.51	1.30	5.30%	9.42%	0.0024						
Nov-05				35.48	33.88	34.68	1.38	5.30%	9.78%	0.0026						
Dec-05				35.78	33.95	34.87	1.38	5.63%	10.10%	0.0026						
Jan-06				36.57	34.54	35.56	1.38	5.30%	9.67%	0.0024						
Feb-06				35.83	32.83	34.33	1.38	5.30%	9.83%	0.0048						
Mar-06				35.49	33.08	34.29	1.38	5.30%	9.83%	0.0048						
Apr-06				35.79	33.79	34.79	1.38	5.38%	9.85%	0.0034						
May-06				36.00	33.30	34.65	1.38	5.38%	9.87%	0.0028						
Jun-06				37.04	34.23	35.64	1.38	5.96%	10.35%	0.0031						
Jul-06				38.43	35.81	37.12	1.38	5.96%	10.17%	0.0030						
Aug-06				38.53	36.70	37.62	1.38	5.96%	10.11%	0.0030						
Sep-06				40.08	37.67	38.88	1.38	5.96%	9.98%	0.0030						
Oct-06				41.94	38.85	40.40	1.42	5.88%	9.85%	0.0030						
Nov-06				41.51	38.53	40.02	1.42	4.88%	8.85%	0.0029						
Dec-06				43.69	40.80	42.25	1.42	4.88%	8.64%	0.0028						
Jan-07				42.98	39.89	41.44	1.42	4.88%	8.71%	0.0025						
Feb-07				46.30	39.79	43.05	1.42	4.88%	8.57%	0.0028						
Mar-07				46.34	42.47	44.41	1.42	4.88%	8.46%	0.0030						
Apr-07				51.50	45.57	48.54	1.42	4.88%	8.15%	0.0029						
May-07				52.85	44.05	48.45	1.42	4.88%	8.15%	0.0025						

Month Ending	NUI DCF	OKE High	OKE Low	OKE Average	OKE Dividend	OKE Growth	OKE DCF	OKE DCF	Peoples High	Peoples Low	Peoples Average	Peoples Dividend	Peoples Growth	Peoples DCF	Peoples DCF	PNY High
Jun-04		22.19	20.90	21.55	0.84	6.50%	10.94%	0.0104	42.75	40.80	41.78	2.16	4.50%	10.30%	0.0065	43.18
Jul-04		22.20	20.72	21.46	0.84	6.33%	10.78%	0.0091	43.00	38.79	40.90	2.16	4.13%	10.04%	0.0054	42.92
Aug-04		23.59	20.61	22.10	0.92	6.33%	11.07%	0.0089	41.68	38.50	40.09	2.16	4.33%	10.37%	0.0053	21.73
Sep-04		26.13	23.48	24.81	0.92	6.33%	10.54%	0.0095	43.86	41.21	42.54	2.16	4.33%	10.02%	0.0058	22.55
Oct-04		26.90	25.66	26.28	0.92	6.33%	10.30%	0.0105	43.27	41.05	42.16	2.16	4.26%	10.00%	0.0062	23.03
Nov-04		28.56	26.13	27.35	1.00	6.40%	10.56%	0.0107	45.25	42.55	43.90	2.16	4.26%	9.77%	0.0060	24.35
Dec-04		28.99	26.85	27.92	1.00	6.40%	10.47%	0.0112	45.38	43.46	44.42	2.16	4.26%	9.70%	0.0063	23.89
Jan-05		28.55	26.86	27.71	1.00	6.40%	10.50%	0.0103	44.32	41.95	43.14	2.18	4.26%	9.92%	0.0056	23.50
Feb-05		29.80	27.79	28.80	1.00	6.50%	10.45%	0.0096	44.00	41.80	42.90	2.18	4.26%	9.95%	0.0054	24.27
Mar-05		31.03	29.42	30.23	1.00	6.50%	10.26%	0.0099	45.10	41.11	43.11	2.18	4.26%	9.92%	0.0052	24.44
Apr-05		31.70	27.98	29.84	1.00	6.50%	10.31%	0.0098	42.68	38.75						23.40
May-05		30.88	28.81	29.85	1.12	6.50%	10.77%	0.0102	42.89	39.40						24.70
Jun-05		32.71	30.59	31.65	1.12	7.00%	11.04%	0.0105	44.97	42.62						24.99
Jul-05		35.24	32.05	33.65	1.12	7.00%	10.80%	0.0100	45.52	42.43						24.91
Aug-05		35.85	32.15	34.00	1.12	7.00%	10.76%	0.0100	43.43	39.80						24.82
Sep-05		35.40	32.41	33.91	1.12	7.00%	10.77%	0.0077	43.46	38.71						25.80
Oct-05		35.29	27.78	31.54	1.12	6.00%	10.02%	0.0072	39.90	34.34						25.45
Nov-05		28.70	26.52	27.61	1.12	7.00%	11.64%	0.0088	37.25	35.41						23.62
Dec-05		28.3	26.3	27.30	1.12	6.88%	11.57%	0.0081	37.96	34.88	36.42	2.18	4.47%	11.21%	0.0042	
Jan-06		29.76	26.77	28.27	1.12	6.88%	11.41%	0.0077	37.56	35.11	36.34	2.18	3.97%	10.69%	0.0038	24.94
Feb-06																
Mar-06		32.58	28.89						37.97	35.35						
Apr-06		33.75	32.09	32.92	1.20	6.42%	10.56%	0.0149	37.16	35.33	36.25	2.18	4.53%	11.31%	0.0057	
May-06		33.71	30.04	31.88	1.20	6.88%	11.18%	0.0131	37.59	35.34	36.47	2.18	4.53%	11.26%	0.0047	24.88
Jun-06		35.15	32.10	33.63	1.20	6.88%	10.95%	0.0134								25.4
Jul-06		37.82	32.99	35.41	1.20	6.88%	10.74%	0.0134								26.17
Aug-06		39.25	37.14	38.20	1.20	6.88%	10.46%	0.0130								26.18
Sep-06		39.33	36.63	37.98	1.28	6.88%	10.72%	0.0139								26.46
Oct-06		41.99	37.67	39.83	1.28	7.46%	11.14%	0.0145								
Nov-06		43.68	38.81	41.25	1.28	7.46%	11.01%	0.0151								
Dec-06		44.48	42.71	43.60	1.28	7.46%	10.82%	0.0149								
Jan-07		43.65	41.00	42.33	1.28	7.82%	11.29%	0.0139								27.25
Feb-07		43.85	41.00	42.43	1.36	7.65%	11.33%	0.0144								26.96
Mar-07		46.33	39.26	42.80	1.36	8.20%	11.87%	0.0160								27.31
Apr-07		49.15	44.21	46.68	1.36	8.20%	11.56%	0.0156								27.50
May-07		54.90	47.93	51.42	1.36	8.80%	11.86%	0.0162								27.50

Month Ending	PNY Low	PNY Average	PNY Dividend	PNY Growth	PNY DCF	PNY DCF	Semco High	Semco Low	Semco Average	Semco Dividend	Semco Growth	Semco DCF	Semco DCF	SJI High	SJI Low
Jun-04	40.53	41.86	1.72	4.50%	9.09%	0.0058									
Jul-04	40.30	41.61	1.72	4.50%	9.12%	0.0049									
Aug-04	20.45	21.09	0.86	4.50%	9.06%	0.0049									
Sep-04	21.50	22.03	0.86	4.50%	8.86%	0.0054									
Oct-04	21.92	22.48	0.86	4.63%	8.91%	0.0058									
Nov-04	22.70	23.53	0.86	4.63%	8.71%	0.0057									
Dec-04	22.75	23.32	0.86	5.20%	9.34%	0.0065									
Jan-05	22.01	22.76	0.92	5.20%	9.75%	0.0059									
Feb-05	22.65	23.46	0.92	5.25%	9.66%	0.0056									
Mar-05	22.63	23.54	0.92	5.25%	9.65%	0.0055									
Apr-05	21.76	22.58	0.92	5.25%	9.84%	0.0057								29.68	26.66
May-05	22.84	23.77	0.92	4.98%	9.32%	0.0054								29.00	27.23
Jun-05	23.34	24.17	0.92	4.73%	8.99%	0.0047								31.50	28.42
Jul-05	23.76	24.34	0.92	4.73%	8.96%	0.0046								32.38	28.54
Aug-05	23.22	24.02	0.92	4.73%	9.02%	0.0046								29.85	27.73
Sep-05	24.33	25.07	0.92	5.00%	9.12%	0.0043								29.96	28.46
Oct-05	22.33	23.89	0.92	5.00%	9.32%	0.0044								29.24	25.80
Nov-05	21.52	22.57	0.92											29.61	26.22
Dec-05															
Jan-06	23.83														
Feb-06															
Mar-06														28.84	26.72
Apr-06														27.48	25.80
May-06	23.31	24.10	0.96	4.40%	8.85%	0.0050								27.89	25.63
Jun-06	23.46	24.43	0.96	4.40%	8.79%	0.0051								27.52	25.80
Jul-06	24.3	25.24	0.96	4.40%	8.64%	0.0046									
Aug-06	25.04	25.61	0.96	4.30%	8.48%	0.0046									
Sep-06	24.72	25.59	0.96	4.30%	8.48%	0.0047									
Oct-06														31.33	29.10
Nov-06														33.35	30.35
Dec-06														34.26	32.42
Jan-07	25.78	26.52	0.96	4.15%	8.18%	0.0041								33.95	31.81
Feb-07	24.55	25.76	0.96	4.33%	8.48%	0.0044								35.30	33.05
Mar-07	24.33	25.82	0.96											38.56	33.02
Apr-07	26.22	26.86	0.96											40.28	37.06
May-07	25.74	26.62	0.96											41.27	37.93

Month Ending	SJI Average	SJI Dividend	SJI Growth	SJI DCF	SJI DCF	SWX High	SWX Low	SWX Average	SWX Dividend	SWX Growth	SWX DCF	SWX DCF	UGI High	UGI Low
Jun-04						24.20	22.29							
Jul-04						24.46	22.70	23.58	0.82	3.70%	7.55%	0.0021		
Aug-04						23.82	22.87	23.35	0.82	3.70%	7.59%	0.0020		
Sep-04						24.15	23.15	23.65	0.82	3.70%				
Oct-04						24.68	23.45	24.07	0.82	3.70%				
Nov-04						25.98	24.42	25.20	0.82	3.70%				
Dec-04						26.15	24.46	25.31	0.82	3.70%				
Jan-05						25.68	24.00	24.84	0.82	3.70%	7.35%	0.0022		
Feb-05						25.90	24.00	24.95	0.82	6.47%	10.20%	0.0030		
Mar-05						26.13	23.66	24.90	0.82	6.47%	10.21%	0.0029		
Apr-05						25.59	23.53						25.30	22.20
May-05						25.38	24.35						27.26	25.20
Jun-05						26.35	24.85						27.95	24.62
Jul-05						26.95	25.00						29.66	27.27
Aug-05						27.42	25.64						29.98	25.50
Sep-05						28.07	26.88						29.25	26.88
Oct-05						27.86	25.14						28.64	22.60
Nov-05						27.56	26.00						24.36	21.17
Dec-05														
Jan-06														
Feb-06														
Mar-06														
Apr-06	26.64	0.90	5.30%	9.09%	0.0026									
May-06	26.76	0.90	5.30%	9.08%	0.0021									
Jun-06	26.66	0.90	5.30%	9.09%	0.0022									
Jul-06														
Aug-06														
Sep-06														
Oct-06	30.22	0.90	6.00%	9.36%	0.0024									
Nov-06	31.85	0.90	6.33%	9.53%	0.0027									
Dec-06	33.34	0.90	6.33%	9.38%	0.0026									
Jan-07	32.88	0.98	7.00%	10.40%	0.0027									
Feb-07	34.18	0.98	6.67%	9.93%	0.0027									
Mar-07	35.79	0.98	6.75%	9.86%	0.0029									
Apr-07	38.67	0.98	6.75%	9.63%	0.0028									
May-07	39.60	0.98	7.25%	10.07%	0.0026									

