COMMONWEALTH OF KENTUCKY

BEFORE THE PUBLIC SERVICE COMMISSION

* * * * *

In the Matter of :

THE APPLICATION OF KENTUCKY UTILITIES	)	
COMPANY FOR AUTHORIZATION TO IMPLEMENT	)	
A SAMPLE PHASE METERS PROGRAM IN ITS	)	CASE NO. 9479
1) WESTERN DIVISION	)	
2) BLUEGRASS DIVISION	)	
3) MOUNTAIN DIVISION	)	

O R D E R

IT IS ORDERED that Kentucky Utilities Company shall file an original and 12 copies of the following information with this Commission, with a copy to all parties of record, by December 31, 1985. If neither the requested information nor a motion for an extension of time is filed by the stated date, the case may be dismissed.

(1) Please indicate whether the Sample Meter Testing Plan proposed in the application will be the same as the Sample Testing Plan described in Appendix "A" attached to this Order. If it is not the same, then describe the areas where it differs.

Done at Frankfort, Kentucky, this 19th day of December, 1985.

PUBLIC SERVICE COMMISSION

Richard D. Heman Jr.

For the Commission

ATTEST:

Secretary

KENTUCKY PUBLIC SERVICE COMMISSION

STATISTICAL

SAMPLE TESTING PLAN

FOR

SINGLE PHASE ELECTRIC METERS

January 20, 1984

C.N. 9479

APPENDIX "A"

SAMPLE TEST PLAN IMPLEMENTATION

This plan is currently approved by the Public Service Commission of Kentucky for use in lieu of 100% periodic testing where the utility can demonstrate that the use of sample testing is justified. It is justified in those instances where the utility can realize significant savings in meter testing expense while maintaining or improving the level of accuracy and service to the consumers.

Any utility contemplating the use of sample testing should analyze its situation in light of the above considerations. Should circumstances prove favorable to the use of sample testing the utility should seek authorization from the Commission for its implementation.

In considering a sample testing plan for single phase electric watt-hour meters in Kentucky, some factors other than purely statistical must be taken into account. Specifically, the requirements of the Public Service Commission rules must be integrated into the plan to insure compliance with the rules as well as to provide a plan which will be statistically sound, economical, and effective in providing the necessary standards of service to the customer, however, no request by a utility for permission to institute sample testing of meters will be considered unless the utility is currently on schedule in the eight-year test cycle.

In particular the rules state:

1) Periodic sampling plans apply only to single phase meters.

2) No meter may remain in service without testing longer than 25 years.

3) All meters must be tested at 50% power factor, L.L. and F.L.

4) The overall accuracy of meters for refund and back billing purposes is obtained by averaging the percent accuracy at full load and light load.

Obviously, these and other Commission rules will have some effect on the nature of the sampling plan, i.e.:

Provision Number 4: While averaging the full load (FL) and light load (LL) accuracies is permitted and valid in terms of refunding and back billing, its use exclusively in statistical evaluation of test data will obscure much information about meter performance under different load conditions. Various kinds of meters may exhibit marked variations in registration, particularly at light load. Therefore, it is considered desirable to plot and evaluate data at full load, light load and average load.

Provision Number 2: High degrees of reliability can often be obtained from relatively small samples drawn randomly from a homogenous population. However, every meter must be tested at least once every 25 years regardless of the condition of that particular group as indicated by the yearly sample. Therefore, there appears to be no justification for using minimal sample sizes.

On the average, in order to meet the 25-year requirement, 4% of the meters in each group must be tested annually. Therefore, it is considered desirable to have a 4% sample size for each group. While this figure is larger than is needed in many cases for a good estimate of the group condition, the larger the sample the better the estimate of the group condition.

In addition, if substantially less than this number is tested annually, it is quite possible that a utility could build up a large backlog of untested meters in the latter years of a 25-year period which would be very difficult to complete in the remaining time.