Month Ending	UGI Average	UGI Dividend	UGI Growth	UGI DCF	UGI DCF	WGL High	WGL Low	WGL Average	WGL Dividend	WGL Growth	WGL DCF	WGL DCF	NFG High	NFG Low	NFG Average	NFG Dividend	NFG Growth	NFG DCF
Jun-04						29.42	27.36	28.39	1.30	3.67%	8.76%	0.0049	25.38	24.20	24.79	1.08	3.25%	8.07%
Jul-04						29.04	26.91	27.98	1.30	3.57%	8.73%	0.0041	26.78	24.84	25.81	1.08	4.33%	9.00%
Aug-04						28.97	27.30	28.14	1.30	3.67%	8.80%	0.0040	27.11	25.05	26.08	1.12	4.33%	9.13%
Sep-04						29.67	27.74	28.71	1.30	3.48%	8.50%	0.0043	28.43	26.60	27.52	1.12	4.33%	8.87%
Oct-04						29.18	27.71	28.45	1.30	3.57%	8.64%	0.0047	29.06	27.80	28.43	1.12	4.33%	8.72%
Nov-04						30.97	28.20	29.59	1.30	3.57%	8.44%	0.0046	28.75	27.30	28.03	1.12	4.33%	8.79%
Dec-04						31.43	29.63	30.53	1.30	3.88%	8.61%	0.0050						
Jan-05						31.27	28.85	30.06	1.30	3.88%	8.69%	0.0043						
Feb-05						31.66	29.93	30.80	1.30	3.88%	8.57%	0.0041						
Mar-05						31.97	30.00	30.99	1.30	3.88%	8.54%	0.0041						
Apr-05						31.35	29.66	30.51	1.30	3.88%	8.62%	0.0042	29.33	26.80				
May-05						32.80	30.32	31.56	1.33	3.88%	8.57%	0.0042	28.20	26.20				
Jun-05						33.96	32.40	33.18	1.33	3.80%	8.26%	0.0037	29.49	27.72				
Jul-05						34.79	32.96	33.88	1.33	3.80%	8.16%	0.0035	30.40	28.86				
Aug-05						34.70	31.50	33.10	1.33	3.80%	8.27%	0.0036	30.40	27.74				
Sep-05						33.49	31.39	32.44	1.33	4.00%	8.57%	0.0035	35.04	29.70	32.37	1.16	6.00%	10.06%
Oct-05						32.88	29.10	30.99	1.33	4.00%	8.79%	0.0036	35.27	29.51	32.39	1.16	5.03%	9.05%
Nov-05						31.31	29.80	30.56	1.33	4.00%	8.86%	0.0038	32.66	29.25	30.96	1.16	5.03%	9.23%
Dec-05						31.14	29.74	30.44	1.33	3.75%	8.61%	0.0034	34.1	30.58	32.34	1.16	5.03%	9.05%
Jan-06						31.30	29.77	30.54	1.33	3.75%	8.60%	0.0033	35.43	31.09	33.26	1.16	4.03%	7.90%
Feb-06						31.49	29.61	30.55	1.33	3.75%	8.59%	0.0065						
Mar-06						31.08	29.59	30.34	1.33	3.75%	8.62%	0.0063						
Apr-06						30.74	28.80	29.77	1.35	3.75%	8.79%	0.0046						
May-06						29.93	27.04	28.49	1.35	3.75%	9.02%	0.0037	35.98	33.30	34.64	1.20	5.00%	8.88%
Jun-06						29.39	27.82	28.61	1.35	3.75%	9.00%	0.0038	36.75	33.18	34.97	1.20	5.00%	8.84%
Jul-06						30.32	28.44	29.38	1.35	3.75%	8.86%	0.0036	37.43	34.95	36.19	1.20	5.00%	8.71%
Aug-06						31.18	29.01	30.10	1.35	3.75%	8.74%	0.0035	39.16	36.76	37.96	1.20	5.00%	8.54%
Sep-06						31.82	30.05	30.94	1.35	3.75%	8.60%	0.0036	38.71	35.42	37.07	1.20	5.00%	8.62%
Oct-06						33.02	31.16	32.09	1.35	3.50%	8.16%	0.0037	37.96	35.02	36.49	1.20	5.40%	9.10%
Nov-06						33.41	31.84	32.63	1.35	3.33%	7.90%	0.0037	39.10	36.50	37.80	1.20	4.67%	8.21%
Dec-06						33.55	32.33	32.94	1.35	3.33%	7.86%	0.0036	40.21	37.67	38.94	1.20	5.53%	9.00%
Jan-07						32.98	30.99	31.99	1.35	3.25%	7.91%	0.0032	40.94	36.94	38.94	1.20	4.67%	8.11%
Feb-07						33.00	31.23	32.11	1.35	3.50%	8.16%	0.0034	43.79	40.60	42.20	1.20	4.57%	7.74%
Mar-07						32.52	30.37	31.45	1.35	3.50%	8.26%	0.0035	43.60	40.46	42.03	1.20	4.57%	7.75%
Apr-07						34.61	31.88	33.25	1.37	3.50%	8.07%	0.0034	47.87	43.28	45.58	1.20	4.57%	7.50%
May-07						35.77	33.82	34.80	1.37	3.50%	7.86%	0.0032	47.65	44.91	46.28	1.20	4.57%	7.45%

Month Ending	NFG DCF	STR High	STR Low	STR Average	STR Dividend	STR Growth	STR DCF	STR DCF	AGL	ATO	CGC	EGN	EQT	KSE	LG	NJR	GAS	NI	NWN
Jun-04	0.0064	38.85	36.58	37.72	0.86	8.14%	10.76%	0.0132	1.80	1.30			3.30	6.00					0.75
Jul-04	0.0061	42.06	37.83	39.95	0.86	8.33%	10.81%	0.0124	1.80	1.30		1.80	3.30	6.20		1.10			0.75
Aug-04	0.0059	41.40	39.80	40.60	0.86	8.57%	11.01%	0.0120	2.00	1.60		1.80	3.30	6.20		1.20	1.60		
Sep-04	0.0073	46.40	40.01	43.21	0.86	8.57%	10.86%	0.0133				1.80	3.30	6.20		1.20	1.60		
Oct-04	0.0073	49.70	45.02	47.36	0.86	8.44%	10.53%	0.0153					3.60	6.10		1.20	1.60		
Nov-04	0.0074	51.54	47.36	49.45	0.86	8.56%	10.56%	0.0154					3.60	6.10		1.20	1.60		
Dec-04		52.12	47.40	49.76	0.86	8.69%	10.68%	0.0164					3.60	6.10		1.20	1.60		0.88
Jan-05		51.52	46.73	49.13	0.86	8.64%	10.66%	0.0151	1.80	1.30			3.60	6.30		1.20	1.60		0.90
Feb-05		53.57	49.38	51.48	0.86	8.50%	10.42%	0.0140	2.20	2.15			3.50	6.30		1.10	1.60		0.90
Mar-05		62.75	52.19	57.47	0.86	8.50%	10.22%	0.0144	2.20	2.18			3.60	6.30		1.20	1.60		0.95
Apr-05		61.50	54.49	58.00	0.86	8.81%	10.52%	0.0175	2.75	2.28		2.38	4.05	6.42		1.26	1.76		
May-05		63.19	54.85	59.02	0.86	8.81%	10.49%	0.0175	2.75	2.28		2.38	4.05	6.42		1.26	1.76		
Jun-05		67.19	62.16	64.68	0.90	9.30%	10.91%	0.0201	2.89	2.38		2.81	4.58	6.65		1.29	1.83		
Jul-05		71.47	65.95	68.71	0.90	9.30%	10.81%	0.0194	2.89	2.38		2.81	4.58	6.65		1.29	1.83		1.01
Aug-05		78.24	69.43	73.84	0.90	9.30%	10.71%	0.0192	2.89	2.38		2.81	4.58	6.65		1.29	1.83		1.01
Sep-05	0.0069	88.77	76.00	82.39	0.90	9.71%	10.98%	0.0197	2.74	2.12		2.79	4.69	6.08		1.17	1.78		0.97
Oct-05	0.0062	89.56	71.12	80.34	0.90	9.71%	11.01%	0.0198	2.74	2.12		2.79	4.69	6.08		1.17	1.78		0.97
Nov-05	0.0066	81.35	73.75	77.55	0.90	11.44%	12.81%	0.0242	2.74	2.12		2.79	4.69	6.08		1.17	1.78		0.97
Dec-05	0.0067	84.77	74.43	79.60	0.90	11.71%	13.05%	0.0236	2.75	2.14		2.79	4.43	6.25		1.21	1.81		0.96
Jan-06	0.0056	85.70	75.77	80.74	0.90	11.71%	13.03%	0.0224	2.75	2.14		2.79	4.43	6.25		1.21	1.81		0.96
Feb-06		82.35	71.26	76.81	0.90	11.38%	12.76%	0.0436	2.75	2.14			4.43			1.21			0.96
Mar-06		75.45	67.37	71.41	0.90	11.38%	12.87%	0.0448	2.75	2.15			4.28			1.22			0.95
Apr-06		81.90	68.43	75.17	0.90	11.57%	12.98%	0.0324	2.75	2.15			4.28			1.22	1.75		0.95
May-06	0.0076	82.08	67.48	74.78	0.94	11.16%	12.64%	0.0226	2.88	2.19		2.53	4.01			1.27	1.82		0.98
Jun-06	0.0079	81.00	67.68	74.34	0.94	11.16%	12.65%	0.0236	2.88	2.19		2.53	4.01			1.27	1.82		0.98
Jul-06	0.0080	89.00	75.68	82.34	0.94	13.26%	14.63%	0.0316	2.84	2.34		3.06	4.27			1.37	1.89		1.03
Aug-06	0.0077	91.02	84.85	87.94	0.94	11.59%	12.85%	0.0278	2.84	2.34		3.06	4.27			1.37	1.89		1.03
Sep-06	0.0080	87.00	78.06	82.53	0.94	11.59%	12.93%	0.0291	2.84	2.33		3.06	4.27				1.89		1.03
Oct-06	0.0082	86.88	77.48	82.18	0.94	11.16%	12.50%	0.0250	2.92	2.55		3.15	4.92			1.44	1.89		1.08
Nov-06	0.0077	87.30	79.78	83.54	0.94	11.52%	12.85%	0.0275	3.07	2.62		3.33	5.25			1.44			1.14
Dec-06	0.0084	89.56	82.45	86.01	0.94	11.59%	12.88%	0.0277	3.05	2.61		3.33	5.28			1.43			1.14
Jan-07	0.0074	82.81	75.96	79.39	0.94	11.82%	13.22%	0.0240	3.18	2.59		3.40	5.29			1.31	2.06		1.13
Feb-07	0.0073	86.32	79.33	82.83	0.94	11.82%	13.16%	0.0257	3.16	2.56		3.40	5.11			1.36			1.18
Mar-07	0.0075	91.15	81.65	86.40	0.94	11.18%	12.46%	0.0262	3.41	2.84		4.00	6.35			1.50			1.41
Apr-07	0.0073	50.00	44.61	47.30	0.49	11.18%	12.40%	0.0260	3.41	2.84		4.00	6.35			1.50			1.41
May-07	0.0068	54.32	48.16	51.24	0.49	9.25%	10.35%	0.0238	3.19	2.70		4.10	6.13				1.90		1.23

Month Ending	NUI	OKE	PGL	PNY	SEN	SJI	SWX	UGI	WGL	NFG	STR	Mkt Cap	Ave. DCF
Jun-04		2.40	1.60	1.60					1.40	2.00	3.10	25.25	10.36%
Jul-04		2.50	1.60	1.60			0.83		1.40	2.00	3.40	29.58	10.11%
Aug-04		2.50	1.60	1.70			0.83		1.40	2.00	3.40	31.13	10.08%
Sep-04		2.50	1.60	1.70			0.83		1.40	2.30	3.40	27.83	9.76%
Oct-04		2.80	1.70	1.80			0.90		1.50	2.30	4.00	27.50	9.74%
Nov-04		2.80	1.70	1.80			0.90		1.50	2.30	4.00	27.50	9.62%
Dec-04		2.80	1.70	1.80			0.90		1.50		4.00	26.08	9.70%
Jan-05		2.90	1.66	1.79			0.90		1.47		4.20	29.62	9.90%
Feb-05		2.80	1.65	1.77			0.90		1.44		4.10	30.41	9.79%
Mar-05		3.00	1.63	1.78			0.90		1.49		4.40	31.23	9.79%
Apr-05		3.13		1.90					1.60		5.50	33.04	9.88%
May-05		3.13		1.90					1.60		5.50	33.04	9.81%
Jun-05		3.43		1.88					1.60		6.64	35.98	9.76%
Jul-05		3.43		1.88					1.60		6.64	36.99	9.66%
Aug-05		3.43		1.88					1.60		6.64	36.99	9.69%
Sep-05		2.71		1.79					1.53	2.57	6.78	37.72	9.80%
Oct-05		2.71		1.79					1.53	2.57	6.78	37.72	9.90%
Nov-05		2.71							1.53	2.57	6.78	35.93	10.49%
Dec-05		2.63	1.40						1.49	2.75	6.74	37.34	10.45%
Jan-06		2.63	1.40	1.83					1.49	2.75	6.74	39.17	9.82%
Feb-06									1.49		6.74	19.71	11.24%
Mar-06									1.43		6.84	19.64	11.27%
Apr-06		3.87	1.39			0.77			1.43		6.84	27.42	11.00%
May-06		4.01	1.42	1.92		0.78			1.39	2.94	6.13	34.27	10.56%
Jun-06		4.01		1.92		0.78			1.39	2.94	6.13	32.85	10.49%
Jul-06		4.41		1.90					1.43	3.26	7.65	35.44	10.87%
Aug-06		4.41		1.90					1.43	3.17	7.65	35.36	10.41%
Sep-06		4.41		1.90					1.43	3.17	7.65	33.97	10.53%
Oct-06		4.58				0.91			1.59	3.18	7.05	35.25	10.30%
Nov-06		4.80				0.97			1.62	3.29	7.49	35.02	10.33%
Dec-06		4.81				0.97			1.61	3.26	7.52	35.00	10.35%
Jan-07		4.77		1.97		1.00			1.55	3.56	7.06	38.86	10.13%
Feb-07		4.60		1.88		1.00			1.53	3.43	7.08	36.28	10.18%
Mar-07		5.40				1.16			1.68	3.89	8.41	40.05	10.18%
Apr-07		5.40				1.16			1.68	3.89	8.41	40.05	10.07%
May-07		5.51				1.06			1.62	3.68	9.29	40.40	9.67%

Month Ending	AGL High	AGL Low	AGL Average	AGL Dividend	AGL Growth	AGL DCF	AGL DCF	ATO High	ATO Low	ATO Average	ATO Dividend	ATO Growth	ATO DCF	ATO DCF	Cascade High	Cascade Low	Cascade Average	Cascade Dividend
Jun-07	42.80	39.52	41.16	1.64	4.50%	8.95%	0.0071	32.60	29.11	30.86	1.28	5.10%	9.77%	0.0065				
Jul-07								30.84	28.01	29.43	1.28	6.17%	11.12%	0.0081				
Aug-07								28.90	23.87	26.39	1.28	6.17%	11.70%	0.0085				
Sep-07								28.73	27.28	28.01	1.28	6.17%	11.37%	0.0083				
Oct-07	41.16	36.65	38.91	1.64	4.97%	9.71%	0.0075	29.63	27.54	28.59	1.28	5.63%	10.70%	0.0073				
Nov-07	39.21	35.85	37.53	1.64	4.97%	9.88%	0.0076	28.18	26.01	27.10	1.30	5.63%	11.07%	0.0075				
Dec-07	38.65	35.42	37.04	1.64	4.97%	9.95%	0.0077	28.83	26.10	27.47	1.30	5.63%	10.99%	0.0075				
Jan-08	38.69	35.49	37.09	1.68	4.97%	10.06%	0.0078	28.85	26.00	27.43	1.30	5.63%	11.00%	0.0074				
Feb-08	39.13	34.63	36.88	1.68	5.25%	10.39%		29.29	25.84	27.57	1.30	5.22%	10.54%	0.0066				
Mar-08	35.62	33.45	34.54	1.68	5.25%	10.74%		26.52	25.00	25.76	1.30	5.22%	10.92%	0.0069				
Apr-08	36.05	33.73	34.89	1.68	5.25%	10.69%		28.27	25.55	26.91	1.30	5.22%	10.67%	0.0066				
May-08	36.50	34.06	35.28	1.68	5.25%	10.63%	0.0059	28.64	27.14	27.89	1.30	4.67%	9.90%	0.0054				
Jun-08	36.42	33.46	34.94	1.68	5.25%	10.68%	0.0061	27.84	26.31	27.08	1.30	4.67%	10.06%	0.0056				
Jul-08	35.44	32.66	34.05	1.68	5.25%	10.82%	0.0075	28.00	25.00	26.50	1.30	4.67%	10.18%	0.0066				
Aug-08	34.66	32.20	33.43	1.68	5.25%	10.93%	0.0075	27.80	25.61	26.71	1.30	5.00%	10.48%	0.0071				
Sep-08	35.01	30.60	32.81	1.68	4.83%	10.60%	0.0089	28.66	25.52	27.09	1.30	5.00%	10.41%	0.0091				
Oct-08	32.07	24.02	28.05	1.68	4.83%	11.60%	0.0098	28.25	19.68	23.97	1.30	5.00%	11.13%	0.0089				
Nov-08	31.00	25.95	28.48	1.68	4.83%	11.49%	0.0100	25.23	22.26	23.75	1.30	5.00%	11.18%	0.0096				
Dec-08	31.39	26.90	29.15	1.68	4.25%	10.72%	0.0086	24.97	21.98	23.48	1.30	5.00%	11.26%	0.0083				
Jan-09	32.11	29.67	30.89	1.68	4.25%	10.35%	0.0084	25.22	23.20	24.21	1.32	5.00%	11.16%	0.0084				
Feb-09	34.93	27.13	31.03	1.72	4.25%	10.47%	0.0086	26.17	21.54	23.86	1.32	5.00%	11.25%	0.0086				
Mar-09	27.97	24.02	26.00	1.72	4.25%	11.70%	0.0099	23.94	20.07	22.01	1.32	5.00%	11.79%	0.0098				
Apr-09	31.50	26.00	28.75	1.72	4.25%	10.97%	0.0093	25.30	22.52	23.91	1.32	5.00%	11.24%	0.0094				
May-09	31.97	28.12	30.05	1.72	4.25%	10.68%	0.0119	26.43	23.44	24.94	1.32	5.00%	10.97%	0.0115				
Jun-09	32.38	29.15	30.77	1.72	4.25%	10.52%	0.0117	25.51	24.20	24.86	1.32	5.00%	10.99%	0.0115				
Jul-09	34.43	30.05	32.24	1.72	4.25%	10.23%	0.0098	27.39	24.41	25.90	1.32	5.00%	10.75%	0.0099				

Month Ending	Cascade Growth	Cascade DCF	Cascade DCF	EGN High	EGN Low	EGN Average	EGN Dividend	EGN Growth	EGN DCF	EGN DCF	EQT High	EQT Low	EQT Average	EQT Dividend	EQT Growth	EQT DCF	EQT DCF
Jun-07				59.93	53.54	56.74	0.46	5.00%	5.90%	0.0060	53.70	48.11	50.91	0.88	9.75%	11.76%	0.0179
Jul-07				58.90	51.04	54.97	0.46	7.00%	7.95%	0.0095	53.37	46.31	49.84	0.88	10.00%	12.06%	0.0221
Aug-07				57.46	48.24	52.85	0.46	7.00%	7.98%	0.0096	54.42	44.57	49.50	0.88	10.00%	12.07%	0.0221
Sep-07				57.99	53.01	55.50	0.46	7.00%	7.94%	0.0095	52.46	48.42	50.44	0.88	10.00%	12.03%	0.0220
Oct-07				64.49	56.81	60.65	0.46	11.13%	12.02%	0.0151	56.71	52.12	54.41	0.88	11.20%	13.11%	0.0233
Nov-07				65.53	60.42	62.98	0.46	11.13%	11.99%	0.0151	56.75	51.54	54.15	0.88	11.20%	13.11%	0.0233
Dec-07				70.41	63.05	66.73	0.46	11.13%	11.94%	0.0150	55.58	51.55	53.57	0.88	11.20%	13.14%	0.0234
Jan-08				66.88	57.61	62.25	0.48	11.50%	12.41%	0.0151	57.62	47.16	52.39	0.88	11.38%	13.36%	0.0255
Feb-08				64.34	59.98	62.16	0.48	8.50%	9.38%	0.0115	63.77	55.08	59.43	0.88	14.15%	15.94%	0.0326
Mar-08				64.03	57.97	61.00	0.48	8.50%	9.40%	0.0116	65.05	55.65	60.35	0.88	14.15%	15.91%	0.0326
Apr-08				72.39	61.97	67.18	0.48	10.83%	11.67%	0.0157	69.54	58.94	64.24	0.88	14.15%	15.80%	0.0319
May-08				77.06	67.62	72.34	0.48	10.83%	11.61%	0.0138	76.14	63.04	69.59	0.88	10.50%	11.98%	0.0214
Jun-08				79.57	73.15	76.36	0.48	10.83%	11.57%	0.0142	74.22	66.96	70.59	0.88	10.50%	11.96%	0.0183
Jul-08				79.33	59.54	69.44	0.48	10.25%	11.05%	0.0127	71.33	51.47	61.40	0.88	10.50%	12.18%	0.0225
Aug-08				61.19	51.49	56.34	0.48	10.75%	11.75%	0.0127	54.88	46.79	50.84	0.88	11.67%	13.72%	0.0242
Sep-08				56.75	41.03	48.89	0.48	10.75%	11.67%	0.0114	49.19	33.62	41.41	0.88	11.67%	14.19%	0.0226
Oct-08				45.50	24.59	35.05	0.48	10.75%	11.67%	0.0102	36.70	20.71	28.71	0.88	11.67%	15.32%	0.0225
Nov-08				34.35	23.00	28.68	0.48	10.75%	12.71%	0.0106	34.98	24.73	29.86	0.88	11.67%	15.18%	0.0249
Dec-08				31.65	24.84	28.25	0.48	3.75%	5.62%	0.0040	33.88	26.09	29.99	0.88	11.67%	15.16%	0.0219
Jan-09				33.91	25.93	29.92	0.48	3.50%	5.26%	0.0038	38.27	30.30	34.29	0.88	11.67%	14.72%	0.0236
Feb-09				33.41	25.48	29.45	0.50	3.50%	5.36%	0.0040	39.69	29.12	34.41	0.88	11.67%	14.71%	0.0227
Mar-09				30.89	23.18	27.04	0.50	3.50%	5.53%	0.0050	35.52	27.39	31.46	0.88	11.67%	15.00%	0.0244
Apr-09				36.78	28.21	32.50	0.50	3.50%	5.19%	0.0047	36.21	30.38	33.30	0.88	11.67%	14.81%	0.0241
May-09				38.89	35.01	36.95	0.50	5.00%	6.50%		39.50	33.49	36.50	0.88	12.00%	14.87%	0.0302
Jun-09				41.62	37.25	39.44	0.50	5.00%	6.41%		39.43	33.31	36.37	0.88	12.00%	14.88%	0.0302
Jul-09				42.28	35.38	38.83	0.50	5.00%	6.43%		38.64	31.40	35.02	0.88	9.00%	11.91%	0.0220

Month Ending	Keyspan High	Keyspan Low	Keyspan Average	Keyspan Dividend	Keyspan Growth	Keyspan DCF	Keyspan DCF	LG High	LG Low	LG Average	LG Dividend	LG Growth	LG DCF	LG DCF	NJR High	NJR Low
Jun-07															55.24	49.80
Jul-07															51.82	45.91
Aug-07															52.70	45.50
Sep-07															50.50	46.26
Oct-07															33.47	30.59
Nov-07															32.29	29.62
Dec-07															33.23	30.95
Jan-08																
Feb-08															33.47	30.59
Mar-08															32.29	29.62
Apr-08															33.23	30.95
May-08															34.35	31.47
Jun-08															34.63	32.09
Jul-08																
Aug-08																
Sep-08																
Oct-08																
Nov-08																
Dec-08																
Jan-09																
Feb-09																
Mar-09															35.98	29.95
Apr-09															34.84	30.79
May-09															33.60	30.95
Jun-09															37.57	33.57
Jul-09															40.61	35.99

Month Ending	NJR Average	NJR Dividend	NJR Growth	NJR DCF	NJR DCF	GAS High	GAS Low	GAS Average	GAS Dividend	GAS Growth	GAS DCF	GAS DCF	NI High	NI Low	NI Average	NI Dividend
Jun-07						47.47	42.17	44.82	1.86	4.60%	9.24%	0.0043				
Jul-07	48.87	1.52	5.67%	9.17%	0.0037											
Aug-07	49.10	1.52	5.67%	9.16%	0.0037											
Sep-07	48.38	1.52	5.67%	9.21%	0.0037											
Oct-07																
Nov-07																
Dec-07																
Jan-08																
Feb-08	32.03	1.07	5.50%	9.25%	0.0032	42.62	33.99	38.31	1.86	4.00%	9.42%	0.0040				
Mar-08	30.96	1.07	5.50%	9.38%	0.0032	34.29	32.35	33.32	1.86	4.00%	10.25%	0.0044				
Apr-08	32.09	1.12	5.50%	9.43%	0.0032	36.00	33.33	34.67	1.86	4.00%	10.00%	0.0042				
May-08	32.91	1.12	6.00%	9.85%	0.0029	41.60	36.08	38.84	1.86	4.20%	9.55%	0.0037				
Jun-08	33.36	1.12	6.00%	9.80%	0.0030	44.55	40.20	42.38	1.86	4.20%	9.10%	0.0036				
Jul-08						43.25	38.01	40.63	1.86	4.50%	9.63%	0.0046				
Aug-08						46.84	39.29	43.07	1.86	4.25%	9.07%	0.0051				
Sep-08						51.99	42.00	47.00	1.86	4.25%	8.66%	0.0063				
Oct-08						48.42	35.25	41.84	1.86	4.25%	9.22%	0.0070				
Nov-08						47.60	34.46	41.03	1.86	4.25%	9.31%	0.0065				
Dec-08						39.50	32.53	36.02	1.86	2.85%	8.56%	0.0044	11.97	10.45	11.21	0.92
Jan-09						35.89	31.95	33.92	1.86	2.85%	8.92%	0.0046	11.40	9.60	10.50	0.92
Feb-09						36.34	28.38	32.36	1.86	2.85%	9.22%	0.0050	10.88	8.47	9.68	0.92
Mar-09	32.97	1.24	7.00%	11.30%		34.46	27.50	30.98	1.86	6.00%	12.86%		10.32	7.79	9.06	0.92
Apr-09	32.81	1.24	7.00%	11.32%		34.00	30.78	32.39	1.86	6.00%	12.55%		11.20	9.64	10.42	0.92
May-09	32.28	1.24	7.00%	11.39%		34.03	30.28	32.16	1.86	4.30%	10.80%	0.0076	11.62	10.39	11.01	0.92
Jun-09	35.57	1.24	7.00%	10.98%		35.37	31.73	33.55	1.86	4.30%	10.52%	0.0074	11.82	10.79	11.31	0.92
Jul-09	38.30	1.24	7.00%	10.69%		37.42	32.83	35.13	1.86	4.33%	10.27%	0.0062	13.39	11.41	12.40	0.92

Month Ending	NI Growth	NI DCF	NI DCF	NWN High	NWN Low	NWN Average	NWN Dividend	NWN Growth	NWN DCF	NWN DCF	NUI High	NUI Low	NUI Average	NUI Dividend	NUI Growth	NUI DCF
Jun-07				50.49	44.35	47.42	1.42	4.88%	8.23%	0.0025						
Jul-07																
Aug-07																
Sep-07																
Oct-07				48.45	44.28	46.37	1.50	4.88%	8.50%	0.0030						
Nov-07				50.89	44.62	47.76	1.50	4.88%	8.39%	0.0029						
Dec-07				50.58	46.35	48.47	1.50	4.88%	8.34%	0.0029						
Jan-08				50.74	45.87	48.31	1.50	4.88%	8.35%	0.0029						
Feb-08				48.81	41.88	45.35	1.50	4.90%	8.60%	0.0026						
Mar-08				43.92	41.07	42.50	1.50	4.90%	8.85%	0.0027						
Apr-08				45.74	43.08	44.41	1.50	4.90%	8.68%	0.0026						
May-08				46.50	43.46	44.98	1.50	4.88%	8.61%	0.0023						
Jun-08				48.22	44.36	46.29	1.50	4.88%	8.50%	0.0023						
Jul-08				47.19	43.89	45.54	1.50	4.83%	8.51%	0.0027						
Aug-08				49.56	43.66	46.61	1.50	4.83%	8.43%	0.0029						
Sep-08				78.55	20.00	49.28	1.50	4.83%	8.23%	0.0039						
Oct-08				53.71	36.61	45.16	1.50	4.83%	8.54%	0.0042						
Nov-08				52.39	45.59	48.99	1.50	4.83%	8.25%	0.0041						
Dec-08	3.00%	12.19%	0.0123	49.26	42.13	45.70	1.58	4.75%	8.61%	0.0032						
Jan-09	1.60%	11.30%	0.0106	44.55	40.63	42.59	1.58	4.75%	8.90%	0.0034						
Feb-09	1.60%	12.16%	0.0112	45.66	40.43	43.05	1.58	4.75%	8.86%	0.0037						
Mar-09	3.00%	14.47%	0.0157	45.19	37.71	41.45	1.58	4.75%	9.02%	0.0035						
Apr-09	3.00%	12.91%	0.0140	44.16	39.58	41.87	1.58	4.75%	8.97%	0.0035						
May-09	1.60%	10.84%	0.0159	43.79	39.63	41.71	1.58	4.75%	8.99%	0.0047						
Jun-09	1.60%	10.59%	0.0155	46.07	42.67	44.37	1.58	4.75%	8.73%	0.0046						
Jul-09	3.00%	11.28%	0.0147	46.00	42.23	44.12	1.58	4.75%	8.76%	0.0038						