Most sampling plans which are considered in regard to meters are based on the Gaussian or "normal" distribution. The statistics derived from the curve, i.e., $\bar{X}$ "Bar-X", and "sigma," σ once known, completely describe the curve. In other words, if X and sigma are known the curve can be reproduced. $\bar{X}$ is the arithmetic mean, and sigma is the standard deviation. The first is a measure of central tendency and the later is a measure of the dispersion of the data about the mean.

In order for these statistics to be valid and useful the population under consideration and/or the sample drawn from that population must distribute normally. For example, because σ is a mathematical function of the normal curve, precisely 68.26% of the items comprising the distribution will be contained in $\pm$ one, σ, etc.

If the items do not distribute normally, an error or uncertainty will be introduced, the magnitude of which will depend on the degree of nonconformity of the data from the normal distribution.

If the population is homogeneous, where the quantity measured is a continuous variable and occurs randomly, and where the sample is selected randomly, the sample will distribute approximately normal, with better and better approximations as the sample size increases. But when watthour meters of different age, manufacturer, bearing systems, retarding magnets, etc., are grouped together for purposes of sample testing, the group may no longer be sufficiently homogeneous to produce distributions for which $\bar{X}$ and σ are meaningful.

The experience of some utilities using sample testing has been to get multimodal, and particularly bimodal distributions (Figure 1). Also, some distributions, particularly on light load tests, bear no resemblance whatever to the normal curve.

The question to be answered is what is a good enough approximation of the normal distribution to justify the use of its statistics. This question must be resolved by the users of the sampling plan as the situations occur. When these situations occur the user must be aware of the limitations of the information derived, and he should attempt to determine the cause.

The sample should be drawn randomly. That is, each meter in the group should have an equal chance of being selected. For a given year, the sample should be without replacement. In subsequent years, the sample should not include any meters which have been tested in the previous seven years.

The reliability of normal curve statistics begins to diminish at about sample size 200 or less and is generally considered too low at sample size 30. Consequently, 30 should be the minimum sample size. Below this number other statistical techniques are employed.

In consideration of the preceding arguments, the following sample testing procedure is presented:

Steps:

1) Divide single phase meters into groups (usually five) according to differences in operating characteristics, bearing systems, compensations, etc.
2) Randomly select 4% of each group (minimum of 30). Eliminate from the sample any nonregistering meters and replace.
3) Test selected meters at LL, FL and 50% power factor when applicable. (50% P.F. test will not be used in calculations.)
4) Plot on separate tally sheets, FL, LL, and average of the two. (Note general shape of the distribution.)

5) Compute sample mean and standard deviation for each of the above distributions.

(Perform the following operations only on the distribution for the <u>average</u> of FL and LL.)

6) Standardize variables. (so standard normal curve tables may be used). This is performed as follows:

The allowable error for meters is $\pm$ 2%, so +2% is the upper limit (u) and -2% is the lower limit (L). Then the standardized variables are Z_u for upper and Z_L for lower.

$$Z_u = \frac{u - \bar{X}}{\sigma} = \frac{+2 - \bar{X}}{\sigma}$$

$$Z_L = \frac{\bar{X} - L}{\sigma} = \frac{\bar{X} - (-2)}{\sigma} = \frac{\bar{X} + 2}{\sigma}$$

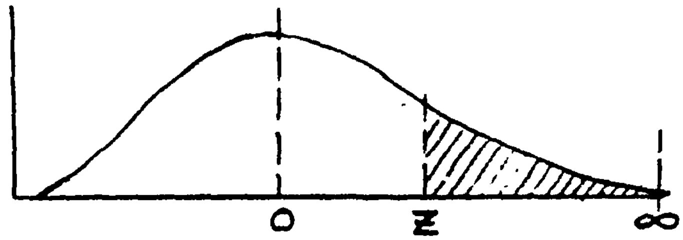
7) Enter table 1 page 7 with $Z = Z_u$ and read the percentage of meters faster than +2%.

Enter table 1 again with $Z = Z_L$ and read the percentage of meters slower than -2%.

These two values are added together. They will both either be positive or zero. This is the estimate of the percentage of meters in the group outside the limits of $\pm$2%.

8) Refer to the table in PSC KAR 5:041E, Sect. 16(4)(a) to determine if additional meters in the group must be tested. (See table 2, page ~~X~~ 7.)

AREAS
UNDER THE
STANDARD NORMAL CURVE
from z to ∞
in percent



z	% area	z	% area
0.0	50.00	2.0	02.28
0.1	46.02	2.1	01.79
0.2	42.07	2.2	01.39
0.3	38.21	2.3	01.07
0.4	34.46	2.4	00.82
0.5	30.85	2.5	00.62
0.6	27.42	2.6	00.37
0.7	24.20	2.7	00.35
0.8	21.19	2.8	00.26
0.9	18.41	2.9	00.19
1.0	15.87	3.0	00.13
1.1	13.57	3.1	00.10
1.2	11.41	3.2	00.07
1.3	09.68	3.3	00.05
1.4	08.08	3.4	00.03
1.5	06.68	3.5	00.02
1.6	05.48	3.6	00.02
1.7	04.46	3.7	00.01
1.8	03.59	3.8	00.01
1.9	02.87	3.9	00.00

TABLE 1

Percent of Meters Within Limits of 2% Fast or Slow (Indicated by Sample)*		Percentage of Meters to be Tested Annually
99.0	100.0	2
98.0	98.9	4
97.0	97.9	6
96.0	96.9	8
95.0	95.9	10
93.0	94.9	12
91.0	92.9	14
Less than	91.0	16

*807 KAR 5:041E Sect. 16(4)(a)

TABLE 2

APPENDIX "I" (~~7 Pages~~)

Example of Distribution Tables,

Computation of $\overline{X}$ and σ^{-}, and

use of Tables I and II

TALLY SHEET

SAMPLE GROUP No. 1 - 1968 LOAD Full

1% Sample Tests

METER TEST RESULTS - PERCENTAGE OF ERROR

	Error	Quantity of Meters Tested	Total
% FAST	2.1	/	1
	2.0		
	1.9	/	1
	1.8		
	1.7		
	1.6		
	1.5		
	1.4		
	1.3		
	1.2		
	1.1		
	1.0	//	2
	.9	/	1
	.8		0
	.7	//	2
	.6	////	4
	.5		18
	.4		21
	.3		15
	.2		16
	.1		8
	.0	///	3
% SLOW	.1	/	1
	.2	////	4
	.3		8
	.4		15
	.5		30
	.6		23
	.7		20
	.8		13
	.9		10
	1.0		6
	1.1	/	1
	1.2	//	2
	1.3	/	1
	1.4	/	1
	1.5		
	1.6		
	1.7		
	1.8		
	1.9		
	2.0		
	2.1		

Figure No. 1

227 Total

TALLY SHEET

SAMPLE GROUP No. 5 - 1968 LOAD Light

1% Sample Tests

METER TEST RESULTS - PERCENTAGE OF ERROR

Error	Quantity of Meters Tested	Total
% FAST		
2.1		
2.0		
1.9		
1.8		
1.7		
1.6		
1.5		
1.4		
1.3		
1.2		
1.1		
1.0		
.9		
.8		
.7		3
.6		3
.5		35
.4		28
.3		69
.2		63
.1		20
.0		12
% SLOW		
.1		28
.2		35
.3		96
.4		54
.5		101
.6		39
.7		41
.8		30
.9		11
1.0		33
1.1		
1.2		1
1.3		
1.4		
1.5		
1.6		
1.7		
1.8		
1.9		
2.0		
2.1		