Month Ending	NUI DCF	OKE High	OKE Low	OKE Average	OKE Dividend	OKE Growth	OKE DCF	OKE DCF	Peoples High	Peoples Low	Peoples Average	Peoples Dividend	Peoples Growth	Peoples DCF	Peoples DCF	PNY High
Jun-07		54.82	47.91	51.37	1.36	8.80%	11.86%	0.0162								27.47
Jul-07		55.27	50.45	52.86	1.44	8.80%	11.95%	0.0168								
Aug-07		52.09	41.85	46.97	1.44	8.80%	12.35%	0.0173								
Sep-07		47.80	45.60	46.70	1.44	8.80%	12.37%	0.0174								
Oct-07		50.20	46.91	48.56	1.44	8.80%	12.24%	0.0153								
Nov-07		52.16	45.96	49.06	1.44	8.80%	12.20%	0.0153								
Dec-07		48.11	43.71	45.91	1.44	8.80%	12.44%	0.0156								
Jan-08		49.38	43.38	46.38	1.52	8.80%	12.60%	0.0168								
Feb-08		49.69	45.76	47.73	1.52	9.07%	12.77%	0.0159								25.95
Mar-08		48.66	43.60	46.13	1.52	9.07%	12.90%	0.0160								27.32
Apr-08		49.63	44.68	47.16	1.52	9.07%	12.82%	0.0157								27.68
May-08		51.33	46.57	48.95	1.52	8.60%	12.19%	0.0124								27.42
Jun-08		50.69	47.15	48.92	1.52	8.60%	12.20%	0.0128								27.95
Jul-08		50.05	44.44	47.25	1.52	9.07%	12.81%	0.0163								27.06
Aug-08		46.59	41.80	44.20	1.52	9.07%	13.07%	0.0161								29.20
Sep-08		45.97	33.00	39.49	1.60	8.60%	13.31%	0.0155								35.29
Oct-08		34.98	21.56	28.27	1.60	9.07%	15.71%	0.0199								33.96
Nov-08		32.52	23.51	28.02	1.60	9.07%	15.78%	0.0182								34.19
Dec-08		30.04	24.19	27.12	1.60	9.07%	16.00%	0.0164								32.94
Jan-09		31.74	27.05	29.40	1.60	9.07%	15.45%	0.0144								31.98
Feb-09		29.72	20.92	25.32	1.60	9.07%	16.51%	0.0149								27.55
Mar-09		23.84	18.10	20.97	1.60	7.25%	16.13%	0.0163								26.74
Apr-09		27.01	21.91	24.46	1.60	7.25%	14.83%	0.0150								26.75
May-09		29.31	25.51	27.41	1.60	7.25%	13.99%	0.0195								24.86
Jun-09		30.50	27.93	29.22	1.60	7.25%	13.57%	0.0189								25.50
Jul-09		33.46	27.50	30.48	1.68	7.25%	13.61%	0.0175								25.18

Month Ending	PNY Low	PNY Average	PNY Dividend	PNY Growth	PNY DCF	PNY DCF	Semco High	Semco Low	Semco Average	Semco Dividend	Semco Growth	Semco DCF	Semco DCF	SJI High	SJI Low
Jun-07	24.37	25.92	0.96											39.28	34.53
Jul-07														36.48	32.37
Aug-07														35.98	31.20
Sep-07														36.41	31.83
Oct-07														37.78	33.80
Nov-07														38.50	35.32
Dec-07														38.03	34.73
Jan-08														38.41	33.82
Feb-08	24.28	25.12	1.00	5.17%	9.65%	0.0047								36.88	34.05
Mar-08	24.05	25.69	1.00	5.17%	9.55%	0.0046								35.71	31.90
Apr-08	26.03	26.86	1.04	5.54%	9.91%	0.0047								37.54	35.31
May-08	25.70	26.56	1.04	5.54%	9.96%	0.0041								39.25	36.36
Jun-08	25.23	26.59	1.04	5.54%	9.95%	0.0042								39.36	36.70
Jul-08	25.00	26.03	1.04	5.75%	10.27%	0.0054								38.90	36.00
Aug-08	26.19	27.70	1.04	5.75%	9.99%	0.0057								37.47	33.10
Sep-08	27.53	31.41	1.04	7.93%	11.74%	0.0099									
Oct-08	20.52	27.24	1.04	7.93%	12.33%	0.0107									
Nov-08	28.85	31.52	1.04	7.93%	11.73%	0.0109									
Dec-08	29.21	31.08	1.04	7.87%	11.72%	0.0080								40.58	33.58
Jan-09	24.77	28.38	1.04	7.13%	11.32%	0.0072								40.78	35.33
Feb-09	23.62	25.59	1.04	7.13%	11.79%	0.0080								38.68	34.66
Mar-09	20.68	23.71	1.08	7.00%	12.22%	0.0076								35.93	31.98
Apr-09	24.11	25.43	1.08	7.00%	11.86%	0.0074								36.20	33.70
May-09	21.65	23.26	1.08	6.77%	12.09%	0.0096								36.20	33.04
Jun-09	22.71	24.11	1.08	6.77%	11.90%	0.0094								35.13	33.23
Jul-09	22.50	23.84	1.08	6.93%	12.12%	0.0080								37.53	33.96

Month Ending	SJI Average	SJI Dividend	SJI Growth	SJI DCF	SJI DCF	SWX High	SWX Low	SWX Average	SWX Dividend	SWX Growth	SWX DCF	SWX DCF	UGI High	UGI Low
Jun-07	36.91	0.98	7.25%	10.28%	0.0027									
Jul-07	34.43	0.98	7.00%	10.24%	0.0030									
Aug-07	33.59	0.98	7.00%	10.32%	0.0031									
Sep-07	34.12	0.98	7.00%	10.27%	0.0030									
Oct-07	35.79	0.98	7.00%	10.12%	0.0029									
Nov-07	36.91	0.98	7.00%	10.02%	0.0029									
Dec-07	36.38	0.98	7.00%	10.07%	0.0029									
Jan-08	36.12	1.03	6.63%	9.87%	0.0028									
Feb-08	35.47	1.03	6.60%	9.90%	0.0027	29.96	25.48	27.72	0.86	5.67%	9.16%	0.0030		
Mar-08	33.81	1.03	6.60%	10.07%	0.0028	28.35	25.14	26.75	0.86	5.67%	9.29%	0.0030		
Apr-08	36.43	1.08	6.60%	9.97%	0.0027	30.05	27.90	28.98	0.86	5.67%	9.01%	0.0029		
May-08	37.81	1.08	6.60%	9.84%	0.0024	31.74	28.90	30.32	0.90	6.00%	9.35%	0.0025		
Jun-08	38.03	1.08	6.60%	9.82%	0.0024	31.35	28.98	30.17	0.90	6.00%	9.37%	0.0026		
Jul-08	37.45	1.08	7.00%	10.29%	0.0030	30.07	27.63	28.85	0.90	6.00%	9.52%	0.0032		
Aug-08	35.29	1.08	6.67%	10.15%	0.0029	30.69	27.56	29.13	0.90	6.00%	9.49%	0.0034		
Sep-08						33.29	28.27	30.78	0.90	6.00%	9.30%	0.0037		
Oct-08						30.78	21.46	26.12	0.90	6.00%	9.90%	0.0046		
Nov-08						26.84	21.11	23.98	0.90	6.00%	10.25%	0.0044		
Dec-08	37.08	1.14	7.00%	10.49%	0.0040	25.80	22.74	24.27	0.90	6.00%	10.20%	0.0037		
Jan-09	38.06	1.14	7.50%	10.92%	0.0039	26.36	23.97	25.17	0.90	6.00%	10.05%	0.0037		
Feb-09	36.67	1.14	7.50%	11.05%	0.0046	26.38	19.35	22.87	0.95	6.00%	10.72%	0.0035		
Mar-09	33.96	1.19	7.00%	11.00%	0.0040	22.28	17.08	19.68	0.95	6.00%	11.49%	0.0036		
Apr-09	34.95	1.19	7.00%	10.89%	0.0039	21.61	19.77	20.69	0.95	6.00%	11.22%	0.0035		
May-09	34.62	1.19	9.67%	13.69%	0.0066	21.15	18.96	20.06	0.95	6.00%	11.39%	0.0051		
Jun-09	34.18	1.19	9.67%	13.74%	0.0066	22.32	21.05	21.69	0.95	6.00%	10.97%	0.0049		
Jul-09	35.75	1.19	9.67%	13.56%	0.0055	24.92	21.58	23.25	0.95	6.00%	10.63%	0.0042		

Month Ending	UGI Average	UGI Dividend	UGI Growth	UGI DCF	UGI DCF	WGL High	WGL Low	WGL Average	WGL Dividend	WGL Growth	WGL DCF	WGL DCF	NFG High	NFG Low	NFG Average	NFG Dividend	NFG Growth	NFG DCF
Jun-07						35.91	31.82	33.87	1.37	3.50%	7.98%	0.0032	46.94	42.75	44.85	1.24	4.57%	7.65%
Jul-07						33.44	29.79	31.62	1.37	3.33%	8.13%	0.0039	46.72	43.19	44.96	1.24	5.23%	8.32%
Aug-07						35.01	29.79	32.40	1.37	3.33%	8.01%	0.0038	46.02	40.95	43.49	1.24	5.23%	8.42%
Sep-07						34.60	31.55	33.08	1.37	3.33%	7.92%	0.0038	47.00	43.20	45.10	1.24	5.23%	8.31%
Oct-07													49.29	45.20	47.25	1.24	5.23%	8.17%
Nov-07													49.06	45.63	47.35	1.24	5.23%	8.16%
Dec-07													50.29	46.56	48.43	1.24	5.23%	8.10%
Jan-08													46.90	38.04	42.47	1.24	5.23%	8.50%
Feb-08						33.38	31.11	32.25	1.37	4.00%	8.74%		48.70	41.56	45.13	1.24	3.65%	6.68%
Mar-08						33.49	30.26	31.88	1.37	4.00%	8.79%		48.78	44.27	46.53	1.24	3.65%	6.59%
Apr-08						33.94	31.84	32.89	1.44	5.50%	10.45%		53.35	47.00	50.18	1.24	3.65%	6.37%
May-08						35.69	33.51	34.60	1.44	5.50%	10.20%	0.0038						
Jun-08						36.22	34.17	35.20	1.44	5.50%	10.12%	0.0039						
Jul-08																		
Aug-08																		
Sep-08																		
Oct-08																		
Nov-08																		
Dec-08																		
Jan-09																		
Feb-09																		
Mar-09						34.32	28.89	31.61	1.47	4.00%	9.19%		32.75	26.67	29.71	1.30	5.00%	9.92%
Apr-09						33.29	30.21	31.75	1.47	4.00%	9.16%		34.17	29.83	32.00	1.30	5.00%	9.56%
May-09						31.70	28.59	30.15	1.47	4.00%	9.44%		34.34	30.56	32.45	1.34	5.00%	9.64%
Jun-09						32.60	29.91	31.26	1.47	4.00%	9.25%		37.61	33.09	35.35	1.34	5.00%	9.25%
Jul-09						33.79	30.37	32.08	1.47	4.00%	9.11%		41.10	33.77	37.44	1.34	8.50%	12.65%

Month Ending	NFG DCF	STR High	STR Low	STR Average	STR Dividend	STR Growth	STR DCF	STR DCF	AGL	ATO	CGC	EGN	EQT	KSE	LG	NJR	GAS	NI	NWN
Jun-07	0.0070	55.84	51.49	53.66	0.49	9.25%	10.30%	0.0237	3.19	2.70		4.10	6.13				1.90		1.23
Jul-07	0.0094	58.75	49.50	54.13	0.49	8.50%	9.54%	0.0242		2.58		4.23	6.47			1.41			
Aug-07	0.0095	52.54	44.42	48.48	0.49	8.50%	9.66%	0.0245		2.58		4.23	6.47			1.41			
Sep-07	0.0094	53.27	48.52	50.90	0.49	8.50%	9.60%	0.0243		2.58		4.23	6.47			1.41			
Oct-07	0.0086	57.36	50.67	54.02	0.49	8.70%	9.74%	0.0250	2.86	2.51		4.66	6.58						1.29
Nov-07	0.0086	57.16	51.46	54.31	0.49	8.70%	9.74%	0.0250	2.86	2.51		4.66	6.58						1.29
Dec-07	0.0085	56.59	53.02	54.81	0.49	8.70%	9.73%	0.0250	2.86	2.51		4.66	6.58						1.29
Jan-08	0.0089	57.48	45.00	51.24	0.49	8.88%	9.98%	0.0242	2.82	2.46		4.45	6.98						1.26
Feb-08		58.00	49.42	53.71	0.49	9.00%	10.05%	0.0271		2.52		4.93	8.21			1.37	1.71		1.20
Mar-08		58.32	52.70	55.51	0.49	9.00%	10.02%	0.0270		2.52		4.93	8.21			1.37	1.71		1.20
Apr-08		65.03	56.17	60.60	0.49	9.00%	9.93%	0.0264		2.52		5.48	8.21			1.37	1.71		1.20
May-08		68.74	60.59	64.67	0.49	9.00%	9.88%	0.0263	2.57	2.50		5.48	8.21			1.36	1.78		1.22
Jun-08		71.64	63.42	67.53	0.49	9.00%	9.84%	0.0270	2.57	2.50		5.48	6.84			1.36	1.78		1.22
Jul-08		74.87	52.02	63.44	0.49	9.00%	9.89%	0.0241	2.57	2.39		4.26	6.84				1.78		1.19
Aug-08		54.64	46.91	50.78	0.49	9.00%	10.12%	0.0246	2.54	2.50		4.00	6.53				2.07		1.29
Sep-08		50.69	36.96	43.83	0.49	9.00%	10.29%	0.0216	2.40	2.50		2.78	4.54				2.09		1.36
Oct-08		40.35	20.66	30.51	0.49	9.00%	10.86%	0.0236	2.33	2.20		2.41	4.06				2.09		1.35
Nov-08		35.26	22.59	28.93	0.49	9.00%	10.96%	0.0230	2.31	2.27		2.21	4.36				1.84		1.32
Dec-08		34.10	24.26	29.18	0.49	9.00%	10.95%	0.0214	2.40	2.20		2.12	4.30				1.52	3.01	1.12
Jan-09		37.70	30.00	33.85	0.49	9.00%	10.68%	0.0212	2.58	2.38		2.30	5.05				1.63	2.95	1.20
Feb-09		37.73	28.14	32.94	0.50	9.00%	10.75%	0.0207	2.13	2.00		1.92	4.02				1.42	2.40	1.08
Mar-09		33.55	24.85	29.20	0.50	8.00%	9.96%	0.0199	2.46	2.43		2.61	4.73					3.15	1.14
Apr-09		32.69	28.51	30.60	0.50	8.00%	9.87%	0.0198	2.46	2.43		2.61	4.73					3.15	1.14
May-09		36.93	28.98	32.96	0.50	7.00%	8.72%		2.44	2.30			4.46				1.54	3.22	1.15
Jun-09		36.52	30.46	33.49	0.50	7.00%	8.69%		2.44	2.30			4.46				1.54	3.22	1.15
Jul-09	0.0150	35.40	27.98	31.69	0.50	1.00%	2.69%		2.60	2.50			5.02				1.65	3.54	1.18

Month Ending	NUI	OKE	PGL	PNY	SEN	SJI	SWX	UGI	WGL	NFG	STR	Mkt Cap	Ave. DCF
Jun-07		5.51				1.06			1.62	3.68	9.29	40.40	9.70%
Jul-07		4.96				1.05			1.70	3.98	8.95	35.33	10.06%
Aug-07		4.96				1.05			1.70	3.98	8.95	35.33	10.21%
Sep-07		4.96				1.05			1.70	3.98	8.95	35.33	10.14%
Oct-07		4.64				1.07				3.89	9.50	37.00	10.80%
Nov-07		4.64				1.07				3.89	9.50	37.00	10.83%
Dec-07		4.64				1.07				3.89	9.50	37.00	10.84%
Jan-08		4.87				1.04				3.84	8.88	36.58	11.13%
Feb-08		4.98		1.94		1.10	1.32				10.80	40.07	11.39%
Mar-08		4.98		1.94		1.10	1.32				10.80	40.07	11.47%
Apr-08		4.98		1.94		1.10	1.32				10.80	40.63	11.67%
May-08		4.69		1.90		1.11	1.23		1.70		12.27	46.03	10.69%
Jun-08		4.69		1.90		1.11	1.23		1.70		12.27	44.66	10.62%
Jul-08		4.69		1.95		1.07	1.23				8.99	36.95	10.86%
Aug-08		4.57		2.12		1.06	1.32				9.00	36.99	11.23%
Sep-08		3.33		2.41			1.14				5.98	28.52	11.30%
Oct-08		3.49		2.39			1.29				5.99	27.58	12.13%
Nov-08		3.07		2.46			1.14				5.58	26.57	12.21%
Dec-08		3.05		2.02		1.13	1.08				5.82	29.77	11.62%
Jan-09		2.93		2.00		1.13	1.15				6.25	31.56	11.31%
Feb-09		2.35		1.77		1.07	0.86				5.00	26.03	11.55%
Mar-09		2.93		1.82		1.05	0.91				5.82	29.04	11.98%
Apr-09		2.93		1.82		1.05	0.91				5.82	29.04	11.46%
May-09		3.06		1.75		1.06	0.99					21.97	12.25%
Jun-09		3.06		1.75		1.06	0.99					21.97	12.08%
Jul-09		3.49		1.80		1.10	1.08			3.23		27.18	11.66%

Line No.	Date	DCF	Bond Yield	Risk Premium	y Risk Premium	x Lag Risk Premium	x A Bond Yield	x Lag Yield	Adjusted Risk Premium	Adjusted Bond Yield
1	Jun-98	0.1154	0.0703	0.0451						
2	Jul-98	0.1186	0.0703	0.0483	0.0483	0.0451	0.0703	0.0703	0.0120	0.0135
3	Aug-98	0.1234	0.0700	0.0534	0.0534	0.0483	0.0700	0.0703	0.0143	0.0132
4	Sep-98	0.1273	0.0693	0.0580	0.0580	0.0534	0.0693	0.0700	0.0150	0.0128
5	Oct-98	0.1260	0.0696	0.0564	0.0564	0.0580	0.0696	0.0693	0.0095	0.0137
6	Nov-98	0.1211	0.0703	0.0508	0.0508	0.0564	0.0703	0.0696	0.0053	0.0141
7	Dec-98	0.1185	0.0691	0.0494	0.0494	0.0508	0.0691	0.0703	0.0083	0.0123
8	Jan-99	0.1195	0.0697	0.0498	0.0498	0.0494	0.0697	0.0691	0.0100	0.0139
9	Feb-99	0.1243	0.0709	0.0534	0.0534	0.0498	0.0709	0.0697	0.0132	0.0146
10	Mar-99	0.1257	0.0726	0.0531	0.0531	0.0534	0.0726	0.0709	0.0100	0.0154
11	Apr-99	0.1260	0.0722	0.0538	0.0538	0.0531	0.0722	0.0726	0.0110	0.0136
12	May-99	0.1221	0.0747	0.0474	0.0474	0.0538	0.0747	0.0722	0.0040	0.0164
13	Jun-99	0.1208	0.0774	0.0434	0.0434	0.0474	0.0774	0.0747	0.0051	0.0171
14	Jul-99	0.1222	0.0771	0.0451	0.0451	0.0434	0.0771	0.0774	0.0101	0.0146
15	Aug-99	0.1220	0.0791	0.0429	0.0429	0.0451	0.0791	0.0771	0.0065	0.0169
16	Sep-99	0.1226	0.0793	0.0433	0.0433	0.0429	0.0793	0.0791	0.0086	0.0154
17	Oct-99	0.1233	0.0806	0.0427	0.0427	0.0433	0.0806	0.0793	0.0077	0.0166
18	Nov-99	0.1240	0.0794	0.0446	0.0446	0.0427	0.0794	0.0806	0.0101	0.0143
19	Dec-99	0.1280	0.0814	0.0466	0.0466	0.0446	0.0814	0.0794	0.0106	0.0173
20	Jan-00	0.1301	0.0835	0.0466	0.0466	0.0466	0.0835	0.0814	0.0090	0.0178
21	Feb-00	0.1344	0.0825	0.0519	0.0519	0.0466	0.0825	0.0835	0.0143	0.0151
22	Mar-00	0.1344	0.0828	0.0516	0.0516	0.0519	0.0828	0.0825	0.0098	0.0162
23	Apr-00	0.1316	0.0829	0.0487	0.0487	0.0516	0.0829	0.0828	0.0070	0.0161
24	May-00	0.1292	0.0870	0.0422	0.0422	0.0487	0.0870	0.0829	0.0028	0.0201
25	Jun-00	0.1295	0.0836	0.0459	0.0459	0.0422	0.0836	0.0870	0.0119	0.0134
26	Jul-00	0.1317	0.0825	0.0492	0.0492	0.0459	0.0825	0.0836	0.0121	0.0150
27	Aug-00	0.1290	0.0813	0.0477	0.0477	0.0492	0.0813	0.0825	0.0080	0.0147
28	Sep-00	0.1257	0.0823	0.0434	0.0434	0.0477	0.0823	0.0813	0.0049	0.0167
29	Oct-00	0.1260	0.0814	0.0446	0.0446	0.0434	0.0814	0.0823	0.0095	0.0150
30	Nov-00	0.1251	0.0811	0.0440	0.0440	0.0446	0.0811	0.0814	0.0080	0.0154
31	Dec-00	0.1239	0.0784	0.0455	0.0455	0.0440	0.0784	0.0811	0.0099	0.0129
32	Jan-01	0.1261	0.0780	0.0481	0.0481	0.0455	0.0780	0.0784	0.0114	0.0147
33	Feb-01	0.1261	0.0774	0.0487	0.0487	0.0481	0.0774	0.0780	0.0098	0.0144
34	Mar-01	0.1275	0.0768	0.0507	0.0507	0.0487	0.0768	0.0774	0.0114	0.0143
35	Apr-01	0.1227	0.0794	0.0433	0.0433	0.0507	0.0794	0.0768	0.0023	0.0174
36	May-01	0.1302	0.0799	0.0503	0.0503	0.0433	0.0799	0.0794	0.0154	0.0158
37	Jun-01	0.1304	0.0785	0.0519	0.0519	0.0503	0.0785	0.0799	0.0113	0.0140
38	Jul-01	0.1338	0.0778	0.0560	0.0560	0.0519	0.0778	0.0785	0.0141	0.0144
39	Aug-01	0.1327	0.0759	0.0568	0.0568	0.0560	0.0759	0.0778	0.0116	0.0131
40	Sep-01	0.1268	0.0775	0.0493	0.0493	0.0568	0.0775	0.0759	0.0034	0.0162
41	Oct-01	0.1268	0.0763	0.0505	0.0505	0.0493	0.0763	0.0775	0.0108	0.0137
42	Nov-01	0.1268	0.0757	0.0511	0.0511	0.0505	0.0757	0.0763	0.0103	0.0141
43	Dec-01	0.1254	0.0783	0.0471	0.0471	0.0511	0.0783	0.0757	0.0058	0.0172
44	Jan-02	0.1236	0.0766	0.0470	0.0470	0.0471	0.0766	0.0783	0.0090	0.0134
45	Feb-02	0.1241	0.0754	0.0487	0.0487	0.0470	0.0754	0.0766	0.0108	0.0136
46	Mar-02	0.1189	0.0776	0.0413	0.0413	0.0487	0.0776	0.0754	0.0020	0.0167
47	Apr-02	0.1159	0.0757	0.0402	0.0402	0.0413	0.0757	0.0776	0.0069	0.0131
48	May-02	0.1162	0.0752	0.0410	0.0410	0.0402	0.0752	0.0757	0.0086	0.0141
49	Jun-02	0.1170	0.0741	0.0429	0.0429	0.0410	0.0741	0.0752	0.0098	0.0134
50	Jul-02	0.1242	0.0731	0.0511	0.0511	0.0429	0.0731	0.0741	0.0164	0.0133
51	Aug-02	0.1234	0.0717	0.0517	0.0517	0.0511	0.0717	0.0731	0.0105	0.0127
52	Sep-02	0.1260	0.0708	0.0552	0.0552	0.0517	0.0708	0.0717	0.0134	0.0129
53	Oct-02	0.1250	0.0723	0.0527	0.0527	0.0552	0.0723	0.0708	0.0082	0.0151
54	Nov-02	0.1221	0.0714	0.0507	0.0507	0.0527	0.0714	0.0723	0.0081	0.0130
55	Dec-02	0.1216	0.0707	0.0509	0.0509	0.0507	0.0707	0.0714	0.0099	0.0131
56	Jan-03	0.1219	0.0706	0.0513	0.0513	0.0509	0.0706	0.0707	0.0102	0.0135
57	Feb-03	0.1232	0.0693	0.0539	0.0539	0.0513	0.0693	0.0706	0.0125	0.0123
58	Mar-03	0.1195	0.0679	0.0516	0.0516	0.0539	0.0679	0.0693	0.0081	0.0120
59	Apr-03	0.1162	0.0664	0.0498	0.0498	0.0516	0.0664	0.0679	0.0081	0.0116
60	May-03	0.1126	0.0636	0.0490	0.0490	0.0498	0.0636	0.0664	0.0089	0.0100
61	Jun-03	0.1114	0.0621	0.0493	0.0493	0.0490	0.0621	0.0636	0.0097	0.0108
62	Jul-03	0.1127	0.0657	0.0470	0.0470	0.0493	0.0657	0.0621	0.0072	0.0166
63	Aug-03	0.1139	0.0678	0.0461	0.0461	0.0470	0.0678	0.0657	0.0081	0.0148