Figure No. 2

702 Total

METER CALBRATION EVALUATION
1% SAMPLE TESTS 1968 GROUP 5

LIGHT		LOAD
AVERAGE	($\bar{X}$)	= - .232 %
STD. DEV	(σ)	= .427 %
NO. OF METERS TESTED		= 702

METER ERROR IN % (X)	NO. OF METERS (N)	(NX)		(X^2)	(NX^2)
2.1				4.41	
2.0				4.00	
1.9				3.61	
1.8				3.24	
1.7				2.89	
1.6				2.56	
1.5				2.25	
1.4				1.96	
1.3				1.69	
1.2				1.44	
1.1				1.21	
1.0				1.00	
.9				0.81	
.8				0.64	
.7	3	2.1		0.49	1.47
.6	3	1.8		0.36	1.08
.5	35	17.5		0.25	8.75
.4	28	11.2		0.16	4.48
.3	69	20.7		0.09	6.21
.2	63	12.6		0.04	2.52
.1	20	2.0		0.01	.20
	TOTAL 2 =	67.9			
.0	12	00.0		00.0	00.00
.1	28	2.8		0.01	.28
.2	35	7.0		0.04	1.40
.3	96	28.8		0.09	8.64
.4	54	21.6		0.16	8.64
.5	101	50.5		0.25	25.25
.6	39	23.4		0.36	14.04
.7	41	28.7		0.49	20.09
.8	30	24.0		0.64	19.20
.9	11	9.9		0.81	8.91
1.0	33	33.0		1.00	33.00
1.1	0	0		1.21	0
1.2	1	1.2		1.44	1.44
1.3				1.69	
1.4				1.96	
1.5				2.25	
1.6				2.56	
1.7				2.89	
1.8				3.24	
1.9				3.61	
2.0				4.00	
2.1				4.41	

TOTAL 1 = 702

TOTAL 3 = 230.0

TOTAL 4 = 165.60

$$\bar{X} = \frac{\text{TOTAL 2} - \text{TOTAL 3}}{\text{TOTAL 1}}$$

$$\bar{X} = \frac{(67.9) - (230.9)}{(702)}$$

$$\bar{X} = \frac{(-163.0)}{(702)} = -.232\%$$

$$\sigma = \sqrt{\frac{\text{TOTAL 4}}{\text{TOTAL 1}} - \bar{X}^2}$$

$$\sigma = \sqrt{\frac{(165.60)}{(702)} - (-.232)^2}$$

$$\sigma = \sqrt{(.2359) - (.0538)}$$

$$\sigma = \sqrt{(.1821)} = .427\%$$

TALLY SHEET

SAMPLE GROUP No. 5 - 1968 LOAD Full

1% Sample Tests

METER TEST RESULTS - PERCENTAGE OF ERROR

	% Error	Quantity of Meters Tested — Total
% FAST	2.1	
	2.0	
	1.9	
	1.8	
	1.7	
	1.6	
	1.5	
	1.4	
	1.3	
	1.2	
	1.1	
	1.0	
	.9	
	.8	1
	.7	4
	.6	1
	.5	15
	.4	14
	.3	20
	.2	45
	.1	10
	.0	14
% SLOW	.1	40
	.2	73
	.3	50
	.4	84
	.5	139
	.6	40
	.7	64
	.8	76
	.9	2
	1.0	10
	1.1	
	1.2	
	1.3	
	1.4	
	1.5	
	1.6	
	1.7	
	1.8	
	1.9	
	2.0	
	2.1	

Figure No. 3

702 Total

METER CALIBRATION EVALUATION
1% SAMPLE TESTS 1968 GROUP 5

FULL LOAD
AVERAGE $(\bar{X})$ = -.348 %
STD. DEV. (σ) = .357 %
NO. OF METERS TESTED = 702

	METER ERROR IN % (X)	(n)	(nx)	(x^2)	(nx^2)
FAST (+)	2.1			4.41	
	2.0			4.00	
	1.9			3.61	
	1.8			3.24	
	1.7			2.89	
	1.6			2.56	
	1.5			2.25	
	1.4			1.96	
	1.3			1.69	
	1.2			1.44	
	1.1			1.21	
	1.0			1.00	
	.9			0.81	
	.8	1	.8	0.64	.64
	.7	4	2.8	0.49	1.96
	.6	1	.6	0.36	.36
	.5	15	7.5	0.25	3.75
	.4	14	5.6	0.16	2.24
	.3	20	6.0	0.09	1.80
	.2	45	9.0	0.04	1.80
	.1	10	1.0	0.01	.10
		TOTAL 2 =	33.3		
SLOW (-)	.0	14	00.0	0.00	00.00
	.1	40	4.0	0.01	.40
	.2	73	14.6	0.04	2.72
	.3	50	15.0	0.09	4.50
	.4	84	33.6	0.16	13.44
	.5	139	69.5	0.25	34.75
	.6	40	24.0	0.36	14.40
	.7	64	44.8	0.49	31.36
	.8	76	60.8	0.64	48.64
	.9	2	1.8	0.81	1.62
	1.0	10	10.0	1.00	10.00
	1.1			1.21	
	1.2			1.44	
	1.3			1.69	
	1.4			1.96	
	1.5			2.25	
	1.6			2.56	
	1.7			2.89	
	1.8			3.24	
	1.9			3.61	
	2.0			4.00	
	2.1			4.41	