64	Sep-03	0.1127	0.0656	0.0471	0.0471	0.0461	0.0656	0.0678	0.0099	0.0109
65	Oct-03	0.1123	0.0643	0.0480	0.0480	0.0471	0.0643	0.0656	0.0100	0.0113
66	Nov-03	0.1089	0.0637	0.0452	0.0452	0.0480	0.0637	0.0643	0.0065	0.0118
67	Dec-03	0.1071	0.0627	0.0444	0.0444	0.0452	0.0627	0.0637	0.0079	0.0113
68	Jan-04	0.1059	0.0615	0.0444	0.0444	0.0444	0.0615	0.0627	0.0086	0.0109
69	Feb-04	0.1039	0.0615	0.0424	0.0424	0.0444	0.0615	0.0615	0.0066	0.0119
70	Mar-04	0.1037	0.0597	0.0440	0.0440	0.0424	0.0597	0.0615	0.0098	0.0101
71	Apr-04	0.1041	0.0635	0.0406	0.0406	0.0440	0.0635	0.0597	0.0051	0.0153
72	May-04	0.1045	0.0662	0.0383	0.0383	0.0406	0.0662	0.0635	0.0055	0.0149
73	Jun-04	0.1036	0.0646	0.0390	0.0390	0.0383	0.0646	0.0662	0.0081	0.0112
74	Jul-04	0.1011	0.0627	0.0384	0.0384	0.0390	0.0627	0.0646	0.0068	0.0105
75	Aug-04	0.1008	0.0614	0.0394	0.0394	0.0384	0.0614	0.0627	0.0085	0.0108
76	Sep-04	0.0976	0.0598	0.0378	0.0378	0.0394	0.0598	0.0614	0.0060	0.0102
77	Oct-04	0.0974	0.0594	0.0380	0.0380	0.0378	0.0594	0.0598	0.0075	0.0111
78	Nov-04	0.0962	0.0597	0.0365	0.0365	0.0380	0.0597	0.0594	0.0058	0.0117
79	Dec-04	0.0970	0.0592	0.0378	0.0378	0.0365	0.0592	0.0597	0.0083	0.0110
80	Jan-05	0.0990	0.0578	0.0412	0.0412	0.0378	0.0578	0.0592	0.0107	0.0100
81	Feb-05	0.0979	0.0561	0.0418	0.0418	0.0412	0.0561	0.0578	0.0085	0.0094
82	Mar-05	0.0979	0.0583	0.0396	0.0396	0.0418	0.0583	0.0561	0.0058	0.0130
83	Apr-05	0.0988	0.0564	0.0424	0.0424	0.0396	0.0564	0.0583	0.0104	0.0093
84	May-05	0.0981	0.0553	0.0427	0.0427	0.0424	0.0553	0.0564	0.0085	0.0098
85	Jun-05	0.0976	0.0540	0.0436	0.0436	0.0427	0.0540	0.0553	0.0091	0.0093
86	Jul-05	0.0966	0.0551	0.0415	0.0415	0.0436	0.0551	0.0540	0.0063	0.0115
87	Aug-05	0.0969	0.0550	0.0419	0.0419	0.0415	0.0550	0.0551	0.0084	0.0105
88	Sep-05	0.0980	0.0552	0.0428	0.0428	0.0419	0.0552	0.0550	0.0090	0.0108
89	Oct-05	0.0990	0.0579	0.0411	0.0411	0.0428	0.0579	0.0552	0.0065	0.0133
90	Nov-05	0.1049	0.0588	0.0461	0.0461	0.0411	0.0588	0.0579	0.0129	0.0121
91	Dec-05	0.1045	0.0580	0.0465	0.0465	0.0461	0.0580	0.0588	0.0093	0.0105
92	Jan-06	0.0982	0.0575	0.0407	0.0407	0.0465	0.0575	0.0580	0.0031	0.0107
93	Feb-06	0.1124	0.0582	0.0542	0.0542	0.0407	0.0582	0.0575	0.0214	0.0118
94	Mar-06	0.1127	0.0598	0.0529	0.0529	0.0542	0.0598	0.0582	0.0092	0.0128
95	Apr-06	0.1100	0.0629	0.0471	0.0471	0.0529	0.0629	0.0598	0.0044	0.0146
96	May-06	0.1056	0.0642	0.0414	0.0414	0.0471	0.0642	0.0629	0.0034	0.0134
97	Jun-06	0.1049	0.0640	0.0409	0.0409	0.0414	0.0640	0.0642	0.0075	0.0122
98	Jul-06	0.1087	0.0637	0.0450	0.0450	0.0409	0.0637	0.0640	0.0120	0.0120
99	Aug-06	0.1041	0.0620	0.0421	0.0421	0.0450	0.0620	0.0637	0.0058	0.0106
100	Sep-06	0.1053	0.0600	0.0453	0.0453	0.0421	0.0600	0.0620	0.0113	0.0099
101	Oct-06	0.1030	0.0598	0.0432	0.0432	0.0453	0.0598	0.0600	0.0066	0.0114
102	Nov-06	0.1033	0.0580	0.0453	0.0453	0.0432	0.0580	0.0598	0.0104	0.0097
103	Dec-06	0.1035	0.0581	0.0454	0.0454	0.0453	0.0581	0.0580	0.0088	0.0113
104	Jan-07	0.1013	0.0596	0.0417	0.0417	0.0454	0.0596	0.0581	0.0051	0.0127
105	Feb-07	0.1018	0.0590	0.0428	0.0428	0.0417	0.0590	0.0596	0.0091	0.0109
106	Mar-07	0.1018	0.0585	0.0433	0.0433	0.0428	0.0585	0.0590	0.0087	0.0109
107	Apr-07	0.1007	0.0597	0.0410	0.0410	0.0433	0.0597	0.0585	0.0061	0.0125
108	May-07	0.0967	0.0599	0.0368	0.0368	0.0410	0.0599	0.0597	0.0037	0.0117
109	Jun-07	0.0970	0.0630	0.0340	0.0340	0.0368	0.0630	0.0599	0.0043	0.0146
110	Jul-07	0.1006	0.0625	0.0381	0.0381	0.0340	0.0625	0.0630	0.0107	0.0116
111	Aug-07	0.1021	0.0624	0.0397	0.0397	0.0381	0.0624	0.0625	0.0089	0.0119
112	Sep-07	0.1014	0.0618	0.0396	0.0396	0.0397	0.0618	0.0624	0.0076	0.0114
113	Oct-07	0.1080	0.0611	0.0469	0.0469	0.0396	0.0611	0.0618	0.0149	0.0112
114	Nov-07	0.1083	0.0597	0.0486	0.0486	0.0469	0.0597	0.0611	0.0107	0.0104
115	Dec-07	0.1084	0.0616	0.0468	0.0468	0.0486	0.0616	0.0597	0.0076	0.0134
116	Jan-08	0.1113	0.0602	0.0511	0.0511	0.0468	0.0602	0.0616	0.0133	0.0105
117	Feb-08	0.1139	0.0621	0.0518	0.0518	0.0511	0.0621	0.0602	0.0106	0.0135
118	Mar-08	0.1147	0.0621	0.0526	0.0526	0.0518	0.0621	0.0621	0.0108	0.0120
119	Apr-08	0.1167	0.0629	0.0538	0.0538	0.0526	0.0629	0.0621	0.0113	0.0128
120	May-08	0.1069	0.0627	0.0442	0.0442	0.0538	0.0627	0.0629	0.0008	0.0119
121	Jun-08	0.1062	0.0638	0.0424	0.0424	0.0442	0.0638	0.0627	0.0067	0.0131
122	Jul-08	0.1086	0.0640	0.0446	0.0446	0.0424	0.0640	0.0638	0.0104	0.0125
123	Aug-08	0.1123	0.0637	0.0486	0.0486	0.0446	0.0637	0.0640	0.0125	0.0120
124	Sep-08	0.1130	0.0649	0.0481	0.0481	0.0486	0.0649	0.0637	0.0089	0.0135
125	Oct-08	0.1213	0.0756	0.0457	0.0457	0.0481	0.0756	0.0649	0.0069	0.0232
126	Nov-08	0.1221	0.0760	0.0461	0.0461	0.0457	0.0760	0.0756	0.0092	0.0150
127	Dec-08	0.1162	0.0654	0.0508	0.0508	0.0461	0.0654	0.0760	0.0136	0.0040

128	Jan-09	0.1131	0.0639	0.0492	0.0492	0.0508	0.0639	0.0654	0.0082	0.0111
129	Feb-09	0.1155	0.0630	0.0524	0.0524	0.0492	0.0630	0.0639	0.0127	0.0115
130	Mar-09	0.1198	0.0642	0.0556	0.0556	0.0524	0.0642	0.0630	0.0133	0.0133
131	Apr-09	0.1146	0.0648	0.0498	0.0498	0.0556	0.0648	0.0642	0.0049	0.0130
132	May-09	0.1225	0.0649	0.0576	0.0576	0.0498	0.0649	0.0648	0.0175	0.0126
133	Jun-09	0.1208	0.0620	0.0588	0.0588	0.0576	0.0620	0.0649	0.0123	0.0096
134	Jul-09	0.1166	0.0597	0.0569	0.0569	0.0588	0.0597	0.0620	0.0094	0.0097

Regression Analysis - Linear model: Y = a + b*X

Dependent variable: Risk Premium
Independent variable: Bond Yield

Parameter	Estimate	Standard Error	T Statistic	P-Value
Intercept	0.0356608	0.00361025	9.87768	0.0000
Slope	0.160561	0.0527725	3.04251	0.0028

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
Model	0.000241579	1	0.000241579	9.26	0.0028
Residual	0.00344483	132	0.0000260972		
Total (Corr.)	0.00368641	133			

Correlation Coefficient = 0.255992
R-squared = 6.55321 percent
R-squared (adjusted for d.f.) = 5.84528 percent
Standard Error of Est. = 0.00510855
Mean absolute error = 0.00417017
Durbin-Watson statistic = 0.482343 (P=0.0000)
Lag 1 residual autocorrelation = 0.738621

Multiple Regression Analysis   Natural Gas

Dependent variable: RiskPremium_1

Parameter	Estimate	Standard Error	T Statistic	P-Value
CONSTANT	0.00674849	0.00282192	2.39145	0.0182
LagRiskPremium	0.80731	0.0529898	15.2352	0.0000
ABondYield	-0.60463	0.133268	-4.53695	0.0000
LagYield	0.637726	0.132834	4.80092	0.0000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
Model	0.00250579	3	0.000835264	91.43	0.0000
Residual	0.00117847	129	0.0000091354		
Total (Corr.)	0.00368426	132			

R-squared = 68.0135 percent
R-squared (adjusted for d.f.) = 67.2696 percent
Standard Error of Est. = 0.00302248
Mean absolute error = 0.00218227
Durbin-Watson statistic = 1.88326 (P=0.2514)
Lag 1 residual autocorrelation = 0.054764

Regression Analysis - Linear model: Y = a + b*X Gas

Dependent variable: AdjustedRiskPremium
Independent variable: AdjustedBondYield

Parameter	Estimate	Standard Error	T Statistic	P-Value
Intercept	0.0130414	0.00150143	8.68599	0.0000
Slope	-0.306783	0.113389	-2.70559	0.0077

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value
Model	0.0000739149	1	0.0000739149	7.32	0.0077
Residual	0.00132276	131	0.0000100974		
Total (Corr.)	0.00139667	132			

Correlation Coefficient = -0.230048
R-squared = 5.29222 percent
R-squared (adjusted for d.f.) = 4.56926 percent
Standard Error of Est. = 0.00317764
Mean absolute error = 0.00242696
Durbin-Watson statistic = 1.82467 (P=0.1569)
Lag 1 residual autocorrelation = 0.0838633

Schedule 4
Comparative Returns on S&P 500 Stock Index and Moody's A-Rated Utility Bonds 1937 - 2009

Line No.	Year	S&P 500 Stock Price	Stock Dividend Yield	Stock Return	A-rated Bond Price	Bond Return
1	2009	865.58	0.0310		$68.43	
2	2008	1,380.33	0.0211	-35.19%	$72.25	0.24%
3	2007	1,424.16	0.0181	-1.27%	$72.91	4.59%
4	2006	1,278.72	0.0183	13.20%	$75.25	2.20%
5	2005	1,181.41	0.0177	10.01%	$74.91	5.80%
6	2004	1,132.52	0.0162	5.94%	$70.87	11.34%
7	2003	895.84	0.0180	28.22%	$62.26	20.27%
8	2002	1,140.21	0.0138	-20.05%	$57.44	15.35%
9	2001	1,335.63	0.0116	-13.47%	$56.40	8.93%
10	2000	1,425.59	0.0118	-5.13%	$52.60	14.82%
11	1999	1,248.77	0.0130	15.46%	$63.03	-10.20%
12	1998	963.35	0.0162	31.25%	$62.43	7.38%
13	1997	766.22	0.0195	27.68%	$56.62	17.32%
14	1996	614.42	0.0231	27.02%	$60.91	-0.48%
15	1995	465.25	0.0287	34.93%	$50.22	29.26%
16	1994	472.99	0.0269	1.05%	$60.01	-9.65%
17	1993	435.23	0.0288	11.56%	$53.13	20.48%
18	1992	416.08	0.0290	7.50%	$49.56	15.27%
19	1991	325.49	0.0382	31.65%	$44.84	19.44%
20	1990	339.97	0.0341	-0.85%	$45.60	7.11%
21	1989	285.41	0.0364	22.76%	$43.06	15.18%
22	1988	250.48	0.0366	17.61%	$40.10	17.36%
23	1987	264.51	0.0317	-2.13%	$48.92	-9.84%
24	1986	208.19	0.0390	30.95%	$39.98	32.36%
25	1985	171.61	0.0451	25.83%	$32.57	35.05%
26	1984	166.39	0.0427	7.41%	$31.49	16.12%
27	1983	144.27	0.0479	20.12%	$29.41	20.65%
28	1982	117.28	0.0595	28.96%	$24.48	36.48%
29	1981	132.97	0.0480	-7.00%	$29.37	-3.01%
30	1980	110.87	0.0541	25.34%	$34.69	-3.81%
31	1979	99.71	0.0533	16.52%	$43.91	-11.89%
32	1978	90.25	0.0532	15.80%	$49.09	-2.40%
33	1977	103.80	0.0399	-9.06%	$50.95	4.20%
34	1976	96.86	0.0380	10.96%	$43.91	25.13%
35	1975	72.56	0.0507	38.56%	$41.76	14.75%
36	1974	96.11	0.0364	-20.86%	$52.54	-12.91%
37	1973	118.40	0.0269	-16.14%	$58.51	-3.37%
38	1972	103.30	0.0296	17.58%	$56.47	10.69%
39	1971	93.49	0.0332	13.81%	$53.93	12.13%
40	1970	90.31	0.0356	7.08%	$50.46	14.81%
41	1969	102.00	0.0306	-8.40%	$62.43	-12.76%
42	1968	95.04	0.0313	10.45%	$66.97	-0.81%
43	1967	84.45	0.0351	16.05%	$78.69	-9.81%
44	1966	93.32	0.0302	-6.48%	$86.57	-4.48%
45	1965	86.12	0.0299	11.35%	$91.40	-0.91%
46	1964	76.45	0.0305	15.70%	$92.01	3.68%
47	1963	65.06	0.0331	20.82%	$93.56	2.61%
48	1962	69.07	0.0297	-2.84%	$89.60	8.89%
49	1961	59.72	0.0328	18.94%	$89.74	4.29%
50	1960	58.03	0.0327	6.18%	$84.36	11.13%
51	1959	55.62	0.0324	7.57%	$91.55	-3.49%
52	1958	41.12	0.0448	39.74%	$101.22	-5.60%
53	1957	45.43	0.0431	-5.18%	$100.70	4.49%
54	1956	44.15	0.0424	7.14%	$113.00	-7.35%
55	1955	35.60	0.0438	28.40%	$116.77	0.20%
56	1954	25.46	0.0569	45.52%	$112.79	7.07%
57	1953	26.18	0.0545	2.70%	$114.24	2.24%
58	1952	24.19	0.0582	14.05%	$113.41	4.26%
59	1951	21.21	0.0634	20.39%	$123.44	-4.89%
60	1950	16.88	0.0665	32.30%	$125.08	1.89%
61	1949	15.36	0.0620	16.10%	$119.82	7.72%
62	1948	14.83	0.0571	9.28%	$118.50	4.49%
63	1947	15.21	0.0449	1.99%	$126.02	-2.79%
64	1946	18.02	0.0356	-12.03%	$126.74	2.59%
65	1945	13.49	0.0460	38.18%	$119.82	9.11%
66	1944	11.85	0.0495	18.79%	$119.82	3.34%
67	1943	10.09	0.0554	22.98%	$118.50	4.49%
68	1942	8.93	0.0788	20.87%	$117.63	4.14%
69	1941	10.55	0.0638	-8.98%	$116.34	4.55%
70	1940	12.30	0.0458	-9.65%	$112.39	7.08%
71	1939	12.50	0.0349	1.89%	$105.75	10.05%
72	1938	11.31	0.0784	18.36%	$99.83	9.94%
73	1937	17.59	0.0434	-31.36%	$103.18	0.63%
74	S&P 500 Return 1937--2009		10.8%			
75	A-rated Utility Bond Return		6.3%			
76	Risk Premium		4.5%			

Schedule 5
Comparative Returns on S&P Utility Stock Index
and Moody's A-Rated Utility Bonds 1937 - 2009