TOTAL 1 = 702 TOTAL 3 = 278.1 TOTAL 4 = 174.68

$$\bar{X} = \frac{\text{TOTAL 2} - \text{TOTAL 3}}{\text{TOTAL 1}}$$

$$\bar{X} = \frac{(33.3) - (278.1)}{(702)}$$

$$\bar{X} = \frac{(-244.8)}{(702)} = (-.348)\ \%$$

$$\sigma = \sqrt{\frac{\text{TOTAL 4}}{\text{TOTAL 1}} - \bar{X}^2}$$

$$\sigma = \sqrt{\frac{(174.68)}{(702)} - (-.348)^2}$$

$$\sigma = \sqrt{(.2488) - (.1211)}$$

$$\sigma = \sqrt{(.1277)} = .357\ \%$$

TALLY SHEET

SAMPLE GROUP No. 5 - 1968 LOAD Average

1% Sample Tests

METER TEST RESULTS - PERCENTAGE OF ERROR

Quantity of Meters Tested

	Error	Total
% FAST	2.1	
	2.0	
	1.9	
	1.8	
	1.7	
	1.6	
	1.5	
	1.4	
	1.3	
	1.2	
	1.1	
	1.0	
	.9	
	.8	
	.7	
	.6	3
	.5	5
	.4	10
	.3	18
	.2	35
	.1	24
	.0	48
% SLOW	.1	79
	.2	70
	.3	49
	.4	78
	.5	87
	.6	89
	.7	70
	.8	20
	.9	14
	1.0	3
	1.1	
	1.2	
	1.3	
	1.4	
	1.5	
	1.6	
	1.7	
	1.8	
	1.9	
	2.0	
	2.1	

Figure No. 4

702 Total

METER CALIBRATION EVALUATION
1% SAMPLE TESTS 1968 GROUP 5

AVERAGE ______ LOAD

AVERAGE ($\overline{X}$) = -.316%
STD. DEV. (σ) = .322%
NO. OF METERS TESTED = 702

	METER ERROR IN % (X)	NO. OF METERS (n)	(nx)	(x^2)	(nx^2)
FAST (+)	2.1			4.41	
	2.0			4.00	
	1.9			3.61	
	1.8			3.24	
	1.7			2.89	
	1.6			2.56	
	1.5			2.25	
	1.4			1.96	
	1.3			1.69	
	1.2			1.44	
	1.1			1.21	
	1.0			1.00	
	.9			0.81	
	.8			0.64	
	.7			0.49	
	.6	3	1.8	0.36	1.08
	.5	5	2.5	0.25	1.25
	.4	10	4.0	0.16	1.60
	.3	18	5.4	0.09	1.62
	.2	35	7.0	0.04	1.40
	.1	24	2.4	0.01	.24
		TOTAL 2 =	23.1		
SLOW (-)	.0	48	00.0	0.00	00.00
	.1	79	7.9	0.01	.79
	.2	70	14.0	0.04	2.80
	.3	49	14.7	0.09	4.41
	.4	78	31.2	0.16	12.48
	.5	87	43.5	0.25	21.75
	.6	89	53.4	0.36	32.04
	.7	70	49.0	0.49	34.30
	.8	20	16.0	0.64	12.80
	.9	14	12.6	0.81	11.34
	1.0	3	3.0	1.00	3.00
	1.1			1.21	
	1.2			1.44	
	1.3			1.69	
	1.4			1.96	
	1.5			2.25	
	1.6			2.56	
	1.7			2.89	
	1.8			3.24	
	1.9			3.61	
	2.0			4.00	
	2.1			4.41	