Line No.	Year	S&P Utility Stock Price	Stock Dividend Yield	Stock Return	A-rated Bond Yield	Bond Return
1	2009				$68.43	
2	2008			-25.90%	$72.25	0.24%
3	2007			16.56%	$72.91	4.59%
4	2006			20.76%	$75.25	2.20%
5	2005			16.05%	$74.91	5.80%
6	2004			22.84%	$70.87	11.34%
7	2003			23.48%	$62.26	20.27%
8	2002			-14.73%	$57.44	15.35%
9						
10	2002	243.79	0.0362		$57.44	
11	2001	307.70	0.0287	-17.90%	$56.40	8.93%
12	2000	239.17	0.0413	32.78%	$52.60	14.82%
13	1999	253.52	0.0394	-1.72%	$63.03	-10.20%
14	1998	228.61	0.0457	15.47%	$62.43	7.38%
15	1997	201.14	0.0492	18.58%	$56.62	17.32%
16	1996	202.57	0.0454	3.83%	$60.91	-0.48%
17	1995	153.87	0.0584	37.49%	$50.22	29.26%
18	1994	168.70	0.0496	-3.83%	$60.01	-9.65%
19	1993	159.79	0.0537	10.95%	$53.13	20.48%
20	1992	149.70	0.0572	12.46%	$49.56	15.27%
21	1991	138.38	0.0607	14.25%	$44.84	19.44%
22	1990	146.04	0.0558	0.33%	$45.60	7.11%
23	1989	114.37	0.0699	34.68%	$43.06	15.18%
24	1988	106.13	0.0704	14.80%	$40.10	17.36%
25	1987	120.09	0.0588	-5.74%	$48.92	-9.84%
26	1986	92.06	0.0742	37.87%	$39.98	32.36%
27	1985	75.83	0.0860	30.00%	$32.57	35.05%
28	1984	68.50	0.0925	19.95%	$31.49	16.12%
29	1983	61.89	0.0948	20.16%	$29.41	20.65%
30	1982	51.81	0.1074	30.20%	$24.48	36.48%
31	1981	52.01	0.0978	9.40%	$29.37	-3.01%
32	1980	50.26	0.0953	13.01%	$34.69	-3.81%
33	1979	50.33	0.0893	8.79%	$43.91	-11.89%
34	1978	52.40	0.0791	3.96%	$49.09	-2.40%
35	1977	54.01	0.0714	4.16%	$50.95	4.20%
36	1976	46.99	0.0776	22.70%	$43.91	25.13%
37	1975	38.19	0.0920	32.24%	$41.76	14.75%
38	1974	48.60	0.0713	-14.29%	$52.54	-12.91%
39	1973	60.01	0.0556	-13.45%	$58.51	-3.37%
40	1972	60.19	0.0542	5.12%	$56.47	10.69%
41	1971	63.43	0.0504	-0.07%	$53.93	12.13%
42	1970	55.72	0.0561	19.45%	$50.46	14.81%
43	1969	68.65	0.0445	-14.38%	$62.43	-12.76%
44	1968	68.02	0.0435	5.28%	$66.97	-0.81%
45	1967	70.63	0.0392	0.22%	$78.69	-9.81%
46	1966	74.50	0.0347	-1.72%	$86.57	-4.48%
47	1965	75.87	0.0315	1.34%	$91.40	-0.91%
48	1964	67.26	0.0331	16.11%	$92.01	3.68%
49	1963	63.35	0.0330	9.47%	$93.56	2.61%
50	1962	62.69	0.0320	4.25%	$89.60	8.89%
51	1961	52.73	0.0358	22.47%	$89.74	4.29%
52	1960	44.50	0.0403	22.52%	$84.36	11.13%
53	1959	43.96	0.0377	5.00%	$91.55	-3.49%
54	1958	33.30	0.0487	36.88%	$101.22	-5.60%
55	1957	32.32	0.0487	7.90%	$100.70	4.49%
56	1956	31.55	0.0472	7.16%	$113.00	-7.35%
57	1955	29.89	0.0461	10.16%	$116.77	0.20%
58	1954	25.51	0.0520	22.37%	$112.79	7.07%
59	1953	24.41	0.0511	9.62%	$114.24	2.24%
60	1952	22.22	0.0550	15.36%	$113.41	4.26%
61	1951	20.01	0.0606	17.10%	$123.44	-4.89%
62	1950	20.20	0.0554	4.60%	$125.08	1.89%
63	1949	16.54	0.0570	27.83%	$119.82	7.72%
64	1948	16.53	0.0535	5.41%	$118.50	4.49%
65	1947	19.21	0.0354	-10.41%	$126.02	-2.79%
66	1946	21.34	0.0298	-7.00%	$126.74	2.59%
67	1945	13.91	0.0448	57.89%	$119.82	9.11%
68	1944	12.10	0.0569	20.65%	$119.82	3.34%
69	1943	9.22	0.0621	37.45%	$118.50	4.49%
70	1942	8.54	0.0940	17.36%	$117.63	4.14%
71	1941	13.25	0.0717	-28.38%	$116.34	4.55%
72	1940	16.97	0.0540	-16.52%	$112.39	7.08%
73	1939	16.05	0.0553	11.26%	$105.75	10.05%
74	1938	14.30	0.0730	19.54%	$99.83	9.94%
75	1937	24.34	0.0432	-36.93%	$103.18	0.63%
76	Return 1937--2009	Stocks	10.5%			
77		Bonds	6.3%			
78	Risk Premium		4.2%			

Ex Post Risk Premium Cost of Equity

Risk Premium Utility Stock Index	4.2%
Risk Premium SP500	4.5%
A-rated Utility Bond Yield	5.97%
Risk Premium Cost of Equity Utilities	10.2%
Risk Premium Cost of Equity S&P500	10.4%
Flotation cost	0.27%
Ex Post Risk Premium Cost of Equity	10.6%

Time	SP500 Risk Premium
2008	-0.3543
2007	-0.0586
2006	0.1101
2005	0.0421
2004	-0.0540
2003	0.0795
2002	-0.3540
2001	-0.2240
2000	-0.1995
1999	0.2566
1998	0.2387
1997	0.1036
1996	0.2749
1995	0.0568
1994	0.1071
1993	-0.0893
1992	-0.0777
1991	0.1221
1990	-0.0796
1989	0.0758
1988	0.0025
1987	0.0771
1986	-0.0141
1985	-0.0922
1984	-0.0872
1983	-0.0053
1982	-0.0751
1981	-0.0399
1980	0.2916
1979	0.2841
1978	0.1820
1977	-0.1327
1976	-0.1417
1975	0.2381
1974	-0.0796
1973	-0.1277
1972	0.0689
1971	0.0169
1970	-0.0773
1969	0.0436
1968	0.1126
1967	0.2586
1966	-0.0200
1965	0.1226
1964	0.1202
1963	0.1820
1962	-0.1173
1961	0.1464
1960	-0.0495
1959	0.1106
1958	0.4535
1957	-0.0967
1956	0.1449
1955	0.2820
1954	0.3845
1953	0.0046
1952	0.0979
1951	0.2528
1950	0.3041
1949	0.0837
1948	0.0479
1947	0.0479
1946	-0.1463
1945	0.2907
1944	0.1545
1943	0.1849
1942	0.1673
1941	-0.1352
1940	-0.1673
1939	-0.0816
1938	0.0842
1937	-0.3199

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.191
R Square	0.037
Adjusted R Square	0.023
Standard Error	0.167
Observations	72

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.074	0.074	2.657	0.108
Residual	70	1.960	0.028		
Total	71	2.035			

	Coefficient	*standard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*pper 95.0%*
Intercept	3.096	1.872	1.654	0.103	(0.637)	6.830	(0.637)	6.830
Time	(0.002)	0.001	(1.630)	0.108	(0.003)	0.000	(0.003)	0.000

Time	SP Util Risk Premium
2008	-0.2614
2007	0.1196
006	0.1856
2005	0.1025
2004	0.1150
2003	0.0321
2002	-0.3008
2001	-0.2683
2000	0.1796
1999	0.0848
1998	0.0809
1997	0.0126
1996	0.0431
1995	0.0823
1994	0.0582
1993	-0.0954
1992	-0.0281
1991	-0.0519
1990	-0.0678
1989	0.1951
1988	-0.0255
1987	0.0410
1986	0.0551
1985	-0.0504
1984	0.0383
1983	-0.0049
1982	-0.0628
1981	0.1241
1980	0.1683
1979	0.2068
1978	0.0636
1977	-0.0004
1976	-0.0243
1975	0.1749
1974	-0.0138
1973	-0.1008
1972	-0.0557
1971	-0.1219
1970	0.0464
1969	-0.0162
1968	0.0608
1967	0.1003
1966	0.0276
1965	0.0225
1964	0.1243
1963	0.0686
1962	-0.0464
1961	0.1818
1960	0.1139
1959	0.0849
1958	0.4248
1957	0.0341
1956	0.1451
1955	0.0997
1954	0.1530
1953	0.0738
1952	0.1110
1951	0.2199
1950	0.0271
1949	0.2010
1948	0.0092
1947	-0.0762
1946	-0.0959
1945	0.4879
1944	0.1731
1943	0.3296
1942	0.1322
1941	-0.3292
1940	-0.2360
1939	0.0121
1938	0.0959
1937	-0.3755

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.113
R Square	0.013
Adjusted R Square	(0.001)
Standard Error	0.151
Observations	72

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.021	0.021	0.912	0.343
Residual	70	1.594	0.023		
Total	71	1.615			

	Coefficients	*Standard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	1.654	1.688	0.980	0.331	(1.713)	5.020	(1.713)	5.020
Time	(0.001)	0.001	(0.955)	0.343	(0.003)	0.001	(0.003)	0.001

Schedule 6
Using the Arithmetic Mean to Estimate
the Cost of Equity Capital

End Year 1

Ending Wealth	*Probability*
$1.30	0.5
$0.90	0.5

End of Year 2

Ending Wealth		*Value*	*Probability*	*Value x Probability*
(1.30) (1.30)	=	$ 1.69	0.25	$ 0.42
(1.30) (.9)	=	$ 1.17	0.50	$ 0.59
(.9) (.9)	=	$ 0.81	0.25	$ 0.20
Expected Wealth	=			$ 1.21

Cost of Equity = $1(1+k)^2 = 1.21$

Cost of Equity = $k = (1.21/1)^{.5} - 1 = 10\%$ 10%

Arithmetic mean = $(30\%)\ (.5) + (-10\%)\ (.5) = 1$ 10%

Geometric mean = $[(1.3)\ (.9)]^{.5} - 1 = .082 = 8.2$ 8.2%

Thus, the geometric mean is not equal to the cost of equity capital.
For an investment with an uncertain outcome, the arithmetic mean is the best measure of the cost of equity capit

Schedule 7
Calculation of Capital Asset pricing Model Cost of Equity
Using SBBI 7.1 percent Risk Premium

Line No.	Proxy Companies			
1	Risk-free Rate	4.38%	20-year Treasury Bond Yield forecast	Jul-09
2	Beta	0.85	Average Beta Proxy Companies	
3	Risk Premium	6.50%	Long-horizon SBBI risk premium	
4	Beta x Risk Premium	5.53%		
6	Flotation cost	0.27%		
7	Cost of Equity	10.2%		

Schedule 7 (continued)
Calculation of Capital Asset pricing Model Cost of Equity
Using SBBI 7.1 percent Risk Premium

Line No.	Company	Beta	Market Cap $
1	AGL Resources	0.75	2,598
2	Atmos Energy	0.65	2,499
3	EQT Corp.	1.15	5,024
4	National Fuel Gas	0.90	3,227
5	Nicor Inc.	0.75	1,648
6	NiSource Inc.	0.85	3,539
7	Northwest Nat. Gas	0.60	1,183
8	ONEOK Inc.	0.95	3,485
9	Piedmont Natural Gas	0.65	1,796
10	South Jersey Inds.	0.65	1,099
11	Southwest Gas	0.75	1,083
12	Market-Weighted Average	0.85	

Betas from The Value Line Investment Analyzer August 2009

Schedule 8
Calculation of Capital Asset pricing Model Cost of Equity
Using DCF Estimate of the Expected Rate of Return
on the Market Portfolio

Line No.				
1	Risk-free Rate	4.38%	20-year Treasury Bond Yield	Jul-09
2	Beta	0.85	Average Beta Proxy Companies	
3	DCF S&P 500	12.7%	DCF Cost of Equity S&P 500 (see following)	
4	Risk Premium	8.4%		
5	Beta * RP	7.1%		
6	Flotation cost			
6	Cost of Equity	11.5%		

Schedule 8
'alculation of Capital Asset pricing Model Cost of Equity
Using DCF Estimate of the Expected Rate of Return
on the Market Portfolio

Company	P_0	D_0	Growth	Cost of Equity
AMERISOURCEBERGEN	18.38	0.20	11.57%	12.9%
AETNA	25.61	0.04	12.60%	12.8%
ALLERGAN	47.14	0.20	13.28%	13.8%
ASSURANT	24.26	0.60	8.75%	11.6%
ALLSTATE	25.15	0.80	9.20%	12.9%
APPLIED MATS.	11.75	0.24	8.71%	11.1%
ABERCROMBIE & FITCH	27.61	0.70	10.98%	14.0%
AON	37.40	0.60	12.35%	14.3%
AMERICAN EXPRESS	25.55	0.72	10.00%	13.3%
BOEING	43.97	1.68	8.29%	12.7%
BECTON DICKINSON	67.82	1.32	11.72%	14.0%
FRANKLIN RESOURCES	70.83	0.84	10.00%	11.4%
BROWN-FORMAN 'B'	44.95	1.15	8.10%	11.0%
BANK OF NEW YORK MELLON	28.69	0.36	11.43%	12.9%
BEMIS	25.01	0.90	8.00%	12.1%
BRISTOL MYERS SQUIBB	20.23	1.24	7.04%	14.1%
CA	18.01	0.16	9.60%	10.6%
CATERPILLAR	36.63	1.68	9.00%	14.4%
CHUBB	40.82	1.40	8.50%	12.5%
COCA COLA ENTS.	17.31	0.32	9.20%	11.3%
COLGATE-PALM.	68.42	1.76	9.75%	12.8%
CLOROX	55.64	2.00	9.67%	13.9%
COMCAST 'A'	14.45	0.27	11.25%	13.5%
CME GROUP	291.33	4.60	10.92%	12.8%
CUMMINS	34.44	0.70	10.33%	12.7%
CMS ENERGY	11.92	0.50	6.75%	11.5%
CONSOL EN.	35.90	0.40	12.03%	13.3%
COSTCO WHOLESALE	47.29	0.72	11.54%	13.3%
CAMPBELL SOUP	28.57	1.00	8.43%	12.5%
CSX	33.21	0.88	9.88%	13.0%
CINTAS	23.53	0.47	11.75%	14.1%
CVS CAREMARK	31.75	0.30	13.05%	14.2%
DOMINION RES.	32.50	1.75	6.36%	12.5%
DEERE	42.30	1.12	7.60%	10.6%
QUEST DIAGNOSTICS	53.12	0.40	12.39%	13.3%
DUKE ENERGY	14.38	0.96	3.50%	11.0%
ESTEE LAUDER COS.'A'	33.17	0.55	12.00%	14.0%
EATON	45.95	2.00	7.25%	12.2%
ENTERGY	74.35	3.00	9.02%	13.7%
FAMILY DOLLAR STORES	30.50	0.54	12.15%	14.3%
FIRSTENERGY	39.49	2.20	6.67%	13.1%
FEDERATED INVRS.'B'	24.16	0.96	9.00%	13.6%
FLUOR	47.91	0.50	12.40%	13.6%
FORTUNE BRANDS	36.46	0.76	8.23%	10.6%
FPL GROUP	56.43	1.89	9.59%	13.5%
GENERAL DYNAMICS	55.12	1.52	8.86%	12.1%
GENERAL ELECTRIC	12.66	0.40	9.07%	12.7%
GENUINE PARTS	33.66	1.60	6.00%	11.4%
GAP	16.37	0.34	10.00%	12.4%
GOLDMAN SACHS GP.	143.65	1.40	12.40%	13.6%
WW GRAINGER	81.86	1.84	11.26%	13.9%
HASBRO	25.19	0.80	9.00%	12.7%
HOME DEPOT	24.20	0.90	9.88%	14.2%

HARTFORD FINL.SVS.GP.	13.78	0.20	9.33%	11.0%
HARLEY-DAVIDSON	18.41	0.40	9.50%	12.0%
HONEYWELL INTL.	32.88	1.21	9.38%	13.7%
HEWLETT-PACKARD	37.47	0.32	10.07%	11.1%
HARRIS	29.42	0.76	11.00%	14.0%
INTERNATIONAL BUS.MCHS.	106.61	2.20	9.92%	12.3%
INTL.GAME TECH.	16.02	0.24	12.50%	14.3%
INTEL	16.61	0.56	10.00%	14.0%
ITT	43.96	0.85	8.50%	10.7%
PENNEY JC	28.39	0.80	10.27%	13.6%
JOHNSON & JOHNSON	56.35	1.96	8.13%	12.1%
JANUS CAPITAL GP.	11.11	0.04	10.67%	11.1%
JP MORGAN CHASE & CO.	35.33	0.20	12.00%	12.7%
NORDSTROM	21.78	0.64	10.00%	13.4%
KELLOGG	45.48	1.50	9.84%	13.7%
KB HOME	15.03	0.25	10.50%	12.4%
KRAFT FOODS	26.03	1.16	8.47%	13.6%
LENNAR 'A'	9.43	0.16	8.67%	10.6%
L3 COMMUNICATIONS	72.36	1.40	10.66%	12.9%
LOCKHEED MARTIN	80.81	2.28	10.56%	13.9%
LINCOLN NAT.	16.66	0.04	11.45%	11.7%
LOWE'S COMPANIES	20.03	0.36	11.75%	13.9%
SOUTHWEST AIRLINES	6.99	0.02	12.67%	13.0%
MCDONALDS	57.06	2.00	8.99%	13.1%
MCKESSON	43.02	0.48	11.27%	12.6%
MOODY'S	27.52	0.40	9.00%	10.7%
MEDTRONIC	33.68	0.82	10.54%	13.4%
3M	60.46	2.04	10.13%	14.1%
MORGAN STANLEY	27.72	0.20	11.60%	12.4%
MICROSOFT	22.15	0.52	10.17%	12.9%
M&T BK.	51.92	2.80	4.72%	10.8%
NISOURCE	11.57	0.92	3.00%	11.9%
NIKE 'B'	54.06	1.00	12.11%	14.3%
NORTHEAST UTILITIES	21.59	0.95	8.33%	13.4%
NEWELL RUBBERMAID	11.08	0.20	9.80%	11.9%
OMNICOM GP.	31.94	0.60	11.63%	13.9%
PEOPLES UNITED FINANCIAL	15.78	0.61	9.33%	13.8%
PACCAR	32.16	0.36	10.25%	11.6%
PG&E	37.52	1.68	7.07%	12.2%
PROCTER & GAMBLE	52.00	1.76	9.50%	13.5%
PROGRESS ENERGY	36.58	2.48	5.36%	13.1%
PARKER-HANNIFIN	44.24	1.00	10.00%	12.6%
PERKINELMER	17.12	0.28	11.75%	13.7%
PINNACLE WEST CAP.	28.90	2.10	5.67%	14.0%
PEPCO HOLDINGS	13.10	1.08	3.67%	13.0%
PRAXAIR	73.12	1.60	9.62%	12.2%
POLO RALPH LAUREN 'A'	54.40	0.20	13.75%	14.2%
ROCKWELL AUTOMATION	33.22	1.16	8.00%	12.0%
RADIOSHACK	13.91	0.25	9.48%	11.6%
RAYTHEON 'B'	45.34	1.24	11.14%	14.4%
SCANA	31.74	1.88	5.34%	12.1%
SCHERING-PLOUGH	24.40	0.26	11.10%	12.4%
SHERWIN-WILLIAMS	54.89	1.42	8.83%	11.8%
SARA LEE	9.49	0.44	8.43%	13.8%
SOUTHERN	30.07	1.75	4.97%	11.6%
STANLEY WORKS	35.98	1.32	8.00%	12.2%
STRYKER	39.44	0.40	12.53%	13.7%
AT&T	24.84	1.64	4.11%	11.5%
MOLSON COORS BREWING 'B'	43.13	0.96	10.82%	13.4%
TIFFANY & CO	27.46	0.68	10.75%	13.7%
TJX COS.	30.80	0.48	12.17%	14.0%
T ROWE PRICE GP.	41.15	1.00	10.75%	13.6%

TOTAL SYSTEM SERVICES	13.49	0.28	9.38%	11.8%
TIME WARNER	24.90	0.75	8.06%	11.5%
TEXTRON	11.10	0.08	11.40%	12.2%
UNITED PARCEL SER.	51.34	1.80	7.65%	11.7%
UNITED TECHNOLOGIES	52.29	1.54	9.00%	12.4%
VERIZON COMMUNICATIONS	30.23	1.84	4.58%	11.4%
WALGREEN	30.32	0.55	12.00%	14.2%
WISCONSIN ENERGY	40.33	1.35	9.03%	12.9%
WELLS FARGO & CO	23.91	0.20	10.75%	11.7%
WINDSTREAM	8.45	1.00	0.82%	14.0%
WESTERN UNION	17.00	0.04	11.64%	11.9%
XCEL ENERGY	18.19	0.98	6.58%	12.8%
DENTSPLY INTL.	30.02	0.20	12.67%	13.5%
XTO EN.	39.15	0.50	11.40%	12.9%
Market-weighted Average				12.7%

TABLE 5

WEIGHTED AVERAGE COST OF CAPITAL

Source of Capital	% of Total	Cost Rate	Weighted Cost
Long-term Debt	48.6%	6.87%	3.34%
Common Equity	51.4%	11.00%	5.66%
Total	100.00%		9.00%

REQUEST:

[Rate of Return] - Please refer to the testimony of Dr. Vander Weide, page 3, lines 15-19. Please provide the data that supports the statement regarding equity ratios and the financial risk of Atmos Energy Corporation versus the remainder of the proxy group.

RESPONSE:

Please see Attachment 1 for the data supporting the statement regarding the financial risk of Dr. Vander Weide's comparable companies. The equity ratio for Atmos Energy's ratemaking capital structure is cited in response to Answer 86 of Dr. Vander Weide's direct testimony. The data supporting the ratemaking capital structure is discussed in the testimony of Company Witness Robert J. Smith.

ATTACHMENT:

ATTACHMENT 1 - Atmos Energy Corporation, Comparable Companies Financial Risk, 1 Page.

Respondent: Dr. James Vander Weide

Line No.	Company	Long-Term Debt	Market Cap $ (Mil)	Total Capital	%Debt	%Equity
1	AGL Resources	1,675	2,598	4,273	39%	61%
2	Atmos Energy	2,120	2,499	4,619	46%	54%
3	EQT Corp.	1,249	5,024	6,273	20%	80%
4	National Fuel Gas	999	3,227	4,226	24%	76%
5	Nicor Inc.	449	1,648	2,096	21%	79%
6	NiSource Inc.	5,944	3,539	9,483	63%	37%
7	Northwest Nat. Gas	512	1,183	1,695	30%	70%
8	ONEOK Inc.	4,113	3,485	7,598	54%	46%
9	Piedmont Natural Gas	794	1,796	2,591	31%	69%
10	South Jersey Inds.	333	1,099	1,432	23%	77%
11	Southwest Gas	1,286	1,083	2,368	54%	46%
12	Composite	19,473	27,182	46,655	42%	58%
13	Average				37%	63%

Sources of Data: The Value Line Investment Analyzer, August 2009, and I/B/E/S Thomson Reuters

CASE 2009-00354
ATTACHMENT 1
TO AG DR SET NO. 1
QUESTION NO. 1-75

REQUEST:

[Rate of Return] - Please refer to the testimony of Dr. Vander Weide, page 3, line 20 through page 4, line 2. Please provide the studies and data that support the statements regarding the forecasted yield on utility bonds, the small size premium for small market capitalization and that CAPM underestimates the cost of equity for companies with betas less than 1.0.

RESPONSE:

a) Please see Attachment 4 to the Company's response to AG DR Set No. 1, Question No. 1-73 for the Blue Chip Financial Forecasts, dated August 1, 2009, that support the statement regarding the forecasted yield on utility bonds.

b) Regarding the request for "studies and data that support the small size premium for small market capitalization companies," please see Dr. Vander Weide's testimony, Answer 79, and Table 4. A description of these studies is contained in Ibbotson SBBI 2009 Valuation Yearbook Market Results for Stocks, Bonds, Bills, and Inflation 1926 - 2008. Dr. Vander Weide does not have the underlying data supporting these studies.

c) The studies supporting the conclusion that the CAPM underestimates the cost of equity for companies less than 1.0 are cited and summarized in response to Question 81, pp. 28 - 29 of Dr. Vander Weide's testimony. Copies of these articles are supplied in the Company's response to AG DR Set No. 1, Question No. 1-73. Dr. Vander Weide does not have the underlying data reported in these articles.

Respondent: Dr. James Vander Weide

REQUEST:

[Rate of Return] - Please refer to the testimony of Dr. Vander Weide. With reference to the proxy group referenced on page 17 of the testimony and in Schedule 1, Pages 33-34.

a. Please explain in detail why Dr. Vander Weide considered it appropriate to include Atmos Energy Corporation in the proxy group in his analysis.

b. Please provide a listing of all companies considered for inclusion in the proxy group but rejected by Dr. Vander Weide including the specific reason(s) for the rejection.

RESPONSE:

a) Dr. Vander Weide considers it to be appropriate to include Atmos in the proxy group because Atmos Energy satisfies the criteria for inclusion in his proxy group. These criteria are cited in Dr. Vander Weide's testimony on p. 17.

b) At the time of his studies, each of the following companies met all of Dr. Vander Weide's selection criteria with the exception of the criterion that a selected company must have at least two analysts included in the I/B/E/S mean growth forecast. The following table indicates the number of I/B/E/S estimates available for each company at the time of Dr. Vander Weide's studies.

Company	Ticker	No. of I/B/E/S Estimates	Selected
AGL Resources	AGL	2	yes
Atmos Energy	ATO	3	yes
EQT Corp.	EQT	2	yes
Nicor Inc.	GAS	3	yes
National Fuel Gas	NFG	2	yes
NiSource Inc.	NI	4	yes
Northwest Nat. Gas	NWN	2	yes
ONEOK Inc.	OKE	2	yes
Piedmont Natural Gas	PNY	3	yes
South Jersey Inds.	SJI	3	yes
Southwest Gas	SWX	2	yes
Energen Corp.	EGN	1	no
Laclede Group	LG	NA	no
MDU Resources	MDU	1	no

New Jersey Resources	NJR	1	no
Questar Corp.	STR	1	no
UGI Corp.	UGI	1	no
WGL Holdings Inc.	WGL	1	no

Respondent: Dr. James Vander Weide

REQUEST:

[Rate of Return] - With respect to page 25, lines 14-25, please provide: (1) the source documents for the 5.97% utility bond yield; (2) copies of the source documents and data used to compute the risk premium; please provide copies of the source documents, workpapers, and data in (1) and (2) both hard copy and electronic (Microsoft Excel) formats, with all data and formulae intact.

RESPONSE:

1) There is no source document for the 5.97 percent average yield on Moody's A-rated utility bonds. Dr. Vander Weide obtained the July 2009 average Moody's A-rated utility bond yield equal to 5.97 percent electronically.

2) Dr. Vander Weide's work papers are supplied in the Company's response to AG DR Set No. 1, Question No. 1-74.

Respondent: Dr. James Vander Weide

REQUEST:

[Rate of Return] - With respect to page 15, lines 14-16, please provide copies of the source documents and data used to compute the flotation cost adjustment of 5%. Please provide copies of the source documents, workpapers, and data in both hard copy and electronic (Microsoft Excel) formats, with all data and formulae intact.

RESPONSE:

Dr. Vander Weide's flotation cost adjustment of five percent in his application of the DCF model is derived as explained in his direct testimony beginning on page 15, Question 44, through page 17, Answer 48, and in Appendix 3. In addition, as discussed in Answer 46 and shown in detail in Schedule 2, Atmos Energy, in fact, has incurred flotation costs equal to approximately five percent of its stock price when it has issued new equity securities. Dr. Vander Weide's work papers are provided in the Company's response to AG DR Set No. 1, Question No. 1-74.

Respondent: Dr. James Vander Weide

REQUEST:

[Rate of Return] - With respect to page 13, lines 14-19, please explain why I/B/E/S Thompson Reuters was used as the sole source of EPS growth rate forecasts as opposed to one of the other sources of analysts EPS growth rate forecasts such as Zacks or Yahoo!.

RESPONSE:

Dr. Vander Weide has purchased the I/B/E/S data for many years. Thus, the I/B/E/S data are a consistent data source for the purpose of his studies. Further, in purchasing the I/B/E/S data, Dr. Vander Weide is also able to obtain earnings growth estimates for all U.S. companies, along with complementary information such as the number of analysts' estimates in the mean estimate, stock prices, dividends, and market capitalization, from a single data source.

Respondent: Dr. James Vander Weide