TOTAL 1 = 702

TOTAL 3 = 245.3

TOTAL 4 = 142.90

$$\overline{X} = \frac{\text{TOTAL 2} - \text{TOTAL 3}}{\text{TOTAL 1}}$$

$$\overline{X} = \frac{(23.1) - (245.3)}{(702)}$$

$$\overline{X} = \frac{(-222.2)}{(702)} = .316\%$$

$$\sigma = \sqrt{\frac{\text{TOTAL 4}}{\text{TOTAL 1}} - \overline{X}^2}$$

$$\sigma = \sqrt{\frac{(142.90)}{(702)} (-.316)^2}$$

$$\sigma = \sqrt{(.2035) - (.0999)}$$

$$\sigma = \sqrt{(.1036)} = .322\%$$

Use of Tables I and II

From the computations for average load, from the previous page.

$$\bar{X} = -.316 \cong -.32$$

$$\sigma = .322 \cong .32$$

Standardize variables:

$$Z_u = \frac{+2-(-.32)}{.32} = \frac{2.32}{.32} = 7.25 = 7.2$$

$$Z_L = \frac{-.32+2}{.32} = \frac{1.68}{.32} = 5.25 = 5.2$$

(round off using standard round of rule, or interpolate)

Enter table I with $Z = 7.2$. Table only extends to $Z = 3.9$, so value for $Z = 7.2$ is zero.

The same is true for $Z = 5.2$. Consequently all meters are within the limits of $\pm$ 2% and no additional meters must be tested.

Suppose Z_u had been 1.4

and Z_L had been 1.7

Then from table I, the value for: Z_u = 8.08%

Z_L = 4.46%

Adding these gives a total of 12.54%. Going to Table II it is seen that 16% of the meters in the group must be tested.

APPENDIX II

Method of Computing Confidence Intervals for $\overline{X}$ and σ

CONFIDENCE INTERVALS

Since the $\bar{X}$ and σ of a sample which is drawn from a population are seldom exactly the same as the mean and standard deviation of the population, it is very helpful to be able to apply some test to determine how much in error they are likely to be.

This can be achieved by means of confidence intervals. The confidence interval provides a range of values within which you have a certain probability (confidence level) that the true population statistics will lie.

Any confidence level for the confidence interval may be computed, but the 95% confidence level is very frequently used. For a 95% confidence level, the confidence intervals for $\bar{X}$ and σ are found from the following formulas:

$$\bar{X} \pm 1.96 \frac{\sigma}{\sqrt{N}} \qquad\qquad \sigma \pm 1.96 \frac{\sigma}{\sqrt{2N}}$$

Where X is the sample size.

Using a confidence interval only slightly larger, 95.44% instead of 95%, permits the use of a factor of 2 instead of 1.96 in the above formulas, thus simplifying the math.

Then:

for a 95.44% $\cong$ 95% confidence interval for $\bar{X}$ and σ, the equations become:

$$\bar{X} \pm 2 \frac{\sigma}{\sqrt{N}} \qquad\qquad \sigma \pm 2 \frac{\sigma}{\sqrt{2N}}$$

Example: $N = 100$, $\bar{X} = .25$, $\sigma = .30$

$$\bar{X} \pm 2 \frac{\sigma}{\sqrt{N}} = .25 \pm 2 \frac{.30}{\sqrt{100}}$$

$$= .25 \pm \frac{.60}{10} = .25 \pm .06$$

Which means that you can be approximately 95% sure that the true population mean is between .19 and .31.

$$\sigma \pm 2 \frac{\sigma}{\sqrt{2N}} = .30 \pm 2 \frac{.30}{\sqrt{200}} = .30 \pm \frac{.60}{14.14}$$

$$= .30 \pm .04$$

Which means that you can be approximately 95% sure that the true population standard deviation is between .26 and .34.